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TOWARDS AN ALGEBRAIC DESCRIPTION OF REALITY

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Abstract : Bohm has recently proposed a new conceptual framework to account for the radically new features implied by the quantum formalism. This novel approach does not depend upon an a priori given space-time continuum, nor is it formulated in terms of Hilbert space although its general structure incorporates the algebraic content of the formalism. In this paper we discuss and develop further this conceptual structure and show how a totally algebraic theory begins to open up the possibility of describing space-time in terms of a pre-geometric structure.

Résumé : Bohm a récemment proposé un cadre conceptuel neuf afin de rendre compte des propriétés radicalement nouvelles impliquées par le formalisme quantique. Cette nouvelle approche ne dépend pas d'un continuum d'espace-temps donné a priori, et n'est pas non plus formulée en termes d'espace de Hilbert bien que sa structure générale incorpore le contenu algébrique du formalisme. Dans cet article nous discutons et nous développons cette structure conceptuelle et montrons comment une théorie totalement algébrique ouvre la voie à la possibilité de décrire l'espace-temps en termes d'une structure pré-géométrique.

1. INTRODUCTION

The persistent conceptual difficulties concerning quantum mechanics, the failures of quantum field theories and the difficulties in quantizing gravity has led to the feeling that there is need for a radical change in the fundamental concepts that are used in physics today. For example, from the quantum theoretic side, Dirac (1973) argues that "... quantum theory as far as it has gone ... is far from satisfactory, because of the failure to fit quantum theory with relativity". On the same theme Migdal (1977) writes : "A consistent description of systems in small regions of space-time will quite probably demand a fundamental revision of our concepts".

A similar view is also expressed by those working mainly with general relativity. For example, Trautman (1973) sees the need to build a quantum theory of space, time and gravitation as one of the most important outstanding theoretical problems. It is not just a matter of quantizing the gravitational field because "gravitation is so closely related to the structure of space-time that it is hard to conceive a profound modification of the description of the former without introducing drastic changes in the nature of the latter" and he goes on to suggest that it will be necessary to abandon the notion of a differential manifold as a model of space-time. Penrose (1972) sees the nature of the continuum itself as presenting fundamental difficulties and suggests that its replacement will call for something more primitive than either quantum mechanics or relativity. Indeed, Wheeler (1976) who has been a pioneer in this somewhat speculative work, has introduced the term "pregeometry" to describe this more primitive structure which probably lies behind both quantum theory and relativity. But what is the nature of this structure ? At present there is no clear answer to this question. Our group at Birkbeck have been discussing these issues for a number of years now and we see as one of the main problems the need to develop a coherent conceptual structure or frame of thought which does not rely on the space-time continuum for its mathematical expression, but which will be rich enough to include some of the features that appear to be necessary to understand quantum and relativistic *phenomena*. Some of our ideas have already been discussed in a series of papers by Bohm, Hiley and their co-workers (1970, 1971, 1973, 1975, 1976). This article, which arose out of a talk given to a meeting of the Séminaire de la Fondation Louis de Broglie in June 1978, is an attempt to give an overall view of the conceptual structure lying behind the various notions that have been developed individually in the above papers and to develop further some new ideas that have emerged in our attempts to move towards

a totally algebraic formulation of quantum mechanics and space-time. Indeed, we have discussed the very strong similarities between our approach and the algebraic approach originally adopted by both Grassmann and Clifford. It is through this type of approach that we can see a possibility of throwing some light on the notion of pregeometry. We discuss these possibilities in this article.

2. NEW CATEGORIES : THE HOLOMOVEMENT

We start by remarking that, while there exists a well established set of mathematical rules for non-relativistic quantum mechanics, any attempt to explain these rules in "intuitive pictures" or "imagery" using concepts of particle, wave, locality, etc. fails because these categories by themselves do not have an overall coherent logical structure that enables one to keep a consistent line of reasoning going without, at some stage, falling back on the formalism itself. We have no coherent conceptual structure, merely a set of rules. Indeed, in order to account for quantum phenomena, one must continually rely on two apparently mutually exclusive themes :

(i) point-like autonomous objects

together with

(ii) some form of extension which uses a wave description to account for the wave-like phenomenon that is exhibited through an ensemble of individual effects.

In spite of the persuasive eloquence of Bohr (1961) the doubts linger particularly if one is pertinently confronted with the question : can electrons, protons, etc. still be conceived as actual material substance with sharply defined attributes such as momentum and position ? One always hesitates to give a definite answer because, although the categories supply the driving force behind physical thinking, they are used in a very loose and almost poetic way. Ultimately what really matters are the mathematical algorithms and their predictions. Any successful piece of mathematics must finally be an explanation in itself despite its lack of intuitive clarity. Such a point of view has been expressed by people like Dyson (1958) and De Witt (1971). While some find this an easy position to defend, others have seen the total reliance on an abstract symbolism as a crisis point in science. Even mathematicians like Rene Thom (1975) for example, claim that an abstract formalism devoid of any intuition, be it physical, geometrical or structural, will result in blind mechanical manipulation which will eventually lead to total indifference(1).

It is not surprising, in view of Thom's remarks that the most ardent critics of quantum mechanics come from France. Indeed, the French physicists have been strongly influenced by de Broglie and his collaborators who have constantly criticised the orthodox views. However, I think it is fair to say that on the whole they have tended to maintain the traditional categories of the continuous space-time arena, with locality and determinism incorporated in a field theory in which the particle-like aspects are carried by singularities in the field (1956). However, I believe this programme still faces a number of fundamental difficulties particularly in view of the recent work on non-locality (see Paty (1977) Clauser and Shimony (1978)).

Bohm and his group at Birkbeck have been considering a more radical approach (1970, 1971, 1973). What seems to have become clear is that the older categories are totally inadequate and therefore ought to be dropped. Naturally there will be a considerable resistance to such a suggestion and it is my purpose in this article to present further arguments in support of Bohm's proposals (1971, 1973). The alternative approach that we are considering uses for one of its primitives, not particles or fields in interaction, but a new notion that we call the holomovement, a notion that takes activity as its basic form. The prefix "holo" is used to indicate a kind of undivided totality, a notion that has been discussed at length by Bohm (1971, 1973) and I will be content to develop this theme in a slightly different context in this article.

Activity itself, used as a basic notion, has a long history going back to Heraclitus and his notion that "all is flux", but it has never been satisfactorily encompassed in a suitable mathematical scheme, as has mechanical atomism. However, quantum theory challenges mechanical atomism and, if we follow the Heraclitean allegory of the flame of a candle which is used to illustrate the illusion of material substance, we can argue that in physics the solidity of matter itself now appears even more of an illusion.

For instance, a closer examination of a solid reveals that the atoms are in constant movement vibrating about their equilibrium positions. When we look closer still at the atoms themselves, we find a nucleus surrounded by a cloud of electrons whose exact positions and momenta cannot be specified and exist, if that is the right word, in a cloud of probabilities. When we enter the nucleus we find not only neutrons and protons but a variety of other particles, πons, kaons, Λs, anti-Λs, etc. all being created and annihilated in time of the order of 10^{-24} seconds. Neutrons turn into protons and

vice versa with apparent ease. Examination of the nucleons themselves reveals yet a finer inner structure consisting of point-like entities which are called partons. The number that are present seems to depend on the energy of the probe one is using to examine the structure. The most likely candidate for the parton seems to be the quark and at lowish energies three quarks suffice to account for many nucleon properties. The appearance of more partons is attributed to the creation of quark-anti-quark pairs in the nucleon itself. While this pair creation occurs with considerable ease, individual quarks have not yet been seen. This is somewhat surprising since one can also produce electro-positron pairs but here the electrons and positrons appear as individual entities. The non-appearance of the quark is causing considerable consternation, which has led to many theories about its non-appearance, including the idea that, as quarks are separated from each other, the attractive force between them increases in such a way that they can never actually be separated. I will point to a similar feature that arises in an unexpected context later in this article but, at this stage, I merely want to emphasize one point. Whenever we look closely at anything in nature we find activity and process, and the ultimate Democritean "ατμος" appears totally illusive and eventually disappears into the vacuum.

Having recognised the need to consider activity as a basic form, we could argue that the idea of a field $\psi(xt)$ is just the way to describe such activity. Unfortunately, it is just this notion of field that presents the difficulties in non-relativistic quantum theory, since it leaves the relation between the individual and the ensemble somewhat ambiguous. But there is also a more serious difficulty which I hope to bring out clearly.

If we are to start with activity as basic, we do not imply, of course, that we must necessarily start from chaos. Nature exhibits order and structure, and our description must be rich enough to talk coherently about this order. Now, although we start with activity and process, we recognise that these processes do manifest themselves in quasi-stable, semi-autonomous forms. It is these forms that we call matter and it is in these forms that the process presents itself as an isolatable system. Just as the inter-galactic matter aggregates to form quasi-stable stars, so the underlying process structures into quasi-stable forms that we call matter. The success of science over a large domain of experience has been to treat these quasi-stable forms as stable and to attempt to explain instability as a failure of interacting stable systems to hold together. These notions have been seriously challenged in high energy physics and it is our contention that it is not only in the ultra-small and at very high energies that these notions are to be called into question. They are to be questioned wherever quantum phenomena occur.

3. NON-LOCALITY, THE MANIFESTING FIELD, & THE FORMATIVE CAUSE

In quantum theory there are two essential novel features that must be kept in mind. Firstly, non-local effects are exhibited in many-body problems and in situations described by Einstein, Podolsky, Rosen. This non-locality has received considerable attention in the last few years both experimentally and theoretically (see for review Paty (1977) and Clauser and Shimony (1978)) and holds a central position in our own thinking but, as this position has been adequately discussed in Bohm and Hiley (1975) and Baracca et al. (1975), I will not elaborate this point further in this article.

The second novel feature of quantum theory is that not all of actuality can be made manifest together[(2)]. This particular feature has been recognised, at least implicitly, in many approaches to quantum theory. In our way of thinking, for example, we regard the holomovement as an implicate order (see Bohm (1971)) and it is impossible to make all aspects of this order explicate together.

To make any one aspect of the holomovement manifest we need some activity, some form of manifesting field, in which we can make a thing manifest. In other words, we must always relevate one aspect of the holomovement against another. In this domain there is no ding-an-sich with all its properties manifest together and displayed independently of the process of manifestation. There exists a kind of immutable relationship -Bohr (1961) called it an "indivisible link"- between that which is being manifested and that with which it is being manifested against.

Bohr talks about the "indivisibility of the quantum of action" which was essentially his way of talking about the uncertainty principle and it is this feature that makes it impossible to say what belongs to the object and what belongs to the observing apparatus. To emphasise this point he uses the analogy of a blind man tapping out the boundaries of a room with either a very flexible rod or a rod that is very loosely held in the hand. The man does not get a very clear outline of the room because he cannot separate the effects of the room from the effects produced by the flexible rod itself.

This type of reasoning led Bohr to talk about the inseparability of subject and object, or of the observer and the observed. It is not just the observer in the form of homo sapiens, but includes his instruments and his bits of apparatus which is regarded as an extension of man. As Wheeler (1976) puts it, "Man no longer gazes at nature through a glass window ; he has to break the window and get involved. He can only give meaning to position (or momentum) through the participatory act of observation. Thus he becomes the observer participator". It is this involvement that calls into question the Descartean dualism which has dominated Western science and which has led to the notion that it is always possible, in principle at least, to make all aspects of nature manifest together so that it becomes exposed to our intellectual gaze[(3)]. Reality in the Descartean view is a given thing which some superhuman intelligence could embrace at once in a moment. The new view requires the activity of the participator which can be included in any description through what we have called a manifesting field.

The very notion of a manifesting field already contains aspects which are inherently non-local and I want to bring this particular aspect out more clearly by utilising an approach to quantum mechanics that was developed by Bohm (1952) and contains features closely related to the earlier work of de Broglie (1927). Later I will indicate how this non-locality arises quite naturally in the algebraic approach that we are developing.

Let us approach the problem of quantum mechanics from the naive realist standpoint by assuming that the electron is a small localised blob of substance with well-defined but perhaps unknown values of position and momentum. Suppose we treat the wavefunction, ψ, as a real physical field acting in some way so as to produce a real force on the electron. Further we will assume the validity of Schrödinger's equation and substitute $\psi = Re^{iS/\hbar}$. The resulting equation can be split into two real equations. One corresponds to a conservation-type equation which involves $|\psi(r)|^2 = R^2(r)$ and can be interpreted as an equation for the conservation of probability. The other equation is essentially the classical one-particle Hamilton-Jacobi equation containing an additional term which is proportional to $\hbar^2$. Hence in the classical limit when $\hbar = 0$, we fully recover the classical theory. This additional term, which is called the quantum potential, has a deceptively simple form, namely, $Q = \frac{-\hbar^2}{2m}\frac{\nabla^2 R}{R}$ and with it we can completely account for all the known quantum effects successfully.

For example, in the two slit experiment, the electron will experience an infinite repulsive force at those points where the electrons do not actually appear. The presence of this potential then removes the mystery which inevitably arises in considering the build-up of an interference pattern in an ensemble constructed from a series of experiments in which only one particle is present at a time. The information that two slits are open is contained in the quantum potential and the electron is simply following one of the allowed geodesics defined by this potential. It is in this way that the quantum potential provides an example of a manifesting field. A detailed calculation of the form of the quantum potential together with an ensemble of individual trajectories for the two-slit experiment has been presented by Philippidis, Dewdney and Hiley (1979).

Before developing the notion of a manifesting field further let us have a closer look at the properties of the quantum potential which actually turns out to have a very different character from the potentials used in classical physics. There are three essential differences.

(1) The force between two particles need not vanish when the separation becomes very large. In fact the force can become infinite thus calling into question the logical validity of treating the two particles as separately existing objects. It is interesting to note, as we remarled earlier, that this is exactly the type of force that is now used to explain the non-appearance of the free quark, but this particular case does not challenge the validity of analysis into parts because when the quarks are close together they appear to behave as independent entities. (However see Heisenberg (1976)).

(2) The quantum potential gives rise to non-local features in the sense that anything you do to particle A will immediately affect the particle B no matter how far A is from B. This, of course, is just what is needed to account for the EPR-type experiment.

(3) The quantum potential depends on the environment in an essentially non-local way. There is no one-particle problem. In the case of a single particle approaching a two-slit arrangement we must regard the apparatus itself as a many-body system and whether one or two slits are open will be reflected in the quantum potential itself. In other words the quantum potential unites the single particle with all the particles in the apparatus in a non-classical and non-local way. It is this feature of the quan-

tum potential that gives mathematical expression to the manifesting field. Indeed, it is quite remarkable that this view supports Bohr's contention that it was necessary to specify the whole experimental arrangement in order to account for quantum phenomena. In view of this close relationship to Bohr's approach it is somewhat surprising to find the quantum potential universally rejected or ignored.

One reason[4] for this reaction appears to stem from the failure to realise that the quantum potential appears in a qualitatively new way and I would like to clarify this particular feature by recalling Aristotle's views on causality. He distinguished four causes, the material cause, the efficient cause, the formal cause and the final cause although today only one is explicitly used.

In classical physics cause is used in the sense that an agent can directly exert some power on an object to produce a given change. In other words, the potentials in the equations of motion provide the driving forces that act on the body to compel it to develop in one way rather than another, resulting in a transference of energy. Aristotle himself, used the word "aition" which, although translated as cause, is actually used in a more general way. X is called an 'aition' in respect of Y, if it is responsible for Y *in any way whatever*. Even given this change in meaning it is his efficient cause that comes closest to describing the causality used in classical physics. The other three causes are never used explicitly and, although global symmetries do appear as constraining the functioning of the efficient cause, any explanation of breaking of that symmetry is always looked for in terms of some force field. So that apart from certain global symmetries like space and time translational and rotational symmetry, all effects are attributed ultimately to some force arising from a classical-type potential.

But the quantum potential cannot be reduced to a classical-type potential and therefore cannot be regarded as the source of an efficient cause. However we can regard the apparatus as providing an environment which conditions the way in which a process actually manifests itself, that is, through the quantum potential, the apparatus provides the formal or, perhaps more appropriately, the formative cause of the process, in such a way that the development of an individual process cannot be reduced to a series of efficient causes.

One can use a very nice analogy by considering an orchestra playing a symphony. The symphony provides a form or theme that slowly develops, each instrumentalist responding to the overall development of the theme. No one would try to explain the behaviour of the individual members of the orchestra in terms of forces between them ! If we are willing to accept the analogy, then what we are suggesting is that we should not look at physical process through mechanical atomism, but rather regard it as an organic process whose development is to be analysed in a more general way in which efficient causality no longer dominates. The introduction of the formative cause is a way of giving expression to the manifesting field[5].

One could continue to develop the quantum potential approach as a particular way of exploring the properties of the manifesting field and indeed such an investigation is being actively pursued at present. But whilst this work is of some interest in its own right, the way in which one is at present forced to construct the quantum potential gives it no calculational advantage over the usual formalism and therefore appears no more than pedagogical. However, the notion of manifesting field has much wider implications than those contained in this particular model and it is to those that I now want to turn our attention.

4. PREGEOMETRY

At the beginning of this article I argued that the search for a material cause did not seem to lead to a Democretean atom out of which all was constructed, but rather what seemed to be primary was activity and energy. Of course such a notion is already implicit in the mass-energy relation which has reached its fullest expression to date through a relativistic field concept although the rest mass problem still remains unresolved. However, as we have already pointed out, this approach leads to interpretative difficulties and it is still plagued with infinities. While we do have prescriptions for removing some of these, it is interesting to note that Migdal (1977) traces these infinities to the fact that one assumes the fields interact locally and concludes that these methods must be inapplicable at short distances. Should we, therefore, continue with the field concept ? Naturally, the easiest answer is that one should continue to use these concepts simply because no alternatives exist. But the doubts remain and one cannot help wondering if the nonlocality exhibited in many-body systems and the EPR situation is not a macroscopic remnant of the difficulties that are so apparent on the small scale. Indeed there is an argument that uses some of the lessons from quantum field theories to call into question the very meaning of

the space and time continuum in the small. Let me recall the argument.

The spectrum of hydrogen shows a small but significant shift in some of the levels from those predicted by the Dirac equation. This Lamb-Retherford shift has been completely accounted for in quantum electrodynamics by making use of vacuum fluctuations. Let us briefly recall how these fluctuations arise. QED requires us to quantize the electromagnetic field and this procedure results in regarding the field as a collection of quantized harmonic oscillators. The important fact about the oscillators is that even when all the oscillators are in their lowest state, where the electromagnetic field will be zero, there remains a zero-point energy. In other words, even if there are no photons present, the zero-point energy remains and we can regard this energy as being associated with the vacuum state. To get some idea of the energy involved in the vacuum consider a small volume a^3 and let the root-mean-square field energy in a^3 be $\Delta E_a = \sqrt{<E^2>}$. Since each oscillator has a zero-point energy $h\nu/2$ and the main contribution comes from the region $\lambda \sim a$, we have $\Delta E_a \sim \frac{hc}{a}$ so that the energy depends on the volume chosen. For an electron confined to a region defined by its Compton wavelength $\left(\lambda_c = \frac{h}{mc} \sim 2.4\text{x}10^{-12}\text{m}\right)$ the fluctuation energy is $\Delta E_{\lambda c} \sim \frac{1}{2}$ MeV, which is about the same energy as the rest mass of the electron.

The qualitative effect of these fluctuations can be understood if we consider, for convenience, the classical picture of the orbiting electron in the hydrogen. A closer examination of the orbit will show the electron undergoing fluctuations so that it experiences different values of the Coulomb field of the nucleus. In fact it can be shown (Weisskopf (1949)) that the electron energy level undergoes a shift of

$$\Delta E = \frac{(\Delta x)^2}{2} <\nabla^2 V>.$$

Using the measured value of the Lamb-Retherford shift we can find $(\Delta x)^2$ and since $<\nabla^2 V>$ is known for the Coulomb field we can check that the results are consistent with the value derived from ΔE[6].

Having seen the need to take the vacuum fluctuations seriously, in the case of the electromagnetic field, one can immediately turn to the question of the quantizing the gravitational field.

That such a procedure is necessary can be quickly seen by examining the form of Einstein's equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$$

This equation connects the geometry expressed through the curvature tensor $R_{\mu\nu}$ and metric field $g_{\mu\nu}$ to the energy-momentum tensor $T_{\mu\nu}$. The latter is expressed in terms of quantum fields and is thus subject to fluctuations which, in turn, implies that the geometry itself must be subject to quantum fluctuations.

A characteristic length for gravitation is the Planck length, $L = \left(\frac{\hbar G}{c^3}\right)^{1/2} \sim 10^{-33}$cm. If ΔE_L is evaluated for a volume L^3 we find an energy of 10^{22} MeV. Since the rest masses of elementary particles like protons, neutrons, etc., are of the order of 10^2 MeV, there appears to be more energy in the vacuum than in the material particles themselves. In other words, the manifestation of energy as matter is almost insignificant when compared with the energy in the vacuum. These gravitational fluctuations have serious implications since, once the energy exceeds twice the rest mass of a particle, the process of creating particle-antiparticle pairs becomes possible. Indeed, it is already well known that high energy particles fired into matter do produce such pairs, and the creation of quark-antiquark pairs to which we referred earlier is an example of such a process. Furthermore one of the correction terms involved in QED referred to in our discussion of the Lamb-Retherford shift also involves considering the spontaneous creation of virtual pairs in the vacuum. Thus in this way the polarisation of the vacuum takes on a form that can give meaning to effects we see in the macroscopic domain and therefore must be taken seriously.

Wheeler does in fact explore the possible consequences of the gravitational fluctuations. He makes the implicit assumption that the vacuum is some form of "plenum" with a latent structure, a structure in which only a small portion is made manifest in the form of material particles. To date we have treated the latent structure as if it were a continuum defining a geometry and we have given absolute significance to the geometry. In Wheeler's hands the quantization of the gravitational field involves a "sum over geometries" following Feynman's (1949) "sum over paths" version of non-relativistic quantum mechanics. In this view the vacuum fluctuations are seen as a fluctuation from one geometry to another and it is the averaging over these various geometries that gives rise to the non-quantized Einstein geometry.

But Wheeler has added one more feature to the geometry that is not normally considered, namely, he suggests that things like charge may be understood in terms of topological features like "wormholes" in space. Once these new topological features are introduced then the fluctuations take on a new significance. Suppose we consider two neighbouring "holes" in the space as shown in fig. 1. Then any violent fluctuation in the neighbourhood of these small holes may cause the two holes to merge, thus forming a single hole.

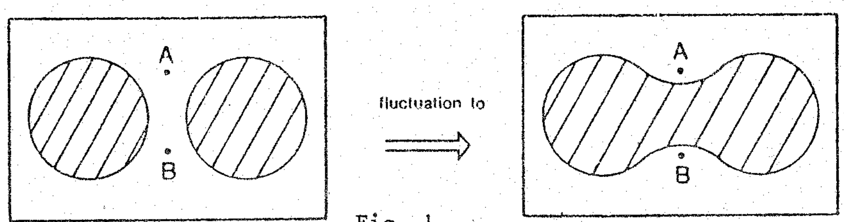

Fig. 1

In other words, the fluctuations will be so violent as to cause the "plenum to tear". Under these fluctuations we no longer have a continuum but something more like a foam. Indeed, the convenience of the continuum disappears because points that are neighbours before the fluctuation are no longer neighbours afterwards. The neighbourhood relation can change suddenly, as a result of a fluctuation, so that the use of overlapping coordinates patches, and hence differential topology, no longer has any meaning.

One might object to Wheeler's notion of a "plenum that tears" but I think that the important point is the nature of the gravitational fluctuations are such as to indicate that we need a new starting point that does not presuppose geometry. In his own words : "the programme for the creation of the universe was not

Day One Geometry, Day Two Quantization but
Day One the Quantum Principle, Day Two Geometry".

In other words, he is arguing that geometry is to be abstracted from something more primitive which, for the sake of a name, we can call "pregeometry". This is also the sentiment that lies behind Trautman's remarks (1973) we quoted in the introduction.

5. THE ALGEBRAIC APPROACH

Already in 1970, inspired partly by Wheeler's earlier work

(1962), we were exploring possible pregeometric forms using combinatorial topology (Bohm, Hiley and Stuart (1970), Hiley (1971)). We did not use the term "pregeometry" but preferred to use a word that carried the notion of activity, as opposed to something essentially static, so we introduced the term holomovement. To us the holomovement is the pregeometric form. By means of this general notion we were able to give partial mathematical expression to the idea of the inseparability of the manifest and the manifesting field (in Bohm et al. (1970) we used the terms "potentiation" and a dual term "copotentiation"), but it became apparent that combinatorial topology itself does not seem to capture completely the active ingredient that we feel to be so essential. Furthermore the connection between the abstract cohomology theory and the differential form language that predominated the old quantum theory seemed to offer a possible clue, but since then our attention has been directed towards the algebraic structure that seems to be more appropriate for quantum theory.

This change of emphasis is not as drastic as it may seem at first sight. One can show that the Clifford algebra, to which we will refer again later, carries an antisymmetric tensor structure and it is this structure that can be linked to the cohomology theory. However the algebraic approach, particularly when supplemented by an extension of the Heisenberg algebra, can be extended to carry a symmetric tensor structure and when combined with the Clifford algebra gives a larger algebraic structure that we wish to exploit. Indeed some features of this structure have already been exploited in super symmetries (see Corwin, Ne'eman and Sternberg (1975)), but we feel that it is more than simply finding Lagrangians which are invariant under these larger groups. The implications go much deeper. Let me try to set out the reasons for this.

For example, let us first consider Heisenberg's original approach to quantum mechanics through matrices. It is well known that, at a certain level, there is an equivalence between the Heisenberg and Schrödinger approach through representations by means of differential operators. However there are two important features which show the limitation of this equivalence.

(1) The matrix mechanics enable us to consider situations which take us beyond Hilbert space (see Dirac (1963)).

(2) By looking at the "complete" algebraisation of quantum mechanics, we can ask questions of the mathematics that cannot be raised in the context of the Hilbert space theory. Some of the points have been discussed in Frescura and Hiley (1980)

although their full significance is still under investigation.

However these are not our only reasons for emphasising the algebraic approach. Bohm (1973) has already discussed the way the notions of the implicate-explicate order point towards the algebraic approach which gives a natural expression to the general forms that we are investigating. For example, in classical terms, motion is described using one-parameter groups and we imagine this as a mapping of one mathematical object into another at neighbouring points of a trajectory. This corresponds to a motion that Bergson calls "les images du caractère cinématographique".

Motion in the holomovement is a continual unfolding and enfolding process, giving rise to a kind of natural recurrent process. All that is manifested is a trace of dots in a bubble chamber. No *object* is seen to pass from one point to another by passing through all the intermediate points. The manifestation of dots can equally be regarded as a continuous unfolding or restructuring under the influence of the manifesting field. In more technical terms, not only do we have the multiplication properties of a group to describe succession, but we also have the addition (or interference) properties which makes the motion algebraic.

Let me try to illustrate some of these points by means of a particular example. Consider the hologram and let us look at a process unfolding in time (see fig. 2). The intensity pattern on the screen is given by $\rho(x,t) = \psi^{*}(x,t)\psi(x,t)$ while at an earlier time it is given by

$$T(x,x',t,t_0)\psi^{*}(x,t)\psi(x,t)T^{-1}(x,x'',t,t_0)$$
$$= \psi^{*}(x',t_0)\psi(x'',t_0)$$
$$= \rho(x',x'',t_0)$$

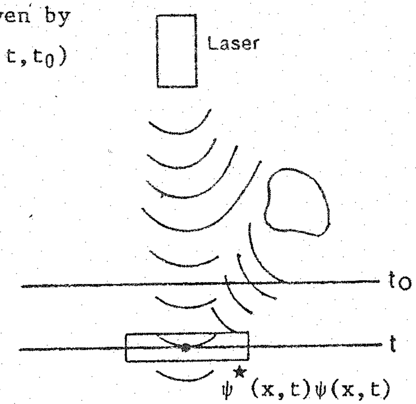

Fig. 2

Here the $\rho(x',x'',t_0)$ already contains in it -"folded up" as it were- the information that a particular black spot will occur at a certain point on the screen at time t, provided the experimental set-up is unchanged. Notice that $\rho(x',x'',t_0)$ already contains non-locality in an essential way since it contains information as to what will be manifest at the photographic plate at time t. This non-locality becomes more evident at low intensities where we have to consider an "individual photon". Its state can be described by $\rho(x',x'',t_0)$ (7), so that the photon can no longer be considered as a localised entity ; it is only the manifestation that is local. In fact the main feature of the algebraic approach is that it regards the manifestation as local, whereas the process itself cannot be understood in terms of local concepts.

We can take the question of locality a little further and argue that, in the new order that we are considering, locality can no longer be regarded as an absolute, but is essentially a relationship between the manifest and the manifesting field. Consider the above hologram again, but now let us consider the relation between the object that is being hologrammed and the hologram itself. We find that the local features of the object no longer appear as local features on the hologram. The locality relation of each region of the object has been transferred to every portion of the hologram. Locality is thus "stored" in a non-local way, but notice the original local relations have not been lost. They can be recovered, using the appropriate laser light (i.e. the appropriate manifesting field).

In optical holograms this particular process is described through Fourier transformations ; it is just these transformations that play a vital role in the present form of quantum mechanics. For example, we can consider the relation between the position representation and the momentum representation. Here, when an instrument is manifesting the momentum aspects of a process, the position-locality relation is carried non-locally, but can be remanifested latter in a way that cannot be described in terms of a local-position unfolding relation (8). There is an essential restructuring which cannot in general be described by a one-parameter group of local transformations as used in classical physics and we are suggesting that an algebraic approach is a more natural way to express this restructuring.

6. ACTIVITY IN THE FOUNDATIONS OF ALGEBRAIC GEOMETRY

Recently I was fascinated to re-read some views expressed by Grassmann (1894), Hamilton (1967) and Clifford (1882) (see also Hankins (1976) and Lewis (1977)). They all took activity as an essential form and, although the quantum theory had not been crea-

ted at that stage, they all expressed sentiments that were very close to the theme that I have been developing. I think it is significant that these men made very important contributions to the foundations of what has now become algebraic geometry and what I think particularly interesting is their attitude to algebra.

They all stressed the role of activity, in the sense that algebras naturally describe a becoming or a basic movement. Both Grassmann and Hamilton were uncompromising : mathematics, they claimed, was about thought, it was not about a material reality. It was a way of studying relationships in thought, not a relationship of content, but rather a relationship of form in which the content can be carried. The fact that these forms could be "useful" for helping us order physical phenomena was, to them, almost incidental. Mathematics is to do with ordering forms that are created through thought and hence is of thought. Now in the process of thought there appears to be a kind of polarisation in which one aspect of thought is being looked at, is being reflected in, or better still, is being co-related with another aspect of thought. In other words, a thought is being made manifest in the manifestation field of thought itself. Thus in thought we have appearing in a very basic form an inseparability between the observer and the observed.

While Grassmann did not direct his attention to this particular problem, he did consider the question of how one thought became another. Can we ever say that we have an entirely new thought which is completely independent of the old ? Or is it that they are distinct but related ? I like to regard them as opposite poles of an essential relationship that exists between the poles themselves. They are then inseparable in the sense that the old thought contains implications of the new and the new thought contains a trace of the old. As Bergson (1922) puts it "... evolution implies a real persistence of the past in the present". It is the relationship that is essential. But one can go further. The relationship itself is also of thought which, in turn, must be one pole of another essential relationship, so that thought itself can be understood as a structure of relationships ordered in some form of hierarchy. By considering this structuring in thought, Grassmann was able to break out of the straight-jacket of our three-dimensional space and, for the first time it became possible to describe mathematically spaces of higher dimensions than three.

Grassmann insisted that his theory of space was but a particular realisation of the more general notion that he was working towards. Nevertheless he saw each point of space as a distinctive form in a continuous process of becoming and motion was conceived of as a continuous generation of one distinctive form followed by another. In its inception this notion is not unlike the folding-unfolding motion referred to earlier. However the distinctive forms to which Grassmann refers are more reminiscent of Spinoza's modes (1954) although in Spinoza's work they have their existence in substance rather than process.

To Grassmann the succession of these distinctive forms were not to be regarded as the generation of a series of independent points, but rather each successive point is the opposite pole of its immediate predecessor so that pairs of points become essentially related and it was this essential relationship that Grassmann called his field of extensive magnitudes. Indeed to strengthen the essential link between two points, he wrote $[p_i p_j]$ the square braces indicating the unity of the poles. Higher order connections of distinctive forms could then be written as $[p_i p_j p_k]$ etc.

These extensives are not yet the vectors, bivectors, trivectors, etc., that present mathematics uses in order to describe what is now called a Grassmann algebra. The original basic forms that motivated Grassmann have long been forgotten or ignored and only those features that are more appropriate for static visualisation have been retained. Thus the very notion of becoming that Grassmann felt to be of considerable importance to his approach to algebras has been lost. In fact this emphasis on the static forms occured very early in the development of algebraic geometry and by the 1880's the notion of activity had already been replaced by the static forms of a vector, bivector, etc., together with the exterior product that we associate with Grassmann algebras to-day. It was for this reason that Clifford (1882), in contrasting Grassmann's approach to that of Hamilton, found it necessary to point out that the quaternion product was based on movement or activity in contrast to the static nature of Grassmann's product. We have recently discovered that if some of Clifford's ideas are applied to the Grassmann extensives rather than the static forms, one can generate the Clifford algebra without the need to associate the underlying vector space with a local Euclidean structure. It is this which provides one of the clues to the relationship between the abstract cohomology theory and the algebraic approach that we mentioned in section 5. Our work in these directions seem to offer some hope of realising an algebraic approach to pregeometry. In the next section we will sketch very briefly an outline of the development.

7. THE ALGEBRAIC REALISATION OF PREGEOMETRY

As we have already remarked in section 5, our first attempt to describe mathematically a pregeometric structure used the language of cohomology theory (Bohm, Hiley and Stuart (1970), Hiley (1971)). Here we represented the structures by abstract simplicial complexes so that the simplexes themselves provided a description of the outward manifestations of the inner process of becoming. In such a scheme the quantum fluctuations could be considered as the creation and annihilation of distinctive relationships and it is this structure that can be statistically ordered into a coherent whole. While the basic relationships do undergo change, this change does not take place in space and time. Rather space and time are themselves relationships which are to be ultimately abstracted from the underlying structure, thus giving rise to what we may call a statistical geometry.

The basic relationships or "connections" as we prefer to call them are neither local nor non-local. They will be neutral or a-local. It is this neutrality that will enable us to include the macroscopic connection that occurs in the EPR situation and appears in that context to be non-local. Hence this type of connection is to be regarded as a macroscopic manifestation of the basic underlying a-local connection. With such connections there is no question of "the passing of information or energy at speeds greater than those of light". The measurement simply breaks the connection and produces a spontaneous localisation. No energy transfer is involved.

What now appears to be a problem is the apparent dominance of local connections in our macroscopic world. However it should be noted that our macroscopic world involves systems of many degrees of freedom and are, therefore, essentially thermodynamic in nature. Our experience at low temperatures suggests that localisation is essentially a thermodynamic property and it is this thermodynamic property that enables us to understand the dominance of locality in the macroscopic world. This particular point has already been discussed at some length in Baracca et al. (1975) and we will not pursue it further here.

In terms of the cohomology language the notion of a manifestation field can be given meaning in terms of duality. For example, one aspect of the complex can be described by p-chains $C_k^{(p)}$, while the structure which was to play the role of a manifesting field can be described by p-cochains, $C^j_{(p)}$. That which is manifest is then described by the intersection matrix g_k^j defined through the duality relation

$$\left[C_k^{(p)}, C_{(p)}^j\right] = g_k^j$$

Some of the physical implications of this work were investigated in Bohm, Hiley and Stuart (1970) where the isomorphism that exists between abstract cohomology theory and the de Rham cohomology was explored and some of the results were applied to electromagnetic theory. Here it was noted that our particular description provides a natural framework for quantized change. But for quantum phenomena, the de Rham cohomology theory in its present form was only adequate to discuss its relation to the old Bohr-Sommerfeld quantum theory. What was missing was the essential algebraic property.

In spite of certain very suggestive ideas implicit in our approach, the means of realising a statistical geometry seemed quite remote although we were able to establish a tentative connection with gauge fields (see Hiley and Stuart (1971)). Here certain rather strong assumptions were made in order to avoid some of the fundamental problems involved in using a pregeometric structure to define a geometric structure.

However our exploration of the more general concepts involved in relationships between the implicate and explicate orders suggested that we should not look for a Euclidean system of order in terms of rigid local connections. For example, the Euclidean order of the object is not mapped by a one-to-one relation into the hologram. As we have already indicated, this order is stored in a non-local, essentially algebraic way and this suggests a modification to the Erlangen programme in the sense that it would be more appropriate to study the group algebras rather than the groups themselves.

In order to examine this possibility an extensive study of the structures underlying the Clifford and simplectic algebras is being carried out (see Frèscura and Hiley (1977) and (1980)). The importance of these particular algebras is related to the fact that the Clifford algebra carries the rotation group, while the simplectic algebra is related closely to the group of canonical transformations and its quantum analogue, the Heisenberg algebra. In the case of the Clifford algebra we have the additional advantage in that the algebra itself carries the spinor structure in a natural way and it is the spinor that plays such a predominant role in quantum mechanics. The usual approach to

spinors through group theory seems to depend on what appears to be an entirely accidental local isomorphism between the rotation group and its covering group (for example SO(3) ∿ SU(2) or SO(1,3) ∿ SL(2C) and even SO(2,4) ∿ SU(2,2)) whereas in the algebraic approach the spin group occurs quite naturally together with the appropriate rotation group. Although the algebraic approach has been known to exist for some time now it is little used and where it has been used it is regarded as a mathematical convenience which does not offer any new physical insights[(9)]. Naturally we claim that it is the algebraic approach that gives rise to the possibility of a new insight into the meaning of the spinor and it is for this reason that we have undertaken a systematic exploration of the Clifford algebra in particular. This work will be reported elsewhere. In the remainder of this article I would like to indicate briefly the relevance of these ideas in discussing the pregeometric structure. In order to simplify the discussion we will not consider the consequences of the quantum fluctuations at this stage.

A realisation theorem of homology theory states that an abstract complex of dimension n has a realisation in a Euclidean space of dimension 2n + 1, so that the simplest complex that we need to consider in order to carry the three-dimensional Euclidean structure would be one-dimensional. Since R^3 will carry an orientation we will assume that the complex will consist of oriented one-simplexes. Furthermore, in order to allow for the abstraction of the full Euclidean order from the complex, we need to distinguish a minimum of three classes of one-simplexes. A collection of three simplexes, one from each class, will be called a "frame". It should be emphasised that we make no assumptions about the nature of these frames and, if need be, we could attribute each member of the frame a different quality such as "colour". I deliberately choose the quality colour inspite of its use in quantum chromodymaics where it is used in a different mathematical structure. Let me stress that each one-simplex is a movement, not a material structure, and we want the possibility of transforming movements into each other. To achieve this we define a multiplication rule which transforms one colour into another so that, for example, the movement "red" into "blue" and on into "red" again reverses the orientation of the original "red" one-simplex in the frame. This set of basic movements connecting the colours generates an algebra that is isomorphic to the quaternion algebra. This suggests that a rotation can be carried topologically by means of an algebra[(10)].

The fact that the quaternion algebra over the reals is the largest division algebra also carries an important implication. If the simplicial complex is to allow the abstraction to R^3 via an algebraic structure then it is essential that the colour transformation be stable under algebraic multiplication. In other words we want to ensure that the colour transformation structure is preserved under all inner automorphisms. Any zero divisors will not satisfy this stability criterion and must therefore be excluded. This means that our stability criterion restricts the maximum number of colours to three. Thus our basic structural assumptions are sufficiently stringent to carry the three dimensions of space.

Further evidence for the relation between the combinatorial features of an irregular one-simplex and the dimensionality of its embedding space has been presented by Hiley, Finney and Burke (1977) in a different context. Their numerical work indicates that it is possible to distinguish whether a given irregular network is embedded in a two -or three- dimensional continuum simply by counting the number of distinct members of a certain class of one-chains and one-cycles. This suggests that dimensionality is intrinsic to the complex in the sense that movements in the network itself are sufficient to distinguish the dimensionality of the embedding continuum. This is reminiscent of the notion that curvature is a property of, say, the surface of a sphere and not a property of the space in which it is embedded.

The above considerations entailed developing an algebra for the transformation between the colours, but the colours are themselves movements and therefore ought to be included in the algebra. The simplest way to achieve this is to extend our field to the complex numbers which, in effect, "doubles" our algebraic structure. If we formally regard both $\mathbb{C}$ and $\mathbb{H}$ as real algebras, then it is easy to show that $\mathbb{C}\otimes\mathbb{H}$ is isomorphic to a Clifford algebra that we have called the Pauli algebra (see Frescura and Hiley (1980)). This is the appropriate Clifford algebra for the rotation group in three dimensions and contains the Pauli spinors.

Frescura and Hiley (1980) have shown in detail how the contragredient spinor is a member of the right ideal, R, while the cogredient spinor is a member of the left ideal, L. This investigation provides a new insight into the meaning of a spinor which we will not discuss here. However, what is more striking from our point of view is the way a Clifford algebra carries a set of antisymmetric tensors (for example, bivectors, trivectors, etc.) and these geometrical objects are obtained through the relation $L\,R = S + V + B + T + \ldots\ldots$ which does, of course, provide the flag plus flag-pole picture that is usually attributed to a spinor, but it clearly shows that the flag-picture appears as a derived quantity. In the algebraic approach the spinor itself appears in a more natural way which indicates that it cannot strictly be represented directly as a geometric object on a manifold. It is a pregeometric structure and the geometric objects only emerge through a generalised concept of duality, a concept that we have already used to carry the basic notion of a manifesting field. What now appears as a geometric object is a co-relation between two dual movements in the pregeometric structure.

In abstracting geometrical objects from the Clifford algebras and their spinors, we find only the antisymmetric tensors. The symmetric tensors seem to be missing, but it was Schönberg (1957) who first noted that, if the Heisenberg algebra used in quantum mechanics is generalised by adding an element which seems to play the role of a vacuum state, it is possible to generate the symmetric tensors from a product of the spinors of this generalised algebra. In this way we can also derive the symmetric tensors from the same type of generalised duality relation. Thus the algebraic structure of the pregeometry carries the conventional tensor objects in a new way which does not depend on the pre-existence of a differential manifold. Furthermore our preliminary investigations indicate that some features of the differential manifold itself can be carried by the algebra and therefore does not depend upon the properties of the continuum. This feature is at present under active investigation and will not be discussed further here.

Finally, I should just like to remark that it is also possible to generate the Fermion and Boson algebras by essentially doubling the Clifford and symplectic structures respectively. Whether the implications of this result are sufficient to enable us to provide a more satisfactory basis for a statistically geometry arising from an algebraic pregeometry remains unanswered.

FOOTNOTES

(1) While I would hesitate to claim that this is the main reason for the recent drift from physics, I certainly do not think it has helped. Furthermore anyone who has witnessed the way the authority of quantum mechanics is used to justify the existence of all sorts of "paranormal" phenomena will see yet another aspect of the inherent confusion that lies within this over emphasis on the mathematics.

(2) Richards (1974, 1976) raises a very similar point in connection with the problem of communication. He writes : "Are there ways of asking 'what does this mean' ? which destroys the possibility of an answer" ? Can the way we investigate the meaning of something or some statement make it impossible for us to understand it ? In fact is there a difference between the message our words convey and the message we intended to convey ? In other words are all meanings context dependent ?
Richards tried to draw attention to this context dependence by introducing a symbolism in the form of a ratio $\frac{\text{Tenor}}{\text{Vehicle}}$.
The tenor or content is riding on, or being carried by a vehicle so that V is a way of presenting T. Richards does not want to go as far as McLuhan in identifying the message and the medium but tries to argue that the ratio $\frac{T}{V}$ is a way of pointing to the possibility of saying something that is independent of both T and V.

(3) Bohr was probably the first to recognise the limitation of this view in science although it has been challenged as far back as Parmenides through to Leibnitz and on to Husserl, Heidegger and the whole existentialist movement. Bohr's main influence seems to be his childhood exposure to the problems that arise in psychology and psychiatry. He recalls how he was allowed to sit and listen to the discussion groups held by his father who was a professor in physiology at Copenhagen University. A constantly recurring theme in those discussions was the problem of the observer and the observed and how there seemed to be a need for some form of complementary descriptions in many domains of intellectual activity except physics. When he became aware of Heisenberg's work he realised that it entailed complementary

descriptions so that complementarity was needed *even in physics*. Thus an impression became moulded into a universal principle concerning man's thought.
Having recognised this aspect of man's apparent limitations Bohr felt that communication now became a problem. Our language was one in which subject and object were always sharply separated, the verb playing a linking role. However the link could be separated out and the subject and object could stand alone and be sharply distinguished. Bohr felt that this sharp separation was essential to unambiguous communication and, since classical physics gave the clearest expression of this sharp separation, he argued that it was necessary to retain the classical concepts. It is at this point that we part company with Bohr.

(4) The reasons for the rejection of the quantum potential approach were many and varied. Apart from the one mentioned in the text there is the confusion that has arisen from the attempts to rule out the possibility of any form of hidden variables. Bohm argued that the positions and momenta of the particles act as hidden variables and this has been taken to mean that the variables could form a phase space which would explain the quantum probabilities in a classical way. What has actually been proved is that such phase space theories cannot be maintained. These results are quite correct and fit the intuition one gets by working with the quantum formalism. The essential point in Bohm's work is that the position and momenta are not hidden variables in the sense of providing a phase-space but are hidden in the sense that they cannot be made manifest together. One needs a manifesting field or a context in which to manifest either the position or the momenta. All the "no-go" theorems assume a non-context dependence for the variables and are therefore irrelevant for the type of approach we are investigating.
A second and perhaps more important reason lies in the nature of the quantum potential itself. As it stands it necessitates solving Schrödinger's equation before the quantum potential can be evaluated and so appears contrived. It also contains this manifestly non-local feature which until recently has been sufficient reason to reject the whole thing out of hand. However a retreat into the field $\psi(r,t)$ in a hope that the non-local aspects will disappear seems to be mere wishful thinking. There are experiments that confirm these non-local features exist for distances up to at least 5m (Paty (1977), Bohm and Hiley (1976)).

(5) In this respect our views come close to those contained in Thom's (1975) morphogenetic field. The quantum potential appears to extend these ideas.

(6) Similar evidence for the existence of zero-point energy can be found in the failure of liquid He^4 to solidify under standard atmospheric pressure at absolute zero and the non-existence of a permanent magnetic moment of the electron gas (see Enz (1974)).

(7) We are using the density matrix to describe the state of a *single* system. See Fröhlich (1973) for discussion on this point.

(8) Mugur-Schachter (1977) has studied the non-local aspects of the Fourier transform for the case of a single particle and has discussed the possibility of an experimental verification of these implications.

(9) Eddington (1936) seems to have been the only physicist to have attempted to exploit the algebraic approach. However he considers exclusively the Clifford algebra and does not bring out the pregeometric nature of the algebra.

(10) Penrose (1971) has also suggested that space-time can be built up from combinatorial principles and uses quantized angular momentum to abstract direction and angle. These notions have been generalised to include displacements and this leads directly to the spinor structure underlying the conformal group. If we take our basic connections to be light rays then we are led to the Clifford algebra H(4) which contains the twistor in a natural way.

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