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ON EINSTEIN'S VIEWS OF THE RELATIVITY AND QUANTUM THEORIES AND THEIR FUTURE PROGRESS I [x]

par M. Mendel SACHS

Department of Physics and Astronomy
State University of New York at Buffalo,
Amherst, N.Y. 14260

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Abstract : The flux of Einstein's ideas and dialogues about the bases of contemporary physics -the relativity and quantum theories- are discussed, with stress placed upon his change of interpretations, from his earlier operational/positivistic approach in special relativity theory to his later epistemological approach of "abstract realism", when his investigations progressed to general relativity theory. The thesis it put forward that his unified field approach, which was the culminating part of his life-work, was logically necessitated by the axiomatic basis of the theory of relativity, which, in turn, rejected the axiomatic basis of the Copenhagen interpretation of quantum mechanics. The requirement of the unified field theory was then to fuse the inertial manifestations of matter with its force manifestations, such as gravitation and electromagnetism, into a single, self-consistent, non-singular field theory. The correspondence principle is applied in requiring the non-relativistic limit of the part of this (non-linear) formalism that relates to inertia to correspond with the linear mathematical structure of quantum mechanics. The author's investigations of this type of unified field theory will be discussed in a succeeding article (Part II).

[x] Dedicated to the memory of Albert Einstein, in honor of the 100th anniversary of his birth (1879 - 1979).

Résumé : On discute l'évolution des idées d'Einstein à propos des bases de la physique contemporaine -la relativité et les théories quantiques- en insistant sur son changement d'attitude, depuis la première approche opérationnelle-positiviste dans la relativité restreinte jusqu'à son approche ultérieure, épistémologique, de "réalisme abstrait", quand ses recherches portèrent sur la théorie de la relativité générale. On avance la thèse que son approche du type champ unifié, qui fut le point culminant de tout son travail, était rendue logiquement nécessaire par les bases axiomatiques de la théorie relativiste, qui, à son tour, rejetait les bases axiomatiques de l'interprétation de la mécanique quantique selon l'Ecole de Copenhague. La théorie du champ unifié exigeait alors de fondre les manifestations de l'inertie de la matière et celles des forces qu'elle crée, comme la gravitation et l'électromagnétisme, en une seule théorie autonome d'un champ non singulier. En appliquant le principe de correspondance, on exige que la limite non relativiste de la partie de ce formalisme (non linéaire) qui se rapporte à l'inertie, corresponde à la structure mathématique linéaire de la mécanique quantique. Les recherches de l'auteur sur ce type de théorie du champ unifié seront discutées dans un article ultérieur (partie II).

1. Introduction

It has been said many times over that the most significant scientist-philosopher of the twentieth century -and, indeed, since the time of Galileo and Newton- is Albert Einstein. There is no question in my mind about the truth of this comment. The major evolutionary developments of twentieth century physics were in large part initiated and pursued significantly by Einstein, not only regarding their mathematical expositions, but more importantly, in regard to the conceptual changes that physics underwent in this century.

A unique occurrence in the history of science took place in the contemporary period. This was the simultaneous appearance of two scientific revolutions -Einstein's theory of relativity and the quantum theory of measurement. Einstein was not only primarily responsible for the seminal ideas and development of relativity theory ; he was also largely responsible, together with Planck, for the introduction of the "old quantum theory" -the concept that the energy of the electromagnetic radiation field is divided up into discrete quanta, each with their energies linearly proportional to their frequencies, with implicit

"wave" and "particle" manifestations.

It is salient that the notion of "wave-particle dualism" of the old quantum theory, later extended to matter by de Broglie, eventually led to the Copenhagen view of the quantum theory of measurement of Bohr and Heisenberg, which was opposed by Einstein, Planck and de Broglie. Einstein's opposition was indeed based on conceptual and logical fallacies that he saw and believed were never actually resolved in his lifetime.

Yet, the consensus of the physics community interpreted this opposition as a sign of extreme conservatism that sometimes accompanies old age ! Further on in this essay I will discuss the logical basis of Einstein's opposition to the Copenhagen school, justifying it on the grounds of sound scientific reasoning. In fact, I feel quite certain from my reading the dialogue of this most important twentieth century debate in physics that, contrarily, it was the extreme conservatism of the followers of the Copenhagen school that led most of the physics community to reject Einstein's critical approach and free thinking style in physics. Einstein was quite sincere when he said (1) that he owed a debt to Mach, who

> "shook dogmatic faith, (and) exercised a profund influence upon me ... in his incorruptible skepticism and independence".

Before discussing this important controversy of modern physics, I will commence with a discussion of Einstein's early ideas about special relativity theory, as well as his early ideas about "atomism" and the old quantum theory. Later on, I will indicate how the earlier ideas took a drastic turn when Einstein was led, in logical fashion, to the general form of relativity theory, and how this conceptual turn strongly influenced his rejection of the Copenhagen view of elementary matter.

It was the philosophy that Einstein came to with the general form of relativity theory that strongly indicated that he should proceed toward a unified field theory. This was not for esthetic reasons alone. It was rather an extension of his thinking, that was logically necessitated by the axiomatic basis of physics, that he was led to.

Thus it was truly unfortunate that Einstein did not fully accomplish the mathematical task of such a unification in his lifetime, at least between the gravitational and the electromagnetic manifestations of interacting matter. But this mathematical failing did not at all refute the validity of the unified field approach to the laws of nature, as many contemporary scientists contend. I will discuss this development in Einstein's thinking in the last section of this essay. I will then review my own work in unified field theory in a succeeding paper (Part II) and show how many of my results are indeed compatible with Einstein's expectations.

An outstanding feature of Einstein's method in physics was his willingness to reject ideas that he had vigorously upheld when he saw that they could not be supported by the requirements of logical consistency or the experimental facts. Such intellectual freedom in his thinking is indeed characteristic of other great thinkers of our time and all earlier times. Another notable example is Bertrand Russell, in philosophy, who, like Einstein, changed some of his most basic views as he progressed in his intellectual career. It seems to me that Einstein and Russell should serve the physics and philosophy communities as prime examples of undogmatic thinkers, whose method of investigation was conducive to the necessary progress in any field of intellectual endeavor.

2. Einstein's early view of special relativity

There has been voluminous literature on the meaning of special relativity theory, since its inception in 1905, when Einstein published his original version of the theory (2). At the present stage of this essay it is important to emphasize the difference between Einstein's view of the Lorentz transformations and that of Lorentz.

In 1895 (3) Lorentz postulated his space and time transformations of the electromagnetic variables, from one inertial frame to another, in order to explain the null result of the Michelson-Morley experiment -an experiment that was designed to determine the magnitude of the speed of light as a function of its direction of propagation relative to the conducting ether. Their null result led Lorentz to infer that the ether physically acts on all measuring devices in such a way that is dependent on their direction of motion relative to the ether -shortening measuring rods in their directions of motion and retarding clocks in such a way as to yield a measured speed of the propagation of light that would appear to be independent of the speed of its source. Thus, Lorentz explained the null

result of the Michelson-Morley experiment as being due to a peculiar velocity-dependent force exerted by the ether on all possible measuring rods and clocks that could be used to measure the speed of light.

In 1905, Einstein took a quite different view of the role of the Lorentz transformations compared with Lorentz' interpretation (even though, as it is currently agreed by historians of science, Einstein was unaware of the Michelson-Morley experiment when he wrote his paper). Einstein saw that the Lorentz transformations, which have the following form when comparing reference frames that move at constant relative speed, v, in the x-direction, parallel to the x' direction :

$$x \to x' = \frac{(x + vt)}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{1/2}} , \quad y \to y' = y , \quad z \to z' = z ,$$

$$t \to t' = \frac{\left(t + \frac{vx}{c^2}\right)}{\left[1 - \left(\frac{v}{c}\right)^2\right]^{1/2}} \tag{1}$$

have the effect of preserving the mathematical formalism (Maxwell's equations) that provide the underlying explanation for the behavior of light. Thus, he asserted that the logical role of the Lorentz transformations is strictly to preserve the forms of all of the laws of nature, when they are compared in all possible relatively inertial frames of reference -that is, reference frames that move relative to each other with constant speed in a straight line. This pronouncement is the principle of special relativity. This is the primary axiom that underlies the theory of special relativity. When extended to cover the correspondence of laws of nature in arbitrarily moving frames of reference, the axiom extends to the principle of general relativity.

In Einstein's original formulation of special relativity theory, he asserted that this theory is based on two fundamental axioms - 1) the principle of special relativity and 2) the universality of the speed of light, c, with respect to all inertial frames of reference. However, according to Minkowski's argumentation (4), it appears that one can logically derive the universality of the speed of light from the principle of relativity -thereby reducing the number of independent axioms to only one. I have discussed this view further in an article published in 1969 ([5]).

We see, then, that Einstein's original formulation of special relativity theory implied that the Lorentz transformations are not to be viewed as cause-effect relations, since they do not, in any way, entail a physical effect of a background ether on material rods and clocks. He then concluded that the ether concept in a theory of light is indeed superfluous.

Still, at this early stage of development of relativity theory, Einstein defined lengths and times "operationally". It was then claimed that the physical length of a measuring stick for one observer, correlated with a measure he may call "one meter" in his own frame of reference, would not be the physical length of this material stick if he should observe it in a reference frame that moves away from him. That is, this would not merely be an illusion because of the motion of the stick, but it would indeed be only 88 cm long if it moves away from the observer at about $v = \frac{c}{2}$. Similarly, the relativity of the time measure implied to Einstein that if a material entity should be seen to endure for 5 minutes in the observer's reference frame, it would indeed endure for perhaps 3 minutes in a reference frame that is in motion relative to this observer. That is, Einstein originally interpreted the Lorentz transformations not only as the proper scale changes of the space and time measures that would ensure the objectivity of the laws of nature, but they also meant to him that, by definition, a physical length and time are variable, relative to the instrument's motion with respect to an observer. Thus, physical length and time of material entities lost all objective connotation -they were now to be defined strictly in operational terms.

This early view of Einstein was strongly influenced by Ernst Mach's epistemological stand of "positivism" -a view that asserts that the only meaningful entities that can be known in science are the reactions of our senses or our instruments to physical measurements. According to the posivistic philosophy, all that would be required of the scientist in his attempt to know about the world is to classify the data as economically

as possible, and then to use a method of inductive logic in order to make further predictions about the outcomes of new experiments. Thus, at this early stage of special relativity theory, Einstein was willing to say that lengths and times have magnitudes to be defined only with respect to a particular observer's frame of reference -i.e. they would not necessarily be the same in other frames of reference that would be in motion relative to the first. Einstein felt that this "operational" view of lengths and times is compatible with the use of the Lorentz transformations that he found necessary to preserve the forms of the laws of nature in all relatively moving inertial frames of reference.

However, in his later years, Einstein did retract from Mach's positivistic view of science, when he said ([1]) :

" ... in my younger years Mach's epistemological position (positivism) also influenced me very greatly, a position which today appears to me to be untenable. For he did not place in the correct light the essentially constructive and speculative nature of thought and more especially of scientific thought."

It became apparent to Einstein that a pursuit of the operational view of space and time to its logical extreme led to paradoxes. The source of these paradoxes was in part due to the confusion between the concept of a "measure of a thing", on the one hand (such as length and time (abstract) measures) and on the other hand, the thing itself (such as a material rod or the evolution of the unwinding spring of a clock). But Einstein did back off from the extreme operational view of lengths and times in his later years, when he said ([6])

" ... strictly speaking, measuring rods and clocks would have to be represented as solutions of the basic equations (objects consisting of moving atomic configurations), not, as it were, as theoretically self-sufficient entities."

Taking this latter view to its logical extreme, it would then follow that indeed one must not interpret the Lorentz transformations as physical cause-effect relations, within the logical structure of the theory of relativity. That is, to know if a clock's actual reading (the locations of its hands), or the length of a stick (the extent of its constituent molecules relative to each other) are altered by virtue of a motion relative to an observer's frame of reference, compared with the clock's reading, or the length of a rod in the different reference frame, one must appeal to the actual physical cause-effect relations that predict the behavior of matter -the basic matter equations that are the laws of nature.

But in the same place where Einstein made this concession, he went on to justify the previous conclusion with the argument (7) :

"However the procedure justifies itself because it was clear from the very beginning that the postulates of the theory are not strong enough to deduce from them sufficiently complete equations for physical events sufficiently free from arbitrariness, in order to base upon such a foundation a theory of measuring rods and clocks."

This statement appears to me to be a non-sequitur. For if the postulates of a theory do not lead to a complete set of equations, that would incorporate the physical properties of measuring rods and clocks, this would not imply that the former interpretation of the Lorentz transformations, as physical changes of matter, must be valid ! It rather implies that one must add further description to the former incomplete theory, in order to make up the complete theory that would actually incorporate the material properties of measuring rods and clocks (or any other sorts of measuring devices). We will see later on that, indeed, the completion of the theory that could fully incorporate the measurements necessitates a "unified field theory" -which would unify the inertial manifestations of matter with its force manifestations (gravitation, electromagnetism, etc.). It is my contention that this was one of the prime motivations for Einstein's attempts, during the last 40 years of his life, to formulate a unified field theory. That is, it wasn't only for esthetic reasons that he had such strong motivation to discover the unified field theory ; it was rather because of his recognition that the unified field approach is logically necessitated by the requirement of self-consistency of the starting axiomatic basis of the theory of relativity.

Nevertheless, after claiming the acceptability of such an incomplete description of matter, Einstein continued (8) :

"If one did not wish to forego a physical interpretation of the coordinates in general (something which, in itself, would be possible), it was better to permit such inconsistency -with the obligation, however, of eliminating it at a later stage of the theory." (my underlining)

Thus, Einstein admitted that an inconsistency is indeed encountered by interpreting the space-time transformations of relativity operationally, as he had done originally. His previous non-sequitur then led him to admit into the theory a logical inconsistency -but "with the obligation of eliminating it at a later stage of the theory". The latter statement then indicates to me that Einstein did not take seriously the interpretation of the Lorentz transformations as having the consequences of physical cause-effect relations. For the inconsistencies implied by such an interpretation would shatter the validity of the theory of relativity -as indeed Einstein recognized in his later years, when he wrote his Autobiographical Notes (8).

3. Logical paradoxes from the operational interpretation of special relativity

What are these inconsistencies that Einstein refers to ? They are the logical paradoxes that one comes to when interpreting the Lorentz transformations as objective physical cause-effect relations, while at the same time admitting that motion, per se, is strictly a subjective aspect in the theoretical description of matter.

The Clock Paradox

The first of these inconsistencies is the well-known "clock paradox". It arises as follows : If one should interpret the time scale change :

$$t'(\text{moving}) = t(\text{stationary})\left(1 - \left(\frac{v}{c}\right)^2\right)^{1/2}$$

as a physical change of the reading of a moving clock (i.e. the locations of its hands), then if initially A and B are synchronized, physically identical clocks, and if A should then move away from B at the velocity v, an observer in B's reference frame would have to say that the reading of clock A is slow compared with the reading of clock B. He would then conclude that when A would return to the inertial frame of clock B, it would be seen to read slow compared with clock B. This result in itself is not a logical paradox. The paradox arises because motion, per se, is a subjective element in the theoretical description, according to the theory of relativity, and because in this theory, the form of the law of nature must be independent of the frame of reference from which it is expressed. Thus, an observer in A's frame of reference, in looking back at B moving away from him (in the opposite direction from the motion observed from B's frame) would similarly have to come to the conclusion that when the clocks would be together again in the same inertial frame of reference, after the round trip journey in completed, B must be slow compared with A -i.e., A would be fast compared with B. Now according to Einstein's principle of relativity, the observer in A's reference frame and the observer in B's reference frame must both be telling the truth -since their claims are both based on the prediction of a law of nature that is independent of the frame of reference from which it is expressed. We would then have to conclude that after the journey is completed, A would be both slow and fast compared with B. This conclusion is then a logical paradox -it is called the "clock paradox". As a logically inconsistent conclusion, it should then be unacceptable as a result of a bona fide scientific theory, and thus it would refute the theory.

Einstein tried to resolve this paradox by taking into account the periods of non-uniform motion during the round trip journey ([9]). These happen when the clock A speeds up from rest, relative to B, to start on its journey, turns around at the end of its path, for the return, and when it comes to rest again relative to B, when it arrives in B's inertial frame. Einstein argued that if one would make use of the equivalence principle, of general relativity theory, describing the periods of non-uniform motion in terms of the static situation in a gravitational field, then he could eliminate the paradox. He discovered that in this way he could derive a compensating factor during the periods of non-uniform motion of A's journey that would nullify the symmetry of the previous description that only considered uniform relative motion. This asymmetry would then lead observers in both A's reference frame and B's reference frame to agree that only the clock A would be slow compared with B, after the journey.

But this is an unacceptable resolution for a number of reasons: One is that it does not take account of the case of two relatively moving bodies that are not in the presence of a much larger mass, such as the earth. Another is that it implies that while velocity is a relative dynamical variable, acceleration is

an absolute dynamical variable (since it acts as the cause of an absolute physical effet). But this is not true, according to Einstein's theory of relativity ! If the spatial and temporal coordinates are all relative to the reference frame in which they are expressed (in contrast with the "absolute" temporal coordinate of classical physics), then any order derivative of any of these coordinates with respect to any other must also be relative. For example, if an observer in B's reference frame describes A's motion in terms of an acceleration away from B, then reciprocally, an observer in A's reference frame would describe B's motion in terms of an acceleration away from A. That is, acceleration is as relative as velocity is.

In bringing in the accelerating portions of the clock's round trip journey, in order to resolve the clock paradox, Einstein was appealing to the principle of equivalence of general relativity theory. This is an assertion that the action of a gravitational force, in causing a body to have weight, is equivalent to the effect on that body if it should be in a state of free fall at an acceleration that is equal to the acceleration due to gravity, g. But the cause of this acceleration is still a force. The body's acceleration is an effect of this cause. That is, acceleration, *per se*, is not the cause of physical effects. It is, rather, a particular effect that itself is due to a particular cause -a force. This means, then, that the acceleration of the clock would not cause it to alter its reading compared with a stationary clock's reading.

The question that then arises is : Does the force that causes the clock A to accelerate away from the clock B also cause a retardation of A's reading, compared with B's reading ? For example, in the case of a rocket ship blasting off from earth, its engines would propel a large amount of fuel toward the earth in order to exert the forward force that could counteract the force of gravity -so as to cause the ship to lift off of the earth (to accelerate). Does this same force then cause the spring of a clock inside of the rocket ship to slow down in its unwinding process ? I do not believe that there is any evidence for the existence of a coupling between this type of force to the action of the spring of a clock, so as to retard its unwinding, or to the cell degeneration of the space pilot in the ship, that would slow down his aging process, or to the weak interaction force field of a radioactive sample inside of the ship, that would slow down its decay process, etc.

Summing up, the error that I see in Einstein's use of the equivalence principle of general relativity theory, to resolve the clock paradox of special relativity, is in his tacit assumption that acceleration, per se, is the cause of a physical effect, rather than being an effect itself. The equivalence principle, rather, relates to the relationship in general relativity theory between the curvature of space-time and the trajectory of a test body. The curvature of space-time, according to general relativity, is a geometrical representation for the existence of other matter in giving rise to a force manifestation -that is the cause of the acceleration motion of the test body.

The resolution that the clock paradox that I have proposed is based on a re-examination of the meaning of the space and time coordinates and their transformations, within relativity theory itself (9). My analysis also provides a mathematical proof that the theory of relativity, in itself, does not predict asymmetric aging, and thus that there is no paradox. My analysis utilized the most general expression of this theory, as based on the principle of general relativity, [discussed briefly in the following article (Part II)]. The reaction of the majority of the physics community to my resolution, and my response, was published somewhat later (10), where I showed that they did not address themselves to my axiomatic starting point on the meaning of the space-time coordinates (which was also implied by Einstein's later stand, as I discussed above) nor did most of this reaction address itself to my mathematical proof.

Briefly, my resolution of the clock paradox starts with the basic axiom of the theory of relativity -the principle of relativity- asserting the objectivity of the laws of nature, that is, the objectivity of the dynamical relations that govern the behavior of matter. This principle asserts that the forms of the laws of nature, as expressed in all possible reference frames, must be in one-to-one correspondence. The space and time coordinates then play the role of the words of a (mathematical) language whose purpose is to facilitate an objective description of the laws that govern the behavior of matter. The objectivity of the laws of nature then implies that there must be a unique set of transformations of the space-time coordinates of one reference frame to those of any other (relativity moving) reference frame. (This would be analogous to the (almost) unique language translation between French and English that could be used in order to express the same physical ideas in one language or the other). When the relative motion between reference frames is uniform, these are the Lorentz transformations (and the underlying group is the Poincaré group) of special relativity theory.

When we examine the details of these transformations that are there to preserve the forms of the laws of nature, we see that they imply "scale changes" in both spacial and temporal measures -in the respective expressions of the laws of nature in the different reference frames, according to any particular observer's view. But scale changes, per se, are not physical changes (shrinking rods or retarding clocks). If the latter "physical changes" should indeed happen to instruments that are in motion relative to an observer, then in accordance with the theory of relativity itself, they would have to be explained in terms of (covariant) cause-effect relations, not in terms of "language transformations", since the latter are descriptive rather than explanatory !

The idea here is that a fundamental approach to the laws of nature, such as Einstein's theory of relativity, focuses on the features of the explanatory relations (the dynamical laws of nature), rather than stopping at the descriptive level (the mathematical language) as a phenomenological approach might do. The space and time transformations of relativity theory are only a descriptive aspect of the theory, while the laws of nature that they apply to are the explanatory aspect. Thus, the transformations of the space-time coordonates must not, in themselves, be interpreted as physical changes of material systems. This is analogous to Galileo's initial introduction of a kinematic description of moving bodies, and then his explanation in terms of a dynamical law, such as his principle of inertia (and later, Newton's dynamical laws of motion). The kinematics was not meant to explain material behavior in terms of motion ; it was only meant to provide a proper language to describe it.

Relative Simultaneity

Another paradox that this view of space and time resolves is concerned with the meaning of "relative simultaneity", in Einstein's theory. Once more, we must make a distinction between the scale changes of space and time measures in the language structure of the theory and the underlying interactions of matter. Now, according to the transformations of special relativity theory, and, in particular the temporal transformation between relatively moving reference frames (eq. 1), if a measure of time difference between two events should be zero in one frame, the corresponding time interval would not generally be zero in other relatively moving frames. That is, the events would not be called "simultaneous" in the latter frames while they would be simultaneous in the first frame. But relative simultaneity refers here to a comparison between time measures according to the language of a physical law, expressed in one frame or another. It does not refer, directly, to the physical effects of matter on matter and their ordering.

To demonstrate this difference, consider the situation whereby a dog is crossing a road intersection. A man leaning against a building, in observing the dog's motion, may also see that just as the dog steps down toward a manhole cover in middle of the intersection, a workman simultaneously pulls a lever that releases the manhole cover. He would then see the dog disappear below the street. On the other hand, a helicopter pilot, flying at a high speed over the street, may conclude from his observations that the event of the dog stopping down toward the manhole cover and the workman pulling the lever are not simultaneous, and he would decide that the dog would not fall below the street.

Nevertheless, the pilot would see that in fact the dog does seem to disappear before reaching the other side of the road ! The disappearance of the dog would then be an indisputable, absolute fact, not subject to the relativity of coordinates. In further analyzing the situation, the pilot may then conclude that his original determination was wrong because his observations of the interaction of the dog and the earth were made from a reference frame that is in motion with respect to the reference frame of the earth. When he would then learn how to transform away the parameters of his motion from the description, thereby projecting himself into the reference frame of the interacting objects (the Earth-dog interaction, which is, in fact, the cause of the physical event of the dog's fall) then he would realize that in the proper frame that would determine the physical event, the dog would indeed approach the manhole cover simultaneously with the workman's pulling the lever that releases it.

Thus, we must recognize that there is something special about the simultaneity in the reference frame of the interacting

objects -since this is the only reference frame in which the physical consequence of the interaction would follow. The outcome of this interaction is independent of who happens to be observing it, from one moving platform or another. In our example, the helicopter pilot would have discovered the Lorentz transformations in order to predict the actual outcome of the events.

On the other hand, to claim that the parameter "t" is an actual physical duration, and not only a parameter that expresses a measure of duration from a particular reference frame, leads to a paradox -that the initial claims of the observer in the reference frame of the dog and the earth, and that of the pilot, are both correct, according to the principle of relativity itself. That is, we would have to conclude that the dog both does and does not fall below the street. The theory would then fail, since it would lead to a logical paradox. But there is no paradox in the predictions of the theory of relativity, regarding relative simultaneity, when one interprets the time (and space) coordinates as only relative parameters of a mathematical language -whose only purpose is to express physical laws in a covariant way, for any particular observer of physical phenomena that he may compare in different relatively moving frames of reference.

4. General relativity and matter

Einstein's original "operational" view of the space and time coordinates and their transformations in special relativity was abandoned and gave way to a philosophic view of "abstract realism" when he logically extented his theory to encompass the requirement of the objectivity of the laws of nature with respect to any sort of relative motion. This was the extension to "general relativity theory". The corresponding change in his philosophical outlook, from the positivistic operational view of his early days to the view of "abstract realism" of the later approach, is reflected in Einstein's ideas about space and time, that he took until the end of his life.

With this change in his philosophy, Einstein came to a stand whereby the essence of the universe -i.e. its underlying reality- must be expressed by the scientist as a set of fundamental relations- as universals. Some of the logical implications of these universals are the particulars that may be associated with the physical observables. The procedure to investigate the scientific truths, according to this view, is then to compare the physical observables with the logical implications that follow deductively from the universals that one starts with, as hypotheses. So long as the comparison would remain successful, one may then claim the scientific validity of the universals. The method of gaining new scientific knowledge is then hypothetico-deductive. In contrast, the method of gaining new knowledge, according to the earlier operational view, was based on inductive logic.

Atomism versus Continuity

An important implication of general relativity theory has to do with the age old debate on whether matter is intrinsically atomistic or intrinsically continuous. Even though we do sense the "thingness" of the world, an abstract view in physics would assert that this does not necessarily imply that there is true individuation. For example, our eyes and ears sense the individuation of the breakers at the ocean front. But we know all the while that these are not separable, individual entities. They are rather "modes" of the entire, continuous ocean.

It has been my contention that if one should assume at the outset that space-time is a language that entails a continuous and continuously differentiable (analytic) set of coordinates, then the basic "field variables" that relate to the behavior of matter must also be continuous and analytic, everywhere. This idea was reflected in the 1935 article by Einstein and Rosen, who said (11) :

"Every field, in our opinion, must therefore adhere to the fundamental principle that singularities of the field are to be excluded."

Their conclusion was based on a demonstration that if a solution should admit singularities as representing particles, then the "particles" so-described could not be accelerated by their mutual interaction.

My argument in favor of Einstein's and Rosen's conclusion is much more general and perhaps closer to Einstein's later reason for continuity. This is a reason not tied to any special solution or model, such as the assumption in Einstein and Rosen's analysis that space-time must be Euclidean at infinity -a requirement that is generally not true for the real world !

I start, rather, with the observation that a general relative motion between reference frames in which laws of nature are compared must be characterized by the 16 continuous and analytic parameters, $\frac{\partial x^{\mu'}}{\partial x^{\nu}}$ -the underlying symmetry group of general relativity is then a 16-parameter Lie group. With the covariance defined in this way, it then follows that the variables that are to represent matter are the field solutions of differential equations that are covariant with respect to such a group, and thus cannot be singular anywhere, nor can one, in principle, admit a discrete cut-off in the spatial representation of matter.

On the requirement that the underlying symmetry group of general relativity theory must be continuous, Einstein made the following remark (12) :

"If, then, one must give up the attempt to give the coordinates an immediate metric meaning (differences of coordinates = measurable lengths, viz., times), one will not be able to avoid treating as equivalent all coordinate systems, which can be treated by the continuous transformations of the coordinates. The general theory of relativity accordingly proceeds from the following principle : Natural laws are to be expressed by equations which are covariant under the group of continuous coordinate transformations."

With this requirement of general relativity, then, matter cannot be represented as point singularities nor as extended particles with discrete boundaries. With this view, what appear to us as individuated objects with sharp boundaries are, more fundamentally represented in terms of continuous field variables that are continuous and analytic, everywhere. These are the regular solutions of the laws of nature -the field equations that are to underlie the observable features of matter.

The word "underlie" signifies that the basic truths in nature that we seek are not directly observable entities ; they are rather the objective relations that predict the full set of causes and effects of the matter of the physical universe. This epistemology then rejects the operationalist/positivist stand, whereby the only meaningful entities of a "theory" are those that have to do with what is directly perceivable. Since Einstein's relativity theory now starts with universals (that are not directly perceivable) -the underlying laws of nature-though it implies by logical deduction particulars that can be compared with observations, this approach takes the epistemological stand of "abstract realism".

5. The quantum theory

The contemporary view of elementary matter is based on the ideas of the Copenhagen School -the "new quantum theory"- that was interpreted, primarily, by N. Bohr and W. Heisenberg (though these two scholars did not agree on all points), and later refined by P.A.M. Dirac. Theirs was a view that went far beyond the initial ideas of Einstein's and Planck's "old quantum theory", moving in a direction that Einstein found it impossible to accept as scientifically true. Other great theoreticians of the 20th century who found it difficult to accept the Copenhagen view were Planck, Schrödinger and de Broglie, the latter two who were architects of the formal mathematical structure of the new quantum mechanics. I have discussed Schrödinger's and de Broglie's objections to (and their respective replacements for) the Copenhagen view, in a previous publication ([13]). In this chapter I will concentrate primarily on the conflict between the Bohr-Heisenberg approach and Einstein's approach to the problem of matter. But first I will review his contributions to the ideas that led up to quantum mechanics.

Einstein's studies of the "old quantum theory" and early 20th century atomic theory were seminal in the later developments of modern physics. His 1906 analysis of Brownian motion ([14]) demonstrated verification of Boltzmann's atomic model of matter, deriving Boltzmann's constant from the analysis of the random motion (macroscopically observed) of bits of matter as they collide in a downward motion with the (microscopic) molecules of an emulsion.

Einstein's use of the "quantum of light" -the "photon"- to explain the photoelectric effect ([15]) was compatible with Planck's earlier discovery that the spectral distribution of blackbody radiation is predictable from the view of electromagnetic radiation as a "gas" of distinguishable vibrational modes, with the energy of each mode proportional to its frequency, in a linear fashion -

$$E_n = h\nu_n \qquad (2)$$

It is interesting to note, in retrospect, that Planck derived his spectral function for blackbody radiation by using the classical Maxwell-Boltzmann distribution for a set of distinguishable vibrational modes ([16]), with the energy of each mode obeying the quantization rule (2). But later on Einstein (and simultaneously N. Bose) ([17]) re-derived Planck's formula by abandoning classical statistics and assuming the rule of "quantum statistics" -that the separate "photons" of the radiation gas are indistinguishable, and further, that any number of them may have the same momentum and spatial location (i.e. that they may occupy the same cell of phase space) at any given time. This was to be the basic degeneracy that would characterize a photon gas, or a gas of any other quantum articles that have angular momenta that are integral multiples of $\frac{h}{2\pi}$, where h is Planck's constant.

I will not pursue the details of these investigations in this paper except to emphasize that Einstein's "particle" theory of light was certainly not the normal atomistic view (as was, for example, Newton's corpuscular theory of light). In losing their quality of distinguishability, the constituent "particles" of light take on a peculiar characteristic in which the continuous wave aspect and the discrete particle aspect of light strongly interplay. Of course, this "wave-particle dualism" concept was already strongly suggested by Planck's discovery of the quantization of light. Yet, the concept of the "degeneracy" of a photon gas (as an example of a "Bose-Einstein" (or boson) gas did not yet appear in Planck's analysis. Such degeneracy of the boson gas indeed gave a strong hint about the new concepts of the Copenhagen school that were to be used in interpreting "wave-particle dualism" in terms of "measurement".

The new interpretation of the data of atomic physics that was formulated by Bohr and Heisenberg, and then refined from the conceptual view by Dirac, was that the wave aspect of wave-particle dualism is related to the act of measurement of the physical properties of an elementary particle of matter, carried out by a macroscopic apparatus. It was to represent a probability amplitude, prescribing, according to a particular probability calculus, how the magnitude of the physical property of elementary matter is weighted throughout all of space.

It is important to note, at this stage of the discussion, that in the new quantum theory, probabilities, per se, are not simply used because the "knower" does not have complete knowledge of the system, so that he uses a probability calculus in order to estimate magnitudes. It is rather that the probability calculus, here, is to play a fundamental role in the laws of nature -i.e. the laws of nature became laws of chance ! Einstein's famous response to this view was that he did not believe that when God created the universe he was playing with dice !

A further implication of the new quantum theory, that seemed to clash with Einstein's epistemological approach to nature was the idea that any particular value and qualitative features of a property of microscopic matter depends on the way in which it would be measured. Thus, the Copenhagen approach to the basic nature of matter is to start out with a coupled system -a macroscopic measuring apparatus (observer) and microscopic matter (observed), carefully specifying beforehand where the line of demarcation between "observer" and "observed" must be placed. The properties of the "observed" that are then determined must depend intimately on the location of the demarcation line. If this line should be moved, ever so slightly, then, in principle, the predictions about the "observed" matter must change. Such a view then leads to the conclusion that the nature of elementary matter is in a way "subjective" -i.e. it depends on the way in which it would be observed ! This view denies the validity of the philosophic notion of realism (whether "naive realism" or "abstract realism") that asserts that there is objective matter with well defined properties, that are independent of any measurement that may be carried out on them, in one way or another.

In Einstein's early "operationalist" view, in special relativity theory, he seemed to take a similarly subjective approach when he interpreted the Lorentz transformations to mean that the lengths and times that characterize the physical properties of matter, to any particular observer, are a function of the state of motion of the observer when he observes this matter. That is, with the latter view, Einstein was denying the meaningfulness of a reality that may underlie the data. But when his theory evolved to general relativity theory, he rejected the latter approach to physics, that entailed operationalism and positivism, as an incomplete view. He then extended his new philosophy to reject any fundamentally incomplete theory, particularly the quantum theory according to the Copenhagen approach.

The principle of relativity, when followed to its logical extreme, implies that the relation, which is the law that governs the behavior of interacting matter, must indeed be independent of which component of an interaction is called "observer" and which component is called "observed". Thus, it implies that the formal description of a law of nature must be invariant with respect to an interchange of the respective reference frames of the interacting components of a system. It is then only a matter of convenience as to which aspect of an interaction would be called "observer" and which would be called "observed". This view is clearly in sharp contrast with the philosophy of the new quantum theory because relativity theory then implies a totally objective description must exist -i.e. if the interchange of the observer (subject) with the observed (object) does not change the overall description, then it must be entirely objective. But, as I have discussed above, the quantum theory is not symmetric with respect to an interchange of the observer and the observed, and thus it is not totally objective in nature. Another way of putting this is to remark that in the quantum theory, matter is characterized by the elementarity of the "thing", though the latter does not have a predetermined description, independent of measurement (thus it is a "non-deterministic" theory). On the other hand, the implication of relativity theory is that it is the "interaction relation" that is elementary. I have referred to this concept as a theory of "elementary interaction" -that is, interaction, per se, is not in principle decomposable into separate interacting "parts" ([18]).

Finally, an important mathematical implication of the axiomatic basis of the quantum theory is its linearity. This is the requirement that a solution of the equations of the theory can always be made up of a sum of other solutions of the same equations. This requirement is called "the principle of linear superposition". The wave functions that were discovered in the 1920's to correctly prescribe the data of atomic physics indeed seemed to be in accordance with this principle. One more requirement of the mathematical equations of quantum mechanics is that its equations are homogeneous in their solutions. Thus, the quantum mechanical equations must, in principle, be such that their solutions appear linearly, in all terms. On the other hand, if one should fully exploit the idea that matter must be described by a closed system, as is the case in a fully relativistic theory, then the basic equations can not be linear, nor is there any requirement that they be homogeneous in their solutions. Einstein thus remarked that the theory of general relativity automatically excludes the type of basic equation implied by quantum mechanics, when he said ([19]) :

"The group of general relativity is the first one which demands that the simplest invariant law be no longer linear or homogeneous in the field variables and in their differential quotients."

The Einstein-Podolsky-Rosen Paradox

With this view of Einstein's anticipation for the future development of the atomic theory, as based on the conceptual and mathematical framework of relativity theory, it is clear that the thought experiment he proposed with Podolsky and Rosen ([20]), to be discussed below, was not intended as an alternative to the ideas of Bohr and Heisenberg. The analysis (which I will henceforth refer to as EPR) was rather to examine the logical consistency of the Copenhagen school on its own terms, and then to demonstrate in this way that the quantum mechanical formalism is indeed not complete. They showed that this result would follow if one interpreted quantum mechanics as describing a single atom of matter. If then quantum mechanics should be claimed as a (fundamental) theory of elementary matter, it would follow that if this theory is complete then it is incomplete. This, of course, is a logical paradox, and cannot be accepted as a feature of a bona fide scientific theory. [A different view of the EPR paradox, whereby it is not a bona fide logical paradox, was recently proposed by O. Costa de Beauregard ([21])].

A possible resolution of the EPR paradox was a generalization to complete the description of elementary matter by introducing extra parameters to the underlying space and time parameters, that might relate to the real (though unobservable) trajectories of the elementary particles. This is a generalization that would maintain the probability calculus form of quantum mechanics by maintaining its formal linear and homogeneous structure of equations, though the solutions would now depend on extra (hidden) independent parameters. Because the extra independent variables in this theory are not to be directly related to observables, such as the trajectories of the elementary particles, this theory has been referred to as a "hidden variable" theory.

The essential idea of the hidden variable theory, that the extra (unobservable) parameters must relate to the actual trajectories of the constituent elementary particles of a material system, was first introduced in the 1920's by de Broglie, not long after his fundamental discovery of wave-particle dualism for material particles. This was expressed in terms of his "double solution" resolution of the problem of quantum mechanics, where he attempts to restore determinism to the fundamental theory of matter ([22]). In later years, hidden variable theories were investigated again by D. Bohm and his collaborators ([23]).

Many members of the physics community believed that Einstein was proposing a hidden variable theory as a resolution of the EPR paradox. But this was not at all the case, as the quotation from Einstein's Autobiographical Notes, indicated above ([19]), clearly reveals.

Reviewing the idea of the EPR paradox, it starts with the attempt to refute the assertion of the Copenhagen school that the precision of our knowledge about one variable of an elementary atom of matter necessarily limits any fundamental knowledge about another (canonically conjugate) variable of that matter, because of an interference that must accompany the measurement of the first variable. It is to be emphasized that this is supposed to be a fundamental limitation, and not only one that is restricted by our skill in constructing a sufficiently precise apparatus.

Consider, then, a coupled two-particle system, such as a diatomic molecule AB, in which the atoms A and B each have a spin angular momentum. According to quantum mechanics, the wave functions for this system are correlated in terms of the respective spin angular momenta of A and B. Suppose now we choose to dissociate this molecule with a spin-independent force. The spin correlation between A and B would then remain -no matter how far apart they should now be removed. Thus, if one should then measure the spin angular momentum of A, σ_A, with infinite precision, then according to the uncertainty relations of quantum mechanics, and their interpretation in the Copenhagen view, the observer would automatically sacrifice any precision in his knowledge about the orientation, Φ_A, of this angular momentum -because of the interference that would be introduced in making the former precise measurement.

On the other hand, because of the persisting correlation between the spin angular momenta of A and B, an arbitrarily precise knowledge of σ_A would automatically imply an equally precise knowledge of σ_B. Yet this precise knowledge of σ_B was obtained without measuring any property of B ! Thus there was no apparatus here to probe B that would have caused an interference effect that would then have destroyed precise knowledge of other properties of B, at the same time.

Similarly, if the orientation of the spin angular momentum of A, Φ_A, had been measured initially, this would have interfered with our knowledge of σ_A. But if this was a measurement with arbitrary precision, then the persisting correlation between A and B would then have implied the orientation, Φ_B, with equal precision -without in any way interfering with atom B.

Einstein, Podolsky and Rosen then concluded that a complete knowledge of B must exist independent of any measurement that may be carried out on it. On the other hand, if the quantum mechanical equations do indeed exhaust the fundamental description of the atoms -i.e. if this is the limiting form of the fundamental description of the atoms, then such a description is intrinsically incomplete. This is because quantum mechanics is a particular form of a probability calculus, and thus it does not entail information that pertains to all of the dynamical variables, simultaneously. It is a formalism that only leads to average values of the atom's properties (including transition rates) -at the fundamental level of explanation. We must then conclude that if indeed quantum mechanics is a fundamental theory of an atom of matter, the paradoxical result is reached that if it is a complete description then it is an incomplete description. This is the Einstein-Podolsky-Rosen paradox. This is not a "paradox" in the sense of being "an unusual, unexpected result" ; it is, rather, a bona fide logical paradox -implying a logical inconsistency in the Copenhagen interpretation of quantum mechanics.

A possible resolution of this paradox, as I have discussed above, may entail the "hidden variable theory" -where, to complete the description one must widen the underlying parameter space in

which the wave functions are mapped. The hidden parameters are then the "unobservable" trajectories of the atoms of matter, and the ordinary spatial parameters are the locations where the probability amplitudes are defined -that relate directly to a measurements of properties of the matter. It is only the latter amplitudes that are supposed to solve linear, homogeneous differential equations, that are consistent with a probability calculus. Nevertheless, as de Broglie pointed out in his analysis [22], it may be necessary to adjoin to this formalism a non linear formalism that would underlie the mapping of the "hidden variables" -that are the actual trajectories of the particles of matter. But as I have indicated above, Einstein did not anticipate that the problem would indeed be resolved in this way.

Another way out of the EPR paradox would be to deny that the wave function that solves the quantum mechanical equations does indeed represent a single atom of matter, e.g. an electron or a hydrogen atom, etc. An alternative view would be that the wave function is a sort of distribution function, analogous to Boltzmann's distribution function of the kinetic theory of gases, whose only role is to weight the average properties of an ensemble of atoms. With such a view, the quantum mechanical wave function would have no meaning within the context of a fundamental description of a single atom. With this interpretation, the incomplete description, at this stage of the analysis, is entirely expected, since this is a form of a probability calculus that is useful in describing average properties of matter, and it is nothing more. However, with this view, it is still anticipated that underlying the probability calculus of quantum mechanics there is indeed a complete dynamical description of the system of atoms, prescribing all of their physical properties with any precision required, and independent of measurements that may be carried out. The latter would be analogous to the Newtonian dynamical laws that underlie the complete description of a gas of atoms, even though Boltzmann's equation is useful in order to determine a distribution function that is utilized in order to calculate the average properties of this gas, if the underlying complete knowledge would not be at hand.

That is, with the classical view, Boltzmann does not deny that there is a prescribed, predetermined description of a gas of atoms. He merely uses a probability calculus in order to adjust for his incomplete knowledge at hand and make some predictions that relate to the average properties of the gas. Probability, per se, is then not a fundamental concept here.

On the other hand, with the Copenhagen view, probability is indeed at the fundamental level of explanation of the behavior of matter. Bohr and Heisenberg postulated that there does not exist a prescribed, predetermined description of the gas of atoms, that would be independent of measurements that may be carried out on this gas. Thus, while Boltzmann was still assuming that the laws of nature pertaining to elementary matter are indeed deterministic and totally objective, the Copenhagen view assumes that the fundamental laws of elementary matter are non-deterministic, with an irreducible element of subjectivity in them.

Einstein rejected the latter view, and maintained the view that the laws of nature must be deterministic and completely objective. However, in the 1930's he came to the conclusion that fully exploiting his general relativity theory implies that the basic elements of matter are not the discrete particles that Boltzmann had assumed, and that he postulated in his earlier days. Matter, in general relativity, is rather described most primitively in terms of continuous and analytic field variables (11). As he proceeded toward fully incorporating matter in his field theory he was led naturally to the concept of a unified field.

6. Toward a unified field theory and fusion with microscopic physics

A prime motivation for Einstein's drive, during the last 40 years of his career, to fuse the electromagnetic and gravitational field theories was to unify the force manifestations of matter with its inertial manifestations. In the low energy limit, the inertial properties are represented most primitively in terms of the mathematical equations of quantum mechanics. Thus, Einstein anticipated that a completely unified field theory would correctly incorporate the equations of quantum mechanics, at least as a non-relativistic approximation for covariant equations, in correctly representing the empirical data in that limit. Thus, Einstein anticipated that a complete theory of matter would be a unified theory that would properly incorporate the "particle equations" of quantum mechanics in a linear limit of a fundamentally non-linear field theory.

In his articles in 1945 and 1946, with E. Straus,(24) which discussed his latest attempt to construct a unified field theory, Einstein explained that what he would do in that paper (which was to add to the symmetric tensor field formalism, that had described gravitation, an antisymmetric tensor field formalism, in order to incorporate electromagnetism with gravitation) was an artificial, preliminary attempt at a truly unified field theory. He advised that the way to fully carry out the program of a unified field theory must instead start with a total field, that is not generally the sum of fields for the different phenomena to be explained, but with such a structure that under special physical conditions this field would reveal the predictions of one sort of phenomena, such as electromagnetism, and under different sorts of physical conditions it would reveal other predictions of other sorts of phenomena, such as gravitation. This would be the type of unified field that was discovered by Faraday, when he found that the electromagnetic field reveals electricity alone when the observer would be at rest relative to electrical matter, while the field formalism would reveal magnetic phenomena when the observer would be in motion with respect to this same matter. In this case, the electromagnetic "unified field" is not the sum of an electric and a magnetic field amplitude. It is rather a fully unified electromagnetic field amplitude of which the electric and magnetic fields are implicit at all times and under all conditions -even though one type of field reveals itself or predominates under particular physical conditions.

In this same sense Einstein anticipated that the gravitational and electromagnetic aspects must be implicit in the solutions of the truly unified field equations.

Einstein also suggested in these papers (24) that a significant starting point in formulating such a unified field theory would be to examine the underlying symmetry group of general relativity. That is, the group of continuous transformations between the space-time coordinate systems that describe the relatively and nonuniformly moving reference frames.

It is commonly believed within the physics community, because Einstein's particular attempts at constructing a unified field theory did not fully succeed, that the program of research toward a unified field theory has generally failed. This is certainly a false conclusion. It would be analogous be believing that an algebraic attempt to solve a differential equation, in failing, would thereby refute the meaningfulness of the differential equation and its relation to physical problems !

7. Summary

The major developments of contemporary physics -the theory of relativity and the quantum theory- were together influenced by Albert Einstein more than by any other single scientist of the 20th century. His contributions were seminal, not only in providing initial ideas of modern physics, but also in his continued dialogue and debate on the logical and mathematical consistency of the ongoing ideas, and the most reasonable directions for the future progress of our understanding of nature. He saw the ideas of physics as dynamic, rather than static, continually subject to refinement and, if need be, to change. According to this anti-dogmatic philosophy of science, conflict is an essential ingredient in the evolution of ideas that constitutes scientific progress. Since we must assume that man is a finite being and, therefore, can never become omniscient, he must continually challenge the validity of the ongoing ideas in order to gain bona fide progress.

Conflict was indeed implicit within Einstein's own research program. His earlier view of matter in terms of atomism and his atomistic view of radiation in accordance with the "old quantum theory", conflicted with his later continuum view that emerged with his theory of general relativity. His earlier operational epistemological approach, that initially led him to a formulation of special relativity theory, conflicted with his later philosophy of "abstract realism" that emerged with general relativity.

It seems clear to me, also, that Einstein's continued debate with Bohr on the Copenhagen view of quantum mechanics, in the early period of the latter theory, helped to sharpen Bohr's own views of quantum mechanics, leading to its present epistemological approach of non-determinism and subjectivity(25). Along with two of the architects of quantum mechanics, de Broglie and Schrödinger, Einstein disagreed with the philosophical stand of the Copenhagen school, and each of these scholars sought, in his own way, to restore determinism and objectivity to the fundamental laws of nature.

Einstein's way was to fully exploit the theory of relativity -leading to a unified field theory of matter. His primary motivation for seeking a unified field theory, in my view, was

his conviction that the problem of matter had a solution in the unification of the inertial, electromagnetic and gravitational manifestations of matter -or, generally, the fusion of the inertial features of matter with its force manifestations. He suggested that since the theory of relativity is, essentially, a theory of symmetry, prescribing the covariance of the laws of nature with respect to the group of continuous space-time transformations that distinguish different frames of reference from which the laws may be represented, the underlying symmetry group is an important starting point for the investigation of a unified field theory.

I have tried to fully exploit Einstein's suggestions in my research program and I have found that indeed a great deal of progress could be achieved toward the structuring of a unified field theory of matter -beyond the point that was reached by Einstein in his own published investigations -not only in terms of the mathematical structure of the generalized theory, but also in terms of new physical predictions that emerge from the theory. I will discuss this research program and its accomplishments in more detail in succeeding article (Part II).

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