Annales de la Fondation Louis de Broglie - Vol. 3, n° 1, 1978

SUR LE PROBLEME DE LA REGLE DE CORRESPONDANCE EN THEORIE QUANTIQUE

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(manuscrit reçu le 10 Mars 1977)

Abstract : In the present paper a class of correspondence rules (rules of constructing quantum operators) is determined, in which Neumann's requirement (an operator of the function is equal to the function of the operator) has been replaced by a weaker one. This class may contain both unique and non-unique correspondence rules and includes every Neumann correspondence rule.

The main consequences, to which the application of the correspondence rules from the determined class to quantum theory leads, are considered.

Résumé : On définit la classe des règles de correspondance (règles de la construction des opérateurs quantiques) dans laquelle la condition de Neumann (l'opérateur de la fonction est égal à la fonction de l'opérateur) est remplacée par une condition plus faible. Cette classe peut contenir les règles de correspondance aussi bien univoques que non univoques, y compris toutes les règles de correspondance de Neumann.

Des conséquences essentielles découlant de l'utilisation de ces règles de correspondance en théorie quantique sont examinées.

INTRODUCTION

The problem of constructing quantum operators O(A) which represent the physical quantities A in quantum theory was formulated at the time of the birth of quantum mechanics. ([1-4]). In the course of the development of quantum theory quite a few works have been published (see ([1-9]) for example) which offered various recipes of constructing quantum operators (i.e. various correspondence rules).

The best known of them are the symmetrization rule ([5]) and the correspondence rules of Neumann ([1]), Dirac ([2]), Weyl ([3]), Born-Jordan ([4]).

The number of the correspondence rules alone, which are currently under consideration, indicates that the problem has neither complete nor conventional solution so far. Moreover, a detailed analysis ([10-15]) shows that all generally accepted correspondence rules have essential drawbacks. Thus, in the case of non-unique rules ([1,2]) the technique of constructing quantum operators has not been fully defined and all attempts to eliminate non-uniqueness inevitably lead to contradictions ([10-12]). Most of the unique correspondence rules do not guarantee positive definiteness of operators for the non-negative functions of physical quantities, such as dispersions ([11-12]).

The operators of the current quantum theory satisfy the well-known Neumann set of requirements ([1]), presupposing in particular, that

$$O(f(A)) = f(O(A)).$$

The above requirement is incompatible with those of linearity and uniqueness of the correspondence rule ([14]), i.e. the Neumann set of requirements cannot determine the unique correspondence rule.

In this connection it seems reasonable to sacrifice the above requirement and replace it by its sequence

$$\langle O(f(A)) \rangle \geqslant 0, \quad \text{if} \quad f(A) \geqslant 0.$$

The resulting set of requirements is essentially the same Neumann rule, though slightly less restricted, and it can determine the unique rule of correspondence. (As an example, one can cite the principle of constructing quantum operators in the

so-called quantum mechanics with a non-negative quantum distribution function ([16])). In a general case, however, $O(A^2) \neq O(A)\,O(A)$ which gives rise to a number of mathematical and physical consequences. The study of these consequences is the principal aim of the present work.

I . The Neumann rule of correspondence and its consequences

The Neumann correspondence rule, which is the basis of the current (modern) quantum theory, includes the following requirements for quantum operators :

(Ia) $$O(1) = \hat{I},$$

(Ib) $$O(A+A_1) = O(A) + O(A_1),$$

(Ic) $$O(\alpha A) = \alpha\, O(A),$$

(Id) $$O^+(A) = O(A),$$

(Ie) $$O(f(A)) = f(O(A)).$$

Here, $\hat{I}$ is a unit operator, A_1, A are arbitrary physical quantities, α is a real constant, $O^+(A)$ is an operator, conjugated on O(A), f is an arbitrary rational function of A.

Let any correspondence rule, in which operators of physical quantities satisfy the set of requirements, listed in (I), be called the Neumann correspondence rule. The properties of operators (I) involve a few physical consequences of the quantum theory based on the Neumann correspondence rule, such as :

1) The value <A> of the physical quantity A, characterizing a system in the state $|\Psi\rangle$ and calculated by the formula

(2) $$\langle A\rangle = \langle\Psi|O(A)|\Psi\rangle,$$

can be interpreted as some average value, i.e. as the mathematical expectation for the quantity A.

2) The average values <A> of physical quantities are real. The specific Neumann requirement (Ie) in the particular case,

when $f(A) = A^2$ takes the form

$$O(A^2) = O^2(A) \qquad (3)$$

and leads to the three following consequences :

3) The average value of the square of any physical quantity in an arbitrary state is non-negative.

4) The dispersion $\langle(\Delta A)^2\rangle$ of the quantity A, i.e. the average value of the quantity's squared deviation from its average value, which is a functional of the state $|\Psi\rangle$

$$\langle(\Delta A)^2\rangle = \langle\Psi|O([A-\langle A\rangle]^2)|\Psi\rangle,$$

is a positive number. Indeed, in conformity with (3) and the condition of self-adjoint operators (Id), we have :

$$\langle(\Delta A)^2\rangle = \langle\Psi|O([A-\langle A\rangle]^2)|\Psi\rangle = \langle\Psi|O^2(A-\langle A\rangle)|\Psi\rangle \geqslant 0. \qquad (4)$$

5) The value $\langle A\rangle$ is exact for the state, represented by the eigenvector of the operator $O(A)$, i.e. the dispersion of the quantity A in this state is zero.

The property of the operators (3) and consequence 5) are of equal value in the sense, that we inevitably obtain equality (3), provided that the statement 5) is postulated (I).

Thus, the set of requirements (I), which determine the class of the Neumann correspondence rules, brings about reasonable physical consequences of the quantum theory 1) - 5).

However, due to the requirement (of the uniqueness) of the one-to-one correspondence of quantum operators and physical quantities, the properties (Ib) and (Ie) become incompatible ([14]). Therefore, we propose that certain changes be introduced into the set of requirements for the correspondence rules, so that they gain the property of uniqueness while retaining as many as possible of the above-mentioned consequences 1) - 5).

2. General formulation of the Non-Neumann correspondence rule

Let us introduce the following set of requirements for quantum operators :

(5a) $\quad O(1) = \hat{I},$

(5b) $\quad O(A+A_1) = O(A) + O(A_1),$

(5c) $\quad O(\alpha A) = \alpha\, O(A),$

(5d) $\quad O^+(A) = O(A)$

(5e) $\quad O(f(A))$ a positive operator for any non-negative function $f(A)$

and let any correspondence rule, in which operators of physical quantities satisfy the set of requirements (5), be called the Non-Neumann rule.

The requirements, imposed on quantum operators in (5) differ from those in (I) only in that the requirement (5e) is less restricted than (Ie). Any Neumann correspondence rule, therefore, can be considered as a Non-Neumann one. However, the class of Non-Neumann correspondence rules includes both the unique and the non-unique rules of quantum operator construction (in contrast to the class of the Neumann rules which comprises only non-unique ones).

Attempts have been made to use the Non-Neumann unique rule for quantum operator construction. Paper ([16]) offers the so-called quantum mechanics with non-negative distribution function, based on the unique correspondence rule, satisfying the non-Neumann definition (5).

Let us formulate the physical consequences of the quantum theory with the Non-Neumann correspondence rule resulting from the set of requirements for operators.

1. The value $\langle A\rangle$ of the physical quantity A, characterizing the quantum system in the state $|\Psi\rangle$ which is calculated by using (2), can be interpreted as an average value. The requirements (5a), (5b) and (5c) make it possible to consider the value $\langle A\rangle$ as the mathematical expectation of the quantity A.

2. The average values $\langle A \rangle$ of physical quantities are real, which is obvious from the self-adjointness of quantum operators (property (5d)).

The requirement (5e), imposed on quantum operators in a special case when $f(A) = A^2$, means a positive definiteness of the operator $O(A^2)$, which we can symbolically write in the form

(6) $$O(A^2) \geqslant 0.$$

In a general case instead of Eq. (3), we have

(7) $$O(A^2) = O^2(A) + \mathcal{D}(A).$$

The last formula should be considered as a definition of some operator $\mathcal{D}(A)$.

3. The average value of the square of any physical quantity in an arbitrary state is non-negative due to the inequality (6) which follows from the requirement (5e).

4. The dispersion of the quantity A

(8) $$\langle\Psi|O([A-\langle A\rangle]^2)|\Psi\rangle = \langle\Psi|O^2(A-\langle A\rangle)|\Psi\rangle + \langle\Psi|\mathcal{D}(A)|\Psi\rangle,$$

is positive due to the requirement (5e). Indeed, when $f(A) = (A-\langle A\rangle)^2$ the operator $O((A-\langle A\rangle)^2)$ is positive and its average value in any state is non-negative.

There is no analogue of consequence 5) in this case. Moreover, the dispersion of the quantity A in the eigenstates of the operator O(A) is not equal to zero.

$$\langle(\Delta A)^2\rangle = \langle\Psi|\mathcal{D}(A)|\Psi\rangle \neq 0.$$

Now the physical meaning of operator $\mathcal{D}(A)$ is quite clear : $\mathcal{D}(A)$ is the dispersion operator in the eigenstates of the operator O(A). Since the dispersion of the quantity A in the eigenstates of the operator O(A) is not zero, the value A in these states can no longer be determined exactly. Thus, the problem of definiteness of physical quantities aquires a particular significance.

3. The states of maximum certainty

Let us set the task of finding the states $|\Psi\rangle$ in which the value $\langle A\rangle$ is determined with maximum accuracy. The degree of certainly of the value $\langle A\rangle$ has also a particular meaning because of the general statistical nature of quantum theory.

One can consider dispersion, i.e. the conditional functional of the state $|\Psi\rangle$ (8), as the quantitative measure of certainty. The requirement of the minimum uncertainly can be written in the form

(9) $$\delta\{\langle(\Delta A)^2\rangle\} = 0.$$

The functional's minimum (8) is guaranteed by its zero restriction from below, since the dispersion of any physical value in an arbitrary state is positive (see §2).

The condition of the normalization of quantum states can be taken into account by transforming the formula the calculation of average values (2)

(10) $$\langle A\rangle = \frac{\langle\Psi|O(A)|\Psi\rangle}{\langle\Psi|\Psi\rangle}.$$

Then, by using the definition (8) and the operator properties (5a), (5b), (5c) problem (9) with a conditional extremum can be reduced to the following unconditional one :

(11) $$\delta\left\{\frac{\langle\Psi|O(A^2)|\Psi\rangle}{\langle\Psi|\Psi\rangle} - \left(\frac{\langle\Psi|O(A)|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)^2\right\} = 0.$$

Considering variations of $|\Psi\rangle$ and $\langle\Psi|$ independent let us fulfil the procedure of variating analogous to that made in book [17]. By varying $\langle\Psi|$, and making some simple transformations, we arrive at

(12) $$\frac{1}{\langle\Psi|\Psi\rangle}\langle\delta\Psi|O(A^2) - 2\frac{\langle\Psi|O(A)|\Psi\rangle}{\langle\Psi|\Psi\rangle}O(A) + 2\left(\frac{\langle\Psi|O(A)|\Psi\rangle}{\langle\Psi|\Psi\rangle}\right)^2 - \frac{\langle\Psi|O(A^2)|\Psi\rangle}{\langle\Psi|\Psi\rangle}|\Psi\rangle = 0.$$

Taking into account the arbitrary choice of variation $\langle\delta\Psi|$, from Eq. (12) we get

(13) $$\{O(A^2) - 2\alpha O(A) + \alpha^2\}|\Psi\rangle = d^2|\Psi\rangle,$$

where

$$\langle A\rangle = \alpha, \qquad \langle A^2\rangle - (\langle A\rangle)^2 = d^2.$$

Thus, the physical meaning of α and d^2 is obvious. The nonlinear equation (13) (α depends on $|\Psi\rangle$) defines some family of quantum states $|\Psi\rangle$, related to the physical quantity A in the sense, that the uncertainty $\langle(\Delta A)^2\rangle$ increases with the transition $|\Psi\rangle \to |\Psi\rangle + |\delta\Psi\rangle$.

It is, therefore, natural that these states be called the states of maximum certainty of the physical quantity A.

Note should be made, that Eq. (13) defines the extremum states, i.e. the family of its solutions may contain, apart from the states of maximum certainty of the quantity A, some other extremum states.

4. Some properties of the equation of maximum certainty

In the present section we consider some properties of the above equation of maximum certainty (13) and its eigenvalues.

1. In spite of the non-linearity of Eq. (13), the state vector $C|\Psi\rangle$ (C = const) is a solution of Eq. (13) if $|\Psi\rangle$ is a solution in conformity with (10). That is why any solution of Eq. (13) can be normalized to unity.

2. In accordance with the Neumann correspondence rule Eq. (13) is transformed into an eigenvalue equation of the operator O(A)

$$O(A)|\Psi\rangle = \alpha|\Psi\rangle. \tag{14}$$

Indeed, let us rewrite Eq. (13) using the definition for the operator $\mathcal{D}(A)$

$$\{[O(A) - \alpha]^2 + \mathcal{D}(A)\}|\Psi\rangle = d^2|\Psi\rangle. \tag{15}$$

The Neumann correspondence rule implies $\mathcal{D}(A) \equiv 0$ (property (Ie)). One can, therefore, easily extract the root of the operator in the left-hand-part of Eq. (15).

$$O(A)|\Psi\rangle = (\alpha + d)|\Psi\rangle.$$

Multiplying both sides of this equation by $\langle\Psi|$, we find that d = 0 and obtain Eq. (14). Thus, for the Neumann correspondence rule the states of maximum certainty of the physical quantity A are the eigenstates of the operator O(A).

3. Eqs. (13) and (14) are equivalent when

$$[O(A^2), O(A)] = 0 \tag{16}$$

From the commutator (16) and the definition (7) follows the commutation of the operator $\mathcal{D}(A)$ and O(A), and, consequently the coincidence of the solution of Eqs. (13) and (14).

Really, commutation of the operators O(A) and O(A^2) means

$$O(A)|\phi_m\rangle = a_m|\phi_m\rangle,$$
$$O(A^2)|\phi_m\rangle = b_m|\phi_m\rangle,$$

where $\{|\phi_m\rangle\}$ is a complete orthonormalized system of vectors, $a_n = a_m$ if and only if m = n. Let $|\psi_\mu\rangle$ be the solution of (13) and $|\psi_\mu\rangle$ does not satisfy Eq. (14). After substitution of $|\psi_\mu\rangle$ in the form

$$|\psi_\mu\rangle = \sum_n C_n^\mu|\phi_n\rangle$$

to Eq. (13) one can easily get

$$C_n^\mu(b_n - 2\alpha_\mu a_n + \alpha_\mu^2 - d_\mu^2) = 0.$$

For any n if $C_n^\mu \neq 0$ the last equality transforms to

$$b_n - 2\alpha_\mu a_n + \alpha_\mu^2 - d_\mu^2 = 0$$

and

$$\alpha_\mu = a_n \pm\sqrt{d_\mu^2 - d_n^2} \overset{\text{def}}{===} a_n + \chi_n$$

The obvious correlation $b_n = d_n^2 + a_n^2$ has been used here. Let us substitute this value of α_μ in Eq. (13)

$$\sum_m C_m^\mu[d_m^2 - d_n^2 + (a_m - a_n)^2 - 2(a_m - a_n)\chi_n]|\phi_m\rangle = 0$$

If some $m \neq n$ exists for which $C_m^\mu \neq 0$, then

$$\chi_n = \frac{d_m^2 - d_n^2}{2(a_m - a_n)} + a_m - a_n.$$

Comparing now the two equal expressions $a_n + \chi_n$ and $a_m + \chi_m$ we obtain $a_m = a_n$; this is possible only when m = n. Therefore the state $|\psi_\mu\rangle$ coincides with $|\phi_n\rangle$, i.e. $|\psi_\mu\rangle$ satisfies Eq. (14).

The commutation (16) can, naturally, be achieved only for some concrets realizations of the Non-Neumann correspondence rule.

4. Theorem. If the family of solutions of the equation of maximum certainty (13) forms a complete orthonormalized system of vectors in the state-space, and $\alpha_k \neq \alpha_\ell$ for $k \neq \ell$, then equations (13), (14) are equivalent.

To prove the theorem, let us consider the functional $\langle\psi_\ell|O(A^2)|\psi_k\rangle$, here $|\psi_k\rangle$, $|\psi_\ell\rangle$ are solutions of Equ. (13). By using (13), the self-adjointness of the operators (5d) and their orthonormalization, we obtain :

$$\langle\psi_\ell|O(A^2)|\psi_k\rangle = 2\alpha_k\langle\psi_\ell|O(A)|\psi_k\rangle = 2\alpha_\ell\langle\psi_\ell|O(A)|\psi_k\rangle$$

and, since $\alpha_k \neq \alpha_\ell$

(17) $$\langle\psi_\ell|O(A)|\psi_k\rangle = 0.$$

In view of the completeness of the system of vectors $\{|\psi\rangle\}$ correlation (17) can be rewritten as

(18) $$O(A)|\psi_k\rangle = \alpha_k|\psi_k\rangle,$$

i.e. any solution of Eq. (13) satisfies Eq. (14). Using (18) it is easy to obtain from the equation of maximum certainty (13) the following :

(19) $$O(A^2)|\psi_k\rangle = (d_k^2 + \alpha_k^2)|\psi_k\rangle.$$

Eqs. (18) and (19) imply the commutation of the operators $O(A^2)$ and $O(A)$ on a complete system of vectors, i.e. condition (16)

is fulfilled. The commutation of operators (16) makes it possible, by means of a mere substitution, to see that any solution of Eq. (14) satisfies Eq. (15).

5. Provided the conditions of the above theorem are fulfilled, the operator $D(A)$ is positively defined. Indeed, let $|\psi\rangle$ be an arbitrary state, then

$$|\psi\rangle = \sum_k C_k|\psi_k\rangle$$

and

$$\langle\psi|D(A)|\psi\rangle = \sum_k |C_k|^2\, d_k^2 \geq 0.$$

At an arbitrary value of $|\psi\rangle$ the last correlation indicates positive definiteness of the operator $D(A)$.

6. If the uncertainty of the physical quantity A reaches its minimum in the states $|\psi_k\rangle$, $|\psi_\ell\rangle$, then the following inequality is true

(20) $$|d_k^2 - d_\ell^2| \leq (\alpha_k - \alpha_\ell)^2.$$

To prove this statement we must make use of the identity

(21) $$\frac{\langle\psi_k|O((A-\alpha_\ell)^2)|\psi_k\rangle}{\langle\psi_k|\psi_k\rangle} = d_k^2 + (\alpha_\ell - \alpha_k)^2.$$

The dispersion minimum in the state $|\psi_\ell\rangle$ implies

(22) $$\frac{\langle\psi_\ell + \gamma\psi_k|O((A-\alpha_\ell)^2)|\psi_\ell + \gamma\psi_k\rangle}{\langle\psi_\ell + \gamma\psi_k|\psi_\ell + \gamma\psi_k\rangle} - d_\ell^2 \geq 0$$

where $\gamma \ll 1$. Using identity (20) and making a few transformations we obtain

$$d_\ell^2 - d_k^2 \leq (\alpha_k - \alpha_\ell)^2.$$

Replacing $|\psi_k\rangle$ by $|\psi_\ell\rangle$ and vice versa in expression (22) we immediately find that

$$d_k^2 - d_\ell^2 \leq (\alpha_k - \alpha_\ell)^2.$$

The last two inequalities constitute the required correlation (20).

7. After finding a family of solutions for Eq. (13) $\{|\psi_k>\}$ and the corresponding values of α_k and d_k^2 one may state that for any state

$$(23) \qquad <(\Delta A)^2> \geqslant (\delta A)^2 \overset{def}{===} \min_k \{d_k^2\}.$$

Hence, the accuracy of determining even a simple physical quantity in the quantum theory with the Non-Neumann correspondence rule is limited. The quantity (δA) is determined by a concrete realization of the correspondence rule.

It has no analogue either in classical or in quantum theories, so it will further be called the "subquantum uncertainty" of the quantity A.

Since the quantities (δA) are contained in the experimentally verified values of physical quantities, "subquantum uncertainties" should reflect some physical reality, which nature was considered briefly in paper ([18]). Generally values of physical quantities are not strictly determined, that is why the appearence of "subquantum uncertainties" may be interpreted as a result of interaction between the regarded physical system and something surrounding it. The idea of considering a physical system interacting with some environment is not new. Present investigation shows that hypophesises of existence of the "hidden thermostat" ([19]), the "imaginary thermostat" ([20]), the "subquantum medium" ([21]) and so on with necessary changes may find application in the quantum theory with the Non-Neumann correspondence rule.

8. Let $|\psi_0>$ be the state of the absolute minimum of the dispersion of the quantity A, i.e. $(\delta A)^2 = d_0^2$; then, inequalities (20) and (23) form the following correlation :

$$(24) \qquad (\delta A)^2 \leqslant d_k^2 \leqslant (\delta A)^2 + (\alpha_k - \alpha_0)^2.$$

Hence, the eigenvalues of the dispersion are founded both from below and from above.

5. The correlation of uncertainties

The appearance of the "subquantum uncertainty" (23) of a separate physical quantity in the quantum theory with the Non-Neumann correspondence rule leads to the obvious correlation

of uncertainties for two physical quantities

$$(25) \qquad <(\Delta A)^2> <(\Delta B)^2> \geqslant (\delta A)^2(\delta B)^2.$$

Expression (25) has no analogue in the conventional quantum theory. But it can be shown that the product of uncertainties of two physical quantities in the quantum theory based on the Non-Neumann correspondence rule also depends on the commutator of their respective operators.

Indeed, let $O(A)$, $O(B)$ be the operators representing the physical quantities A and B. The self-adjoint operator $O(C)$ is defined by the equality :

$$(26) \qquad i[O(A), O(B)] = O(C).$$

Let the physical system under consideration be in an arbitrary state $|\psi>$. For the operators of the deflection of the physical quantities from their average values $O(\Delta A) = O(A - <A>)$ and $O(\Delta B)$ we have

$$(27) \qquad i[O(\Delta A), O(\Delta B)] = O(C).$$

Let us consider now the auxiliary expression

$$(28) \qquad <\psi|\,|O(\Delta A) + i\alpha O(\Delta B)|^2\,|\psi> \geqslant 0,$$

where α is a real parameter. Commutator (27) makes it possible to rewrite the inequality (28) in form

$$<\psi|O^2(\Delta A)|\psi> - \alpha<\psi|O(C)|\psi> + \alpha^2<\psi|O^2(\Delta B)|\psi> \geqslant 0.$$

The last expression implies, due to the arbitrariness of α, that

$$(29) \qquad <\psi|O^2(\Delta A)|\psi> <\psi|O^2(\Delta B)|\psi> \geqslant \frac{1}{4}(<\psi|O(C)|\psi>)^2.$$

With the aid of the operator $D(A)$ (7) we obtain an expression for one of the factors of (29)

$$<\psi|O^2(\Delta A)|\psi> = <\psi|O((\Delta A)^2)|\psi> - <\psi|D(A)|\psi>.$$

The same equality is also true for the quantity B. Therefore, inequality (29) is transformed into the uncertainty correlation for two physical quantities in the quantum theory with the Non-Neumann correspondence rule

(30) $[\langle(\Delta A)^2\rangle - \langle D(A)\rangle][\langle(\Delta B)^2\rangle - \langle D(A)\rangle] \geqslant \frac{1}{4} (\langle O(C)\rangle)^2.$

The general analysis of the above expression is a thorough investigation of the properties of the operator $D(A)$, which is outside the limits of the present work, since it is not concerned with the results of any concrete realization of the Non-Neumann correspondence rule.

It should be stressed, however, that in the particular case, when the conditions of the theorem (§4.4.) are fulfilled, the minimum of the dispersion is reached in the eigenstates of the corresponding operators. Thus, for the operator O(A) it is $\langle D(A)\rangle$, and the inequality

$$\langle D(A)\rangle \geqslant (\delta A)^2$$

is true.

Hence, the inequality (30) will become stronger if we write it in the form :

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geqslant \frac{1}{4} (\langle O(C)\rangle)^2 + \langle(\Delta A)^2\rangle(\delta B)^2 + \langle(\Delta B)^2\rangle(\delta A)^2 - (\delta A)^2(\delta B)^2.$$

Finally, with (23) taken into account the last expression gives :

(31) $\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geqslant \frac{1}{4} (\langle O(C)\rangle)^2 + (\delta A)^2(\delta B)^2.$

The uncertainty correlation for two physical quantities, together with the appearence of the "subquantum uncertainty" shows that, in the quantum theory with the Non-Neumann correspondence rule, the concept of certainty becomes more general and fundamental than it is in the conventional quantum theory and, of course, includes the uncertainty principle of the generally excepted quantum theory, as a particular case.

<u>Conclusion</u>

The class of the Non-Neumann correspondence rules introduced in the present paper includes every Neumann rule of constructing quantum operators. The set of the requirement for quantum

operators (5) formulated here, determining the Non-Neumann class of correspondence rules may be extended to provide a concrete unique recipe for constructing quantum operators ([16]). Apparently, other concrete realizations of the Non-Neumann correspondence rule are possible. However, the investigations made in the present work show that the following statements in the theory with any Non-Neumann correspondence rule are true.

I. Out of all the possible states of the physical system, two families of states relative to the physical quantity A can be picked out :

a) The family of the eigenstates of the operator O(A).

b) The family of the states, in which the physical quantity A is determined with maximum accuracy. These states satisfy the non-linear equation, constructed on the operators O(A) and $O(A^2)$.

II. The sets of states a) and b) are the same for :

1) The Neumann correspondence rule.

2) The completeness of the system of states of maximum certainty.

3) Such a realization of the Non-Neumann correspondence rule, when the operators O(A) and $O(A^2)$ commute.

III. The accuracy of determining even a simple physical quantity in the quantum theory with the Non-Neumann correspondence rule is limited. It, naturally, leads to the generalization of the uncertainty correlation for two physical quantities. Thus, the concept of certainty in this case becomes more general and fundamental than it is in the conventional quantum theory.

"Appearance of the "subquantum uncertainty" (δA), excluding in general the dispersionless states, admits the idea of presence of some unknown "subquantum medium" interacting with a physical system."

The results predicted by the theory with the Non-Neumann correspondence rule will not, apparently, be in contradiction with the experimental data for the case when the operators O(A) and $O(A^2)$ commute, and the "subquantum uncertainties" (δA) are small enough for physical quantities measured experimentally.

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