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FIELD-THEORETICAL SPACE-UNCERTAINTY DESCRIPTION

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Abstract : An approach has been given to define both the nonzero minimum value of the space-uncertainty evaluation and of the upper rest-mass bound of the involved particles. In this respect there are analysed the space-uncertainties wich emerge both from the regularised quantum field-theory and high-energy behaviour. In such conditions there are involved particles wich are effectively nonpoint ones. It can be then concluded that the dualism broglien between waves and nonpoint particles is actually involved, now in more general terms.

Résumé : On donne une méthode pour définir les valeurs minima non nulles de l'incertitude spatiale et de la limite supérieure des masses au repos des particules impliquées. Dans ce but les incertitudes spatiales sont analysées dans la théorie quantique des champs régularisés, ainsi que dans la physique des hautes énergies. On parvient ensuite à établir que dans ces cas les particules impliquées sont effectivement nonponctuelles. On conclut que le dualisme broglien onde-corpuscule-nonponctuel est confirmé d'une manière assez générale.

1. Introduction

The aim of this paper is to analyse the space-time structure as it emerges from the high-energy field-theory. The research tool which we intend to use is to perform the space-uncertainty evaluations and more exactly the finite nonzero lower bounds of these ones. The main reason to be interested in the evaluation of the lower bounds of the space uncertainties is the emerging possibility to establish the upper rest-mass bounds involved by the theory. Although the existence of space and/or time-uncertainties is questionable from the rigorous relativistic point of view, we shall pass over such difficulty by restricting ourselves, at least for the moment, to the center-of-mass system. There is also the possibility to perform an invariant description of the space-uncertainty within certain co-ordinate spaces [1]. However, such approaches involve "new" co-ordinates, thus also altering the direct validity of the correspondence principle. In this respect we shall then maintain the usual co-ordinates. So, there is the probability functional method, which permits to define both the space-distribution function of an interacting particle and the corresponding space-uncertainty evaluation in terms of the Källen-Lehmann (K-L) spectral function [2]. Along this line we shall now use the extension of the standard nonrelativistic limits of the charge-distribution approach. Such an extrapolation is merely a suitable starting research tool to analyse the existence of the space-uncertainties within the high-energy field-theory. The use of the above mentioned extrapolation shall be then subsequently justified by the mutual agreement of the involved results. In this way there emerges an effective field-theoretical approach to the evaluation of the space-uncertainty. It turns out that the above mentioned minimum of the space-uncertainty is determined essentially by the existence of an upper rest-mass bound of the involved intermediary particle, or alternatively, by a large-value of the effective mass. Throughout this paper we are thus interested rather in the evaluation mechanism of the minimum space uncertainty than on the concrete nature of the underlying interacting particles, thus also considering the minimum space-uncertainty as a general physical entity which should be rather independent from the starting interaction.

We shall consider that the mathematical-methods which are needed to manipulate the divergent evaluations of the field-theory are able to be reinterpreted as physical-methods of the nonpoint particle description. In this sense the nonpoint-particle description which is raised effectively by the Pauli-Villars regularization shall be analysed for a massless scalar field in Section 2. In Section 3 a proof is given that the charge distribution approach to the evaluation of the space-uncertainty is formally equivalent to the one performed in Section 2. In this respect there are considered again massless scalar-particles. The calculations which are performed in Section 4 show that the above results are in agreement with the scale-invariance behaviour. A general functional minimization method is proposed in Section 5.

The effective mass-emerging -within the zero rest-mass limit- from the power-like behaviour of the K-L function is established in Section 6. Units will be chosen so that h = c = e = 1. The conventions of Bjorken and Drell shall be also used.

2. The Pauli-Villars regularization method

Let us analyse as an example the modification of the particle-structure description which is raised effectively by the Pauli-Villars regularization method. This regularization replaces the free propagator

$$D_0(p^2) = \frac{1}{(2\pi)^4}\frac{1}{p^2}\ , \qquad (2.1)$$

by the expression

$$\mathrm{reg}\ D_0(p^2) = \frac{1}{(2\pi)^4}\left(\frac{1}{p^2} - \frac{1}{p^2-M_0^2}\right)\ , \qquad (2.2)$$

where M_0 is the rest-mass parameter and p^2 the square of the four-momentum. Performing for convenience the Wick-rotation $p_0 \to i\ \bar{p}_4$ there is

$$p^2 \to -\bar{p}^2 = -\bar{p}_1^2 - \bar{p}_2^2 - \bar{p}_3^2 - \bar{p}_4^2\ , \qquad (2.3)$$

where $\bar{p}_\mu$ are the components of the euclidean four-momentum. Using the relations

$$\frac{1}{4\pi^2}\int d\xi\ \frac{1}{r} M_0\ K_1(M_0 r)\exp(-i\bar{p}\xi) = \frac{1}{\bar{p}^2+M_0^2}\ , \qquad (2.4)$$

and

$$\frac{1}{4\pi^2}\int d\xi\ \frac{1}{r^2}\exp(-i\bar{p}\xi) = \frac{1}{\bar{p}^2}\ , \qquad (2.5)$$

one finds

$$\mathrm{reg}\ D_0(\xi) = \frac{1}{4\pi^2}\left(-\frac{1}{r^2} + \frac{M_0}{r}K_1(M_0 r)\right)\ , \qquad (2.6)$$

where $r^2 = \xi^2$ is the square of the euclidean four-position vector ξ_μ and where $K_1(M\ r)$ is the Bessel function of imaginary argument. Consequently

$$\Box_\xi\ \mathrm{reg}\ D_0(\xi) \equiv \rho(\xi) = \frac{M_0^3}{4\pi^2 r}K_1(M_0 r)\ , \qquad (2.7)$$

whereas for the free-propagator there is

$$\Box_\xi D_0(\xi) = \Box_\xi \left(-\frac{1}{4\pi^2 r^2}\right) = \delta(\xi). \qquad (2.8)$$

Interpreting now the propagators $D_0(\xi)$ and reg $D_0(\xi)$ as classical fields which are raised by certain source-distributions, we can see that whereas the point source is associated to the free-propagator, it is the extended source $\rho(\xi)$ which corresponds to the modified propagator. The short distance behavior of the source function $\rho(\xi)$ is given by

$$\lim_{r\to 0} \rho(\xi) = \frac{M_0^2}{4\pi^2 r^2} - \frac{M_0^5 r}{8\pi^2}\left(1 + \psi(2)\right) \quad , \qquad (2.9)$$

where $\psi(2) \cong 0.42$ is the Euler-function of argument 2. Consequently

$$\lim_{r\to 0} r^2 \rho(\xi) = \frac{M_0^2}{4\pi^2} , \qquad (2.10)$$

which shows that the rest-mass parameter M_0 establishes the source intensity at the origin. Computing now the space-uncertainty in terms of the so implied source fields, one can see that the zero-value comes from the point source $\delta(\xi)$, whereas the nonzero square-uncertainty evaluation

$$\Delta\xi^2 = \langle r^2\rangle = \frac{\int_0^\infty dr\, r^4\, K_1(M_0 r)}{\int_0^\infty dr\, r^2\, K_1(M_0 r)} = \frac{8}{M_0^2} , \qquad (2.11)$$

establishes the extension of the source $\rho(\xi)$. In these conditions the uncertainty of a single space-time co-ordinate is

$$\Delta\xi_\mu = \frac{1}{2}\sqrt{\langle r^2\rangle} = \frac{\sqrt{2}}{M_0} . \qquad (2.12)$$

Assuming now that the source-intensity at the origin cannot be indefinitely large, we have to consider the existence of an upper-bound of the rest-mass

$$M_0 \leqslant M_0^{(max)} , \qquad (2.13)$$

which leads formally to the existence of a lower-bound of the space-uncertainty evaluation

$$\Delta\xi_\mu \geqslant \Delta\xi_\mu^{(min)} \equiv \frac{\sqrt{2}}{M_0^{(max)}} . \qquad (2.14)$$

The so obtained result shows that the Pauli-Villars regularization method implies effectively the existence of the nonpoint particle-description. Conversely, proofs have been given that the Pauli-Villars regularization is a special consequence of the so-called stochastic space-method(3). We have now to notice that the stochastic space-method expresses in fact a special way to account for the existence of the intrinsic space-time dispersions. In such conditions we can interpret the existence of the nonzero minimum value of the space-uncertainty as a consequence of the same dispersions, too. Generally, these dispersions can be expressed quite naturally by the imaginary part of the average-value of the non-hermitean space-time operators(4).

3. Extension of the charge distribution approach

The K-L representation of the modified propagator

$$\text{reg}\, D_0(p^2) = \frac{1}{(2\pi)^4}\int_0^\infty dM^2\, \rho(M^2)\, \frac{1}{p^2 - M^2 + i\varepsilon} , \qquad (3.1)$$

results from the spectral function

$$\rho(M^2) \equiv \delta(M^2) + \sigma(M^2) = \delta(M^2) - \delta(M^2 - M_0^2) . \qquad (3.2)$$

Because

$$\int_0^\infty d\, M^2\, \rho(M^2) \leqslant 0 \quad , \qquad (3.3)$$

it is also the indefinite metric which is actually implied(5). For generality both the cases of $\delta(M^2)$ and $\delta(M^2-\varepsilon^2)$, where $\varepsilon \to 0$ have been considered. Using the well-known connection between the spectral function and the charge-distribution(6)

$$\rho(\vec{x}) = f(\infty)\, \delta(\vec{x}) + \frac{1}{4\pi|\vec{x}|}\int_0^\infty ds\, \sigma(s)\exp(-|\vec{x}|\sqrt{s}) \quad , \qquad (3.4)$$

one obtains

$$\rho(\vec{x}) = f(\infty)\, \delta(\vec{x}) - \frac{1}{\pi|\vec{x}|}\exp(-2M_0|\vec{x}|) , \qquad (3.5)$$

where $f(t)$ is the form-factor of the electromagnetic current-operator. The square uncertainty of the position vector is now given by

$$\Delta\vec{x}^2 = \langle\vec{x}^2\rangle = \frac{3}{2M_0^2\left(1 - M_0^2\, f(\infty)\right)} \quad , \qquad (3.6)$$

where the average has been performed with respect to the charge-distribution $\rho(\vec{x})$.

Choosing $f(\infty) > 0$, it can be proved that the so obtained space-uncertainty evaluation possesses -with respect to the rest-mass parameter M_0- the finite nonzero minimum value only when the metric is an indefinite one :

$$\int d\,\vec{x}\,\rho(\vec{x}) < 0 \quad , \tag{3.7}$$

which reads as

$$f(\infty) < \frac{1}{M_0^2} \quad . \tag{3.8}$$

In such conditions the required minimum takes the value

$$\min\langle\vec{x}^2\rangle = \frac{6}{M^{(\max)2}} \quad , \tag{3.9}$$

where

$$M^{(\max)} = \frac{1}{\sqrt{f(\infty)}} \quad , \tag{3.10}$$

possesses by itself the meaning of the upper bound of the rest-mass. The natural space-unit can be now defined as the minimum value of the uncertainty of a single space-co-ordinate, thus obtaining the result

$$\delta\, s \equiv \min\sqrt{\langle x^2{}_i\rangle} = \frac{\sqrt{2}}{M^{(\max)}} \quad , \tag{3.11}$$

which is in a formal agreement with the previously performed evaluation. This agreement is also able to support the validity of the so used charge-distribution approach. The relation (3.9) shows also that the upper rest-mass bound of the virtual particles which couple to the (hypothetical charged) massless-particle is nonequivocally determined in terms of the asymptotic limit.

If relation (3.8) is not fulfilled there are generally implied complex values of the space-uncertainty evaluation. Indeed, taking as an example

$$f(\infty) = \frac{5}{4M_0^2} \quad , \tag{3.12}$$

one obtains

$$\int d\,\vec{x}\,\rho(\vec{x}) = \frac{1}{4M_0^2} \quad , \tag{3.13}$$

so that

$$\langle\vec{x}^2\rangle = -\frac{6}{M_0^2} \quad . \tag{3.14}$$

In the present case the single space-uncertainty takes the imaginary value :

$$\sqrt{\langle\vec{x}^2{}_i\rangle} = i\,\frac{\sqrt{2}}{M_0} \quad . \tag{3.15}$$

Such result shows that generally there is needed to describe the space-time measurements by complex numbers [4]. The imaginary part of the so raised complex number can be then interpreted as an expression of the intrinsic dispersion of space-time measurements, too.

4. Agreement with the scale-invariance behaviour

We shall now prove that there is the possibility to perform a scale-invariance approach to the definition of the natural space-unit. Thus, the canonical scale-invariance of the K-L spectral function is expressed by the relation

$$\sigma(M^2) = \frac{\text{const}}{M^2} \quad , \tag{4.1}$$

which shall be now considered within the interval

$$M^2 \in \left[M_0^2 - \frac{\Gamma^2}{4} \,,\; M_0^2 + \frac{\Gamma^2}{4}\right] \quad . \tag{4.2}$$

For simplicity we shall assume that the spectral function takes the zero value outside of this interval. We shall also consider that the width of this interval is sufficiently small, so that we shall restrict ourselves to the first approximation calculations with respect to Γ^2/M_0^2. The normalization to unity of the spectral function

$$\int_{M_0^2 - \frac{\Gamma^2}{4}}^{M_0^2 + \frac{\Gamma^2}{4}} d\,M^2\,\sigma(M^2) = 1 , \tag{4.3}$$

leads to

$$\text{const} = 2\,\frac{M_0^2}{\Gamma^2} \quad , \tag{4.4}$$

where the approximation

$$\ell n\,\frac{1 + x}{1 - x} \cong 2x \quad , \tag{4.5}$$

has been used. It can be now easily remarked that the corresponding charge-distribution function is

$$\rho(\vec{x}) = f(\infty)\ \delta(\vec{x}) + \frac{4M_0^2}{\pi\Gamma^2|\vec{x}|}\left(E_i(-|\vec{x}|\sqrt{4M_0^2+\Gamma^2}) - E_i(-|\vec{x}|\sqrt{4M_0^2-\Gamma^2})\right), \quad (4.6)$$

which takes approximately the form

$$\rho(\vec{x}) = f(\infty)\ \delta(\vec{x}) + \frac{1}{\pi|\vec{x}|}\exp(-2M_0|\vec{x}|). \quad (4.7)$$

As a consequence there is

$$\langle\vec{x}^2\rangle = \frac{3}{2M_0^2\left(1 + M_0^2\ f(\infty)\right)}, \quad (4.8)$$

which is analogous to the relation (3.6). The finite nonzero minimum of the above uncertainty square can be now obtained only when $f(\infty)$ takes negative values :

$$f(\infty) < 0. \quad (4.9)$$

In such conditions one obtains

$$\min\sqrt{\langle x^2_i\rangle} = \frac{\sqrt{2}}{M^{(max)}}, \quad (4.10)$$

where

$$\frac{1}{M^{(max)2}} = |f(\infty)| < \frac{1}{M_0^2}, \quad (4.11)$$

and where the metric is a positive valued one. Limiting ourselves to the very existence of the natural space-unit, we can conclude that the above obtained evaluation is in a formal agreement with the previously calculated expressions, too. In this sense all the above used approaches are able to imply uniquely the formal existence of the same minimum space-uncertainty. Conversely, the physical uniqueness of the so defined space-uncertainty is able to establish the formal consistency of the proposed methods.

5. The general nonzero minimum of the proper-time uncertainty

The computation of the space-uncertainty in terms of the previously used source-functions can be easily extended. Indeed, considering as an example the massive scalar field, there is

$$(\Box_x + m^2)\,\Delta_F(x\ ;\ m^2) = -\delta(x), \quad (5.1)$$

and

$$(\Box_x + m^2)\Delta_F'(x) = -\delta(x) - \int_{4m^2}^{\infty} dM^2(M^2 - m^2)\sigma(M^2)\Delta_F(x\ ;\ M^2), \quad (5.2)$$

where $\Delta_F(x\ ;\ m^2)$ and $\Delta_F'(x)$ are the propagators of the free field and of the interacting one respectively. Following the prescriptions of Sec.2 one obtains that the x^2-average is

$$\langle x^2\rangle = \frac{I_1}{1 + I_2}, \quad (5.3)$$

where the average has been performed with respect to the source function

$$\rho(x) = \delta(x) + \int_{4m^2}^{\infty} dM^2(M^2 - m^2)\ \Delta_F(x\ ;\ M^2). \quad (5.4)$$

As a consequence there is

$$I_1 = \int dx\ x^2\ \rho(x), \quad (5.5)$$

and

$$1 + I_2 = \int dx\ \rho(x), \quad (5.6)$$

respectively. Minimalizing the square uncertainty with respect to the spectral function $\sigma(M^2)$ and imposing for this purpose the condition

$$\frac{\delta}{\delta\ \sigma(\overline{M}^2)}\langle x^2\rangle = 0, \quad (5.7)$$

one obtains

$$\mathrm{Min}\langle x^2\rangle = \min\frac{\int dx\ x^2\ \Delta_F(x\ ;\ \overline{M}^2)}{\int dx\ \Delta_F(x\ ;\ \overline{M}^2)} = \min\left(-\frac{2}{\overline{M}^2}\right), \quad (5.8)$$

where on the left there is the minimum of a functional, whereas on the right the one of a function. On the other hand there is the formal equality

$$\min\frac{2}{\overline{M}^2} = \frac{2}{M^{(max)2}}, \quad (5.9)$$

so that the minimum value of the so raised proper-time uncertainty evaluation takes the modulus

$$|\min \Delta\tau| = |\min \sqrt{\langle x^2\rangle}| = \frac{\sqrt{2}}{M^{(max)}} \quad , \tag{5.10}$$

and respectively the imaginary value

$$\min \Delta\tau = i \, \frac{\sqrt{2}}{M^{(max)}} \quad , \tag{5.11}$$

which agrees with the previously performed evaluations. In such conditions the formal existence of the minimum uncertainty evaluations is assured, thus also proving again the physical meaning of the above formulated approach.

6. The power-like behavior of the K-L spectral function

The results obtained in the previous Sections show that there is of sense to seek for the existence of the minimum space-uncertainty. In this respect let us analyse the special peculiarities of the space-uncertainty evaluation corresponding to an inverse power-behavior ($s^{-\delta}$) of the K-L spectral function. For convenience, the computation of the space-uncertainty shall be performed with respect to the involved charge-distribution. To perform the calculations, we shall not impose the normalization condition of the spectral function, but rather the more general normalization-condition of the implied charge-distribution function.

The $\vec{x}$-space distribution function corresponding to the K-L spectral function

$$\sigma(s) = \frac{const}{s^{\delta}} \quad , \quad \delta > 0 \quad , \tag{6.1}$$

is given by

$$\rho(\vec{x}) = f(\infty)\ \delta(\vec{x}) + const\ \frac{|\vec{x}|^{2\delta-3}}{2\pi}\ \Gamma(2-2\delta\ ;\ 2m|\vec{x}|) \quad . \tag{6.2}$$

As a consequence there is

$$\int d\vec{x}\rho(\vec{x}) = f(\infty) + \frac{const}{\delta(2m)^{2\delta}} \quad , \tag{6.3}$$

which takes bounded values only when $\delta > 0$. For simplicity the $\vec{x}$-integration has been performed prior to the s-one. On the other hand

$$\int d\vec{x}\ \vec{x}^2\ \rho(\vec{x}) = \frac{6const}{(1+\delta)(2m)^{2+2\delta}} \quad , \tag{6.4}$$

so that the raised space-uncertainty is

$$\langle\vec{x}^2\rangle = const\ \frac{3}{2m^2}\ \frac{\delta}{\delta+1}\ \left(f(\infty)\ \delta(2m)^{2\delta} + const\right)^{-1} \quad , \tag{6.5}$$

which is now expressed in relative units. It can be now proved that the nonzero minimum value of the above square space-uncertainty evaluation can be mathematically defined only within the interval

$$m^2 \in \left(0,\ m^{(max)2}\right) \quad , \tag{6.6}$$

where

$$m^{(max)2} = \frac{1}{4}\left(\frac{1}{\delta}\ \left|\frac{const}{f(\infty)}\right|\right)^{\frac{1}{\delta}} \quad , \tag{6.7}$$

when also $f(\infty) < 0$. For the moment it has been considered that $const > 0$. The square space-uncertainty becomes infinity at the extremities of the interval (6.6), so that the mathematical formulation of the minimum problem inside the interval (6.6) is uniquely defined, in this way also. Moreover, outside of the interval (6.6) there is only the zero minimum which can be defined. Consequently, the very mathematical description of the minimum problem implies the existence of the finite upper-value of the particle rest-mass. This result is also supported by the one obtained in Sec.5. In these conditions the minimum is obtained when

$$(2m)^{-2\delta} = \beta\ \delta(1+\delta) \quad , \tag{6.8}$$

which leads to

$$\delta^2 s(\delta) \equiv \frac{1}{3}\min \langle\vec{x}^2\rangle = \frac{1}{2m^{(max)2}}(1+\delta)^{\frac{1}{\delta}} = 2\left(\beta\ \delta(1+\delta)\right)^{\frac{1}{\delta}} \quad , \tag{6.9}$$

where

$$\beta \equiv \beta(\delta) = \left|\frac{f(\infty)}{const}\right| \quad . \tag{6.10}$$

It can be easily remarked that when $const < 0$, there is

$$\langle\vec{x}^2\rangle = |const|\ \frac{3}{2m^2}\ \frac{\delta}{\delta+1}\left(-f(\infty)\ \delta(2m)^{2\delta} + |const|\right)^{-1} \quad , \tag{6.11}$$

so that one reobtains the previous results as soon as $f(\infty) > 0$. It can be thus concluded that within the interactions which are described by the K-L spectral function (6.1), the existence of the minimum space-uncertainty can be established in the above mentioned manner only when the parameters $f(\infty)$ and const take opposite signs.

Using now the relation (6)

$$\int d\vec{x}\, \rho(\vec{x}) = f(0) \quad , \tag{6.12}$$

and

$$\int d\vec{x}\, \vec{x}^2\, \rho(\vec{x}) = 6\, f'(0) \quad , \tag{6.13}$$

one obtains

$$\frac{f(0)}{f'(0)} + \left|\frac{f(\infty)}{f'(\infty)}\right| = \frac{1+\delta}{\delta}\, 4m^2 \quad , \tag{6.14}$$

so that

$$\beta(\delta) \lesssim \left(\delta (2m)^{2\delta}\right)^{-1} \quad , \tag{6.15}$$

as soon as

$$\frac{f(0)}{f'(0)} \gtrless 0 \quad , \tag{6.16}$$

where account has been made of the relation (6.4). Consequently, the existence of the upper rest-mass can be assured only when $f(0)/f'(0) > 0$, thus also establishing the validity of the inequality

$$m^{(max)2} > m^2 \quad . \tag{6.17}$$

Generally, the above relation shows the possibility of the dynamical mass-production. Accounting for the results of the previous chapter we have also to define the mass which raised effectively by the minimum space-uncertainty (6.9) as

$$\delta^2\, \tilde{s}(\delta) \equiv \frac{1}{u^2}\, \delta^2\, s(\delta) = \frac{2}{M^2(\delta)} \quad , \tag{6.18}$$

where u is the involved mass-unit. Consequently, the effective-mass takes the form

$$M(\delta) = u\left(\beta\, \delta(1+\delta)\right)^{-\frac{1}{2\delta}} \quad , \tag{6.19}$$

whereas

$$\lim_{\delta\to 0} M(\delta) = \infty \quad . \tag{6.20}$$

This latter relation shows that the relevant range of the δ-spectrum is essentially the one of the small-values. Let us now establish in some more details the relevant range of the δ-values. Indeed, analysing the relation (6.15) we can see that when

$$\ln 2m < 0 \quad , \tag{6.21}$$

the β-function is able to take a constant value which is included in the interval

$$\beta \in \left(0, 2|\ln 2m|\, (2m)^{-\frac{1}{|\ln 2m|}}\right) \quad . \tag{6.22}$$

In such conditions the relative space uncertainty $\delta s(\delta)$ takes the zero-value at the origine, increases then to the maximum value $\delta s(\delta_0)$, where δ_0 is the single root of the equation

$$\ln \beta\, \delta(1+\delta) = \frac{2\delta+1}{\delta+1} \quad , \tag{6.23}$$

and subsequently decreases to two at infinity. However, the condition (6.21) is satisfied -irrespective of the used mass-unit- only within the zero rest-mass limit. Consequently, the relation (6.22) becomes

$$\beta \in (0, \infty) \quad , \tag{6.24}$$

which shows that the assumption about a constant value of the β-function is quite meaningful. In such conditions the δ-domain of interest is

$$\delta \in (0, \delta_0) \quad , \tag{6.25}$$

because only in such a case the emerging effective-mass is able to take the needed large-values :

$$M(\delta) \in (M(\delta_0), \infty) \quad . \tag{6.26}$$

The so obtained result possesses similiarities with the dynamical mass-generation in field theory (7) : in both the cases the raised "dynamical" mass -which is able to take sufficiently large-values- emerges from a massless field. In this respect we have to notice that it is even by means of the zero mass-limit that the relevant range of the δ-spectrum can be determined explicitly by means of the relations (6.25) and (6.23). In terms of a renormalization group formulation it can be also proved that the effective mass possesses, for $\delta \to 0$, a fixed point at infinity. Indeed,

$$\left(\frac{\partial \ \ell n \ \delta}{\partial \ \ell n \ M(\delta)} = \left(\ell n \frac{u}{M(\delta)}\right)^{-1}\right) \quad , \qquad (6.27)$$

which possesses the stable fixed point $M(0) = \infty$, as one would expects. Some other informations concerning the non-zero lower bounds of the δ-spectrum should be established in terms of the constraints raised by the high-energy experimental data.

7. Conclusions

Using several methods proofs have been given that there is a theoretical prescription to define the minimum space uncertainty. The physical background of such a result should be the existence of an upper rest-mass of the involved intermediary particles. In other terms, such a result shows the existence of an upper value of the dynamically generated mass, too. Further data concerning the lower bounds of the δ-spectrum should be established in terms of the high-energy experiments. In this sense the high-energy behaviour is viewed to probe the existence of certain effective masses, or alternatively, of certain relevant lengths. We have also to mention that we would not have the occasion to establish the mutual consistency of the obtained space-uncertainty evaluations if we would not had placed the imaginary space-time dispersions (3.15) and (5.11) on the same footing with the real ones. So, the complex structure of space-time appears to be actually a consistency condition of the very space-time description of the high-energy physics [8].

Finally we have to notice that the actual physical existence of the nonpoint particles is mutually connected with the theoretical validity of the broglien dualism between waves and nonpoint particles. It is quite evident that the dualism wave-point-like-particle is only an incomplete particular form of the above one. Indeed, the above uncertainty-calculations show that the point-particle of the "nonphysical" nonregularised standard theory is replaced effectively -within the regularised theory- by a nonpoint particle, thus also establishing the above mentioned validity of the actual physical dualism. In this respect there is of significance that the high-energy methods are able to probe the validity of the same dualism, too.

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