ON WAVE-PARTICLE DUALISM

by Mendel Sachs

Department of Physics and Astronomy,

State University of New York, Buffalo, N.Y.

ABSTRACT : The interpretations of wave-particle dualism of Schrödinger, de Broglie and Einstein are discussed and compared, and contrasted with its interpretation in the quantum theory of measurement, according to the Copenhagen school.

Résumé : Les interprétations de la dualité onde-corpuscule de Schrödinger, de Broglie et Einstein sont discutées et comparées, et contrastées avec leur interprétation dans la théorie quantique de la mesure, selon l'école de Copenhague.

1. INTRODUCTION.

To increase one's scientific understanding of any particular set of phenomena in nature, it is useful to periodically return to the embryonic stages of their accepted explanations, and re-search the ideas and experimental evidences upon which they were built. This is all the more important a procedure of investigation when after a long period of time a particular law of nature remains controversial in regard to its logical, mathematical and experimental implications.

An important case in point is the present-day explanation of the behavior of microscopic matter in terms of the quantum theory of measurement - its equations and rules by which its solutions are to relate to the observables, and the basic interpretation of this formal mathematical structure according to the Copenhagen school. In this regard, it should be very pertinent to physicists that two of the chief architects of the formal structure and initial ideas of quantum mechanics, Louis de Broglie and Erwin Schrödinger, never accepted the meaning attributed to this formalism by the Copenhagen school. Neither were Max Planck nor Albert Einstein, who were involved in the discovery of the old quantum theory of radiation, which in turn led to the notion of wave-particle dualism, ever convinced by the ideas of the Copenhagen school.

Over 50 years ago, de Broglie wrote his remarkable doctoral thesis, proposing the idea that a fundamental wave-particle dualism must underlie the behavior of elementary matter. The

particle variable - its momentum p - was to relate to its wave variable - its wavelength λ - in accordance with the reciprocal relation, $p = h/\lambda$, where h is Planck's constant. Three years later, de Broglie's idea was confirmed when Davisson and Germer, and independently, G.P. Thomson, observed the diffraction pattern in the electrons scattered from a crystal lattice. In these experiments, the electron beam exhibited interference maxima and minima on an absorbing screen, just as diffracting X-rays would do. Was it possible, then, that these were indeed X-rays produced by the projectile electrons, and not the primary electrons themselves ? This possibility was ruled out when it was observed that the entire diffraction pattern shifted its position in a magnetic field.

Shortly after de Broglie's proposal, Schrödinger suggested the form of a non-relativistic wave equation, whose solutions and the rules for their use to predict the physical properties of elementary particles, provided a mathematical formalism to underlie the wave aspect of de Broglie's wave-particle dualism. The Schrödinger wave equation was remarkably successful in predicting the hydrogen spectrum, in better agreement with the data than was provided by Bohr's semi-classical discrete orbital model. But in spite of the agreement with the experimental data that substantiated the idea of wave-particle dualism, there still remained the task of interpreting the wave nature of the elementary particle.

The interpretation that was to be accepted by the great majority of the physics community - to this day - was that which was formulated by Niels Bohr, Max Born, Werner Heisenberg and Paul Dirac. Though there were differences between the interpretations of these four scholars, they all agreed that the wave aspect of an elementary particle - say, an electron - entails a field of chance, continuously distributed in space, pertaining to the act of a macroscopic measuring apparatus in discerning a physical property of this electron with one probable value or another[4]. It is this interpretation that has been identified with the Copenhagen school.

In this paper I should like to discuss some of the ideas that de Broglie and Schrödinger proposed as an interpretation of the wave nature of matter-ideas that were in contrast with the Copenhagen view. I will then discuss what I feel to be the full logical implication of Einstein's theory of relativity with respect to an interpretation of Schrödinger's non-relativistic wave equation for elementary matter, and how this compares with the respective interpretations of Schrödinger and de Broglie.

In order to lead into this discussion, it will be necessary first to review Bohr's introduction of the quantum idea in his model of the hydrogen atom , indicating its success in explaining the emission spectrum of hydrogen.

2. BOHR'S MODEL OF THE ATOM.

To explain Ernest Rutherford's pioneering experimental discovery of the separation between the positive and negative electrical charge distributions in atoms, Niels Bohr postulated the planetary model of the atom, with the assumption that the angular momenta of the orbiting electrons are quantized in units of $\hbar$ (Planck's constant). This idea then led to a prediction of an explicit form for the discrete energy spectrum of atoms. Together with the further postulate that accompanying each discrete 'quantum jump' between the energy levels of an atom - when it loses (or gains) any energy - there is simultaneously created (or annihilated) a quantum of radiation with the same amount of energy, Bohr successfully predicted most of the spectra of radiating hydrogen that were already measured in his day.

Nevertheless, Bohr's model was not entirely successful. It did not account for some of the lines in the fine structure of hydrogen, nor the extra lines that appear in the spectrum when the radiating hydrogen gas is subjected to an external magnetic field (the anomalous Zeeman effect). Neither was the model successfully applied to the explanation for the spectra of non-hydrogenic atoms. Still, the Bohr model of the atom did better than any other theory of its day, and it seemed to blend in with the earlier discoveries that substantiated the quantization of radiation (e.g. blackbody radiation, the photoelectric effect, etc.). Most physicists then felt, at that time, that while more mathematical work would have to be done to extend the model, Bohr's concept of quantized planetary orbits of atoms, and the explanation of the observed quantized radiation from a radiating gas, must have been correct. Thus, when the wave nature of electrons was discovered in the 1920's, it was then felt that it should be necessary for the Schrödinger wave equation to incorporate these ideas, though with the view of the orbital electrons as waves rather than discrete trajectories.

If the wave description of matter is more accurate than the discrete trajectory model, what additional information might one expect to come from the wave equation for hydrogen ? One important piece of information that seemed to be missing from the Bohr prescription was the way in which a transition occurs, from one energy state to another, in a causal fashion. That

is, the question was asked by some : What is the physical cause for the effect whereby an excited orbital electron of hydrogen de-excites ? Further, how can this energy change occur discretely, that is, without the electron continously changing to intermediate energy values ? Bohr would have answered that the intermediate energy levels do not exist, according to his model, and therefore the atom's energy can only change discretely. But the question is : How can the atomic electron change its energy discretely, if we define 'energy' as the capability of the body to do work ? For with this definition, energy change must relate to a force exerted on the electron over a continuously distributed space - from the spatial extent of its (excited state) orbit to the spatial extent of the orbit into which it goes in the transition.

Another question that arose from Bohr's model was : How long does it take for a transition to occur ? With Bohr's view, when the orbital electron is in the process of transition from one allowed energy level to another, no radiation is yet emitted. There are times, then, before the whole quantum of radiation can be emitted, when the electron changes its potential energy without yet having changed this energy into a different form. That is, during the time of transition of the electron's state, from one orbit to another, energy is not conserved ! Thus, another question that arises is this : Does the law of energy conservation only apply at specific times and not at other times ? There is a logical problem with this because a necessary and sufficient condition for the derivation of the law of energy conservation is the invariance of (any) law of nature with respect to continuous changes of the time coordinate, $t \rightarrow t + \delta t$.

3. INTERPRETATIONS OF WAVE MECHANICS - SCHRODINGER VERSUS BOHR.

The difficulties in answering the questions about Bohr's atom were considered by Schrödinger. He felt that they were unresolvable, concluding that Bohr's model was logically inconsistent. This was in spite of the fact that one of Schrödinger's most striking successes was the prediction of the hydrogen spectrum from his wave equation, while most physicists continued to interpret this success with the same concepts that Bohr assumed in his model - discrete electron energy levels, acausally described transitions, etc. Because of the logical difficulties encountered, Schrödinger could not accept Bohr's interpretation of his wave equation in the general description of matter, and particularly applied to hydrogen. Schrödinger rejected Bohr's idea that atoms of matter have discrete energy levels, and he rejected the notion of the discrete 'quantum jump'.

Schrödinger's conclusions were clearly summarized in some of his later publications, in the 1950's[5]. However, he did form some of those views much earlier in the history of quantum mechanics. For example, see the 1927 Schrödinger correspondence with Planck (ref. 3, p. 19). In regard to Schrödinger's original interpretation of his wave function, as relating to a factorization of the electrical charge density (to be discussed further below), rather than the Bohr-Heisenberg view in terms of waves of probability in measurement processes, or Born's probabilistic interpretation, see the 1926 Schrödinger correspondence with Lorentz (ref. 3, p. 55).

The need for the wave nature of matter in a complete description convinced Schrödinger that the idea of localized elementary particles must be abandoned altogether. He interpreted the fact of transfer of discrete quantities of radiation between emitting atoms and absorbing atoms, not as a transfer from a single atom to another single atom, but rather as a resonance, in which one of the frequency modes of a matter system resonates with that of another matter system - just as a set of oscillators in vibration, when weakly coupled to another set of oscillators, will transfer the vibration to the latter. It is this vibration that has its full expression in the wave aspect of matter, in Schrödinger's view.

Guided by de Broglie's speculation about wave-particle dualism for matter, Schrödinger initially saw the wave aspect of matter to be related in a fundamental way to the electric charge of matter. Recall that in the Maxwell field theory of electromagnetism, there are two types of continuous field variables that appear. There is the charge density field, and its motion :

$$j_\mu = (j_0 = c\rho,\ j_k) \qquad (k = 1, 2, 3)$$

These are the charged matter 'source terms' of Maxwell's equations. Secondly, there are the electric and magnetic field intensities, $F_{\mu\nu}$, which are the potential fields of force that would be exerted on a test charge. The latter are the solutions of the field equations, that relate one of these types of field variable to the other.

$$\partial^\nu F_{\mu\nu} = \frac{4\pi}{c} j_\mu \qquad (1)$$

The actualized (Lorentz) force density is then given by the product of these two types of field variable, as follows :

$$K_\mu = j^\nu(\text{test body})\ F_{\mu\nu}$$

where $F_{\mu\nu}$ is the potential field of force, throughout space and time, that is due to all electrically charged matter, except the test body. This was Faraday's original interpretation of the field $F_{\mu\nu}$.

It was Schrödinger's idea that the (real number) electric charge density, $\rho(r,t)$, must be factorizable into the product of a (complex) field $\psi(r,t)$ and its complex conjugate. That is, he saw the density field ρ as decomposable into the more primitive wave fields, according to the prescription, $\rho = e\psi^* \psi$, where e is the charge of the electron - a fundamental constant. It was the complex field variable ψ that Schrödinger then identified with the de Broglie wave.

In the Maxwell theory, the conservation of electrical charge follows from the continuity equation :

$$\partial\rho/\partial t + \nabla\cdot j = 0 \qquad (1')$$

The connection between the de Broglie wave ψ, and a current density $\underline{j}$ then follows from the replacement $\rho = e\psi^* \psi$ and the actual wave equation whose solutions are the de Broglie waves ψ. The wave equation was, in turn, arrived at by starting with the Hamilton-Jacobi equation of ordinary particle dynamics.

As in classical mechanics, then, we start with the definition of the action functional S as the line integral $\int L dt$ whose extremum value corresponds to the equation of motion of a test body which is subjected to an external potential energy field, U(r). The law of energy conservation implies that the total energy of a moving test body is the sum of kinetic and potential energies, $E = p^2/2m + U(r)$. Using the result that the extremum values of the action function with respect to the spatial and temporal coordinates yield :

$$\delta S/\delta x_k = 0 \Rightarrow d/dt(\partial S/\partial x_k) = 0 \Rightarrow \partial S/\partial x_k = \text{constant in time}$$

$$\delta S/\delta t = 0 \Rightarrow d/dt(\partial S/\partial t) = 0 \Rightarrow \partial S/\partial t = \text{constant in time}$$

and defining the conserved physical properties, $\partial S/\partial x_k$ and $-\partial S/\partial t$, as the momentum components and the energy, respectively, the energy conservation equation converts into the following nonlinear partial differential equation, whose solutions are the action function S :

$$E = p^2/2m + U \rightarrow -\partial S/\partial t = |\nabla S|^2/2m + U \qquad (2)$$

This is the Hamilton-Jacobi equation.

To convert this nonlinear partial differential equation in S, into the wave equation in the complex variable ψ, we take :

$$\psi = \alpha \exp(i\Theta) \qquad (3)$$

where the amplitude of this wave, α, and its phase Θ, are both dependent on the space and time coordinates. It is then assumed that the phase of this wave, Θ, relates to the action functional as follows $\Theta = 2\pi/(h/S) = S/\hbar$. Thus, if S is much greater than $\hbar$, the phase Θ would correspond to many de Broglie wave periods, and ψ would correspond to the trajectory that resembles a geometrical ray, rather than a wave. But if S is the order of magnitude of $\hbar$, or less, then the phase Θ would be a fraction of one de Broglie wavelengh - such a wave would readily exhibit its nature in terms of interference in combination with other waves with similar magnitude phases. The latter is analogous to the "physical optics" limit in the description of light while the former is analogous to geometrical optics.

Taking derivatives with respect to the spatial coordinates x_k and the temporal coordinate t, we have :

$$\partial\psi/\partial t = (i/\hbar)(\partial S/\partial t)\psi \qquad \partial\psi/\partial x_k = (i/\hbar)(\partial S/\partial x_k)\psi \qquad (4)$$

One then looks on these relations as mathematical operations on the wave function ψ, as follows :

$$\partial S/\partial t \rightarrow \partial\hat{S}/\partial t = -i\hbar\partial/\partial t \qquad \partial S/\partial x_k \rightarrow \partial\hat{S}/\partial x_k = -i\hbar\partial/\partial x_k \qquad (5)$$

Thus,

$$(\partial\hat{S}/\partial x_k)^2 = -i\hbar\partial/\partial x_k(-i\hbar\partial/\partial x_k) = -\hbar^2\partial^2/\partial x_k^2 \qquad (6)$$

Converting the Hamilton-Jacobi equation (1) in this way, into the operator equation, to "act on" the wave function ψ, this nonlinear differential equation is converted into the linear differential equation in ψ :

$$\hat{\mathcal{H}}\psi \equiv ih\partial\psi/\partial t = (-h^2/2m)\nabla^2\psi + U_\psi \qquad (7)$$

This is the nonrelativistic Schrödinger wave équation. Combining it with the continuity equation (1') and Schrödinger's factorization $\rho = e\psi^*\psi$, it follows that the electromagnetic current density has the form :

$$j = (\hbar/2im)(\psi^*\nabla\psi - \psi\nabla\psi^*) \qquad (8)$$

Finally, assuming that no current flows out of a sufficien-

tly large spatial volume, it follows from the combination of the wave equation (7) and the integral form of the continuity equation (using Gauss' theorem) that :

$$\int \psi^* \psi dr = \text{constant in time} = 1 \qquad (9)$$

where the constant in time has been normalized to unity.

The combination of the linear wave equation (7) and the boundary condition (9) yields the feature of eq. (7) that its solutions form a discrete set of functions $\{\psi_1, \psi_2, \ldots, \psi_n, \ldots\}$ with $\int \psi_m^* \psi_n \, dr = \delta_{mn}$. Such a set is called a "Hilbert space". Since $\psi^* \psi$ is a positive-definite function of the space and time coordinates, it may be interpreted as a weighting function. The continuous sum (integration) of "weights", $\psi^* \psi dr$, adding up to unity, then expresses the condition for a probability calculus.

In the case where the system may be found in "stationary states", such as the stationary orbit of an electron bound in an atom, each of the wave solutions $\psi(r,t)$ are separable into a product of a space-dependent and time-dependent function, as follows : $\psi_n(r,t) = u_n(r)\exp(-iE_n t/\hbar)$. The Schrödinger wave equation then reduces to the form of the following "eigenfunction equation":

$$\hat{\mathcal{H}} u_n = (-(\hbar^2/2m)\nabla^2 + U)u_n = E_n u_n \qquad (10)$$

where the set of numbers $\{E_1, E_2, \ldots, E_n, \ldots\}$ are the energy eigenvalues to be associated with the states of motion of the elementary particle, that are characterized by the corresponding eigenfunctions $\{\psi_1, \psi_2, \ldots, \psi_n, \ldots\}$. These are the measurable discrete energy levels of the atomic system that is described. Within this calculus, they are equivalently expressed in the form :

$$E_n = \int \psi_n^* \hat{\mathcal{H}} \psi_n dr$$

In addition to the Hamilton , corresponding to the measurable energy values for an atomic system, there are other 'linear operators' that correspond to other measurable properties. These appear in similar eigenfunction equations :

$$\hat{A}\psi_n = a_n \psi_n \qquad (10')$$

$$\hat{B}\psi_n = b_n \psi_n \qquad (10'')$$

where, equivalently, the eigenvalues are the integrals $\int\psi_n^* A\psi_n dr$, $\int\psi_n^* B\psi_n dr$, etc. In these integrals we see how the complex wave function takes on the appearance of the ('square root of a) weighting function. But there are also operators, $\hat{D}$, $\hat{E}$, ..., corresponding to other sets of measurable properties of the atomic system, that correspond to different complete sets of eigenfunctions $\{\phi_1, \phi_2, \ldots, \phi_m\}$, in the sense that $\int\psi_n^*\phi_m dr \neq 0$ $(n \neq m)$. Because $\{\phi\}$ and $\{\psi\}$ each form complete sets of eigenfunctions and solve linear differential equations, each of the elements of one of these sets can be expressed as a linear combination of all of the elements of the other set. This result then implies the following property of the product of two such linear operators, $\hat{A}\hat{D}\psi_n \neq \hat{D}\hat{A}\psi_n$ - that is, their product is noncommutative. In particular, when $\hat{A}$ is the Hamiltonian operator , it was discovered by Heisenberg that :

$$(\hat{\mathcal{H}}\hat{D} - \hat{D}\hat{\mathcal{H}})\psi_n = (i\hbar\partial\hat{D}/\partial t)\psi_n \qquad (11)$$

This is the Heisenberg equation of motion. It is equivalent to the Schrödinger representation of quantum mechanics, according to Schrödinger's equation (7).

The noncommutability of linear operators (in a Hilbert space) that correspond to different respective measurable properties of a microscopic system was then interpreted by Heisenberg to mean that not all of the properties of a particle of matter are simultaneously measurable to arbitrarily high accuracy. He thus placed a limitation on the fundamental knowledge that could be acquired by a macrosopic observer, in performing a set of measurements on the physical properties of microscopic matter. The nonsimultaneity appears because of the nonzero time derivative on the right hand side of eq. (11).

According to Schrödinger's initial idea of the role of the de Broglie wave, it is a primitive matter field that underlies the electric charge density variable in Maxwell's equations. But as a continuously distributed wave, to represent, say, electrons,it does not at all relate to a localized particle. With his view, the atomistic concept must be given up altogether at the fundamental level of explanation. This concept is then replaced with the idea that the basic type of field variable that explains the behavior of microscopic matter is a statistical distribution function - in the same sense that the Boltzmann distribution function relates to the statistics of an ensemble of atomic particles in classical mechanics. However, in the latter case, we still have Newtonian physics to complete the description, entailing the cause-

effect relations that predict the individual trajectories of the constituent atoms of the ensemble. But with Schrödinger's view, the equation for the statistical functions is the primitive law of nature of atomic matter.

Thus, Schrödinger's 'matter field' was not intended to relate to a single particle of matter. Therein lies his chief disagreement with the Copenhagen interpretation of his wave equation. Schrödinger, then, did not agree with the commonly accepted view of this weighting function which identifies it with the probability that a measuring apparatus of macroscopic dimensions would determine one of the physical properties of microscopic matter, with one probable value or another. He made the following critical comments about the 'single particle' Copenhagen view[5]: 1) As long as a particle (a localized entity) is permanently identifiable as such, it cannot be a wave, for the latter must eventually disperse into all of space - thereby ceasing to be identifiable as a particle. 2) The wave-mechanical model of hydrogen is not self-consistent because the electron cloud (replacing Bohr's discrete orbit) shields the nuclear charge to the outside, yet in computing the electron orbit inside, its own field should be taken into account. But it is not so in the usual prescription of quantum mechanics - where the only contribution to the electric potential U(r) comes from the nuclear electric field.

With the Copenhagen view, it is said that an atom must always be found at one of its energy eigenvalues, when you measure it. But Schrödinger emphasizes that according to the logic of the Copenhagen school, one must not say that the atom has any particular energy value, if you do not measure it. Thus to say that there are sharp values for the entire energy level spectrum of the atom is inconsistent with the Copenhagen view unless all of these levels would be measured - at the time that they are claimed to exist ! But, in contrast with Schrödinger's interpretation of his wave equation, Bohr would assert that the eigenvalue equations (10), (10'), (10"), etc. can be interpreted only as the act of measuring a particular property of microscopic matter by a macroscopic apparatus. Thus, e.g., the Hamiltonian operator $\hat{\mathcal{H}}$, represents the act of measuring the energy of a microscopic bit of matter, thence yielding the measured value E_n, when the matter is in the state of motion characterized by the wave function ψ_n. The similarity between Schrödinger's formalism and a probability calculus (of a special type) then indicated to the Copenhagen group that all statements about microscopic matter must be in terms of probabilities, as the fundamental variables to replace the space and time coordinates of classical mechanics of particles.

Schrödinger's rejection of this view, because of the logical difficulties encountered when insisting on the single particle view, led him to the conclusion that the problem could be resolved only by giving up the idea of individual particles, and going to a 'second quantized' description, viewing an ensemble of electrons in a continual process of appearance and disappearance. That is, the wave nature, in his view, has to do with a noncountable set of elementary particles, rather than with a countable set of things. The statistical wave that underlies this noncountable set of particles then has wave-like features, such as frequency and wavelength, but these are not supposed to relate to a single particle in terms of its physical properties, such as the permissible energy levels where one may find it, if a measurement should be carried out.

Then how does Schrödinger explain the process wherein radiation is transferred, discretely, between atoms of matter ? He sees this as a resonance process, whereby a continuous matter field couples electromagnetically to another matter field. In a strictly deterministic way, their coupling then induces the vibrations of one of these systems to transfer to the other. Thus, the transferred vibrational mode dies down in one system (the 'emitter') and builds up in the other (the 'absorber') in some finite time - to be identified with the line width of the resonance. He does not view this process, then, in terms of the acausally described "quantum jump" of Bohr's theory. Schrödinger's model is not different, in principle, than the classical example of a large number of oscillators, vibrating in one of its normal modes, weakly coupled to another set of oscillators, causing the latter to build up its vibrational mode while the former ceases it, in a finite time. The actual transfer of this vibration is subject to a resonance condition, as it is in Schrödinger's model.

If ν_e is the frequency of an emitting system of charged matter (not a single atom), before the transfer occurs, and if ν'_e is its frequency afterwards, and if ν_a and ν'_a are the corresponding frequencies of the absorbing matter system, then Schrödinger's resonance condition for maximum energy-momentum transfer beween the coupled vibrating charged systems is :

$$\nu_e - \nu'_e = \nu_a - \nu'_a$$

Or, if there would be n separate absorbers, with each suffering a frequency change due to the electromagnetic coupling within the entire system, the resonance condition is :

$$\nu_e - \nu'_e = \nu_{a_1} - \nu'_{a_1} + \nu_{a_2} - \nu'_{a_2} + \ldots + \nu_{a_n} - \nu'_{a_n} \qquad (12)$$

Of course, eq. (12) can also be re-expressed as follows :

$$\nu_e + \nu_{a_1} + \nu_{a_2} + \nu_{a_3} + \ldots + \nu_{a_n} = \nu'_e + \nu'_{a_1} + \nu'_{a_2} + \nu'_{a_3} + \ldots + \nu'_{a_n} \quad (12')$$

If one now multiplies both sides of this equation by Planck's constant h, and then calls $E = h\nu$, then the resonance condition (12) takes the form :

$$E_{total}(\text{before}) = E_e + E_{a_1} + E_{a_2} + E_{a_3} + \ldots + E_{a_n}$$

$$= E'_e + E'_{a_1} + E'_{a_2} + E'_{a_3} + \ldots + E'_{a_n}$$

$$= E_{total}(\text{after})$$

While this is a mathematical expression for the conservation of energy for the closed system described, it is not to be interpreted here as a "quantum jump" between single interacting atoms. It is rather the resonance condition (12) between large numbers of coupled vibrating systems that is the fundamental relation to Schrödinger. In his view, the relation $E = h\nu$ does not entail the energy of a single electron because the frequency in this equation has to do with the vibration of a matter field that entails a (noncountable) indefinite number of singular entities. The individuation of particles in terms of their respective locations in space is simply without meaning in this description of matter, at this level of microscopic physics. Rather, the set of frequencies, $\{\nu_n\}$ are the meaningful variables that relate to the system of atoms.

4. INTERPRETATIONS OF THE UNCERTAINTY RELATIONS - SCHRODINGER VERSUS HEISENBERG.

Schrödinger saw the Heisenberg uncertainty principle as an attempt by the Copenhagen school to modify the classical single particle concept in order to also accommodate its proven wave nature. This is an epistemological principle in the sense that it asserts that there is a fundamental limitation on the knowledge that we can acquire about microscopic matter. This limitation is expressed in the form of reciprocal relations between the accuracies with which one can prescribe one of the dynamical variables of a single atom of matter at the expense of another. For example, if one of the dynamical properties is the location of a particle, along the x-direction, and the other is its momentum in this direction, then denoting the accuracies of measurement of these variables as δx and δp_x,

the uncertainty relations say that if δx_1 and δx_2 are two different accuracies of measurement position, and if δp_{x_1} and δp_{x_2} are the corresponding accuracies of momentum measurements where the locations were determined, then it is a fundamental law of nature that the ratios are reciprocally linearly proportional to each other, i.e. $\delta x_1 / \delta x_2 = \delta p_{x_2} / \delta p_{x_1}$. Or, $\delta x = h/\delta p_x$, where h is the constant of proportionality in the reciprocal relation. It was taken to be Planck's constant.

According to Heisenberg's rule, then, if in some measurement we should attain infinitely precise knowledge of the whereabouts of an electron along the x-direction, i.e. $\delta x = 0$, then we could have no knowledge at all about the electron's momentum when it is at that location, i.e. $\delta p = \infty$. The stress here on the meaning that should be given to δx and δp is that they should relate to the precision of measurement of the electron's position or momentum at the same point in space and time. Then, to assert the Heisenberg relation $\delta x \delta p_x \simeq h$, as a fundamental principle alleges that 1) this is a fundamental limitation on our knowledge of the coordinates and momenta of an elementary particle of matter - not merely a limitation on the skill we have to build a piece of equipment with great resolving power - and 2) it is "meaningless" to assert that there can be any knowledge about the elementary bit of matter that would transcend our (or our instrument's) perceptions about the behavior of that matter. This is a view that is in line with the philosophic stand of positivism, a view that corresponds to the approach of Bohr and Heisenberg.

According to the Bohr-Heisenberg stand, then, it would not be meaningful to talk about an objective world of matter - at least not so within the domain of science. One must rather adopt a subjectivist view that the way a bit of matter is is in part a function of the way in which it is observed. Thus, their view of the electron wave as a continuous distribution of measurement probability leads them to the interpretation of the 'interference' of such matter waves - say in the electron diffraction experiments - in terms of epistemological ideas. They say that the more accurately a measurement is made of one particular dynamical variable of a particle, say its position, the less accurately can one ever know another of its variables atthat place, such as its momentum, because the former information interferes with the latter knowable information. This idea is usually illustrated with the famous Young double slit experiment, applied to matter waves rather than to light waves. What is puzzling about this

application is that the electron is still thought of as a single, localized particle, even though it is characterized by a wave motion. This goes as follows :

Consider a screen S_1 with two holes in it, H_1 and H_2, each covered with a shutter. A second (absorbing) screen S_2 is a certain distance beyond S_1. Assume that the de Broglie wavelength of the electron is small compared with the hole diameters in S_1. Now if the shutter over H_1 is opened, then when the electron beam propagates perpendicular to S_1, the image of H_1 would be seen on S_2, just opposite this hole (though it would be slightly fuzzy rather than sharp because of some diffraction occuring). Similarly, if the shutter over H_1 is closed, and the one over H_2 is opened, one should see a (fuzzy) image of H_2 on S_2, just opposite H_2. But if the shutters of both H_1 and H_2 would be opened, the peculiar result would be obtained that maximum electron absorption would occur on S_2 opposite the solid section of S_1 between the two open holes ! And on either side of this maximum absorption, one should see alternate maxima and minima with decreasing intensities. In the case of light waves, such a diffraction pattern is well understood. What does it mean, though, when these waves refer to the observation of individual electrons - localized entities (before one looks at them) ?

If we interpret the electron wave as a wave of probability, then the diffraction minima must be interpreted as the interference of our knowledge (or the 'knowledge' of the screen S_2) about whether the single electron went through hole H_1 or through H_2 - this is a cancellation (destructive interference) if the electron is "looked for' at the place where this minimum occurs because here the knowledge that the electron would have gone through H_1 is entirely eliminated by the knowledge that it would have gone through H_2, and vice-versa. With this view, the interference pattern is then a matter of our looking for the electron (or the screen S_2 'looking for us') at one place or another in space.

Let us call the probability wave that describes the electron's state of motion to pass through H_1, ψ_1 (with no possi-

bility of passing through H_2 - because the shutter over this hole is closed), and call the probability wave that predicts a state of motion of the electron passing through H_2 only, ψ_2. The linear combination $(a_1\psi_1 + a_2\psi_2)$ is then a probability wave representing the electron going through either H_1 or H_2 (i.e. when both shutters are open in S_1). Now, according to the Copenhagen view, the actual probability is the absolute square of the probability wave amplitude. Thus, in the first case above, $P_1 = |(\psi_1 + 0)|^2 = |\psi_1|^2$; in the second case, $P_2 = |(0 + \psi_2)|^2 = |\psi_2|^2$. But in the third case,

$$P_{12} = a_1^2|\psi_1|^2 + a_2^2|\psi_2|^2 + a_1a_2(\psi_1^*\psi_2 + \psi_1\psi_2^*)$$

Only in the case of P_{12} is there an interference term exhibited. It is the cross-product term that gives rise to the diffraction pattern with alternate maxima and minima. This is the form of the probability calculus that is evoked by the quantum theory of measurement. It is important to note that the reason for the probability wave in the third case above, having the form of a linear sum of waves, ψ_1 and ψ_2, is that the Schrödinger wave equation is linear - i.e. the Hamiltonian operator $\hat{\mathcal{H}}$ is independent of the solutions $\{\psi\}$ that it is 'operating on'. Conceptually, this is so because of the assumption that the 'observer' and the 'observed' are not coupled in a dynamical way, in principle; it is for this reason that precise predictions cannot be made on the outcome of a measurement, according to this theory.

As I have indicated earlier, Schrödinger rejected this view of identifying probabilities with a single particle; he rather insisted on identifying his wave function with a physically objective field for a (noncountable) ensemble of elementary particles, rather than a probability wave to 'guide' a single electron. He saw this as an objective field in the sense that it should not merely relate to the chance of a human knower finding the bit of matter to be in one state of motion or another. He objected to the Copenhagen view, which asserts that before the 'knower" would look at the physical situation, the electron is in all of the possible states of motion predicted by the Schrödinger equation, ψ_1, ψ_2, ..., ψ_n ..., simultaneously, with various degrees of probability. Only after the observer looks at the electron would it 'project into' one of its states or anoter, in correspondence with the place where he looked for the electron.

Schrödinger's interpretation of the uncertainty relations, then, has to do with a description of objective matter waves in a space of 3N(t) dimensions, where N is the noucountable number of electrons, variable with respect to the time coordinate. It was Schrödinger's main contention that the uncertainty relations are a feature of linearly superposed waves, defined in terms of a particular statistics, and the definitions of the standard fluctuations from the mean locations and momenta that are defined in terms of these statistical functions. But he did not believe that the uncertainty relations have anything to do with a single particle, or with limitations on the knowledge that a human being could acquire about the dynamical properties of matter. To Schrödinger, the matter wave is a solution for a new type of statistics that is basic to the nature of matter in a way that the atomistic concept must be abandoned. But with this view, the uncertainty relations have nothing to do with incomplete knowledge, per se. They do not reduce the amount of information that might be available with a more objective theory.

In regard to the mathematical expression of the uncertainty relations, it was known for many years before the onset of the quantum mechanical formalism, that if $k_1 = 2\pi/\lambda_1$, $k_2 = 2\pi/\lambda_2, \ldots$ are the wave numbers for a group of linearly superposed waves, and if $x_1, x_2, \ldots$ are the locations of the peaks of the corresponding waves $\Delta k \Delta x \geq 1/2$, where Δ denotes the root mean square deviations of the quantities it applies to. This is the 'Schwarz inequality'. Now one simply has to multiply both sides of this inequality by $h/2\pi$, and use the de Broglie relation $p = hk/2\pi = h/\lambda$, to get the Heisenberg form of this relation $\Delta p \Delta x \geq h/4\pi$. But just as the empirical data do not compel one to multiply both sides of the resonance condition (10) by h to yield the quantum energy jump interpretation, so the Schwarz inequality that agrees with the linear wave description, according to the empirical data, does not compel one to interpret this inequality epistemologically in terms of a single particle, as Heisenberg did. Indeed, one can multiply both sides of either of these equations by anything he pleases, microscopic or macroscopic, without altering these relations at all !

5. A THOUGHT EXPERIMENT TO REFUTE THE COPENHAGEN VIEW OF LINEAR SUPERPOSITION.

Einstein suggested the following thought experiment : A constant energy electron beam, as a plane wave with a fixed wavelength, approaches a screen. There is a hole in the screen whose diameter is small compared with the de Broglie

wavelength of the electron. In this case, the part of the wave front at the hole generates a spherical wave (Huygens' principle) which then propagates beyond the screen. Now place a spherical absorbing screen about the first screen with its geometric center at the hole of the first screen. According to the Copenhagen theory, if one does not look at the second screen, the electron will propagate equally in all radial directions from the hole. On the other hand, if one should look at the spherical absorbing screen, he would see where the electron had hit it - at the special location in space where it was when it was absorbed. Thus, the act of a human observer in looking for the electron, causes all of its other possible states of motion to collapse into the particular state that is identified, according to the place where it is seen to arrive. It then seemed to Einstein, that apart from a peculiar assumption about interference between the knower's brain and the electron beam, there also seemed to be a violation of the rule of relative simultaneity of special relativity theory. For the very act of looking at one spot on the absorbing screen, and seeing the electron there, must spontaneously spread this information to all other parts of the film - i.e. at infinite speed !

6. DE BROGLIE'S DOUBLE SOLUTION INTERPRETATION.

Contrary to Schrödinger's multi-particle, statistical matter field interpretation of the wave function, de Broglie continued to identify the wave aspect of matter with a single particle. But he did agree with Schrödinger and Einstein that the Copenhagen interpretation is logically unsound, and therefore unacceptable. De Broglie saw the wave behavior of matter in terms of an objective wave, and not merely a subjective expression of limitation of our knowledge about an elementary particle of matter. But, clearly, the Schrödinger wave function does not provide a complete description for a single particle. To complete the description, de Broglie then suggested that accompanying the Schrödinger wave, there must be a second solution that (deterministically) relates to the actual (unobservable) discrete trajectory of a particle. It must be this extra solution, he argues, that would direct the electron of Einstein's thought experiment to the definite place where the viewer sees it to be on his spherical film.

Mathematically, de Broglie wished to introduce his second solution from the same differential equation that yields the Schrödinger wave function, $\psi = \alpha\exp(iS/\hbar)$. The second solution was to have the form $\zeta = f\exp(iS/\hbar)$. It is important in this formulation that the phase of the two solutions, $S/\hbar$, is the same. But the amplitude, f, is different than α. It is f that is to represent a point singularity in motion. The idea, then,

was to find a wave equation - most likely a generalization of the ordinary Schrödinger equation - that has two different types of solutions. One of these is the continuous (and analytic) Schrödinger ψ - function, the other is the singular solution, whose phase correlates with the ψ- function, but whose amplitude predicts the actual motion of the elementary particle.

Because the motion entailed in the ζ - function is influenced by the ψ - function, it follows that ζ and ψ must satisfy coupled wave equations, with the general form $O(\psi)\zeta = 0$ and $\hat{P}(\zeta)\psi = 0$. The latter equation is that of Schrödinger plus a term that depends on ζ. De Broglie identified the latter with a "quantum force" = $-\nabla Q$, with $Q = (-\hbar^2/2m)(\nabla^2 f/f)$, where f is the amplitude of the singular ζ - wave. His analysis further imposed the following relation between f and ζ : $(\nabla^2 f/f) = (\nabla^2\zeta/\zeta)$. Because of this relation, the ψ - function may be considered as uncoupled from the ζ - function, thereby maintaining its character as the solution of a linear differential equation and allowing the use of the linear superposition principle for the ψ - functions. One reason for de Broglie's insistence on the linear character of the Schrödinger type equation for a particle was that Pauli's proof of the exclusion principle, giving the relation between spin and statistics for a many-electron system, was based on linearity (in addition to several other axioms). However, because of their coupling, the differential equation in ζ should remain nonlinear. This nonlinearity is then due to the coupling which, in turn, is that part of the formalism that relates to a completion of the particle description, yielding a totally objective explanation for the behavior of the elementary particles of matter.

To summarize de Broglie's position in his own words, he made the following comment in a recent issue of this journal ([1a]):

"... en théorie de la double solution... il faut alors retrouver ... l'ensemble des conclusions exactes de la Mécanique quantique et, en particulier, justifier dans cet espace la symétrisation de la fonction d'onde pour un ensemble de bosons et l'antisymétrisation de la fonction d'onde pour un ensemble de fermions".

de Broglie explained further (in ref. 1b, pp. 46, 47) that the double solution is written as the sum of a singular solution - the 'objective part' that relates to the particle's actual (deterministic) trajectory - and the ψ - wave. He explains that "the ψ - wave must satisfactorily conform everywhere to the normal linear equation. Moreover, the ψ - wave would not

conform to a non-linear equation because the principle of (linear) superposition appears to be a necessary condition of the normal statistical interpretation, as Pauli had previously emphasized in his article on wave mechanics in "Handbuch der Physik". It is important to note that de Broglie does strongly emphasize that the 'objective part' of the total (double) solution must necessarily satisfy a non-linear wave equation, in principle, and therefore this part of the wave solution cannot conform to the linear superposition principle.

The idea of the double solution - to complete the description of microscopic matter in an objective, deterministic way - was introduced by de Broglie in 1927, just three years after his proposal of wave-particle dualism for matter. Very similar ideas in this direction of yielding a deterministic theory of elementary matter were proposed later, in the early 1950's, by David Bohm and his collaborators, in terms of his 'hidden variable theory'. Bohm's idea, generally, was to expand the underlying parameter space (ordinary three-dimensional space) into a larger dimensional space that would entail extra independent variables. One would then consider a Schrödinger-type wave function to be mapped in this larger-dimensional space. As in de Broglie's analysis, this idea also led to an extra 'quantum force' in the wave equation-which would be the term that completes the description of real bits of elementary matter, so as to yield an objective, deterministic theory.

Summing up, de Broglie and Schrödinger both object to the epistemological view of the Copenhagen school, mainly because of their contention that the elementary particle of matter is there, whether or not one should happen to look at it. Thus, they insist that the basic variables which relate to the motion of elementary matter must not be defined in a subjective way, that is, the properties of matter must not depend on the way in which it happens to be 'looked at'. The formal success of the de Broglie-Schrödinger wave theory of matter led Schrödinger to define the wave nature of matter in a deterministic way, as relating to an ensemble of elementary particles in which individuation is not one of their physical attributes. He sees the next step in the analysis, that would fully exploit his view, as a 'second quantization" of the Schrödinger wave function - which in turn is to be identified with a statistical distribution function that relates to the actual charge density of matter (or the density of other of its physical properties).

De Broglie attempted to resolve the problem of indeterminism and incompleteness in the Schrödinger wave formalism by asserting that it leaves out an essential aspect of the single particle motion having to do with its actual trajectory

in space. The latter completion is in terms of a second solution that necessarily accompanies the Schrödinger wave. The two solutions relate to a single particle, though the second solution, describing the motion of a singularity in space, must solve a nonlinear differential equation, while the Schrödinger solution solves an equation that maintains the usual form, though with the addition of an extra force term in the Hamiltonian operator.

Both de Broglie's and Schrödinger's views imply the existence of a richer mathematical structure for quantum mechanics than the conventional formalism. But they have not yet been extended sufficiently so as to establish their validities in terms of better explicit correspondences with the experimental facts. Schrödinger's extension entails second quantization - a formalism that is also required by the relativistic extension of quantum mechanics to relativistic quantum field theory. This type of formalism has been studied also since the onset of quantum mechanics, in the late 1920's, in the latter context. Unfortunately, there has not yet been a successful formulation in terms of providing a demonstrably mathemtically consistent set of equations, nor has it yet been shown that the formalism of quantum field theory goes smoothly into that of ordinary quantum mechanics, in the nonrelativistic limit.

7. THE DIFFICULTY OF FUSING QUANTUM MECHANICS WITH RELATIVITY THEORY.

The extension of ordinary quantum mechanics to relativistic quantum field theory is a logically necessary generalization. The main reason, as recognized in the early stages of quantum mechanics by Paul Dirac, is that the Schrödinger wave equation does not incorporate the electromagnetic radiation field. Our knowledge about the atoms of matter comes from data that is usually interpreted in terms of one atom losing energy, with the simultaneous emission of electromagnetic radiation, subsequently absorbed by other atoms of matter. The entire system to be explained by a proper theory then entails both the atoms of matter and the transmitted radiation. The part of the system identified with the emitter and absorber has a nonrelativistic description, which we can assume is an approximation for a relativistic description of matter. But the radiation component of this system does not have any nonrelativistic limit. It propagates only at the speed of light under all conditions ! Thus, for a mathematically and logically consistent treatment, with predictive capacity, it is necessary to represent the atoms and the radiation together in a way that is consistent with the invariance requirements of the theory of

relativity. If this could be done - yielding a mathematically consistent quantum field theory - then the next step would be to take its nonrelativistic limit (as quantities of energy-momentum transfer between atoms becomes small compared with the rest masses, mc^2 of the atoms), thereby hoping to recover the standard Schrödinger formalism.

But since 1927, there has been no real success in formulating such a (demonstrably consistent) mathematical formalism for 'quantum field theory'. For when the Schrödinger-type equation is extended in the proper way, according to the rules of this theory, the resulting equations have no solutions at all ! This is because terms appear in these relativized equations, in a natural way, that are infinite. About 30 years ago, a 'renormalization method' was invented for the purpose of subtracting off these infinities, in order to yield finite numbers from quantum field theory, to be compared with the experimental data. While this technique has given a few predictions that were in remarkable agreement with the experimental facts, it still is not a demonstrably mathematically consistent scheme of calculation - and no bona fide solutions of the theory have in fact been demonstrated[4d]. It is also important, as I indicated earlier, that the formalism of quantum field theory has never been proven to even contain the formalism of Schrödinger's nonrelativistic wave mechanics - which is, at least, a mathematically consistent formalism. Still, there is a strong similarity between the formal structure of quantum field theory and Schrödinger's own anticipation for the extension of his wave theory.

However, a basic difference stands between Schrödinger's view of how his formalism should be extended and the formalism of quantum field-theory. This is that Schrödinger, like de Broglie, recognized the need to depart from the requirement of the linear superposition principle - because of the actual coupling in the real system of charged matter. The mathematical difference between linearity and nonlinearity indeed strikes at a conceptual difference between the positivistic Copenhagen view and the realistic views of matter, according to Schrödinger, de Broglie and Einstein. For the linearity of the Copenhagen formulation of quantum mechanics is tied to the separability of the measuring apparatus (in a dynamical sense) from the atomic matter that is observed. Such separability is similar to that which is found in Faraday's field theory, where there is a field of force to influence a test body, but where it is assumed that the test body can have no influence on the field of force. In contrast, in de Broglie's and Schrödinger's view, interaction plays a crucial role in the logical description of matter, necessitating nonlinearity at least in a part of the formal structure of the theory.

From fully exploiting Einstein's conceptual view of matter, in terms of the axiomatic basis of the theory of relativity ([4b]), the aim toward a nonlinear wave theory would indeed be in the right direction. However, contrary to Schrödinger's position, second quantization, statistical waves and noncountable matter fields would not be acceptable ideas. And contrary to de Broglie's view, the single (singular) particle with an underlying (nonobservable) motion would also be unacceptable. On the other hand, the approach of the theory of relativity would be in full accord with the deterministic, objective features of matter, as required by de Broglie and Schrödinger.

Taking the relativistic view to its logical extreme, it follows that (contrary to de Broglie's view, but in agreement with Schrödinger), the atomistic concept must be abandoned. But (contrary to Schrödinger) neither can the fundamental continuous field variables relate to a noncountable set of elementary particles. Rather, in accordance with the logical structure of this theory, the continuous field solutions of the theory must relate to manifestations of a single closed system[4a]. This is in contrast with both de Broglie an Schrödinger, whose theories entail an open system. The laws of nature pertaining to this closed system are generally in terms of an infinite set of nonlinear, self-consistent field equations, with all solutions mapped in the space-time coordinate system. Asymptotically, the principle of correspondence would require that these coupled nonlinear field equations must approach the standard linear quantum mechanical formalism for a many-particle system, in the limit of nonrelativistic coupling. However, it is important to note that within the logical structure of this theory, the actual limit cannot be reached, in principle, though it can be approached arbitrarily closely.

Now if the coupled nonlinear fields of this theory do not relate to statistical distribution functions, or to discrete particle trajectories, what would the theory of relativity imply that they should signify ? It has been my contention that these fields must relate to a total 'interaction field' for the closed system. At each space-time point that one might wish to consider, the amplitudes of each of the set of n coupled nonlinear fields, for an n-component system, would specify the weighting of the interaction of the closed system considered. In the asymptotic limit of very weak coupling within the system, these n nonlinear fields then approach a set of n linear elements of a Hilbert space. The latter is the quantum mechanical limit which automatically entails the probability calculus of ordinary quantum mechanics. However, it is important to note that, generally, these fields solve a set of coupled nonlinear differential equations, whose solu-

tions do not form the elements of a Hilbert space, nor does their formal structure relate to a probability calculus. Nevertheless, this does not rule out their interpretation in terms of 'weighting' the total amplitude of interaction at each point of a single space-time. In this case, the physical properties of the material system would be expressed in terms of 'weighted averages' of corresponding mathematical operators - though these would be nonlinear operators. The latter only approach the linear operators of the standard formulation of quantum mechanics (eqs. (10), (10'), (10"),...) in the linear limit of this theory, i.e. the nonrelativistic limit of the theory. The explicit expressions for these 'weighted' operators, in the nonlinear field theory, follow from the standard application of Noether's theorem in the nonlinear field theory.

From the conceptual view, the elementarity of the particle of matter, of ordinary quantum mechanics, as well as in de Broglie's and Schrödinger's views, is replaced here with the elementarity of the interaction - relating to the mutuality of the entire closed system, rather than to separate parts in interaction. That is, what it is that is elementary in this view, is 'ordered relation' rather than 'thing'. The word 'interaction' or the word 'relation' is only used here for want of a better word ! This is because these words normally signify that there are independent relata, and then there is a relation between them. But here there are no elementary relata, per se. It is the mutual relation of a closed system that is elementary, and therefore not decomposable into more elementary entities. Still, in the linear limit of the theory, there is an approximate form of its equations that makesit appear as though there were indeed separate atoms of matter. This is the quantum mechanical limit of the theory. However, without approximation, the full structure of the theory gives a complete description of a closed system - therefore it is a deterministic theory of matter, though, with Schrödinger, it gives up the notion of atomism. It is in the latter limit where the Heisenberg uncertainty relations appear in the mathematical structure of the theory. But this happens only when well-defined (i.e. non-hidden) parts of the formal structure of the theory are left out, for purposes of approximating its formalism. In such an approximation one only has a partial description of a system, which nevertheless does have a complete underlying formalism. Thus, while the Heisenberg uncertainty relations are useful for computational purposes, when particular approximations would be valid, they are not to be considered as a valid principle, in the way that they are conventionally interpreted by Heisenberg, and by the Copenhagen school.

It should be noted that even when the linear approximation is used, there are residual features of the nonlinear formalism that are very important in its linear (quantum mechanical) approximation. One of these concerns the Pauli exclusion principle, and the relation of this principle to the spin-statistics connection for a many-fermion system. In my research program, I have found that the total interaction field of this theory, for the closed system, applied to equally massive components, described by spinor fields, vanishes identically if any two out of n coupled fields would correspond to the same state of motion and a mutually repulsive interaction. This means that the total 'weighting' for such an interacting system is identically zero, which, in turn, implies that under these conditions no observable physical property is predicted. This conclusion is equivalent to the implications of the Pauli exclusion principle, even though the fields of this theory relate to the elementarity of interaction, rather than the elementarity of particle. It is important in this derivation that the result was sensitive to 1) the symmetry in the coupled equations with respect to the field variables of the interacting components (i.e. the symmetry between the 'observer' and the 'observed') and 2) the exact result predicted here was sensitive to the nonlinear structure of the formalism. Both of these features are absent from the conventional quantum mechanical formalism - in principle !

Finally, in the asymptotic limit of this theory, it is found that the total interaction field for the closed system approaches the totally antisymmetrized many-particle wave function that is conventionally evoked in quantum mechanics for the many-fermion system. The latter is the mathematical expression for the connection between spin and statistics for such a system in quantum mechanics. Thus, it is concluded that this nonlinear field theory, which totally exploits the concepts of relativity theory, predicts a feature in nature that is not fully predicted by the quantum theory itself ([4e]) - the Pauli exclusion principle. This result refutes Pauli's claim that its derivation requires the use of the principle of linear superposition.

REFERENCES

([1]) L. de Broglie,

a) Annales de la Fondation Louis de Broglie 1, pp. 7-21 (1975).

b) Etude critique des bases de l'interprétation actuelle de la mécanique ondulatoire, Gauthier-Villars, 1963.

([2]) A. Einstein, "Autobiographical Notes", in Albert Einstein Philosopher-Scientist (P.A. Schilpp, ed.) Library living Philosophers, 1949.

([3]) A. Einstein, E. Schrödinger, M. Planck, H.A. Lorentz, Letters on Wave Mechanics (K. Przibram, ed.) Philosophical library, 1967.

([4]) M. Sachs,

a) "Une Théorie Continue de la Matière Elémentaire", La Recherche 6, pp. 1008-1017 (1975).

b) Ideas of the Theory of Relativity, Keter-Wiley (Halsted Div.), 1974.

c) The Field Concept in Contemporary Science, Thomas, 1973.

d) "Is Quantization Really Necessary ?", Brit. Jour. Phil. Sci. 21, pp. 359-390 (1970).

e) "The Pauli Exclusion Principle from a Self-Consistent Field Theory of Quantum Electrodynamics", Nuovo Cimento 27, pp. 1138-1150 (1963).

([5]) E. Schrödinger,

a) "Are there quantum jumps ?" Brit. Jour. Phil. Sci. 3, pp. 3 -28 (1952).

b) "The Meaning of Wave Mechanics" in Louis de Broglie, Physicien et Penseur, Albin Michel, 1952.

c) "What is an Elementary Particle ?" Smithsonian Institution, n° 4028, pp. 183-196 (1950).

d) "The Philosophy of Experiment", Nuovo Cimento 1, pp. 5-15 (1955).