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ELECTRIC NETWORK ANALOGUES OF LARGE SYSTEMS

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Abstract : We consider a "large system", wether in physics, biology or socio-economics, to be a thermodynamic system in a stationary state of nonequilibrium according to the laws of thermodynamics of irreversible processes. We associate the change of entropy $S_o - S$ *with the structural organization of the system and the entropy flow* d_iS/dt *which occurs between the system and its environment, with the function performed by the system. Entropy flows can be interpreted as information currents and on this basis we suggest a method of modelling the behaviour of "large systems" by electric network analogues.*

Résumé : On considère un "grand système", que ce soit en physique, en biologie ou en socio-économie, comme étant un système thermodynamique dans un état stationnaire de nonéquilibre suivant les lois de la thermodynamique des processus irréversibles. Nous associons le changement d'entropie $S_o - S$ *avec l'organisation structurelle de ce système et le flux d'entropie* d_iS/dt *qui existe entre le système et son milieu ambiant avec la fonction que ce système exerce. Le flux d'entropie est interprété comme étant un courant d'information et à partir de cette remarque on indique une méthode de simulation du fonctionnement d'un grand système par un circuit électrique équivalent.*

INTRODUCTION.

Since the advent of cybernetics and information theory the study of "large systems" is a topic of growing interest in biology, socio-economics, communication theory, etc... Usually the word "large system" suggests that we are in the presence of a complex structure whose various components coordinates their activity in order to fulfill a specific function and one of the main problems is to understand the dependence between "function" and "structure organization" under the tacit assumption that, in view of the complexity of the system, we want to avoid the necessity of describing various mechanisms at the level of single components. We know that thermodynamics succeeded very well in this approach, as far as physical systems are concerned, by introducing a set of variables (pressure, volume, temperature, etc...) appropriate for the description of "macroscopic" states and only later on statistical mechanics tried to relate these variables to the behaviour of the system at a deeper level by applying the laws of mechanics to molecular components. However, in dealing with systems outside the restricted domain of physics, one has the feeling that variables like information, organization, redundancy, etc..., should be given the status of independent variables since concepts like energy, fields and so on, are not sufficient to express what one might call the cybernetic features of the system. Another general feeling is that the concept of entropy S introduced in the second law of thermodynamics, the Carnot-Clausius principle, can play an important role in the study of large systems whatever is the scientific discipline to which they belong and there are several reasons for that. In fact entropy provides us with the most general criterium of evolution of the natural world. Moreover, the statistical interpretation of S by Boltzmann establishes a remarkables relation between entropy and a certain notion of complexiry which applies to any large system, thus transforming the Carnot-Clausius principle into a physical law of the order - disorder type. The celebrated work of L. Brillouin (1) on negentropy has further evidenced the application of Boltzmann formula in linking science and information. Finally modern developments in thermodynamics of irreversible processes due to I. Prigogine (2) and several co-workers have shown the importance of the temporal derivative $\frac{dS}{dt}$ of entropy in the treatment of nonequilibrium states, in particular the so-called stationary states.

On the basis of these considerations we shall propose a method of simulating stationary states of large systems by electric network analogues where the basic electric variables are directly

related to S and $\frac{dS}{dt}$ in such a way that they can be interpreted in terms of information currents and structure organization. We believe that one of the interesting features of this technique is that it emphasizes the duality between "function" and "structure organization" in the sense of the old chicken-and-the egg problem. More precisely we shall see that equivalent circuits dramatize the usual difficulty of distinguishing between "cause" and "effect" when we consider simultaneously information processing and internal structure of the system. We shall discuss first the equivalent circuit of a communication channel of Shannon information theory and then we shall examine the problem of a biological structure which transforms random excitations into information according to the model developed by H. Atlan (3) in a recent work. The interesting result, in this example, is that the performance of the biological structure is simulated quite well by the characteristics of a transistor and this suggests that semiconductor devices could play an important role in the modelling and understanding of large systems.

It is certainly very exciting to anticipate that further progress in the thermodynamics proposed by L. de Broglie (11) for the study of particles in microphysics will definitively allow to extend the concept of "large system" even to a single particle. It will then be tempting to try to interpret the quantization of physical observables as a form of structure organization resulting from a flux of information that the microsystem exchanges with a hidden heat bath. Results pointing towards the reasonableness of this viewpoint can already be found in the work of G. Lochak (12) and his co-workers. Paraphrasing the title of L. de Broglie's book cited before we could eventually speak of a "self-organizing theory of the isolated particle" and this would certainly be in agreement with one of the last ideas expressed by N. Wiener (13) when he was asked to make suggestions as to the way that, in his opinion, fundamental science is going now.

Stationary states in the thermodynamics of irreversible processes.

According to the Carnot-Clausius principle the entropy S of an isolated system in which irreversible processes are taking place increases with time : $\frac{dS}{dt} > 0$. Since every natural process always involves a certain degree of irreversibility, a corollary to this principle is that for an isolated system, equilibrium is a state of maximum entropy S_0. Therefore, if we want to avoid that a system reaches a state of equilibrium in which all activity has ceased, we must allow a certain interaction between the system and its environment that is to say exchanges of energy and/or matter with the exterior. Thermodynamics of irreversible processes is the science of states of nonequilibrium corresponding to entropies $S < S_0$. A basic postulate of this thermodynamics is that the temporal derivative of the system entropy can be written as :

$$\frac{dS}{dt} = \frac{d_iS}{dt} + \frac{d_eS}{dt}$$

where $\frac{d_iS}{dt}$ is the production rate of entropy due to irreversible processes occurring in the interior of the system, and $\frac{d_eS}{dt}$ is a term expressing a flux of entropy between the system and the environment associated to various forms of interaction. The Carnot-Clausius principle applies only to $\frac{d_iS}{dt}$, thus $\frac{d_iS}{dt} > 0$, however the sign of $\frac{d_eS}{dt}$ depends on the nature of the interactions. Let us write :

$$\frac{d_eS}{dt} = \left(\frac{d_eS}{dt}\right)_1 + \left(\frac{d_eS}{dt}\right)_2$$

where we assume that $\left(\frac{d_eS}{dt}\right)_1 < 0$ is the part of the entropy flux which tends to reduce the system entropy, whereas $\left(\frac{d_eS}{dt}\right)_2 > 0$ is the part corresponding to interaction phenomena which tend to increase that entropy. The balance equation for entropy becomes :

$$\frac{dS}{dt} = \frac{d_iS}{dt} + \left(\frac{d_eS}{dt}\right)_1 + \left(\frac{d_eS}{dt}\right)_2$$

The system will be said to be in a stationary state of nonequilibrium if it maintains a constant entropy level $S < S_0$. This implies that $\frac{dS}{dt} = 0$ and :

$$\frac{d_iS}{dt} + \left(\frac{d_eS}{dt}\right)_1 + \left(\frac{d_eS}{dt}\right)_2 = 0$$

Let us now introduce negentropy (-S) and consider fluxes of information $I = -\frac{dS}{dt}$ instead of fluxes of entropy. The condition of stationarity becomes :

$$I_1 = I_i + I_2$$

where $I_1 = -\left(\frac{d_eS}{dt}\right)_1$ is an input flux of information, $I_i = \frac{d_iS}{dt}$ and $I_2 = \left(\frac{d_eS}{dt}\right)_2$ are output fluxes of information. Notice that according to our sign conventions, an input flux of information increases the negentropy of the system and, vice versa, an output flux decreases it.

We thus arrive to the idea of representing a stationary large system as a "black box" which processes information "currents" as in fig. 1, where the conditions are $I_1 = I_i + I_2$ and constant entropy $S < S_0$.

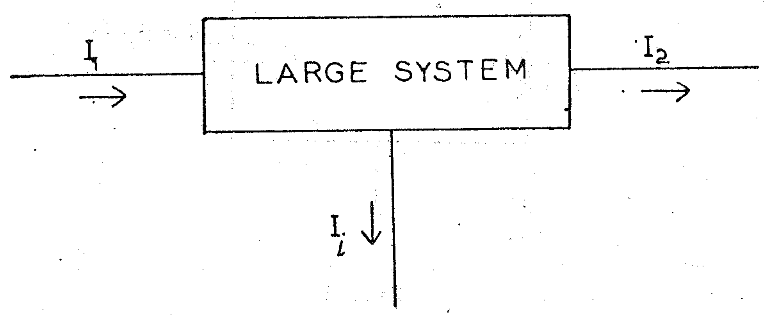

Fig. 1

Electric network analogues.

Let us for convenience express entropy in information units that is bits. In fig. 1, wa can consider that I_1, I_i and I_2 represent electric currents measured in bits/second, and we want now to build the equivalent circuit of the black box standing for the large system. Our starting point will be Boltzmann formula $S = \log P$ (in information units) relating entropy S to the number P of possible configurations of a system, when these configurations are assumed to be a priori equiprobable. The equilibrium S_0 is a state of maximum disorder with $S_0 = \log P_0$. A departure from that state represents a certain amount of organization which we shall measure by the quantity $S_0 - S = \log P_0/P$

with $P < P_0$. Let us adopt the quantity $S_0 - S > 0$ as a definition of structure organization, whether this is due to a decrease in the number of possible configurations or, more generally, to any other physical reason. We now observe that if we suddenly isolate the system, $S_0 - S$ will go to zero in accordance with the Carnot-Clausius principle. On the other hand let us imagine that in an electric circuit we have stored q positive charges.

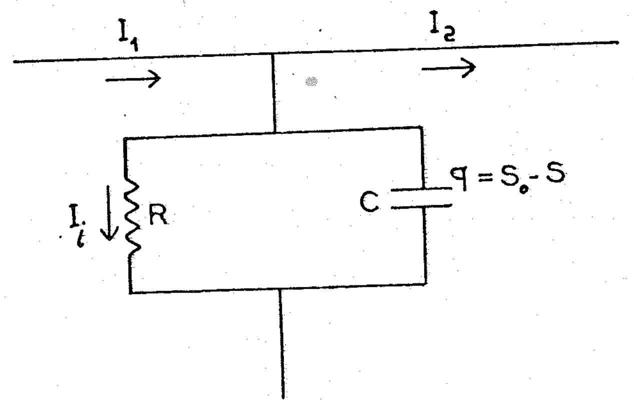

Fig. 2

Due to various poor insulations, we usually observe that when cut from all external electric sources, these positive charges gradually disappear by recombination with electrons : equilibrium is reached with electrical neutrality. This suggests that we model structure organization by the electric variable $q = S_0 - S$ expressed in bits, one bit of organization for one positive charge. The diagram of fig. 2 is the simplest example of an electric circuit which can illustrate everything we have said until now :

1°) Under the stationary regime indicated, the capacitor C stores $q = CRI_i = S_0 - S$ bits. If the circuit is suddenly isolated, that is if we let $I_1 = I_2 = 0$, electrons will flow through the resistor R and neutralize q according, in this particular example, to the law $q = A \exp(-t/_{RC})$. Thus irreversible processes are accounted for by the resistor R; moreover, one might say

that the electric counterpart of the Carnot-Clausius principle itself is found in the action of electrostatic forces attracting electrons onto the positive charges.

2°) This example also shows the duality between function and structure organization mentioned in the introduction. In fact, one can say either that the polarization q is the effect of the potential $V = RI_i$ across the capacitor C, due to the presence of the flux of information I_i or, equivalently, that an initial structure organization $q = S_0 - S$ has caused I_i because of the same induced potential $V = \frac{q}{C}$. Of course in our modelling technique, electric potentials are supposed to be electric analogues for thermodynamic "affinities" (temperature gradients, mass concentration gradients, chemical potential differences, etc ...), but we leave this particular problem for future investigations.

Before we go into the specific examples treated in the next sections, we can derive another fact from our general discussion. As we know, electric currents in a conductor are due to electrons moving in opposite direction. We can adopt the same picture for fluxes of information : an input flux of information I is wiewed as I negative bits/second leaving the circuit thus corresponding to an increase of the positive polarization q at the same rate which means a reduction of S : this is consistent with what we have said previously about the respective effect of input and output fluxes of information on the negentropy of the system. To sum up : positive charges represent bits of structure organization and electrons serve as carriers of information flowing in and out the system. Later on, when dealing with semiconductors we shall find an intersting interpretation also for hole currents, as we did here for electron currents.

The network analogue of a Shannon communication channel.

Let us apply our modelling technique to a simple communication system as encountered in Shannon ([4]) information theory. Let X be a discrete memoryless source producing messages written in an alphabet consisting of n different symbols and assume that the source operates at a constant rate of r symbols/second. If nothing else is known about the source, we must assign to it its maximum entropy $H_0(X) = \log n$ bits/symbol and a message represents a flux of information equal to $I_1 = r\,H_0(X)$ bits/second. If, however, the source is constrained to produce messages in a certain language where the symbols are emitted according to a probability distribution $(p_1, \ldots, p_n)$, instead of being assumed to be equally distributed, the entropy becomes $H(X) = -\Sigma_i p_i \log p_i < H_0(X)$. The particular distribution of the symbols in the message reflects an additional structure organization of the source which, according to our previous definitions, is measured by $q = H_0(X) - H(X)$ and is associated with the flow of information $I_i = r(H_0 - H)$.

In fact the available flux of information is now $I_2 = I_1 - I_i = rH(X)$. Let us suppose that there is an observer Y connected to X by a noisy channel. The existence of Y further reduces the entropy of X to the value $H(X/Y)$; this additional structure organization $q' = H(X) - H(X/Y)$ corresponds to a flux of information $I'_i = r\big(H(X) - H(X/Y)\big)$ and the remaining flux of information is now $I_3 = rH(X/Y)$. The equivalent circuit of this system is given in fig. 3. Since, in this example, the relation between currents and charges is always linear and of the form $I = rq$, we need only RC circuits all having the same time constant $\tau = RC = R'C' = \frac{1}{r}$. Finally we notice that quantities like $H_0 - H$ or $H(X) - H(X/Y)$ which we generally called structure organization are usually called redundancies, in information theory.

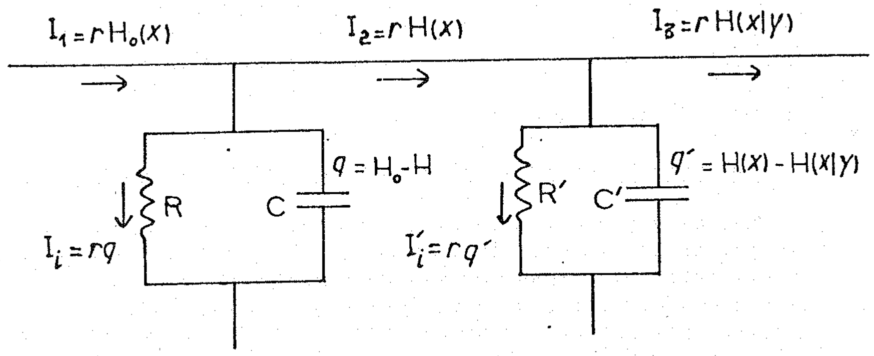

Fig. 3

Information processing by a biological structure according to H. Atlan model.

In a recent work H. Atlan [3] considered the quantity of information H that a biological structure, under certain simplifying assumptions, can produce when it is exposed to various doses θ of external random excitations. The general shape of the curve $H = f(\theta)$ is reproduced in fig. 4(a). For small values of θ, H is increasing; this phase is interpreted as a self-organizing response of the system during which internal redundancy is used to transform disordered excitations into ordered information. After an optimal value H_{max} has been reached, further increases of θ reduce the performance of the structure; this is the aging phase where H decreases with θ.

Some years ago we did a mathematical analysis of the dependence of the current amplification coefficient β of a transistor on the leval θ of minority carriers injected by the emitter into the base [5]. While reading Atlan's work we were struck by the remarkable similarity between the shapes of $H = f(\theta)$ and $\beta = f(\theta)$ shownin fig. 4 (b). Actually it is on the basis of this curious analogy that we developed the idea of simulating large systems by electric analogues.

We don't need to recall here the theory of the transistor and the explicit form of $\beta = f(\theta)$; these can be found in the original paper [5]. For a given θ, the transistor is in a stationary state defined by an emitter current $I_e = I_1$, a base current $I_b = I_i$ and a collector current $I_c = I_2$. The current gain is by definition $\beta = \frac{I_c}{I_b}$. We consider a PNP transistor and we assume that the biological structure we want to model corresponds to the base B of the transistor, and that its environment is represented by the emitter E and the collector C. Hence, in this example, we are using typical current-voltage characteristics of semiconductor junctions to simulate relations between fluxes of entropy (information currents) and thermodynamic affinities.

In doing so we should keep in mind that an electric current I through a semiconductor junction is always the sum of a current of holes I_p and a current of electrons I_n. We want to assign a specific role to a current of holes with regard to structure

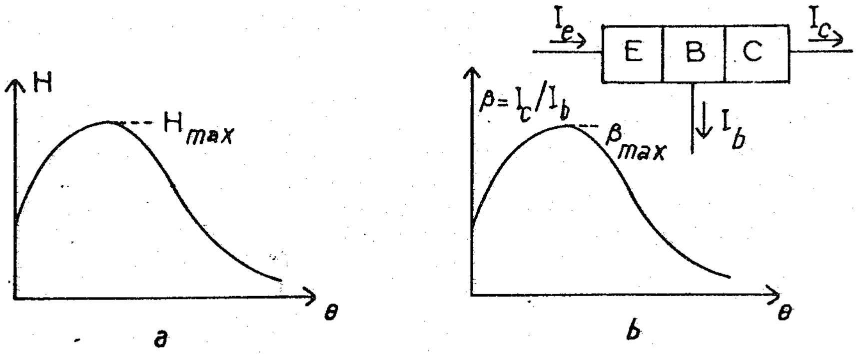

Fig. 4

organization $q = S_0 - S$ as we did previously for electrons. In the expression $q = S_0 - S$, we assumed until now that S_0 is a constant depending on the size and complexity of the system according to Boltzmann formula $S_0 = \log P_0$. Since holes in a semiconductor behave like free positive charges, we shall imagine that an input current of holes in the circuit represents a positive increase of S_0. Therefore, if $I = I_p + I_n$ is an input current, both of its components will tend to increase structure organization : the injection of holes I_p represents size and/or complexity increase of S_0 whereas the outflow of electrons I_n represents reduction of the system entropy S. This is precisely what the emitter current $I_e = I_p + I_n$ is doing in our PNP transistor. The base current $I_b = I_i$ corresponds to irreversible processes occurring in B; this is in fact a current of electrons $I_b = I_n + I_r$ entering the base, where I_n is the electron component of I_e and I_r is a recombination current of electrons due to the fact that injected holes in the base are minority carriers with a finite lifetime. Notice that lifetime of minority carriers in a semiconductor could be a convenient analogue for simulating concepts as reliability and lifetime of various elements of a large system. Finally the output flux or information is the collector current $I_c = I_e - I_b$.

All these currents are increasing functions of θ and $\beta = \frac{I_c}{I_b} = f(\theta)$ measures the efficiency of the structure in the

sense that a large value of β means a large output of information I_c for a relatively small flux I_b necessary to maintain the system in a state fo low entropy. The interesting result is that as the activity level θ increases, β begins first to increase up to an optimal value β_{max}, but then I_b grows faster than I_c and β goes down; thus the aging phase is due to oversize and excess of activity.

It is certainly worthwhile to stress the fact that Atlan's model is essentially a communication channel treated in terms of Shannon information theory while our model relies heavily on the laws of solid state physics, but the results are similar and can be used in a biological context.

Conclusion.

The theory of modelling large systems by electric network analogues as suggested in this paper could be applied whenever the system behaviour can be tied up in some way or another to the laws of thermodynamics. This has been done in communication theory and biology, and according to recent works [6,7] it seems that economics also is a discipline which could profit from a thermodynamic approach; in fact one can already find a few applications of circuit theory to the study of some specific problems in economics [8,9]. We hope that our contribution will stimulate research in that direction.

Finally we also hope that the set of variables we derived from thermodynamics of irreversible processes and the idea of using semiconducteur devices for simulation will represent a useful addition to the general system theory of F. Evans [10] and several co-workers who succeeded in incorporating thermodynamics, network and variational concepts, thus obtaining a powerful tool for the understanding of systems belonging to seemingly far apart disciplines.

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