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ON THE APPLICATION OF THE WEIZSÄCKER-WILLIAMS METHOD TO DETERMINE THE PHOTOELECTRIC EFFECT PRODUCED BY A RELATIVISTIC CHARGED PARTICLE INCIDENT ON A METAL SURFACE

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Abstract : *The Weizsäcker-Williams method is used to calculate the number of electrons emitted by a metal surface by photoelectric effect, in the case of a charged particle moving with velocity nearly equal to c impinging on the surface.*

The interest of the present application consists in the fact that it provides, in principle, a method of measuring high velocities of charged particles, which can be calculated from the number of the emitted electrons.

Résumé : *La méthode de Weizsäcker-Williams est appliquée au calcul du nombre d'électrons émis d'une surface métallique par effet photoélectrique, en cas d'une particule chargée voyageant à une vélocité presque égale à c, qui va frapper la surface.*

L'intérêt de cette application est du au fait qu'elle représente, en principe, une méthode pour mesurer les grandes vélocités de particules chargées, qui peuvent être calculées du nombre d'électrons émis.

This article presents a simple application of the Weizsäcker-Williams method[1-3] to the evaluation of the number of photoelectrons produced by the electromagnetic field generated by a charged particle moving near a metal surface. The method has already been applied to study the structure of the tracks of ionizing particles in the Wilson cloud chamber[4], and to calculate the energy loss of very fast charged particles in solid materials (see for example ref. 5). It has also been used, more recently, to evaluate the radiation produced by a relativistic charge near a metal surface of a particular shape [6,7].

Its importance, as far as the present application is concerned, is in the fact that it provides, in principle, a method of measuring the velocity of the particle, since this can be calculated from the number of the emitted electrons. This has been pointed out by Bortolani[3], who performs a calculation completely based on first principles by using Thompson cross sections, for the two cases of a charged particle passing through a cylindrical hole in the metal and moving parallel to the metal surface, getting an effect much higher than ours.

Here experimental curves for the quantum yield are used to determine the magnitude of the effect in the simple case of a charge impinging on the surface, when the whole particle energy is supposed to be spent in the photoelectric effect. One starts from the fact that the electromagnetic field due to a charged particle moving with a velocity very near to c is almost equal to that of a set of electromagnetic waves of various frequencies, and calculates the action of these "virtual photons" on other charged particles.

Let us then consider a particle of charge Ze in rectilinear motion with velocity v close to c. The number of virtual photons with frequency between ν and $\nu + d\nu$ is given by[3,4]

$$q(\nu)d\nu = -\frac{2Z^2e^2}{\pi\hbar c}\frac{d\nu}{\nu}\left\{\frac{b^2}{2}[K_1(b)^2 - K_0(b)^2] - bK_1(b)K_0(b)\right\} \quad (1)$$

with $b = \frac{\hbar\nu}{\gamma mc^2}$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. m is the electron mass, K_0 and K_1 are respectively the modified Hankel functions of order zero and one[9].

For $b \ll 1$ (1) reduces to the simpler expression

$$q(\nu)d\nu = -\frac{2Z^2e^2}{\pi\hbar c}\frac{d\nu}{\nu}(\ln b + 0.384) \quad , \quad (2)$$

valid, e. g. for $\gamma = 10$, for $h\nu$ ranging from zero to 10^6 eV.

For $b \gg 1$ $q(\nu)$ goes to zero as $\frac{Z^2e^2}{2\hbar c\nu}e^{-2b}$.

The number of photoemitted electrons is given by

$$\int_0^\infty E(\nu)q(\nu)d\nu \quad (3)$$

where the function $E(\nu)$, different from zero only above the threshold frequency, is the efficiency of the metal, that is the ratio between emitted and incident photons. In the calculation $E(\nu)$ has been set equal to 1 from 1 eV to infinity. The result for $Z = 1$ is reported in fig.1. It may be considered an upper limit, because the most sensitive photocathodes (alcaly-antimony compounds) have actually a threshold not below 1.4 eV, and they reach the maximum of emission of about 0.30 emitted electrons per photon at a few electronvolts[10,11]. Instead Ag-O-Cs photocathodes may have a threshold even lower than 1 eV, but below 1 eV their efficiency is lower than 10^{-5} electrons per photon and the maximum of emission (for $h\nu$ higher than 1.5 eV) is about 10^{-2} electrons per photon[11].

It seems therefore doubtful that the effect could be used, as it has been argued, in order to measure the velocity of a relativistic charged particle.

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