

Annales de la Fondation Louis de Broglie,  
Vol. 8, n° 4, 1983

## NEO-HERTZIAN ELECTROMAGNETISM (I)

by Thomas E. Phipps, Jr  
908 South Busey Avenue  
Urbana, Illinois 61801

(manuscrit reçu le 28 Février 1983)

A recurrent question in physics is why Maxwell's equations are not invariant -at least to first-order approximation in velocities- under the Galilean transformation. We point out the little-known fact that Heinrich Hertz discovered a "covering theory" of Maxwell's that *was* Galilean invariant. Hertz's theory was rejected by physicists because he gave a new velocity parameter contained in it an unfortunate (ether-related) interpretation that brought the theory into conflict with observation. We show that a more operationally meaningful interpretation of this parameter -as *velocity of the field detector relative to the observer*- removes the observational conflict and opens the way to a generally-invariant "neo-Hertzian" formulation of electromagnetism. Notable features include (a) a predicted influence of detector motion on light speed and (b) the breaking of spacetime symmetry. These accord precisely with a previously-proposed radical kinematics, of which the invariants are length and proper time. The main elements thus emerge of a consistent "test theory" of Maxwell-Einstein physics.

### 1. Introduction

Maxwell's equations of electromagnetism are foundational to

the current world view in physics. Historically, they supplied the first instance of Lorentz covariance, hence the impetus to Einstein's kinematics of 1905. Yet there remains a curious aspect of Maxwell's equations, directly connected with their transformation properties, that was the subject of much inquiry during the last century. Though today largely forgotten, the question is still unresolved, and is in a sense more pointed now than ever. This is: Why are Maxwell's equations not invariant under the Galilean transformation *to first order in velocities*? Since at that order of approximation the Galilean and Lorentz transformations are practically the same, and since the equations of classical mechanics are Galilean invariant and are in agreement with observation at this same order of approximation, the answer to this question is far from obvious. It was this noninvariance feature of the accepted mathematics that bolstered the nineteenth century's persistent hope of optically detecting (at first order) the earth's motion with respect to an "ether".

Indeed, first-order noninvariance of the equations of optics presented a direct challenge to the relativity principle, even as it was known in the nineteenth century. As Tonnelat [1] notes, by 1874 "Mascart [2] was suggesting that in optics, as in dynamics, it was impossible to distinguish any special Galilean system by any experimental means whatsoever". When Maxwell's four field equations are subjected to a Galilean transformation, two of them fail of invariance by terms of first order in velocity. So either something is wrong with those two equations or something is wrong with the relativity principle ... which if it holds at all orders (as Einstein postulated) certainly must hold at first order.

The modern physicist would prefer to dismiss such questions by ruling out of order the demand for *invariance* of physical laws and substituting *covariance*; but in fact relativity theory contains true invariants of its own (the proper-space and proper-time intervals), the utility of which for constructing "invariant laws" of physics can hardly be ruled out of order *a priori*. The abandonment of genuinely invariant description as the physicist's goal (and as the mathematical key to "relativity") lacks any rationale apart from expediency. Similarly, the *necessity* to invoke second-order considera-

tions to answer first-order questions is difficult to rationalize from any first principles.

Physicists of the nineteenth century were no better able to cope with the question of electromagnetic invariance than their modern counterparts: Most of them shrank from the only logical conclusion (in lack of the "covariance" device, which had not yet been invented) to be drawn from failure of all attempts to detect ether drift at first order—namely, that the first-order-*noninvariant* equations of optics were wrong at first order. This fact may baffle future historians of science, for it was not a case of bad theory being better than none. A better mathematical theory did exist, as we shall see, associated with one of the most distinguished names in the history of electromagnetism.

To Heinrich Rudolph Hertz, who was not only the experimental discoverer of electromagnetic waves but also a powerful theorist, the answer to the question posed above was both obvious and radical: Maxwell's equations were *wrong at first order*, in the sense not of error within their limited scope, but in the sense of demanding to have that scope so expanded as to impart to the laws of electromagnetism invariance properties identical to those of the laws of mechanics. In seeking expanded theoretical scope, Hertz recognized the need for what today would be termed an invariant "covering theory" of Maxwell's electromagnetism. In his classic book, *Electric Waves*, he published just such a theory.

Hertz judged a first-order difference in the transformation properties of electromagnetic and mechanical laws to be intolerable. (Even today, it is hard to disagree with him.) He therefore sought and readily found a modified mathematical formulation of Maxwell's equations that, like the mechanical laws, was Galilean invariant. (We repeat: invariant, *not covariant*!) We shall begin by reviewing Hertz's important mathematical discovery and the reason it has been disregarded and forgotten by physicists. Next, we propose certain further modifications and reinterpretations needed to modernize the Hertz theory and to improve its operational significance, higher-order accuracy, and invariance. We also explain from

first principles why "covariant scrambling" of field components is not a necessary feature of physical description. In a second part of the paper some consequences of the modified equations are examined, with particular reference to solutions of the neo-Hertzian wave equation, and crucial experiments to distinguish neo-Hertzian from Maxwellian electromagnetism are briefly discussed. The observational differences, though far-reaching, are apparently not such as to have been revealed by experiments hitherto performed. This conclusion must remain tentative in the absence of a fully-developed matching kinematics. (The present paper deemphasizes kinematics, insofar as possible, in order to focus on electromagnetism). The reader will perhaps share the writer's surprise to discover so much life in a subject long accorded the honors of the dead.

## 2. Maxwell's vs. Hertz's Equations under Galilean Transformation

We simplify at the outset by postulating with Lorentz (1892) a micro-scale on which all physical materials are idealized as particles in free space. This enables us to suppress the constitutive relations and to express electromagnetism via the Maxwell-Lorentz "microscopic equations",

$$\vec{\nabla} \times \vec{H} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} - \frac{4\pi}{c} \vec{u}_s = 0 \quad (1a)$$

$$\vec{\nabla} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{H}}{\partial t} = 0 \quad (1b)$$

$$\vec{\nabla} \cdot \vec{H} = 0 \quad (1c)$$

$$\vec{\nabla} \cdot \vec{E} - 4\pi\rho = 0 \quad (1d)$$

Maxwellian

Here  $\rho$  and  $\vec{u}_s = \rho \vec{v}_s$  are field-source terms that may be either continuous or discrete charge distributions. Nothing essential is lost by this reduction from the traditional four field quantities ( $\vec{E}, \vec{D}, \vec{B}, \vec{H}$ ) to two. In this and the next section we limit our concern to considerations of first order in velocity parameters.

Let inertial system  $S'$  move with constant velocity  $\vec{v} = (v_x, v_y, v_z)$  with respect to system  $S$ , and let  $(\vec{r}', t') = (x', y', z', t')$  denote the coordinates in  $S'$  of the physical event point designated  $(x, y, z, t)$  in  $S$ . The Galilean transformation

$$\vec{r}' = \vec{r} - \vec{v}t, \quad t' = t \quad (2)$$

implies that

$$\frac{\partial}{\partial t} = \frac{\partial x'}{\partial t} \frac{\partial}{\partial x'} + \frac{\partial y'}{\partial t} \frac{\partial}{\partial y'} + \frac{\partial z'}{\partial t} \frac{\partial}{\partial z'} + \frac{\partial t'}{\partial t} \frac{\partial}{\partial t'}$$

$$\text{or} \quad \frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \vec{v} \cdot \vec{\nabla}' = \frac{\partial}{\partial t'} + \vec{v}' \cdot \vec{\nabla}', \quad (3a)$$

where  $\vec{v}' = -\vec{v}$  is the velocity of  $S$  with respect to  $S'$ . Similarly,

$$\frac{\partial}{\partial x} = \frac{\partial x'}{\partial x} \frac{\partial}{\partial x'} + \frac{\partial y'}{\partial x} \frac{\partial}{\partial y'} + \frac{\partial z'}{\partial x} \frac{\partial}{\partial z'} + \frac{\partial t'}{\partial x} \frac{\partial}{\partial t'} = \frac{\partial}{\partial x'},$$

etc., so

$$\vec{\nabla}' = \vec{\nabla}. \quad (3b)$$

A Galilean transformation of field-source velocity, valid at least to first order in  $v_s/c$ , implies that

$$\vec{v}'_s = \vec{v}'_s(x', y', z', t') = \vec{v}_s(x, y, z, t) - \vec{v}. \quad (4)$$

If we assume scalar invariance of charge density,

$$\rho'(x', y', z', t') = \rho(x, y, z, t), \quad (5)$$

then

$$\vec{u}'_s = (\rho \vec{v}_s)' = \rho' \vec{v}'_s = \rho \vec{v}_s - \rho \vec{v} = \vec{u}_s - \rho \vec{v}, \quad (6a)$$

or

$$\vec{u}_s = \vec{u}'_s + \rho \vec{v} = \vec{u}'_s - \rho' \vec{v}'. \quad (6b)$$

With one further set of assumptions we are ready to study the transformation properties of Maxwell's equations under Galilean transformation. Misguided as it may seem to today's mathematical physicist steeped in covariance, we postulate the simplest thing, *viz.*, that the field quantities transform as scalar invariants :

$$\vec{E}'(x', y', z', t') = \vec{E}(x, y, z, t) \quad (7a)$$

$$\vec{H}'(x', y', z', t') = \vec{H}(x, y, z, t) \quad (7b)$$

Consider first Eq. (1c). By Eqs. (3b) and (7b) we establish invariance :

$$\vec{v} \cdot \vec{H} = \vec{v}' \cdot \vec{H}' = 0 \quad (8)$$

Similarly, with Eqs. (3b), (7a), (5), Eq. (1d) yields

$$\vec{v} \cdot \vec{E} - 4\pi\rho = \vec{v}' \cdot \vec{E}' - 4\pi\rho' = 0 \quad (9)$$

This equation, too, is invariant. Thus, given assumptions (5) and (7), two of Maxwell's equations are invariant under Galilean transformation. The other two are not. With the help of Eqs. (3) and (7), Eq. (1b) yields

$$\begin{aligned} \vec{v} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{H}}{\partial t} &= \vec{v}' \times \vec{E}' + \frac{1}{c} \left( \frac{\partial}{\partial t'} + \vec{v}' \cdot \vec{\nabla}' \right) \vec{H}' \\ &= \vec{v}' \times \vec{E}' + \frac{1}{c} \frac{\partial \vec{H}'}{\partial t'} + \frac{1}{c} (\vec{v}' \cdot \vec{\nabla}') \vec{H}' = 0 \end{aligned} \quad (10)$$

Here, invariance is spoiled by the appearance of an extra first-order term,  $\frac{1}{c} (\vec{v}' \cdot \vec{\nabla}') \vec{H}'$ . Similarly, Eq. (1a) yields, with use of Eq. (6),

$$\begin{aligned} \vec{v} \times \vec{H} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} - \frac{4\pi}{c} \vec{u}_s &= \vec{v}' \times \vec{H}' - \frac{1}{c} \left( \frac{\partial}{\partial t'} + \vec{v}' \cdot \vec{\nabla}' \right) \vec{E}' - \frac{4\pi}{c} (\vec{u}'_s - \rho' \vec{v}') \\ &= \vec{v}' \times \vec{H}' - \frac{1}{c} \frac{\partial \vec{E}'}{\partial t'} - \frac{4\pi}{c} \vec{u}'_s + \left( \frac{4\pi}{c} \rho' \vec{v}' - \frac{1}{c} (\vec{v}' \cdot \vec{\nabla}') \vec{E}' \right) = 0 \end{aligned} \quad (11)$$

Again, invariance fails by terms of first order in  $v'/c$ , shown

in curly brackets.

So, it is necessary either (A) to discard the scalar invariance assumptions embodied in Eqs. (5) and (7), or (B) to modify two of Maxwell's equations, (1a) and (1b), at first order to make them invariant. Lorentz, Einstein, and all modern theorists follow alternative (A), the route of *covariance*. Hertz, preferring the route of *invariance*, chose (B). By definition, invariance of an equation means invariance of each of its terms individually, so there is no evading the necessity of field transformation law (7), once the goal of invariance is chosen. We shall follow Hertz.

Let us introduce an arbitrary velocity-dimensioned parameter  $\vec{v}_d = (v_{dx}, v_{dy}, v_{dz})$ , purely as a mathematical artifice, without attempting as yet to endow this "velocity" with any specific physical significance. We suppose  $\vec{v}_d$  to have vector character in S and S', hence (at least at first order) to obey the Galilean velocity addition law,

$$\vec{v}'_d(x', y', z', t') = \vec{v}_d(x, y, z, t) - \vec{v} \quad (12a)$$

or

$$\vec{v}_d = \vec{v}'_d - \vec{v}' \quad (12b)$$

In place of Eq. (1b), we postulate the following in system S :

$$\vec{v} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{H}}{\partial t} + \frac{1}{c} (\vec{v}_d \cdot \vec{\nabla}) \vec{H} = 0 \quad (13)$$

Here use has been made of a technique from the theory of algebraic forms to insert an extra first-order term, formally the same as that which spoiled the invariance in Eq. (10), but with  $\vec{v}'$  replaced by our arbitrary parameter  $\vec{v}_d$ . We assert that

Eq. (13) is formally invariant under Galilean transformation, regardless of the physical interpretation of  $\vec{v}_d$ . Proof : From Eqs. (3) and (12),

$$\begin{aligned} \vec{v} \times \vec{E} + \frac{1}{c} \frac{\partial \vec{H}}{\partial t} + \frac{1}{c} (\vec{v}_d \cdot \vec{\nabla}) \vec{H} &= \vec{v}' \times \vec{E}' + \frac{1}{c} \left( \frac{\partial}{\partial t'} + \vec{v}' \cdot \vec{\nabla}' \right) \vec{H}' \\ + \frac{1}{c} [(\vec{v}'_d - \vec{v}') \cdot \vec{\nabla}'] \vec{H}' &= \vec{v}' \times \vec{E}' + \frac{1}{c} \frac{\partial \vec{H}'}{\partial t'} + \frac{1}{c} (\vec{v}'_d \cdot \vec{\nabla}') \vec{H}' = 0 \end{aligned} \quad (14)$$

q.e.d. Applying the same technique (replacing  $\vec{v}'$  by  $\vec{v}'_d$  in Eq. (11)), we postulate as our invariant replacement for Eq. (1a):

$$\vec{v} \times \vec{H} - \frac{1}{c} \frac{\partial \vec{E}}{\partial t} - \frac{4\pi}{c} \vec{u}_s + \frac{4\pi}{c} \rho \vec{v}_d - \frac{1}{c} (\vec{v}_d \cdot \vec{v}) \vec{E} = 0 \quad (15)$$

Verification of invariance with the help of Eqs. (3), (6), (12) is immediate.

The newly-postulated equations (13), (15), together with Eqs. (1c) and (1d) (unaltered), constitute the Hertz equations. A few manipulations will bring them into more perspicuous form. Introducing a Galilean-invariant current-dimensioned quantity  $\vec{u}_m$  by the definition

$$\vec{u}_m = \vec{u}_s - \rho \vec{v}_d, \quad (16)$$

and defining an invariant "convective derivative" by

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}_d \cdot \vec{\nabla}, \quad (17)$$

we may rewrite Eq. (15) as

$$\vec{v} \times \vec{H} - \frac{1}{c} \frac{d\vec{E}}{dt} - \frac{4\pi}{c} \vec{u}_m = 0, \quad (18)$$

and Eq. (13) as

$$\vec{v} \times \vec{E} + \frac{1}{c} \frac{d\vec{H}}{dt} = 0 \quad (19)$$

The invariance of this total or convective time derivative is readily verified. From Eqs. (3a), (12):

$$\begin{aligned} \frac{d}{dt} &= \frac{\partial}{\partial t} + \vec{v}_d \cdot \vec{\nabla} = \left( \frac{\partial}{\partial t'} + \vec{v}'_d \cdot \vec{\nabla}' \right) + (\vec{v}'_d - \vec{v}') \cdot \vec{\nabla} \\ &= \frac{\partial}{\partial t'} + \vec{v}'_d \cdot \vec{\nabla}' = \left( \frac{d}{dt'} \right)' \end{aligned} \quad (20)$$

Similarly, by Eqs. (16), (6a), (12a) the invariance of  $\vec{u}_m$  is established:

$$\begin{aligned} \vec{u}'_m &= (\vec{u}_s - \rho \vec{v}_d)' = \vec{u}'_s - \rho \vec{v}'_d = (\vec{u}_s - \rho \vec{v}) - \rho (\vec{v}'_d - \vec{v}) \\ &= \vec{u}_s - \rho \vec{v}_d = \vec{u}_m \end{aligned} \quad (21)$$

Eqs. (3b), (7), (20), (21) show each term in Eqs. (18) and (19) to be individually invariant. Hence the Hertz equations, which we collect here for reference, are the *Galilean-invariant expression of the laws of electromagnetism*:

$$\vec{v} \times \vec{H} - \frac{1}{c} \frac{d\vec{E}}{dt} - \frac{4\pi}{c} \vec{u}_m = 0 \quad (22a)$$

$$\vec{v} \times \vec{E} + \frac{1}{c} \frac{d\vec{H}}{dt} = 0 \quad (22b)$$

$$\vec{v} \cdot \vec{H} = 0 \quad (22c)$$

$$\vec{v} \cdot \vec{E} - 4\pi\rho = 0 \quad (22d)$$

Hertzian

It will be observed that the Hertz equations (22) are formally identical to the Maxwell equations (1) except that

(a) The source current  $\vec{u}_s$  in Eq. (1) is replaced by  $\vec{u}_m = \vec{u}_s - \rho \vec{v}_d = \rho (\vec{v}_s - \vec{v}_d)$ .

(b) The partial time derivative  $\partial/\partial t$  in Eq. (1) is replaced by the convective time derivative,  $d/dt = \partial/\partial t + \vec{v}_d \cdot \vec{\nabla}$ .

To verify that the Hertz equations constitute a "covering theory" of Maxwell's theory, observe that when  $\vec{v}_d = 0$  by definition  $\vec{u}_m$  becomes identical to  $\vec{u}_s$  and  $d/dt$  identical to  $\partial/\partial t$ , so Eq. (22) reduces to Eq. (1) and Maxwell's theory is recovered as a special case within Hertz's. For  $\vec{v}_d \neq 0$  the Galilean invariance of Eq. (22) has been shown, so Hertz's theory is an *invariant covering theory* of Maxwell's. The validity of the scalar-invariant transformation of fields, Eq. (7), has been established incidentally. There is no covariant "scrambling" of fields in Hertz's theory, nor is there any "spacetime symmetry" (since  $d/dt$  is not mathematically symme-

trical with partial spatial derivatives,  $\partial/\partial x, \partial/\partial y, \partial/\partial z$ ).

So far, we have performed purely mathematical manipulations, in emulation of steps that Hertz may have followed. We now come to the crucial question of physical interpretation of the new parameter  $\vec{v}_d$ . A starting point of discussion is the interpretation Hertz himself gave to this parameter; but we emphasize the necessity to distinguish Hertz's equations from Hertz's interpretation. In his day the attention of all physicists was riveted on a luminiferous medium, the ether. Possessed of a new velocity-dimensioned parameter in the equations of electromagnetism, Hertz understandably identified  $\vec{v}_d$  with the velocity of the ether. Not satisfied to deal with an operationally undefined quantity, he went on to adopt the assumption (due to Stokes) that ether was 100% "dragged" by ponderable bodies, hence that  $\vec{v}_d$  was the measurable velocity of bodies in the laboratory. He consequently presented his theory as the "electrodynamics of moving bodies".

A few years after Hertz's early death, which occurred in 1894, experimentalists [3], [4] disconfirmed predictions, based on Hertz's theory, that a dielectric moving in the laboratory would generate a magnetic field measurable by a stationary detector. So, Hertz's theory was discredited and discarded. By contrast, Lorentz and "covariance" (Minkowski's term of 1908) stormed on to a conquest so total that the very existence of Hertz's invariant covering theory has been forgotten.

Had Hertz lived, the story might have been different, for it is obvious from his writings [5] that he adopted the Stokes hypothesis about ether drag exactly as a scientist should: very tentatively. Balked empirically in that direction, Hertz might have realized that the whole idea of an ether was of dubious compatibility with the mathematics he himself had developed. For his new law, Eq. (22), put electromagnetism for the first time on the same group-invariance footing as mechanics. It certified a mechanical-optical relativity principle and legitimized (at least at first order) the defining of "inertial system" as kinematically equivalent to "Galilean system". Since mechanics gets along perfectly without "ether", the necessity of that concept on the side of optics becomes

dubious (in view of the existence of a mechanical-optical dualism that goes back to Hamilton). If ether is unnecessary, why not abolish it—at least *pro tempore*? That Hertz did not reason thus from the start is proof of the power of a fixed idea over even the greatest of minds.

In this section we have rederived in simplified form and modern notation (with invaluable help from the work of Miller [6] in translating Hertz's archaic notation) the Galilean-invariant equations of Hertzian electromagnetism, Eq. (22). The Hertz theory can never conflict with that of Maxwell because the former is a covering theory. Suitably interpreted, it can yield additional physical predictions (including most startling ones, as we shall see in Part II), for right or wrong—but it cannot contradict deductions from Maxwell's theory. The weakness of Hertz's work was that he achieved only half a theory. He produced vital mathematics, but not valid interpretation of the key parameter,  $\vec{v}_d$ , contained in the mathematics. One gets physics by combining mathematics and interpretation, only the combination being subject to empirical test. The task of our next section is to discover an alternative interpretation of  $\vec{v}_d$  that will enable Hertz's equations to become physics. Thus far, Hertz. Now we commence the "neo-" part of this paper.

### 3. Reinterpretation of $\vec{v}_d$

The irreducible elements for producing observable electromagnetic phenomena are (a) the observer, (b) the field source, (c) the field detector. Among tangibles (*i.e.*, omitting "ether") there are no others. The observer plays his part by specifying an inertial system  $S$ . That is, his presence is manifested in the theory through parameters  $(x, y, z)$  measured in  $S$ . The field source and its degrees of freedom are manifested through parameters  $\rho, \vec{v}_s$ . But in Maxwell's theory there are no parameters expressing degrees of freedom of the field detector. There are, to be sure, quantities  $\vec{E}, \vec{H}$  representing measurements by a detector *stationary* with respect to the observer (*i.e.*, at rest in  $S$ ). But there is no velocity-dimensioned parameter related to field sinks in the same manner  $\vec{v}_s$  is related to field sources. Thus *one of the phy-*

sically essential elements is parametrically unrepresented in the mathematics.

Since field detectors are actual instruments, *i.e.*, compositions of matter possessing physical degrees of freedom -and since "fields" cannot exist, or at least cannot be measured, without them- it seems that the absence from Maxwell's theory of a velocity-dimensioned parameter descriptive of field-detector motions might have been noticed as a deficiency even without hints from Hertz's rival mathematics. This would seem to have been doubly true of the relativity-conscious climate of the early part of this century, when physicists became acutely aware of the fallacy of "privileged observers". What (someone might have asked) should an observer *defined* as "at rest" with respect to a particular composition of matter be called but a "privileged observer" with respect to that bit of matter? And if all observers are so defined with respect to their own bits of matter (in the manner of Einstein) is such replication of privilege the same as elimination of privilege?

In light of such questions it is clear that an intellect sufficiently conscious of the relativity *idea* might have been dissatisfied with Maxwell's equations from the start and would have welcomed any method of injecting into those equations a velocity-dimensioned parameter capable of describing field-detector degrees of freedom in  $S$ ; *i.e.*, field-detector velocity relative to the observer.

We have used a great many words in this section to prevent the reader from evading a conclusion to which he has doubtless long since jumped; namely,

*Interpretation of  $\vec{v}_d$* : The parameter  $\vec{v}_d$  in the Hertz equations, Eq. (22), is the velocity of the field detector (or radiation absorber, etc.) relative to the observer.

As a check, we note that when  $\vec{v}_d = 0$  the field detector is at rest in the observer's inertial system  $S$  and we recover from Eq. (22) exactly Maxwell's equations, Eq. (1). This is con-

sistent with the basic assumption of Maxwell's theory, that the field detector is at rest with respect to the coordinate-defining observer. The aforementioned experiments [3], [4] that disproved Hertz's interpretation of  $\vec{v}_d$ , as velocity of an ether-convecting "body" in the laboratory, merely serve to confirm the present interpretation of  $\vec{v}_d$ , as velocity of the field detector in the laboratory. For in those experiments the field detector remained in all cases *at rest* in the laboratory, so  $\vec{v}_d = 0$ , even though the dielectric "body" moved. Since  $\vec{v}_d = 0$ , Hertz's equations reduce to Maxwell's, and the experiments in question fail to test the distinction between the Maxwell and Hertz formalisms.

It is true that a field detector idealized as a "unit test charge" must in principle acquire some motion in order to manifest its detection of a field; so our claim that Maxwell's theory "freezes out" detector degrees of freedom may be challenged. But this test-charge motion can safely be ignored, since in Maxwell's theory (a) the initial state of motion of such a detector is always the rest state in  $S$ , (b) microscopic motions suffice in principle, and (c) any field-induced change in state of motion of the test charge is outside the purview of the theory, which contains no parameters to describe it. It may be convenient to think of a *field detector* either broadly, as any object susceptible to electromagnetic influence, or narrowly, as a small "black box" with a pointer and scale on it, calibrated to read numerical "field value", whether electric or magnetic. The black-box idealization relieves us of the need to be explicit about the dynamical details of the field measurement process, and is consonant with the spirit of the Maxwell-Hertz approximation to physical description.

It is a truism that the "field" is nothing more nor less than what a field detector detects. Still, this is worth emphasizing because it provides the key to the counter-intuitive result obtained in the previous section, *viz.*, that the  $\vec{E}$  and  $\vec{H}$  fields of Eq. (22) transform invariantly (Eq. (7)), instead of covariantly, as do the fields of Eq. (1). Elucidation lies just in the above truism -that "*field*" means different things in Hertz's and in Maxwell's theories. Different laboratory procedures or "operations" are performed in the two cases.

According to Maxwell the "field" is what is measured by an instrument defined to be in a state of rest with respect to the observer. Such a field is a simple quantity with rather complex (covariant) transformation properties. In Hertz's theory (modified as above in respect to interpretation of  $\vec{v}_d$ ) the "field" is what is measured by an instrument defined to be in a state of motion specified by three velocity components,  $\vec{v}_d = (\vec{v}_{dx}, \vec{v}_{dy}, \vec{v}_{dz})$ , with respect to the observer. Such a "field" is a rather complex quantity with simple (invariant) transformation properties. From these considerations it is at once apparent that covariant "scrambling" (linear combining) of electromagnetic field components is by no means a law of nature but an artifact of definition. (On the side of kinematics, of course, the same is true in respect to "scrambling" of space and time descriptors -as has been argued elsewhere [7], [8] without reference to electromagnetic symmetry-breaking).

Clearly one is not getting something for nothing from Hertz's theory. One is paying something (in increased complexity of "field" definition) to get something supremely worth having in the relativity context, *viz.*, elimination of "privilege" of observers in respect to their motions relative to *all* physical compositions of matter. In Hertz's theory we acquire for the first time the basis -though only at first order- for a genuine relativity theory in which true equality exists among all inertial observers, for optical observers on the same group *invariance* basis as for mechanical observers. Even if the reader remains convinced by his schooling that the covariance demanded by Eq. (1) is "just as good" as the invariance offered by Eq. (22), he still must acknowledge that there is some mismatch between a classical electromagnetism covariant at second order and a classical mechanics invariant at first order. Hertz's Eq. (22) eliminates all mismatch by making everything explicitly invariant at first order. Surely one ought to get first-order physics right before essaying the first step toward higher orders. On the supposition that we have now belatedly got first-order electromagnetism right, what about the higher orders? It is to this interesting question that we next turn attention, proceeding to what might be termed the (neo-)2 stage of our inquiry. We confine

ourselves here to electromagnetism, analogous considerations for mechanics having been touched on elsewhere [7].

#### 4. Proper-time Formulation

Thus far we have claimed no more than first-order accuracy for our results, in the spirit of getting first-order things right first. At higher orders, corresponding to speeds readily attainable in the laboratory with high-energy particles, there is every reason to disbelieve Eq. (22) and to seek a more exact (higher-order invariant) formulation, which reduces to Eq. (22) in the low-speed limit.

It is necessary to begin with a digression into kinematics, which we shall keep as brief as possible because a full discussion would require more space than can be given to it here. The fact that an alleged "spacetime" symmetry is broken (or never arises) in Hertzian electromagnetism implies that the Lorentz transformation is irrelevant to Hertzian physics, hence that the kinematics of high-speed motion must be totally reappraised. Such a reappraisal has been under way for some time and preliminary results have been reported [7], [8], [9]. In brief summary, it is indicated that

(1) Einstein's definition of the invariant proper-time interval,

$$d\tau^2 = dt^2 - \frac{1}{c^2}(dx^2 + dy^2 + dz^2), \quad (23)$$

together with recognition of the inexactness of the differential  $d\tau$ , is a brilliant and lasting contribution to physics with a firm basis of empirical confirmation [10]. Here  $d\tau$  is delimited by two events on the worldline of the *same particle*. Proper time possesses a clear operational definition as the "pocket-watch time of the co-moving observer".

(2) The supposedly invariant proper-space interval,

$$d\sigma^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2 \quad (?) \quad (24)$$

is without empirical confirmation. Its invariance would be valid only if there existed true spacetime symmetry, which



Eq. (22) denies. There is no known counterpart in nature nor operational definition of  $d\sigma$ , an "interval" delimited by events on the worldlines of two different particles. An invariant mathematical relationship between two such events implies the objective "reality" of some *connectedness* between two occurrences that by definition cannot be connected by a light signal or anything else ... a virtual contradiction in terms. Nobody has ever met such a connectedness in the laboratory and every attempt to measure the Lorentz contraction [11], [12], [13] has failed. If one asserts operationalism, then one must discard the "elsewhere" and all its works. All the worldline-relational ("metric") statements of special relativity theory are therefore to be viewed as potentially false. Consequently we reject Eq. (24) and accept Eq. (23), interpreted as referring to events on a single worldline. The simplest alternative to Eq. (24), consonant at first order with the Galilean transformation, is that *length*

$ds = \sqrt{dx^2 + dy^2 + dz^2}$  transforms as an invariant scalar.

(3) The invariants of kinematics are therefore postulated to be *length* and *proper time*. This postulate reflects the Hertzian *asymmetry* of space and time. "Spacetime" possesses no metric geometry and no objective existence. For a reconstruction of the foundations of kinematics consonant with this postulate, Ref. [8] may be consulted.

We now proceed to apply the above conclusions about kinematics to our problem of electromagnetic description. We may interpret  $\tau = \tau_d$  in Eq. (23) as field detector proper time. Let  $\vec{v}_d = (dx/dt, dy/dt, dz/dt)$ , where the  $x, y, z$  differentials refer to detector coordinates. From (23) proper time and frame time are related in the familiar way :

$$d\tau_d = dt \sqrt{1 - \frac{v_d^2}{c^2}} \quad (25a)$$

or

$$\frac{dt}{d\tau_d} = \gamma_d, \quad \gamma_d = \frac{1}{\sqrt{1 - \frac{v_d^2}{c^2}}} \quad (25b)$$

Taking the total proper-time derivative of an arbitrary function of detector coordinates, we obtain

$$\begin{aligned} \frac{d}{d\tau_d} f(x, y, z, t) &= \left(\frac{dx}{d\tau_d}\right) \frac{\partial f}{\partial x} + \left(\frac{dy}{d\tau_d}\right) \frac{\partial f}{\partial y} + \left(\frac{dz}{d\tau_d}\right) \frac{\partial f}{\partial z} + \left(\frac{dt}{d\tau_d}\right) \frac{\partial f}{\partial t} \\ &= \left(\frac{dt}{d\tau_d}\right) \left[ v_{dx} \frac{\partial}{\partial x} + v_{dy} \frac{\partial}{\partial y} + v_{dz} \frac{\partial}{\partial z} + \frac{\partial}{\partial t} \right] f = \gamma_d \left( \frac{\partial}{\partial t} + \vec{v}_d \cdot \vec{\nabla} \right) f \\ &= \gamma_d \frac{df}{dt}, \end{aligned} \quad (26a)$$

the last equality following from Eq. (17). Thus we obtain the operator relation

$$\frac{d}{d\tau_d} = \gamma_d \frac{d}{dt}. \quad (26b)$$

Since  $\gamma_d = 1 + O(v_d^2/c^2)$ , it is apparent that  $d/dt$  and  $d/d\tau_d$  differ only at second or higher order. It therefore becomes natural in up-grading the invariance properties of Eq. (22) to replace  $d/dt$  wherever it appears there by  $d/d\tau_d$ . This will ensure recovery of Eq. (22) at first order, *i.e.*, in the limit of low detector speed. It will also express *time* variation of electromagnetic phenomena in terms of a parameter independent of coordinate or inertial/noninertial system choice.

It is necessary at the same time to up-grade the invariance properties of the source term,  $u_m = u_s - \rho \vec{v}_d = \rho(\vec{v}_s - \vec{v}_d)$ , appearing in Eq. (22a). It will be noted that only the *relative velocity* ( $\vec{v}_s - \vec{v}_d$ ) of source and detector enters the Hertz equations. To achieve an invariant formulation, the most direct approach is to replace ordinary velocities  $\vec{v}$  (derivatives with respect to  $t$ ) with proper velocities  $\vec{V}$ , defined by

$$\vec{V} = \frac{d\vec{r}}{d\tau} = \frac{d\vec{r}}{dt} \cdot \frac{dt}{d\tau} = \vec{v} \gamma = \frac{\vec{v}}{\sqrt{1 - \frac{v^2}{c^2}}}. \quad (27)$$

One thus postulates *vector additivity of proper velocities* -subject to later confirmation of the kinematic consistency

of so doing- and is hence led to define (with subscripts s and d denoting source and detector, respectively) a proper current,

$$\vec{U}_m = \rho(\vec{V}_s - \vec{V}_d), \quad (28)$$

which is an invariant of direct physical significance: It is the source current *measured* at the source position (*i.e.*, numerical value of a "pointer reading") by a current meter that rigidly co-moves with the detector. (For a discussion of such non-rotary distant "rigid co-motion", see Ref. [8]). Note that when field source and sink are in the same state of motion ( $\vec{V}_s = \vec{V}_d$ )  $\vec{U}_m$  vanishes. Eq. (28) must be considered provisional until the kinematics of velocity composition is elucidated.

Our invariant "neo-Hertzian" equations of electromagnetism are thus postulated to be

$$\vec{V} \times \vec{H} - \frac{1}{c} \frac{d\vec{E}}{d\tau_d} - \frac{4\pi}{c} \vec{U}_m = 0 \quad (29a)$$

$$\vec{V} \times \vec{E} + \frac{1}{c} \frac{d\vec{H}}{d\tau_d} = 0 \quad (29b)$$

$$\vec{V} \cdot \vec{H} = 0 \quad (29c)$$

$$\vec{V} \cdot \vec{E} - 4\pi\rho = 0 \quad (29d)$$

Neo-Hertzian

The reader should not be misled by the superficial resemblance of Eq. (29) to Maxwell's equations. The only relationship is that Eq. (29) reduces to Eq. (1) at  $O(v_s/c)$  when  $\vec{V}_d = 0$ . Otherwise, the physics is quite different, as we shall show in Part II of this paper.

The full invariance group of Eq. (29) has not been identified. The only question concerns the velocity-dependent source term in Eq. (29a). The remaining three of Eqs. (29) appear to be perfectly generally invariant (*not* covariant), given identification of the invariants of kinematics as length and proper time. But, in the case of accelerated relative motions between source and detector, it requires more discussion than can be given here to deal with misgivings about supposed

effects of causal delay of distant actions. (We shall see presently that ordinary ideas of causality do not enter into Hertzian electrodynamics). To resolve all questions will require a combination of continuing theoretical and experimental investigations.

## REFERENCES

- 1 M. Tonnelat, *The Principles of Electromagnetic Theory and of Relativity* (Reidel, Dordrecht, 1966)
- 2 E. Mascart, *Ann. Sc. Ec. Norm. Sup.* 3, 363 (1874)
- 3 A. Eichenwald, *Ann. d. Phys.* 11, 1 and 421 (1903)
- 4 H.A. Wilson, *Phil. Trans.* 204, 121 (1904)
- 5 H.R. Hertz, *Electric Waves* (MacMillan, London and New York, 1900)
- 6 A.I. Miller, *Albert Einstein's Special Theory of Relativity: Emergence (1905) and Early Interpretation (1905-1911)* (Addison-Wesley, Reading, MA 1981)
- 7 T.E. Phipps, Jr., *Found. Phys.* 6, 263 (1976)
- 8 T.E. Phipps, Jr., *Found. Phys.* 10, 289 (1980)
- 9 T.E. Phipps, Jr., *Found. Phys.* 11, 633 (1981)
- 10 J. Bailey et al., *Nature* 268, 301 (1977)
- 11 D.B. Brace, *Phil. Mag.* 6, 317 (1904)
- 12 F.T. Trouton and A.O. Rankine, *Proc. Roy. Soc.* 80, 420 (1908)
- 13 A.B. Wood, G.A. Tomlinson, and L. Essen, *Proc. Roy. Soc.* 158, 606 (1937).