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SMALL-TIME APPROACH TO THE DESCRIPTION OF THE PERTURBATION FORMALISM

par M. E. PAPP

Polytechnic Institute of Cluj,
Physics Department,
3400-Cluj, Romania

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Abstract : A small-time approach to the description of the time-dependent perturbation theory is proposed. For this purpose we assume that within the interaction-region the particle is not necessarily a point-like one, thus also considering additionally, a large imaginary contribution (Im $V_{ii}^{(1)} \leqslant 0$) of the interaction potential which is confined inside the small-time region. It can be then proved that the validity of the so implied Schrödinger equation with the nonhermitian (hermitian) interaction can be assured only up to a certain upper (lower) bound of the time-parameter. Alternatively, such results are able to probe, in this way also, the existence of a nonzero intrinsic time-dispersion.

Résumé : Nous proposons une méthode liée aux temps courts pour la théorie des perturbations dépendantes du temps. Dans ce but nous supposons que dans la région de l'interaction la particule n'est pas nécessairement ponctuelle. On vient ainsi à considérer une contribution additionnelle imaginaire (Im $V_{ii}^{(1)} \leqslant 0$) du potentiel d'interaction confiné dans la région des temps courts. Dans ces conditions la validité de l'équation du Schrödinger avec interaction nonhermitienne (hermitienne) est établie jusqu'à une

certaine limite supérieure (inférieure) du temps. Nous parvenons ainsi à démontrer l'existence d'une dispersion intrinsèque non-nulle du temps.

I. Introduction

Following the ideas of the short-distance behavior of the field theory (1) there is of interest to analyse such a behavior within the quantum-mechanical time-dependent perturbation theory. The traditional way of computing the transition rate of the perturbation theory consists in considering sufficiently large values of the time parameter. In this respect we want to extend the perturbation approach, thus also considering in some more details the small t-values. For this purpose we shall conjecture that in the region of the small time-values the interacting particle is not a pointlike one, thus also considering for convenience that the interaction potential takes a complex value. Here we assume that the basic physics of the nonpoint particle description is rather connected with the nonhermitian (energy) operators, like some relevant results which have been obtained for the space-time operators (2). There is also in such a way that a smearing out of the interaction could be achieved a-priori in terms of the imaginary parts of the interaction-potential, like the non local or nonlinear methods. In other words it is assumed implicitly that in the region of the small time-values the standard Schrödinger equation with the hermitian interaction is in fact not basically involved. The imaginary part of the potential shall be subsequently eliminated by minimalizing the upper bound of the total probability. The existence of the so confined imaginary contribution -which shall be considered as a negative one- reveals also some aspects of the compound system description. In these conditions there exists actually the possibility to propose a small-time approach to the description of the perturbation formalism. More exactly, we shall prove that there is an upper value of the time parameter up to which the validity of the so implied Schrödinger equation with the nonhermitian interaction makes sense. In this way it turns out that the time-parameter becomes in fact a dynamically determined quantity (2).

II. Small-time approach to the description of the perturbation theory

The state function takes as usually the form

$$|\phi(t)\rangle = \sum a_k(t)\, \exp\left(-\frac{i}{\hbar} E_k\, t\right)|k\rangle, \qquad (2.1)$$

where $|k\rangle$ are the normalized eigenstates of the free hamiltonian. Generally the potential V can be considered as emerging effectively within several approximations orders (with respect to the involved coupling constants or another expansion-parameters). Performing in such conditions the formal expansion of the so considered potential

$$V = V^{(1)} + V^{(2)} + \ldots, \qquad (2.2)$$

and of the k-momentum representation state function

$$a_k(t) = a_k^{(0)}(t) + a_k^{(1)}(t) + \ldots, \qquad (2.3)$$

it results

$$a_k(t) = \delta_{ki} - \frac{V_{ik}^{(1)}}{E_k - E_i}\left(\exp \frac{i}{\hbar}(E_k - E_i)t - 1\right), \qquad (2.4)$$

where we have limited ourselves to the first approximation order and where

$$V_{ik}^{(1)} = \langle k|V^{(1)}|i\rangle. \qquad (2.5)$$

Account has been made of the initial conditions

$$a_k^{(0)}(t) = \delta_{ki}\ , \quad a_k^{(\alpha)}(0) = 0\ , \qquad (\alpha \geqslant 1). \qquad (2.6)$$

In the region of the small time-values the total probability

$$P(t) = \sum |a_k(t)|^2 = 1 + \frac{2t}{\hbar}\, Im\, V_{ii}^{(1)} + $$

$$+\ 4 \sum_{k \neq i} \frac{|V_{ik}^{(1)}|^2}{(E_k - E_i)^2}\ \sin^2 \frac{(E_k - E_i)t}{2\hbar}\ , \qquad (2.7)$$

takes the form

$$\tilde{P}(t) = 1 + \frac{2t}{\hbar}\, Im\, V_{ii}^{(1)} + \frac{t^2}{\hbar^2} \sum |V_{ik}^{(1)}|^2, \qquad (2.8)$$

where $t \geqslant 0$, $V_{ik}^{(1)}$ takes real values for $k \neq i$ and where in agreement with the previous assumption $Im\, V_{ii}^{(1)} \leqslant 0$. Assuming for convenience that $V_{ik}^{(\alpha)}$ takes real values for $\alpha \geqslant 2$, there is the expansion-independent expression $|Im\, V_{ii}^{(1)}| \equiv |Im\, V_{ii}|$ which shall be denoted by Λ. For convenience we shall consider, at least for the moment, the function (2.8) as the single candidate of the probability function in the small-time region. In this respect we have to consider that

$$t \ll \frac{2\hbar}{|E_k - E_i|}\ , \qquad (2.9)$$

thus also establishing approximately the validity limits of the equality

$$P(t) \cong \tilde{P}(t), \tag{2.10}$$

between the above mentioned probability functions.

III. The upper bound of the time-parameter

Analysing the dependence of the probability function $\tilde{P}(t)$ with respect to the parameter Λ, one finds that there is a minimum for

$$\frac{\hbar}{t} = \Lambda, \tag{3.1}$$

which takes the value

$$p(t) \equiv \frac{t^2}{\hbar^2} \sum (Re\ V_{ik}^{(1)})^2. \tag{3.2}$$

Imposing now the probability conservation relation

$$0 < p(t) \leqslant 1, \tag{3.3}$$

there results the existence of the upper bound of the time-parameter as

$$t \leqslant \tau \equiv \hbar \left[\sum (Re\ V_{ik}^{(1)})^2\right]^{-\frac{1}{2}}, \tag{3.4}$$

when also

$$\Lambda \in \left[\frac{\hbar}{t}(1 - \sqrt{1 - p(t)}),\ \frac{\hbar}{t}(1 + \sqrt{1 - p(t)})\right]. \tag{3.5}$$

On the other hand there is

$$0 < \tilde{P}(t) \leqslant 1, \tag{3.6}$$

only when

$$t \leqslant 2\hbar\Lambda \left[\sum |V_{ik}^{(1)}|^2\right]^{-1}. \tag{3.7}$$

However, the present upper-bound contains explicitly the rather "unphysical" parameter Λ. Noticing now that the presence of this parameter can be eliminated by means of the relation (3.1), one obtains again the inequality (3.4). In this way the inner consistency of the so proposed approach is assured. On the other hand

the relation

$$\tau\left[\sum (Re\ V_{ik}^{(1)})^2\right]^{\frac{1}{2}} = \hbar, \tag{3.8}$$

can be considered, at least qualitatively, as a time-energy uncertainty relation. All these facts led us to consider that the time parameter, which is now submitted to certain additional restrictions has to be reinterpreted as a dynamically determined quantity.

Using the relation (3.1) we can see that

$$\Lambda \in \left(\left[\sum (Re\ V_{ik}^{(1)})^2\right]^{\frac{1}{2}},\ \infty\right) \tag{3.9}$$

when $t \in [0, \tau]$. Consequently, the value of the parameter Λ can be established in terms of the small-time level under consideration. Thus, in order to probe just the point particle we have to consider $t \to 0$, so that

$$\Lambda \to \infty. \tag{3.10}$$

In such conditions the imaginary potential-energy contribution can be identified with the cut-off parameter of the field theory. The so introduced cut-off is able to support the existence of the non-zero upper bound of the time parameter.

IV. The validity limits of the proposed approach

We have to remark that the time-derivative of the exact probability function is

$$P'(t) = \frac{2}{\hbar}\left[-\Lambda + \sum_{k\neq i} Im\ V_{kk}^{(1)}\right], \tag{4.1}$$

so that, within the small-time region, there is an agreement with the inequality (3.6) only when

$$\Lambda > \sum_{k\neq i} Im\ V_{kk}^{(1)}, \tag{4.2}$$

which can be fulfilled particularly taking $Im\ V_{kk}^{(1)} = 0$, $k \neq i$. On the other hand, in order to fulfil the inequality (2.9) it is necessary to consider

$$\left[\sum (Re\ V_{ik}^{(1)})^2\right]^{\frac{1}{2}} > \frac{1}{2}\,|E_j - E_i|, \tag{4.3}$$

for all the involved j-levels which shows the validity limits of the above proposed approach. In this sense we have to mention that the above inequality has to be in agreement, at least in a certain approximation, with the general validity conditions

$$|E_j - E_i| \gg |V_{ij}^{(1)}|, \quad j \neq i, \tag{4.4}$$

of the perturbation expansion itself, too.

It can be also noticed that even if the above proposed formalism does not work within the whole Hilbert-space, there exists a subspace -which is determined by the j-values for which both the requirement (4.3) and (4.4) are fulfilled- within which the validity of the so proposed approach can be assured. Without analysing further details (concerning the meaning of the quantum-mechanical description within subspaces) we would then have to interpret the above involved subspace as a limitation which comes from the short-distance behavior itself and conversely.

We can thus conclude that the existence of the upper bound of the time-parameter is in fact mutually connected with both the non-validity of the standard Schrödinger equation and of the pointlike particle description in the region (3.4) of the small-time values. On the other hand the imaginary potential contribution is confined only within the small-time region. In this respect it is for

$$t \in (\tau, +\infty), \tag{4.5}$$

when the interaction can be considered as a hermitian, or a zero one, without affecting the existence of the upper bound τ. In this latter context τ appears as the lower bound of the time region in which the standard Schrödinger equation is valid.

V. Conclusions

A proof has been given that the description of the short distance behavior raises the existence of certain relevant bounds of the space-time evaluations. It is also the existence of these bounds which is able to be defined as a consequence of the non-validity of the standard Schrödinger equation in the small-time region and conversely. From the quantum-mechanical point of view these bound have to express the existence of the nonzero intrinsic dispersion of the space-time measurements (3), or alternatively, the existence of an intrinsic nonzero particle size (4). These nonzero lower bounds have to be considered as fundamental theoretical entities, so that they have to be consistently included into the quantum theory. Generally there are reasons to consider that a rather complete description of the space-time measurements needs the knowledge of both

the standard observable evaluation and of the corresponding intrinsic dispersion. For this purpose there are quite naturally needed intervals or complex numbers. The intrinsic space-time imprecisions can be then defined by the imaginary parts of the averages of the corresponding nonhermitian ("binary") space-time operators (5). Such an extension of quantum mechanics is in agreement with the above proposed small-time approach to the description of the perturbation theory. Indeed, as a consequence of the nonhermitian interaction ($\Lambda \neq 0$), the usual point description of the moment $t = 0$ is effectively replaced by the interval $[0, \tau]$, thus also raising the meaning of the lower bound τ as an intrinsic time-dispersion (4). It can be also noticed that the so raised dispersion does not emerge from the very statistical peculiarities of the measuring process, but it comes rather from the individual aspects of the measuring interaction. So, the elementary measuring process can be considered as a "coincidence" between two particles (6). The measuring process of space and/or time would then consists in the counting process of the involved chain of successive "coincidences". We have now to mention that in this way there is raised an agreement with the opinions expressed by de Broglie (7) concerning the actual role and meaning of the intrinsic peculiarities of the particle-description. Indeed, by accounting the presence of the intrinsic space-time dispersions, there would be accomplished some essential steps towards the introduction of the actual nonpoint particles in quantum theory. So, the above modified potential possesses effectively certain peculiarities of the so-called quantum potential which is needed in order to account the intrinsic particle-localization requirements.

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