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ON EINSTEIN'S VIEWS OF THE RELATIVITY AND QUANTUM THEORIES AND THEIR FUTURE PROGRESS II *

Toward a Unified Field Theory

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Abstract : Part I of this series, which surveys and analyzes Einstein's views in the relativity and quantum theories, is continued in this paper to a discussion of his views of a unified field theory resolution of the problem of matter, and the author's implementation of these views.

Résumé : A la suite du premier de ces articles, qui analyse l'ensemble des vues d'Einstein sur la relativité et les théories quantiques, celui-ci discute son point de vue sur la solution du problème de la matière par une théorie du champ unifié, et sa mise en oeuvre par l'auteur.

* Dedicated to the memory of Albert Einstein, in honor of the 100th anniversary of his birth (1879-1979)

1. Introduction

This two-part series discusses Einstein's ideas about the relativity and quantum theories that were seminal toward the evolution of twentieth century physics. Emphasis was placed in Part I [1]) on his turn of philosophical bases, from the earlier epistemological view of operationalism and positivism, when the theory of special relativity and the quantum theory were in their initial stages of development, to his later epistemological view of abstract realism, when special relativity theory had evolved to general relativity theory. Along with this change in his philosophical outlook, Einstein looked toward the resolution of the problem of matter in terms of a nonlinear, continuum field approach, requiring a complete, deterministic theory of matter, in which the basic variables must be the regular solutions of the underlying field equations.

The latter view led Einstein to reject the Copenhagen approach to matter, in terms of a linear, nondeterministic, subjective approach, since he saw this as an incomplete representation for elementary matter. He anticipated that the complete theory, guided by what he had already discovered in the general relativistic approach to gravitation, would lead to the eigenfunction, Hilbert space formalism of quantum mechanics only as a nonrelativistic, linear approximation, for generally nonlinear, nonhomogeneous field equations, applicable to any domain of matter and incorporating macroscopic physics and microscopic physics in a single dynamical scheme -extending from the domain of fermis to that of light-years.

Just as Einstein's later view of general relativity logically rejected the idea that the Copenhagen approach could be a fundamental theory of matter, so it equally implied that the complete representation of elementary matter must be a unified field theory -fusing the inertial manifestations of matter with its force manifestations- gravitation, electromagnetism and whatever other forces matter may manifest, under one set of physical conditions or another. In the studies during the latter part of his life, Einstein felt that the initial stage of such a unified field theory might attempt to fuse the gravitational and the electromagnetic force manifestations of matter by generalizing his field equations in general relativity so as to incorporate the implications of Maxwell's field equations. He anticipated that such a

unification may lead to the way of incorporating the inertial manifestations of matter and then the observed features of elementary particle physics might follow.

In this paper I will focus my attention on Einstein's suggestions toward the structuring of a unified field theory, and I will outline my own research program based essentially on these suggestions, showing how it reveals conclusions that are indeed compatible with the data of elementary particle physics and astronomy, from a wholistic unified field structure. A salient feature of these results is that some of them are not predicted, even qualitatively, by the aspects of the conventional approaches in contemporary physics that are logically and mathematically incompatible with the approach taken here toward a resolution of the problem of matter. Thus, there is sufficient confirmation of Einstein's approach to encourage further research along this path of inquiry, which is indeed in opposition to current approaches in physics, such as the Copenhagen school in elementary particle physics and the opposing views of contemporary astrophysicists, such as researches toward quantizing gravity, explanations for the features of "black holes" in terms of quantum dynamics, etc.

Two current texts that outline and clearly discuss attempts to set up a unified field theory are by W. Pauli [2]) and by M.-A. Tonnelat [3]). The latter text also contains an extensive bibliography on published works in this field (including the attempts to quantize gravitation) until 1964.

2. Factorization of Einstein's field equations and unification with electromagnetism

In the Introduction of one of his last attempts to formulate a unified field theory, Einstein suggested the following basic approach (while the theory developed in that paper did not yet take this approach) : [4])

> "Every attempt to establish a unified field theory must start, in my opinion, from the group of transformations which is no less general than that of the continuous transformations of the four coordinates. For we should hardly be successful in looking for the subsequent enlargement of the group for a theory based on a narrower group. It is further reasonable to attempt the establishment of a

unified theory by a generalization of the relativistic theory of gravitation. Such a generalization, which does not seem to have been discovered so far, is described in the following.
If we speak about a unified theory we have two possible points of view, whose distinction is essential for the following :
1°) That the field appear as a unified covariant entity. As an example I cite the unification of the electric and the magnetic fields by the special theory of relativity. The unification here consists in this that the entire field considered is described as a skew-symmetric tensor. The basic group of Lorentz transformations does not enable us to split this field independently of the system of coordinates, into an electric and a magnetic one.
2°) Neither the field equations nor the Hamiltonian function can be expressed as the sum of several invariant parts, but are formally unified entities. Also, this (weaker) criterion of uniformity is satisfied in our example of the special relativistic description of Maxwell's equations.
The theory I shall describe is unified according to criterion 2°), but not according to criterion 1°). Such a theory is to be considered unified only in a limited sense."

My investigation into the structuring of a unified field theory proceeds according to Einstein's criterion 1°). According to Einstein's suggestion, I started out by examining the role of the general transformation group in general relativity theory. I have interpreted this to mean that one should fully exploit the algebraic part of the logic of space-time, in addition to exploiting its geometrical part. That is to say, the space-time of relativity theory does not have any ontological connotation. It rather serves only as a language whose purpose is to facilitate an expression of the laws of nature in a totally objective way [5]. The "syntax" of this mathematical language, analogous to the subject-predicate relation of ordinary language, is in two parts -one is geometrical and the other is algebraic.

The geometrical part of the syntax of the space-time language specifies the logical relations between the space-time points in terms of congruence, mapping, continuity, etc. General relativity theory implies that there must be an intimate connection between these geometrical relations and the detailed variable nature of the matter whose behavior this language is to represent -thus necessitating an abandonment of Euclidean geometry for the Riemannian geometrical system of relations.

The algebraic part of the syntax of the space-time language relates to the relations between the space-time coordinates having to do with countability, rules of combination, associativity, distributivity, commutativity, etc. The essence of the latter part of the logic of space-time is expressed in the form of the irreducible representations of the Einstein group -this is the Lie group that is the set of continuous, analytic transformations that underlie the covariance requirements of the theory of general relativity.

I published the first mathematical results of my investigations of a unified field theory in 1967 [6]. My study was based on Einstein's two suggestions as to the mode of approach : 1) to generate a wholistic, irreducible, covariant unified field from the outset (rather than setting up a sum of fields and adjoining their equations) and 2) to structure the general theory in accordance with the full symmetry group of general relativity theory. These suggestions were also implicit in Pauli's comments on research toward a unified theory [7]. I have found from these investigations that indeed a unified field theory of the type that Einstein anticipated does appear. I will now briefly outline this analysis as I have developed it.

The Einstein group, that is asserted in general relativity to leave all of the laws of nature covariant, is a 16-parameter Lie group. It is characterized by the 16 essential parameters, $\left\{\left(\frac{\partial x^{\mu'}}{\partial x^{\nu}}\right)\right\}$, that relate the coordinates of one space-time frame, $\{x^{\mu'}\}$, to those of another, $\{x^{\nu}\}$. An implication of the covariance requirement of this symmetry group is that the basic field equations must be a set of 16 independent relations at each space-time point [8]. On the other hand, the symmetric second-rank tensor field equations that Einstein first proposed :

$$R_{\mu\nu} - \left(\frac{1}{2}\right) g_{\mu\nu} R = \kappa T_{\mu\nu} \qquad (1)$$

is a set of only 10 independent relations at each space-time point. Thus, there are too few relations in Einstein's field equations to fully satisfy the symmetry requirements of the Einstein group.

Of course, Einstein did not claim that the symmetric tensor field equations (1) must be the final form that relates geometry ($g_{\mu\nu}$) to matter ($T_{\mu\nu}$). In his Autobiographical Notes he said [9] :

"Not for a moment, of course, did I doubt that this formulation was merely a makeshift in order to give the general principle of relativity a preliminary closed expression. For it was not anything more than a theory of the gravitational field, which was somewhat artificially isolated from a total field of as yet unknown structure."

When I examined the symmetry properties of Einstein's tensor field equations (1), in 1966, I came to the realization that the reason there are only 10 relations instead of 16 is that they are covariant with respect to the (discontinuous) reflections in space-time -which is not required by general relativity theory- in addition to their covariance with respect to the continuous space-time transformations -which is required by general relativity. Thus, by removing the reflection symmetry elements from the underlying group of eq. (1), these field equations thereby factorize, yielding 16 independent relations.

This procedure is entirely analogous to Dirac's discovery of the electron equation from a factorization of the Klein-Gordon equation in special relativity. The latter equation is covariant with respect to the full Lorentz group. But special relativity theory only requires covariance with respect to the continuous subgroup of the full Lorentz group -i.e. the Poincaré group. With this reduction of the underlying symmetry group of special relativity, the scalar Klein-Gordon field equation factorizes into a two-component spinor field equation and its conjugate equation. Thus Dirac discovered that to fully exploit the Poincaré group of special relativity theory, while maintaing the Schrödinger form of wave mechanics, extra degrees of freedom must appear in the field variables of the theory. He was then able to identify these degrees of freedom with the components of the (previously empirically determined) "electron spin" angular momentum. The resulting simultaneous spinor and conjugate spinor equations (eqs. (21) below) that come from the factorization of the Klein-Gordon equation, are then not covariant with respect to reflections in

space and time, as the original scalar (Klein-Gordon) equation was. In the next step, to recover reflection symmetry, Dirac combined the pair of conjugated two-component spinor equations in a special way, thereby forming the (more restricted) four-component bispinor equation for the electron -a form that has been most commonly used in relativistic wave mechanics, until the discovery of parity nonconservation in weak interactions, in 1957. However, it is still to be noted that to fully exploit the Poincaré group of special relativity theory one must go back to the two-component spinor formalism.

We see, then, that the reason that Dirac discovered the spin degrees of freedom for the electron is not because of a particular mathematical form (Schrödinger's equation) that he wished to express in a relativistically covariant way. It was rather because of the covariance requirement itself that was imposed on the wave equation. That is to say, "spin" is not a consequence of quantum mechanics, per se. It is rather a consequence of the theory of relativity, when its symmetry requirements (i.e. the algebraic part of the logic of the space-time language) would be imposed on any theory that is to be expressed in its irreducible form.

This fact was discovered explicitly in the early 1930's by Einstein and Mayer [10]), when they examined the algebraic structure of the irreducible representations of the Poincaré group. They discovered that when the full Lorentz group, containing the reflections of the spatial and temporal coordinates, as well as the continuous transformations of special relativity, is reduced to its continuous transformations alone, then the four-dimensional representations of the full Lorentz group reduce to the direct sum of two (hermitian) two-dimensional representations. The basis functions of the latter are the spinor (and conjugate spinor) variables. Thus the spinor variable arises as the irreducible basis function of the most primitive representations of the symmetry group of relativity theory. Thus it is not an expression that is unique to quantum mechanics in particular !

More explicitly, what Einstein and Mayer showed, in effect, was the following. The irreducible representations of the symmetry group that leaves invariant the 4-dimensional metric of spatial relativity theory,

$$ds^2 = dx_0^2 - dr^2 \qquad (x_0 = ct) \qquad (2)$$

are the elements of the real, orthogonal, unimodular group of four-dimensional matrices, $\{V(\theta_\alpha)\}$, defined in terms of the transformations

$$|x_\mu'> = V(\theta_\alpha)|x_\mu> \qquad (3)$$

that leave (2) invariant, where θ_α are the essential parameters that characterize the group. For the Poincaré group of special relativity, which is a 10-parameter continuous group, α runs over the 10 values that correspond to : 3 Eulerian angles for the spatial rotations, 3 components of $\frac{v}{c}$ -the relative velocity of inertial reference frames- and the 4 translations along the temporal and spatial axes. There are further physical reasons for the transformations of the space-time coordinates to be analytic as well as continuous ; for example, the requirement that the theory incorporates conservation laws. Thus this group of transformations is to apply to the set of regular solutions of the field equations. Thus, the Poincaré group of special relativity theory is a 10-parameter Lie group.

Now one can equally arrange the column of coordinates, $|x_\mu>$, in the form of a 2-dimensional hermitian matrix

$$Q = \begin{pmatrix} x_0 - x_3 & -(x_1 - ix_2) \\ -(x_1 + ix_2) & x_0 + x_3 \end{pmatrix} = \sigma^\mu x_\mu \qquad (4)$$

where σ^0 is the unit 2-dimensional matrix and

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

are the Pauli matrices. The set of numbers {Q} then obey the algebra of quaternions, where the Pauli matrices and the unit matrix play the role of (i, j, k ; 1) -the basis elements of Hamilton's quaternion number field [11].

It then follows that, corresponding to each transformation of the space and time coordinates, represented by the 4-dimensional real matrix $V(\theta_\alpha)$, there must correspond the transformations

$S(\theta_\alpha)$ of Q, such that the transformations of the coordinates under the Poincaré group induce the quaternion transformations

$$Q(x_\mu) \rightarrow Q'(x_\mu') = S^\dagger Q(x_\mu) S \qquad (5)$$

The invariant hermitian product of four-vectors

$$<x_\mu|x_\mu> = x_0^2 - r^2 \qquad (6)$$

is then in one-to-one correspondence with the invariant determinant of the quaternion Q

$$\det Q = (x_0^2 - r^2)\sigma^0 \qquad (7)$$

Since the vector transformations (3) entail V linearly, the four-vector is a "first-rank" entity. However, with the transformations S appearing in (5) quadratically, Q is a "second-rank" entity. That is, Q is expressible as the direct product of two first-rank entities ("spinors")

$$|x_\mu> \Leftrightarrow Q \sim \psi \otimes \chi \quad , \qquad (8)$$

one transforming as

$$\psi(x) \rightarrow \psi'(x') = S\psi(x) \qquad (9a)$$

and the other as

$$\chi(x) \rightarrow \chi'(x') = S^{\dagger -1}\chi(x) \qquad (9b)$$

The first-rank spinor fields, ψ and χ, are (conjugate) fields that are the space or time reflections of each other. The fact that they are distinguishable in this factorization is because of the loss of the space-time reflection symmetries in the underlying covariance group.

It follows from eqs. (3) and (5) that the two-dimensional hermitian representations {S} of the Poincaré group relate to the four-dimensional representations {V} according to the relation

$$S^\dagger \sigma^\mu S = V\left(\theta^\mu_\nu\right)\sigma^\nu \qquad (10)$$

The solutions of this equation are the (double-valued) representations :

$$S\left(\theta_{\mu\nu}\right) = \exp\left(\left(\frac{1}{2}\right)\sigma^{\mu}\sigma^{\nu}\theta_{\mu\nu}\right) \qquad (11)$$

It follows further from eqs. (8) and (9) that if ψ and ξ are two spinors that transform the same way (i.e. according to (9a), then in accordance with the invariance in eq. (7),

$$\det Q \sim |\psi_1\xi_2 - \psi_2\xi_1|^2 = \text{inv} \equiv (x_0^2 - r^2)\sigma^0$$

Thus, the square root of this invariant is also invariant, i.e.

$$|\psi_1\xi_2 - \psi_2\xi_1| = |\tilde{\psi}\varepsilon\xi| = \text{inv} \qquad (12)$$

This is the invariant metric of the (first-rank) spinor space ; ε is the Levi-Civita matrix

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

It follows that the correspondence between the vector and the spinor representations of the Poincaré group of special relativity theory entails only the continuous transformations. That is, these representations are not entirely equivalent because there are no solutions S of eq. (10) that correspond to the discrete transformations (the reflections in space or time, $x_0 \to -x$ or $r \to -r$) while the transformations V that leave ds^2 (eq. (2)) invariant do indeed include these reflections. The latter full group of transformations is the "Lorentz group" -it then contains more symmetry than is required by the theory of special relativity, which only requires the continuous transformations of this group (the Poincaré group). It is the latter group that indeed spells out the underlying algebraic logic of space-time, according to the theory of special relativity.

What we have seen, then, is that the removal of the space and time reflections that leave (2) invariant yields a factorization (8) of the vector representation of the symmetry group, yielding the sum of two first-rank spinor representations. The spinor representation in itself is thus the irreducible (most general) way of expressing any field theory that would be compatible with the symmetry requirements of the theory of special relativity.

It is important that the two-dimensional representations of the Poincaré group obey the algebra of a quaternion number field -which, in turn, is a form of a second-rank spinor field. When the Poincaré group of special relativity is globally extended to the Einstein group of general relativity, the geometric implications of the group representations change (e.g. replacing the linear vector transformations in space-time by non-linear transformations). But the algebraic properties of the group representations do not change. Thus, the irreducible representations of the Einstein group are still a quaternion number field whose basis functions are spinor variables, though these field variables are now mapped in a curved space-time, obeying the rules of Riemannian geometry.

With these developments of the theory of relativity in mind, what sort of covariant metrical field should one expect to fully represent the symmetry requirement of the theory of general relativity ? Firstly, it must be a 16 component variable -because the Lie group of general relativity is a 16-parameter group, implying that there must be 16 (irreducible) relations at each space-time point to determine the components of the fundamental metrical field. If one should extract 10 of these and show that they correspond to Einstein's original symmetric tensor field equations (1), physically representing gravitation, then 6 relations would remain. It is salient at this point to note that indeed there are 6 independent components of the electromagnetic force field.

The second feature of the metrical field that is implied by the algebraic properties of the Einstein group is that it would obey the rules of quaternion algebra. This might then be implemented by considering the invariant metric of the Riemannian space-time to have the form :

$$ds = q^{\mu}(x)dx_{\mu} \qquad (13)$$

where $q^{\mu}(x)$ is, geometrically, a four vector ; thus, ds is, geometrically, a scalar. But each of the components of $q^{\mu}(x)$ is a quaternion. Since a quaternion has 4 independent components (it is represented most primitively in terms of a 2-dimensional hermitian matrix), the four-vector field $q^{\mu}(x)$ has 4x4 = 16 independent components.

Since ds is the sum of four quaternions it is itself a quaternion, algebraically. The real number field that corresponds to ds, in accordance with the calculus of quaternions, is the product of ds with its conjugate quaternion, $d\tilde{s} = \tilde{q}^{\mu}dx_{\mu}$. The real number field corresponding to ds is then

$$ds\, d\tilde{s} = -\left(\frac{1}{2}\right)\left(q^{\mu}\tilde{q}^{\nu} + q^{\nu}\tilde{q}^{\mu}\right)dx_{\mu}dx_{\nu} \Leftrightarrow ds^2 = g^{\mu\nu}dx_{\mu}dx_{\nu}$$

This factorization of ds^2 into the product of the quaternion (geometrical) invariant, $ds = q^{\mu}dx_{\mu}$ and its conjugate, $d\tilde{s} = \tilde{q}^{\mu}dx_{\mu}$ corresponds to W.R. Hamilton's four-dimensional extension of the two-dimensional factorization of the real number dr^2 into the conjugated complex numbers, dz and $d\tilde{z}$ [11]).

$$dr^2 = dx^2 + dy^2 \Leftrightarrow dz\, d\tilde{z} \rightarrow \begin{cases} dz = dx + idy \\ d\tilde{z} = dx - idy \end{cases} \qquad (14)$$

The complex number z entails more degrees of freedom than the real number $r = (z\,\tilde{z})^{1/2}$. The extra degrees of freedom are only revealed when the factorization (14) is carried out. Similarly, the spin degrees of freedom are revealed in the quaternion ds when we carry out the factorization

$$ds^2 = g^{\mu\nu}dx_{\mu}dx_{\nu} \Leftrightarrow ds\, d\tilde{s} \qquad \begin{cases} ds = q^{\mu}dx_{\mu} \\ d\tilde{s} = \tilde{q}^{\mu}dx_{\mu} \end{cases} \qquad (14')$$

The factorization (14') of ds then leads to the following factorization of Einstein's field equations (1) : [6])

$$R^{\mu\nu} - \left(\frac{1}{2}\right)g^{\mu\nu}R = \kappa T^{\mu\nu} \rightarrow \begin{cases} \left(\frac{1}{4}\right)\left(K_{\rho\lambda}q^{\lambda} + q^{\lambda}K^{\dagger}_{\rho\lambda}\right) + \left(\frac{1}{8}\right)q_{\rho}R = \kappa\mathcal{T}_{\rho} & (15a) \\ -\left(\frac{1}{4}\right)\left(K^{\dagger}_{\rho\lambda}\tilde{q}^{\lambda} + \tilde{q}^{\lambda}K_{\rho\lambda}\right) + \left(\frac{1}{8}\right)\tilde{q}_{\rho}R = \kappa\tilde{\mathcal{T}}_{\rho} & (15b) \end{cases}$$

In these conjugated quaternion field equations, $K_{\rho\lambda}$ is the "spin curvature", defined with the relation to the difference of the second covariant derivatives of a two-component spinor field :

$$\psi_{;\rho;\lambda} - \psi_{;\lambda;\rho} = K_{\rho\lambda}\psi$$

where the explicit form for $K_{\rho\lambda}$ is

$$K_{\rho\lambda} = \partial_{\lambda}\Omega_{\rho} + \Omega_{\lambda}\Omega_{\rho} - \partial_{\rho}\Omega_{\lambda} - \Omega_{\rho}\Omega_{\lambda} \qquad (16)$$

and Ω_{λ} is the "spin-affine connection" of the curved space, defined in terms of the first covariant derivative of the two-component spinor as follows :

$$\psi_{;\mu} = \partial_{\mu}\psi + \Omega_{\mu}\psi \qquad (17)$$

where

$$\Omega_{\mu} = \left(\frac{1}{4}\right)\left(\partial_{\mu}\tilde{q}^{\delta} + \Gamma^{\delta}_{\tau\mu}\tilde{q}^{\tau}\right)q_{\delta} \qquad (18)$$

and $\{\Gamma^{\delta}_{\tau\mu}\}$ are the components of the ordinary affine connection of the Riemannian space-time. The "dagger" in eqs. (15) denotes the hermitian adjoint.

The factorization of the Einstein tensor field equations (1) is then the quaternion field equation (15a) (or, equivalently, its space or time reflected equation -the conjugated field equation (15b)). The quaternion field equation (15a) then has the algebraic properties of the quaternion q_{ρ} -thus, it corresponds to 16 covariant relations at each space-time point. The right-hand side of eq. (15a) is the matter field quaternion, obtained from the variational derivative of the part of the total Lagrangian density that yields the matter field equations (when taken with respect to the quaternion variables).This source term for the metrical field equations is then analogous to the matter tensor on the righ-hand side of Einstein's equations (1) -which is determined in the samé way, except that the variational derivatives are taken in the tensor theory with respect to the components of the metric tensor $g_{\mu\nu}$.

An important point that has been emphasized in regard to the quaternion field equations (15) is that they are covariant only with respect to the continuous transformations of general relativity theory, while the original tensor field equations are also covariant with respect to the reflections in space and time. This is the reason for the increase in the number of components of the metrical field, from 10 to 16, as required by the algebraic group of general relativity.

What I discovered next in these investigations, [12]) was that by iterating the quaternion field equations with a conjugated quaternion solution, thereby generating a second-rank, nonsymmetric tensor equation, the latter could be expressed as the sum of a symmetric tensor part and an antisymmetric tensor part. It was then shown that the 10 relations of the symmetric tensor part are in one-to-one correspondence with Einstein's tensor field equations (1). Taking the covariant divergence of the remaining 6 antisymmetric tensor field equations, I then showed that they can be expressed precisely in the form of Maxwell's field equations for electromagnetism. This follows because of the feature that the symmetric tensor equations and the antisymmetric tensor equations have opposite reflection properties. Thus, equations emerge from the antisymmetric tensor part of this formalism that are odd under reflections, as one expects of Maxwell's equations since their source is a current density.

Such implicit inclusion of the gravitational and electromagnetic features of matter in a single covariant metrical field variable, q^μ, is indeed the type of unified field theory that Einstein anticipated should emerge when the full unification had been achieved, according to his criterion (1) [4]). It was achieved here by appealing to the most general group structure that underlies general relativity theory, as Einstein suggested. The latter, in turn, was in terms of the group that rejects the reflection symmetry elements in space and time.

3. The geodesic equation in general relativity, motion, time and the equivalence principle

The structure of the metrical field equations that relate the geometrical logic of space-time to the matter field -Einstein's tensor field equations (1) or the quaternion field equations (15)- incorporate a geometrical equation whose solutions prescribe the family of geodesics of the curved space-time. The latter "geodesic equation", which follows from the extrema of the path integral, i.e.

$$\delta\int ds = 0$$

has the form :

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\lambda}\left(\frac{dx^\nu}{ds}\right)\left(\frac{dx^\lambda}{ds}\right) = 0 \qquad (19)$$

The derivation of this form in the 2nd-rank tensor theory is given in most standard texts on general relativity [13]). Its derivation in the quaternion theory of this author is demonstrated in ref. [14]).

Though the formal form of the geodesic equation is the same in the tensor and the quaternion field theories, it is important to note that the latter is a more general expression, implicitly, since ds here is a quaternion, thus depending on a set of four parameters, while ds is a real number in the tensor theory, thus expressed in terms of a single parameter set. Since the quaternion may be represented in its irreducible form in terms of a two-dimensional hermitian matrix, the geodesic equation (19) in the quaternion theory stands for four independent equations

$$\left(\frac{d^2x^\mu}{ds^2} + \Gamma^\mu_{\nu\lambda}\left(\frac{dx^\nu}{ds}\right)\left(\frac{dx^\lambda}{ds}\right) = 0\right)_{ij} \qquad (i,j = 1,2) \qquad (20)$$

The geometrical difference between the (real number) geodesic equation (19) and the quaternion form of the geodesic equation (20) is that (19) is characterized by the single parameter trajectory solutions, $x^\mu(s)$, in space-time, whereas (20) has solutions necessarily characterized by trajectories $x^\mu(s_{ij})$ that entail four parameters at each space-time point. That is to say, one need only specify the change of a single parameter in (19) to indicate how the spatial point of a trajectory proceeds to another (continuously connected) spatial point of the path. But in the more general formulation (20), one must specify four parameters (s_{ij}) at each space-time point in order to prescribe the evolution of the trajectory, unambiguously. One way of interpreting "time" is in terms of the parametric representation of the evolution of such a trajectory. Thus, the independent variable s in $x^\mu(s)$ may be interpreted as "proper time". Then, "proper time" is a one-parameter set in the standard interpretation of the geodesic equation as the equation of motion of a test body in general relativity. But the proper time, "s", in the quaternion theory is a four-parameter set. Thus, we see that the quaternion representation (13) of ds -which is suggested by the group structure of general relativity theory- appears to present a more general expression of the "proper time" concept in the description of the motion of a test body.

In this regard, it is interesting to note that the idea that the quaternion algebra may lead to a more general expression of "time" evolution was expressed more than a century ago by the discoverer of the quaternion algebra, W.R. Hamilton, who said [11] :

"It early appeared to me ... to regard ALGEBRA as being no mere Art, nor Language, nor primarily as science of Quantity ; but rather as the Science of Order in Progression. It was, however, a part of this conception, that the progression here spoken of was understood to be continuous and unidimensional : extending indefinitely forward and backward, but not in any lateral direction. And although the successive states of such a progression might (no doubt) be represented by points upon a line, yet I thought that their simple successiveness was better conceived by comparing them with moments of time, divested, however, of all reference to cause and effect ; so that the "time" here considered might be said to be abstract, ideal, or pure, like that "space" which is the object of geometry. In this manner I was led, many years ago, to regard Algebra as the SCIENCE OF PURE TIME.
... And with respect to anything unusual in the interpretations thus proposed, ... it is my wish to be understood as not at all insisting on them as necessary, but merely proposing them as consistent among themselves, and preparatory to the study of quaternions, in at least one aspect of the latter."

I have applied this more general expression of the geodesic equation in its quaternion form (20) to the problem of planetary motion [14], to the derivation of the Hubble law in cosmology [15] and to the problem of the spiral structures of galaxies in astrophysics [15]. In these applications, it was found that indeed extra predictions are made that transcend those of the usual tensor formulation, and are compatible with the observational facts.

A basic ingredient in the early stages of Einstein's analysis in general relativity theory was his assertion of the principle of equivalence. This was based on the following reasoning. Since the geodesic of a Riemannian space-time is a curve rather than a straight line, relative to any other frame of reference of an observer of a moving body, Einstein said that the effect of a force field exerted by another massive body on the observed moving body, in also causing it to move along a curved, rather than a straight

line path, might be "equivalent" to the "free motion" of the test body in a Riemannian space-time. This is the principle of equivalence. In the early stages of his theory, Einstein found that the metric tensor solutions of his field equations (1), together with the geodesic equation (19), as the equation of motion of a test body, predicted everything that Newton's theory of universal gravitation gave, in addition to three extra predictions, not even qualitatively predicted by the classical theory. These were the well known critical tests of his theory - 1) the advance of the perehelion of Mercury's orbit, (or, generally, the orbit of any planetary body), 2) the bending of the path of light as it propagates past the vicinity of the sun and 3) the "gravitational red shift" of radiation when comparing its wavelength in gravitational potentials of increasing value [13].

It should be noted in regard to the conceptual aspects of the principle of equivalence that this should be understood only in the sense of expressing a feature of the motion of a test body in an approximate way. For the test body itself -an isolated quantity of radiation or matter- is, in general relativity (a continuum field theory) only a useful approximation for a component of closed system, represented most generally in this theory in terms of regular field solutions of the basic equations of the theory. This field must, in principle, prescribe all of the features of the matter system it refers to, with the requirement, nevertheless, that in some asymptotic limit it would appear as though there is a "separate test body" that is acted upon by the field of force of the "remainder" of the material system. The salient point here is that, in principle, one should start with the mathematical representation for the closed system, and then take the asymptotic limit where a component of the closed system appears as though it were a separated entity, rather than starting at the outset with the separated "test body" and the field it responds to. Besides this conceptual difference -which implies that in the general form of the theory, the principle of equivalence is not one of the underlying axioms of general relativity- there is a mathematical difference. For it is important that the asymptotic solutions of nonlinear equations of motion do not generally match the features of linear equations of motion that might describe a free test body in a background field. In the description of the closed system, according to the general structure of this theory, the affine connection, $\Gamma^{\mu}_{\nu\lambda}$, in the geodesic equation (19) or (20), entails all of the matter of the system, including the component that one must identify with a "test body" for practical applications of the theory.

The point I make here is that the geodesic path determined with the general form of the affine connection field -that entails all of the matter of the system, including the "test body"- would not match the geodesic path that would be determined from the affine connection field that does not entail the matter of the "test body". This is because the dependence of the affine connection on the matter fields of the system is not additive, because of the basic nonlinearity of the field theory. Nevertheless, it is most likely a good mathematical approximation, in many cases, to assume such additivity of the test matter to the matter of the rest of the system. But if this is only a mathematical approximation, I do not believe that the equivalence principle should be asserted as an axiom of the theory of general relativity -since, in principle, there is no actual (separable) "test body" in the theory. I believe it would be more accurate to replace the statement of the principle of equivalence as an axiom of general relativity with the statement of the principle of correspondence, as a fundamental assertion [16].

4. Inertia from general relativity

The next step in my research program, based on Einstein's suggestions, was to see if the metrical field might relate in an intimate and precise way to the inertial manifestations of matter. I have analyzed this problem in the following way [17]. The inertia of matter appears most primitively in the field equations that describe microscopic physics. In special relativity, the most general (irreducible) form of these equations is the set of simultaneous, two-component spinor equations of Dirac (with $\hbar = c = 1$) :

$$\begin{aligned} \sigma^{\mu}\partial_{\mu}\psi + \mathfrak{I}\,\psi &= -m\chi \\ \tilde{\sigma}^{\mu}\partial_{\mu}\chi + \mathfrak{I}\,\chi &= -m\psi \end{aligned} \qquad (21)$$

where $\mathfrak{I}$ is the "interaction field" that couples to the spinor fields, and ψ and χ are the reflections of each other, with $\chi = \epsilon\psi^{*}$.

The insertion of the mass parameter m into eq. (21) presupposes the existence of a discrete particle, in the usual quantum

mechanical formalism, with this amount of inertial mass. But according to Einstein's conclusions, one must replace the singular, discrete variable with continuous and analytic (regular), though peaked, fields. The first step in deriving the inertia of matter from the field theory would then be to remove this parameter from eq. (21). The remainder of the equation must then be mapped in a curved (Riemannian) space-time, rather than the Euclidean space-time of special relativity.

With the global extension (17) of the derivative in a flat space-time to the covariant derivative in a curved space-time, and the global extension discussed previously,

$$\sigma^{\mu} \rightarrow q^{\mu}(x) \qquad (22)$$

the spinor field equation (21), without the mass term, becomes

$$q^{\mu}(\partial_{\mu}\psi + \Omega_{\mu}\psi) = -\mathfrak{I}\,\psi \qquad (23)$$

What I found next was that when one further imposes gauge invariance (of the first and second kinds) on the matter field equations (23), they may be re-expressed in the form [17] :

$$q^{\mu}\partial_{\mu}\psi + \lambda\chi = -\mathfrak{I}\,\psi \qquad (24a)$$

where

$$\lambda = \left(\frac{1}{2}\right)[\,|\det\Lambda_{+}| + |\det\Lambda_{-}|\,]^{1/2} \qquad (25)$$

is the modulus of a complex function, and therefore it is a positive-definite function of the space-time coordinates, and

$$\Lambda_{\pm} = q^{\mu}\Omega_{\mu} \pm \text{h.c.}$$

where "h.c" is the "hermitian conjugate" function.

The quaternion conjugate of eq. (24a) -corresponding to its (spatial or temporal) reflection- is

$$\tilde{q}^{\mu}\partial_{\mu}\chi + \lambda\psi = -\mathfrak{I}\,\chi \qquad (24b)$$

Comparing eqs. (24a,b) with Dirac's equation (21), it is seen that the positive-definite function λ plays the role of the inertial mass associated with the spinor matter field (ψ,χ).

Three important features of this definition of inertial mass then emerge : First, since λ is a positive-definite function, it follows that in the Newtonian limit of general relativity, gravitational forces can have only one sign -they are either always attractive or always repulsive in this limit. Since it is observed in one case, e.g. the force that creates "weight", to be attractive, the prediction then follows that in the Newtonian limit of General Relativity, gravitational forces can only be attractive. This result, which was never derived from first principles from the standard forms of general relativity theory, is in agreement with all of the empirical facts. It was revealed here because of the factorization from a reducible form of the theory (which masks some of its physical predictions) to the irreducible form.

In regard to this force, it should be pointed out that in the general form of the theory, gravitational forces are not necessarily attractive under all conditions (of matter density, energy-momentum transfer, etc.). This is because the "force" exerted by matter on a "test body" is defined in this theory in terms of the affine connection terms, $\Gamma^{\mu}_{\nu\lambda}\left(\frac{dx^{\nu}}{ds}\right)\left(\frac{dx^{\mu}}{ds}\right)$, of the geodesic equation (19) (or (20)). The idea is that the latter term in the geodesic equation -which plays the role of the external force/mass of the "test body"- is, generally, a non-positive-definite function. But in the linear (Newtonian) limit of general relativity, where the relative separations and speeds of interacting matter are sufficiently small, this equation of motion does asymptotically approach the classical form, F = ma, where $F = \frac{GMm}{r^2}$, and the macroscopic masses, m and M are the sums of positive-definite numbers, $\Sigma\lambda_i$, and thus must have only one polarization -in this limit of the theory. An examination of the implications of the repulsive components in the gravitational force, in general relativity, in the problem of stellar collapse, was recently published [18].

It should also be noted that in the domain of elementary particle physics, where the mass density and the momentum transfer between interacting matter is sufficiently high, the terms that play the role of mutual forces in the equations of motion still have a repulsive term, as well as the attractive component, even though the mass terms λ are positive-definite. But in this domain the Newtonian form for the gravitational force is not valid, and

there is no inconsistency. A well know empirical example of mutual interaction that has repulsive as well as attractive components is the nuclear force field [19].

Secondly, if all of the matter of a physical system, other than the matter represented by the spinor field (ψ,χ), should be depleted, then the spin-affine connection Ω_μ and the hermitian and anti-hermitian fields $\Lambda_\pm$ would correspondingly become null fields. According to the relation in eq. (25), the inertial mass would (in this limit) be zero. This result is due to the feature of this theory that it automatically incorporates the Mach principle -the assertion that the inertia of matter is a measure of its coupling to other matter that it interacts with. That is, if the other matter would disappear, the coupling must vanish and therefore the inertial mass of the observed matter would correspondingly go to zero.

This relativistic interpretation of inertia that was proposed by Mach, strongly influenced Einstein in his initial thinking in general relativity theory. In his "Autobiographical Notes" Einstein made the following comment [20] :

> "Mach conjectures that in a truly rational theory inertia would have to depend upon the interactions of masses, ..., a conception which for a long time I considered as in principle the correct one."

Einstein then went on to say, however, that he gave up the idea of the Mach principle later on, commenting further that [20] :

> "It presupposes implicitly, however, that the basic theory should be of the general type of Newton's mechanics : masses and their interaction as a general concept. The attempt at such a solution does not fit into a consistent field theory."

However, it is not true that the Mach principle does necessarily presuppose a Newtonian particle theory ; for example, this is not the model followed when interpreting inertial mass in the way in which it appears in the most primitive formal expression for the laws of microscopic physics -a generally covariant form of wave mechanics. In this way, the Mach principle can indeed be incorporated into a consistent field theory, as I have outlined above.

Thirdly, in the asymptotic limit, as the description of microscopic matter in a curved space-time approaches its mathematical description in the flat space-time corresponding to spinor field equations in a Hilbert space, the continuous spectrum of λ approaches a discrete set of values -a mass spectrum. This is a feature that emerges from the correspondence of this nonlinear field theory of matter in general relativity with the quantum mechanical formalism, in the nonrelativistic limit.

Finally, it was found in my researches that the gauge invariance imposed on this theory, that was necessary in order to yield the spinor field structure (24), also yields the analytical result that the electromagnetic forces between interacting matter can be attractive or repulsive, depending on the nature of the geometrical fields, $\Lambda_{\pm}$, and their derivatives, under different sets of physical conditions [17]. This prediction, which is also in agreement with the experimental facts, has not been derived from first principles from other field theories or particle theories of matter.

Correspondence with Quantum Mechanics and "Particles"

For an actual material system, represented theoretically in accord with the Mach principle, the functional $\mathcal{J}$ on the right-hand side of eq. (24) plays the role of the coupling of "other matter" to the "observed" matter. Along with this principle we also invoke the "principle of correspondence", asserting that, asymptotically, the solutions $\{\psi^{(i)}\}$ approach the distinguishable "free fields" that are associated with the composite particles of a material system, according to the formalism of quantum mechanics. With this "correspondence" imposed on the generally relativistic, nonlinear formalism, in the appropriate limit, the general form of the matter field equations (24) must be expressible as a set of n coupled spinor field equations, for an n-component system (i.e. a system that in the limit appears as n free bodies, weakly coupled in some perturbation approximation) :

$$q^{\mu}\partial_{\mu}\psi^{(i)}(x) + \lambda\chi^{(i)}(x) = -\mathcal{J}_{i}\psi^{(i)}(x) \quad (i = 1,2,\ldots,n) \qquad (24c)$$

where $\mathcal{J}_i = \mathcal{J}_i\left[\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(i-1)}, \psi^{(i+1)}, \ldots, \psi^{(n)}\right]$ for an n-component material system -corresponding in the asymptotic limit where there appear to be "free bodies" to the n-body formalism of quantum mechanics. In the coupled spinor field equations (24c), the independent variables, x, stand for the four parameters

of a single space-time, common to all of the field components $\psi^{(i)}(x)$, and $\mathcal{J}_i$ is the interaction functional that couples the spinor matter fields other than $\psi^{(i)}$ -corresponding to all of the matter of the closed system, except for the ith component- to the latter field, in accordance with the Mach principle.

In accordance with the axiomatic basis of this theory, and its definition of relation (coupling) as an essential ingredient of its conceptual structure, it is salient that there is no "free field" limit for the solutions of the field equations (24c). It is also significant within this theory that there is no representation for a field to "act on itself" -i.e. the concept of "self-energy" of the particle theories (whether classical or quantum mechanical) is automatically excluded in this approach. Finally, it should be noted that the spinor field equations (24c) are intrinsically nonlinear, by virtue of the appearance of the coupling functional $\mathcal{J}_i$, that in principle cannot be "turned off" -in accordance with the axiomatic structure of the theory.

The coupled field equations (24c), which I have referred to as the "matter field equations", and the metrical field equations (15a), in terms of the generalized quaternion field solution, then form a self-consistent set of relations that entail the gravitational, electromagnetic and inertial manifestations of elementary matter. Such a field unification is indeed in accord with Einstein's anticipation for a unified field theory -resulting from fully exploiting the basis of general relativity.

Also in accordance with another fundamental principle underlying this theory -a principle that has indeed played an important role throughout the history of science, and expressed explicitly in contemporary physics by N. Bohr- is the principle of correspondence. With the application of this principle, the part of the formalism (24c) that directly entails the inertial manifestation of matter reduces, in the linear limit of the theory -corresponding to nonrelativistic momentum-energy transfer between interacting components of a system- to the standard linear eigenfunction form of quantum mechanics. In accordance with Einstein's expectations, then, the generally covariant, nonlinear field theory of matter that evolves here, from fully exploiting the algebraic as well as the geometrical logic of space-time, in general relativity, does successfully incorporate all of the successful results of nonrelativistic quantum mechanics -though as a mathematical approximation to a theory that is based on entirely different principles than those of the Copenhagen school [21].

I have found in my research program that in the asymptotic limit of the theory, as the spinor solutions of (24) approach the elements of a Hilbert space, the expectation values of the inertial mass field, $<\lambda(x)>$ correspondingly approach a discrete set of values [21c]. Thus, in agreement with the prediction of Einstein and Rosen [22], this theory predicts the existence of a nonsingular mass spectrum for elementary matter. It is important to take note, nevertheless, that the actual limit of linearity does not exist in this theory -so that the limit of actually discrete values for the masses also does not exist- though for sufficiently rarefied matter, one can approach discreteness as closely as one pleases. That is to say, all measured values must have an irreducible "line width", because of the irreducible nonlinearity in structure of the field equations, representing the irreducible coupling that cannot "turn off". But the latter prediction is not in disagreement with any experimental fact, though the quantum theory interprets the irreducible line width in terms of the action of the Heisenberg uncertainty principle, rather than predetermined field coupling.

A further prediction of the mass field, as represented in this theory in terms of $\lambda(x)$, is that, because the latter is a two-dimensional matrix field, there are two mass eigenvalues associated with any given spinor field solution, ψ. Thus, the theory predicts that for every matter component of a system, described with a spinor field solution, there must be a twin matter field that is physically identical, except for mass (and lifetime). Indeed, it was found that a close fit could be found with the well known mass doublet of this type -the electron-muon doublet. The theoretical mass ratio, for the electron to muon, was predicted and found to be in close agreement with the data. Then, taking account of the physical vacuum of real pairs, that "creates" the space-time curvature that accounts for the mass of an elementary particle, the absolute mass value was determined from the theory, in terms of a particular density of pairs [23]. The physical mechanism that explains these results are as follows. As the "observed" electron moves through the sea of electron-positron pairs, it can occasionally excite a pair in its environment. The spin-affine connection field is then altered and, in turn, the electron becomes more inertial -by a factor

that is the order of $3/2\alpha \sim 206$, where α is the fine structure constant. When the pairs in the neighborhood of the more inertial electron eventually de-excite, by transferring this excitation energy and momentum to the other pairs of the "physical vacuum", the inertial mass of the "observed" electron then returns to its normal (minimum) value. The transferred energy-momentum is identified in this theory with a two-component spinor field associated with the neutrino in the particle theories, when interpreting the decay of the muon. Then using the method of time-dependent perturbation theory, the decay time of the more inertial electron (the muon) was calculated and found to be within the order of magnitude of the measured lifetime of the muon [24].

It is also noted that the prediction follows here that any spinor field should have an associated mass doublet. Thus the "proton" must also have a mass twin that would be the order of 200 times more inertial -corresponding to the observation of a particle with $m \sim 200$ Gev. Such an elementary particle has not yet been observed in high energy physics experimentation.

Other recent results of this research program in general relativity that relate to elementary particle physics are :
1) a derivation of the mass of a pion, as a composite of spinor field particles, and a calculation of the mass ratio, $\frac{m(\pi^0)}{m(\pi^{\pm})}$, in close numerical agreement with the data [25], and
2) the quantization of electrical charge of elementary matter from an asymptotic form of the representations of the Einstein group [26].

5. Concluding remarks

As Pauli has pointed out [27], if this were a true quantization of electrical charge there would indeed be an incompatibility with a continuum field theory, such as that of general relativity, and the unified field theory discussed in this article. However, my investigation of this problem reveals that with the global generalization of the irreducible representations of the Poincaré group of special relativity to the irreducible representations of the Einstein group of general relativity,

$$S(\theta_{\mu\nu}) = \exp\left[\left(\frac{1}{2}\right)\sigma^{\mu}\sigma^{\nu}\theta_{\mu\nu}\right] \rightarrow S\left[\theta_{\mu\nu}(x)\right] = \exp\left[\left(\frac{1}{2}\right)q^{\mu}(x)q^{\nu}(x)\theta_{\mu\nu}(x)\right] \quad (25)$$

the prediction of the empirical "quantization" of charge follows from the asymptotic form of the latter representations, as the non-Euclidean (nonlinear) space-time approaches the Euclidean (linear) space-time. What was found [26]) was that the asymptotic form of the metrical field solutions of eq. (15) in eq. (25) lead, in order e^2, to the "quantization" of e^2 ; that is, to the possible values : e^2, $2e^2$, $3e^2$, ... ne^2, where n is any integer $\geqslant 1$. This spectrum of constants characterize the electromagnetic coupling between interacting matter. Thus, this field theory implies an asymptotic quantization of the electromagnetic coupling rather than the quantization of the charge, e, itself : e, 2e, 3e, ..., ne. But both types of quantization are indistinguishable as far as experimental measurements are concerned -since all measurements that reveal the electrical charge must couple it to another charge, giving the measurement in terms of e^2. It is only that the unified field theory does not extrapolate from this to the theoretical value, e. Further, as in our previous discussion of the "mass quantization" in the linear limit of this field theory, the measurements that reveal the spectrum e^2, $2e^2$, ... must also entail an irreducible line width, due to the irreducibility of the nonlinearity in the theory, even though it can become arbitrarily small.

In regard to the gravitational manifestations of matter, recent attempts to quantize gravity proceed by postulating a linearization of Einstein's field equations in terms of a massless, spin-2 field operator in a Hilbert space, that would be compatible with the tensor structure of eqs. (1). An excellent account of this type of program is given in the text by Tonnelat, who also includes an extensive bibliography [28]).

It is my view that the quantization of general relativity is logically necessary if the underlying axioms of the quantum theory of measurement are to be maintained. For the same logical reasons, the maintainance of the axiomatic basis of Einstein's unified field theory of matter, in terms of a truly continuum, deterministic field theory, would require that in the final analysis quantization cannot play a fundamental role.

The mathematical difficulties that have been encountered in all of the attempts thus far to quantize general relativity are essentially due to the fact that when the nonlinear features of

Einstein's field equations (1) are removed (later to be treated as a perturbation, as it is done in quantum electrodynamics), there is nothing left ! The nonlinearity here, in contrast with quantum electrodynamics, is not a small perturbation, later to be treated in expanding a "free, quantized gravitational field" in a Hilbert space, in a perturbation series. The nonlinearity is here the entire essential field. It is interesting to note that the nonlinearity of the field in Einstein's theory, that is irreducible, is because of the elementarity of relation contained in his approach, in contrast with the elementarity of particle (though without a predetermination of its trajectory) of the Copenhagen view.

But even if the formalism of quantum field theory had been in a satisfactory state (including the incorporation of a massless, spin-2 field operator to relate to gravitation) one should not rule out the possibility that Einstein's unified field approach could represent the facts of nature in a more complete way -unless this theory had been pursued further and demonstrated to be scientifically false. But the latter has not been done, and indeed quantum field theory, even when it is applied to quantum electrodynamics, has never been expressed in a form that is demonstrably mathematically consistent ! Thus, there is no reason yet to believe that the Copenhagen approach has established itself as a truth of nature, and that thus Einstein's unified field approach must be excluded from a scientific point of view.

In contrast, my research of the unified field approach to the problem of matter, based on initial studies that make use of Einstein's suggestions toward a unified field theory, has given me a great deal of confidence in the idea that Einstein's approach may be, after all, the correct one toward a resolution of the mysteries of elementary particle physics, as well as astrophysics and cosmology, in a single unified scheme.

It is significant, then, that the full exploitation of the basic axioms of one of the revolutionary developments of 20th century physics - the theory of relativity- can only be accomplished by abandoning the underlying axioms of the second revolutionary development of this period -the quantum theory. Einstein fully anticipated that this would indeed become necessary since (along with de Broglie and Schrödinger) he continually emphasized the logically dichotomous nature of a theory that would combine the relativity and quantum theories under one umbrella.

Of course, one must recognize that, in science, what we see in one light during one period might be viewed in an entirely different light in a later period, when further progress in experimental and theoretical science had been achieved. Thus one should never say that one explanation in science is definitely true and another is definitely false. But one can say that if one theory should definitely be true then a second (logically dichotomous) theory must definitely be false. The possibility also must always be left open that in the long run, based on future experimentation and theoretical analyses, both theories would have to be rejected as scientifically false ! Still, I do have an intuitive feeling that a great deal of the conceptual basis of the theory of relativity will remain in future scientific theories. For this achievment, science must always recognize its indebtedness to Albert Einstein.

RÉFÉRENCES

1) M. Sachs, Annales Fond. L. De Broglie, vol. 4, n° 2 (1979)

2) W. Pauli, Theory of Relativity (Pergamon, 1958), p. 224

3) M.-A. Tonnelat, Les Théories Unitaires de l'Electromagnétisme et de la Gravitation (Gauthier-Villars, 1965)

4) A. Einstein, Ann. Math. 46, 578 (1945)

5) For further discussion of this view, in terms of the theory of relativity, see H. Reichenbach, The Philosophy of Space and Time (Dover, 1968) ; M. Sachs, Physics Today 22, 51 (1969) ; M. Sachs, Ideas of the Theory of Relativity (Israel Universities Press, 1974)

6) M. Sachs, Nuovo Cimento 47, 759 (1967)

7) W. Pauli, ibid., p. 226

8) M. Sachs, Nature 226, 138 (1970)

9) A. Einstein, in Albert Einstein - Philosopher-Scientist, (P.A. Schilpp, editor), (Lib. Liv. Phil., 1949), p. 75

10) A. Einstein and W. Mayer, Preussiche Akademie der Wissenschaften, Phys.-math. Klass. Sitz., 522 (1932)

11) H. Halberstam and R.E. Ingram, The Mathematical Papers of Sir William Rowan Hamilton, Vol. III (Cambridge, 1967)

12) M. Sachs, Nuovo Cimento 55B, 199 (1968)

13) See, for example, R. Adler, M. Bazin and M. Schiffer, Introduction to General Relativity (McGraw-Hill, 1975), Second edition

14) M. Sachs, Nuovo Cimento 66B, 137 (1970)

15) M. Sachs, Int. Jour. Theoret. Phys. 14, 115 (1975)

16) I have elaborated on this discussion of the principle of equivalence in general relativity in Brit. Jour. Phil. Sci. 27, 225 (1976)

17) M. Sachs, Nuovo Cimento 53B, 398 (1968)

18) M. Sachs, Ann. Inst. Henri Poincaré 28, 399 (1978)

19) See, for example, H.A. Bethe, Elementary Nuclear Theory (Wiley, 1947), Chapter 14

20) A. Einstein, "Autobiographical Notes", ibid., p. 29

21) I have reviewed this work in the following articles : Int. Jour. Theoret. Phys. a) 4, 433 ; b) 4, 453 (1971) ; c) 5, 35 ; d) 5, 161 (1972). The Pauli exclusion principle is derived, within the context of this field theory in Nuovo Cimento 27, 1138 (1963) ; 44B, 289 (1978). A general (non-mathematical) discussion of this approach and its consequences is given (in French) in La Recherche 6, 1008 (1975)

22) A. Einstein and N. Rosen, Phys. Rev. 48, 73 (1935)

23) M. Sachs, Nuovo Cimento 7B, 247 (1972)

24) M. Sachs, Nuovo Cimento 10B, 339 (1972)

25) M. Sachs, Nuovo Cimento 43A, 74 (1978)

26) M. Sachs, Lettre al Nuovo Cimento 21, 123 (1978)

27) W. Pauli, ibid., Chapter 63

28) M.-A. Tonnelat, ibid., Chapters 13, 14