Annales de la Fondation Louis de Broglie,
Vol. 9, n°1, 1984

AN EXPLANATION FOR THE DEVIATION OF SCHRÖDINGER QUANTUM THEORY FROM SIMPLE CLASSICAL DYNAMICAL THEORY IN TERMS OF THE RELATIVISTIC MASS ENERGY EQUIVALENCE

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(manuscrit reçu le 24 Janvier 1983)

Abstract : Using ideas from an electromagnetic, thermal fluid theory which is an alternative to Schrödinger quantum theory but which contains that theory exactly as a subspace structure, it is shown that the "quantum" potential V_Q *is generated by induced "thermal" mass according to the usual relativistic relation* $V_Q = m^{(th.)}C^2 = \kappa T$.

Résumé : En employant des idées dérivées d'une théorie d'un fluide thermique et électromagnétique qui est une alternative à la théorie de Schrödinger de la mécanique quantique, mais qui contient cette théorie exactement comme une structure de sous-espace, il est démontré que le potentiel, "quantique", V_Q, *est engendré par la masse "thermique" produite selon la relation relativiste usuelle,*

$$V_Q = m^{(th.)}C^2 = \kappa T.$$

INTRODUCTION

In recent papers [1,2], the present author has shown that Schrödinger quantum theory for one dimension space can be expressed in terms of classical thermal and electronic fluid processes taking place on a two dimensional surface. This theory supplies an alternative point of view with philosophically radical implications for the "visualisation" and understanding of fundamental microscopic processes. An important feature of this new formulation of quantum theory is that it uncovers the chain of interactions whereby a "quantum particle" induces in its immediate vicinity electrical and thermal currents external to itself, but which act back on to it, in terms of a potential which turns out to be the "quantum" potential. However, although the mathematical structure is very clear, it is not obvious what carries the induced currents that are responsible for this "extra" self influencing force. This paper will be devoted to showing that the extra potential is a consequence of the classical relativistic relation between mass and energy and also of the way in which the quantum density $\rho(x,t)$ is defined. It turns out that the induced currents have their origin in "gradients" associated with the induced thermal mass.

The utilisation of an extra space dimension with coordinate y, as with analytic continuation when $x \rightarrow x + iy$, is an important aspect of this alternative theory. This particular feature appears in similarly motivated work by Blaquière [3] which relates quantum theory with systems theory and it also appears in work by Harper [4].

The parallel with complex function theory arises because the two dimensional velocity field, $\vec{v} = v_1\vec{i} + v_2\vec{j}$ satisfies the two vector differential equations, $\nabla.\vec{\bar{v}} = \nabla\wedge\vec{\bar{v}} = 0$, where $\vec{\bar{v}} = v_1\vec{i} - v_2\vec{j}$, in analogy with the superfluid momentum field, $\vec{p}^{\,s}$.

Work connected with the idea of forming a classical basis for quantum mechanics has been discussed in some detail recently by Cavalleri [5]. The fluid point of view seems to have originated in work by Madelung [6]. Many authors have contributed to this line and, in particular, Takabayasi [7] and Janossy [8] have written important articles. Much of the history of attempts to reformulate quantum mechanics can be found in the very comprehensive account given in Max Jammer's book [9], where can also be found references to a large number of contributors to the analysis of the problems of the fundamental significance of quantum mechanics and its relation to classical theories. Further references to related, or work similarly motivated to that which is about to be discussed in this paper are [10...12].

2. Induced Density

Given a wave function, $\psi(x,t)$, solution to the Schrödinger equation, the orthodox quantum density $\rho(x,t)$ is defined by,

$$\rho(x,t) = \psi^*(x,t)\psi(x,t). \qquad (2.1)$$

This function ρ has the dimension L^{-1} provided ψ is suitably dimensioned. We shall make much use of a real positive function $\rho^{(1)}(x,y,t)$ defined by,

$$\rho^{(1)}(x,y,t) = \psi^*(x-iy,t)\psi(x+iy,t). \qquad (2.2)$$

The bracketed superscript will be used to indicate "minus the dimensionality" of the ρ to which it is attached. Thus $\rho^{(1)}$ has, in common with the ρ in (2.1), the dimension L^{-1}. The $\rho^{(1)}(x,y,t)$ obtained by analytic continuation of $\psi(x,t)$ and formula (2.2) is not a two dimensional density. A two dimensional density is basic to the fluid process that we shall be studying and so we introduce $\rho^{(2)}(x,y,t)$ by

$$\rho^{(2)}(x,y,t) = \ell_0^{-1}\,\rho^{(1)}(x,y,t), \qquad (2.3)$$

where ℓ_0 is a constant with the dimension of length. Thus, apart from the factor ℓ_0, the orthodox ρ is the boundary value on $y = 0$ of $\rho^{(2)}(x,y,t)$. We take this superscript notation further by also introducing a $\rho^{(3)}(x,y,t)$ such that

$$\rho^{(3)}(x,y,t) = \ell_3^{-1}\,\ell_0^{-1}\,\rho^{(1)}(x,y,t), \qquad (2.4)$$

where ℓ_3 is another constant with the dimension length. We need $\rho^{(3)}$ in order to relate the formalism to the three dimenstional charge density of Maxwell's electromagnetic theory. Thus, if we denote three dimensional charge density by $\sigma(x,y,t)$, we have

$$\sigma(x,y,t) = e'\rho^{(3)}(x,y,t), \tag{2.5}$$

where $e' = -e<0$ and e is the magnitude of the charge on one electron. The three densities $\rho^{(1)}$, $\rho^{(2)}$ and $\rho^{(3)}$ differ in dimensionality. However, in the work to be described here, they will all only depend on the two coordinate variables x and y and on the time t. Our first concern is with $m\rho^{(2)}(x,y,t)$, where m is the mass of one electron. This quantity is a mass density which arises from the alternative theory as a basic mass distribution over the x,y plane. It can be taken to represent the degree of mass polarization over the x,y plane or alternatively to represent the strength of the positive pole mass density distribution over the x,y plane resulting from a ± mass polarization. This particular interpretation of $m\rho^{(2)}$ leads, as will be shown, to some very illuminating images concerning the movement of a quantum particle in relation to the locally disturbed vacuum. However, this mass density only represents the primary particle mass distribution and does not contain, as a direct and additive contribution to its magnitude, information about mass which may have been induced by the passage of the primary particle. This interpretation of the significance of $m\rho^{(2)}(x,y,t)$ is, of course, quite consistent with the orthodox interpretation of $m\rho(x,t)$ from which it is derived by formulae (2.2) and (2.3). The quantum density $\rho(x,t)$ is, in the orthodox theory, also assumed only to represent the "primary" particle probability density.

Suppose that the passage of the primary particle actually "stirs up" the polarizable background and so induces a local thermal energy field $\kappa T(x,y,t)$, where κ is Boltzmann's constant and $T(x,y,t)$ is a local temperature. This thermal energy field can be expected to be equivalent to a local thermal mass field, $m^{(th)}(x,y,t)$, by the relativistic law, $\kappa T = m^{(th)}C^2$.

Consequently, the "bare" mass density $\rho^{(2)}(x,y,t)$ should be supplemented by a density contribution, $\delta\rho^{(2)}$, from the induced mass $m^{(th)} = \kappa T/C^2$, to give a dressed mass or total mass density, $\rho_T^{(2)}$.

In order to make this connection between the induced contribution, $\delta\rho^{(2)}$ and $m^{(th)}$, we shall make use of a temperature, T_0, introduced in the work of de Broglie [10],

$$T_0 = m\,c^2/\kappa\ . \tag{2.6}$$

We shall assume that T_0 is the local temperature associated with the absorption of one electron rest mass per area ℓ_0^2 from the rest mass distribution to become the uniformly distributed thermal energy field κT_0. Thus, in the case when the local temperature has the value T_0 in formula (2.6), the bare density should be supplemented by a contribution

$$\delta\rho_0^{(2)} = -1/\ell_0^2 \tag{2.7}$$

to account for the loss of one electron rest mass to the thermal field κT_0.

In general, a local temperature $T(x,y,t)$ will be assumed to cause a supplementation of $\rho^{(2)}$ by a quantity denoted by $\delta\rho^{(2)}$. We now make the simple and plausible assumption that the induced densities are proportional to the inducing temperatures. That is

$$\frac{\delta\rho^{(2)}}{\delta\rho_0^{(2)}} = \frac{T}{T_0}\ . \tag{2.8}$$

Substituting (2.6) and (2.7) into (2.8) gives the formula,

$$\delta\rho^{(2)} = -\frac{\kappa T}{mc^2}\,\ell_0^{-2} \tag{2.9}$$

or

$$m\delta\rho^{(2)} = -\,m^{(th)}\ell_0^{-2} \tag{2.10}$$

In order to proceed further by making use of formula (2.9) we shall use an expression for the temperature T derived from the alternative theory [1]. This formula is,

$$\kappa T = -m\nu^2 \frac{\partial^2 \ln\rho^{(2)}}{\partial x^2} , \quad (2.11)$$

where $\nu = \hbar/2m$. (2.12)

Thus the total density, $\rho_T^{(2)}$, is given by,

$$\rho_T^{(2)} = \rho^{(2)} + \delta\rho^{(2)} = \rho^{(2)} + \left(\frac{\nu}{c}\right)^2 \ell_0^{-2} \frac{\partial^2 \ln\rho^{(2)}}{\partial x^2} , \quad (2.13)$$

having used (2.9) and (2.11).

We now make an "aesthetic" choice for ℓ_0 by taking it to be given by

$$\ell_0 = \frac{\nu}{c} = \frac{\hbar}{2mc} \quad (2.14)$$

and so obtain the rather simple expression,

$$\rho_T^{(2)} = \rho^{(2)} + \frac{\partial^2 \ln\rho^{(2)}}{\partial x^2} . \quad (2.15)$$

We shall continue to explore this structure on the basis of the choice (2.14) and its consequence, (2.15). However, clearly, other possibilities do seem to be open but they will not be considered in this article.

It is reasonable to expect that the ± mass polarization over the x,y plane carries with it a ± charge polarization and also a ± magnetic "pole" polarization. However, it is likely that this latter possible polarization will be the manifestation of the "distributed" magnetic dipole moment of the electron. In order to make use of the three dimensional Maxwell electromagnetic theory formalism, we multiply (2.15) through by ℓ_3^{-1} and obtain,

$$\rho_T^{(3)} = \rho^{(3)} + \ell_3^{-1} \frac{\partial^2 \ln\rho^{(3)}}{\partial x^2} . \quad (2.16)$$

Hence with the total "three" dimensional mass density $m\rho_T^{(3)}$ we can expect also to find a total charge density, $e'\rho_T^{(3)}$ and a total distributed magnetic induction field,

$$\vec{B}_T = \mu_0 \rho_T^{(3)} \beta\vec{k} \quad (2.17)$$

where $\vec{k} = \vec{i} \wedge \vec{j}$ and β has the value of one Bohr magneton,

$$\beta = -\hbar e'/2m . \quad (2.18)$$

Thus multiplying (2.16) through by $(\mu_0 \hbar e/2m)\vec{k}$, we obtain,

$$\vec{B}_T = \vec{B}_0 + \vec{B} , \quad (2.19)$$

where

$$\vec{B}_0 = \mu_0\rho^{(3)} \frac{\hbar e}{2m} \vec{k} \quad (2.20)$$

and

$$\vec{B} = -\mu_0 \frac{\partial^2 \ln\rho}{\partial x^2} \frac{\hbar e'}{2m} \ell_3^{-1} . \quad (2.21)$$

We can express (2.21) in a more illuminating form by making use of further formulae from the alternative theory [1],

$$v_1 = -\nu \frac{\partial \ln\rho^{(3)}}{\partial y} \quad (2.22)$$

$$v_2 = -\nu \frac{\partial \ln\rho^{(3)}}{\partial x} \quad (2.23)$$

and $\nabla^2 \ln\rho^{(3)} = 0$. (2.24)

It follows that

$$\vec{B} = \frac{\partial v_2}{\partial x} e'\mu_0\ell_3^{-1} \vec{k} \quad (2.25)$$

$$= -\frac{1}{2} e \mu_0\ell_3^{-1} \vec{\nabla}\wedge\vec{v} \quad (2.26)$$

$$= -e \mu_0\ell_3^{-1}\vec{\Omega} , \text{ say,} \quad (2.27)$$

where $\vec{\Omega} = \frac{1}{2} \vec{\nabla}\wedge\vec{v}$ is the local fluid angular velocity. We now make another "aesthetic" choice. This time by taking the

the length parameter ℓ_3 in (2.27) to be the classical electron radius multiplied by 4π.

That is, we let,

$$\ell_3 = 4\pi\ell_c = \frac{e^2\mu_0}{m} . \qquad (2.28)$$

Then, (2.27) becomes,

$$\vec{B} = -\frac{m}{e}\vec{\Omega} = -\frac{m}{2e}\vec{\nabla}\wedge\vec{v} . \qquad (2.29)$$

Which, apart from the minus sign, is the cyclotron resonance frequency relation between $\vec{B}$ and $\vec{\Omega}$ and, an attractively simple relation between fluid and magnetic characteristics. The minus sign in (2.29) is a result of the relevant internal equilibrium equations in this context being between osmotic and Lorentz forces rather than between Lorentz and centrifugal forces as in the cyclotron context. Here the configuration of balancing forces is rather more complicated and the details of this can be found in reference [2]. We shall confine our attention in this paper only to the case given by the choice (2.28). Maxwell's electromagnetic theory can now be used to obtain the induced current $\vec{j}^{(t)}$, say, associated with the magnetic induction vector $\vec{B}$,

$$\mu_0\,\vec{j}^{(t)} = \vec{\nabla}\wedge\vec{B} \qquad (2.30)$$

Hence $$\vec{j}^{(t)} = -\frac{m}{2e\mu_0}\vec{\nabla}\wedge\vec{\nabla}\wedge\vec{v} . \qquad (2.31)$$

It follows that

$$\vec{j}^{(t)} = -\frac{m}{2e\mu_0}\vec{\nabla}(\vec{\nabla}.\vec{v}), \qquad (2.32)$$

because $\nabla^2\vec{v} = 0$ as a result of (2.22), (2.23) and (2.24). The magnetic induction $\vec{B}_0$ carried by the primary particle exerts a Lorentz force $\vec{L}_\ell$ per unit volume on the induced current $\vec{j}^{(t)}$.

This Lorentz force is given by,

$$\vec{L}_1 = \vec{j}^{(t)} \wedge \vec{B}_0 = -\vec{R}_Q, \text{ say}, \qquad (2.33)$$

and R_Q will be the Newtonian third law reaction force experienced by the primary particle from the induced back-ground current $\vec{j}^{(t)}$.

From (2.11), (2.32) and (2.33), we see that this extra force, $\vec{R}_Q$, experienced by a quantum particle per unit volume, is

$$\vec{R}_Q = -\vec{j}^{(t)} \wedge \vec{B}_0 = -\rho^{(3)}\nabla(\kappa T) \qquad (2.34)$$

with the quantum potential being $V_Q = \kappa T$. The force, $\vec{R}_Q$, is additional to the force $\vec{F}_e = -\rho^{(3)}\nabla V_1$, derived from the prescribed external potential,

$$V_1(x,y) = \mathrm{Re.}\ \frac{V^*(\bar{z}) + V(z)}{2} . \qquad (2.35)$$

Thus the one dimensional Schrödinger equation is, in all respects, equivalent to the boundary value, when $y = 0$, of the Newtonian equation,

$$\rho^{(3)}\frac{d\vec{p}}{dt} = -\vec{j}^{(t)} \wedge \vec{B}_0 - \rho^{(3)}\nabla V_1, \qquad (2.36)$$

where $$\vec{p} = m(v_1\vec{i} - v_2\vec{j}) = m\nu\vec{k}\wedge\nabla\ell n\rho \qquad (2.37)$$

and $$\frac{d\vec{p}}{dt} = \frac{\partial\vec{p}}{\partial t} + \vec{v}.\nabla\vec{p} . \qquad (2.38)$$

3. Conclusions

This work supplies a clear physical picture of how the passage of the primary particle through the "vacuum" is accompanied by a polarizing disturbance which is both electromagnetic and thermal and has the effect of modifying the expected "simple" Newtonian behaviour of the primary particle. The derivation, in this paper, of the quantum potential, V_Q, shows how this extra potential is generated from the thermal mass, $m^{(th)}$, which in turn has its origin in the local thermal energy κT, with these two quantities being connected by the usual classical relativistic relation between mass and energy.

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