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THE MISSING ELEMENTS OF REALITY IN THE DESCRIPTION OF QUANTUM MECHANICS OF THE E.P.R. PARADOX SITUATION

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Abstract : We show that quantum mechanics is not a complete theory. We do not as in the case of Einstein Podolsky and Rosen derive this incompleteness by a logical reasoning ex absurdum, but indicate explicitely which are the missing elements of reality in the description by quantum mechanics of separated physical systems.

Résumé : Nous démontrons que la mécanique quantique n'est pas complète. Cette incomplétude n'est pas déduite ici par un raisonnement par l'absurde comme l'ont fait Einstein Podolsky and Rosen ; nous indiquons explicitement les éléments les éléments de réalité manquants dans la description par la mécanique quantique des systèmes physiques séparés.

1. Introduction

Einstein, Podolsky and Rosen show that quantum mechanics is not a complete theory [1]. In a recent study of the description of separated physical systems in a more general theory than quantum mechanics we were able to show that quantum mechanics cannot describe separated physical systems [2][3] and that this incapacity of quantum mechanics is at the origin of the incompleteness proof of E.P.R. [4][5]. Indeed in this incompleteness proof E.P.R. use the physical situation of two separated physical systems and they apply quantum mechanics to describe these two separated physical systems. In doing so they make the hypothesis that quantum mechanics describes correctly two separated systems and from this hypothesis they construct elements of reality of the subsystems that are not contained in the quantum mechanical description of these subsystems. Since these subsystems are arbitrary they can conclude that quantum mechanics is not a complete theory *or* that quantum mechanics does not describe correctly separated systems. Since they say in the beginning of their paper that they suppose quantum mechanics to be correct, and hence also quantum mechanics to give a correct description of separated systems, they can conclude that quantum mechanics is not complete. We think that E.P.R. have touched in their reasoning at a serious deficiency of quantum mechanics. The deficiency of quantum mechanics is however not in the description of the subsystems as is indicated by the reasoning of E.P.R. but in the description of the joint system of two separated systems.

It is the description of this joint system of separated systems by means of the tensorproduct of the Hilbert spaces of the subsystems which is not correct, as we show in [4] and [5]. In this paper we shall show that quantum mechanics is not a complete theory, because it cannot describe separated physical systems. And we will not as in the case of E.P.R. derive this incompleteness by a logical reasoning, but we will explicitely indicate which are the missing elements of reality in the description of separated physical systems.

2. Completeness of a theory

Let us recall the definition of element of reality given by Einstein, Podolsky and Rosen : "*If without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity*". The condition of completeness putted forward by E.P.R. is the following : "A theory is complete if every element of reality has a counterpart in the theory".

Clearly E.P.R. did not mean that a theory should describe all possible elements of reality of the physical system. Indeed, if this was what they meant, then of course every theory is incomplete, because a theory only gives a model for the physical system and this model describes a well defined set of elements of reality of the physical system. Therefore we would like to put this criterium of completeness in a different way. We would say that : "*A theory is complete if it can describe every possible element of reality of the physical system, without leading to contradictions*". This completeness criterium should be satisfied by a reasonable physical theory. It means in fact that the theory is flexible enough to provide a model for any well defined set of elements of reality of the physical system. This is not the case for quantum mechanics as we show, because quantum mechanics cannot provide us a model for the description of separated physical systems.

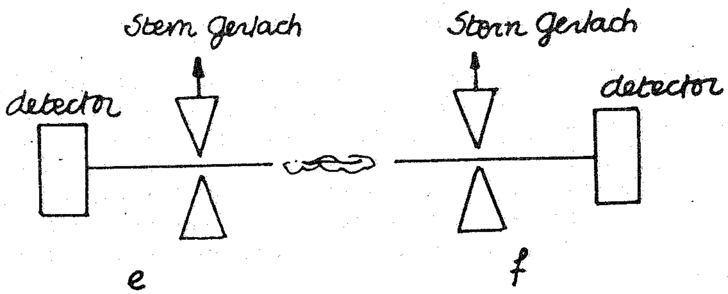
3. Separated physical systems

Two physical systems S_1 and S_2 are separated if a measurement performed on one of the systems does not disturb the other system. This of course does not mean that there is no interaction between S_1 and S_2. In general there is an interaction between separated systems and by means of this interaction the dynamical change of the state of one system is influenced by the dynamical change of the state of the other system. In classical mechanics for example almost all two body problems are problems of separated bodies (e.g. the Keplerproblem).

Two systems are non separated if a measurement on one system disturbs the other systems. We can even say, two systems are non separated, if it is possible to define an element of reality of one system by means of a measurement performed on the other system. For two classical bodies this is for example the case when they are connected by a rigid rod. Let us try to express this idea of separated physical systems in an operational way.

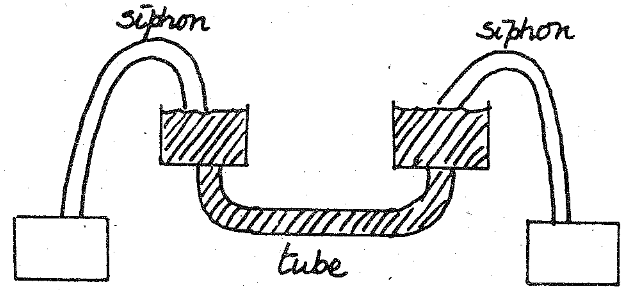
Suppose we consider two experiments e and f on a physical system S with outcome sets E and F. In general it is not possible to perform e and f together. This because often the performance of one of the experiments changes the state of the system in such a way, that it becomes impossible to perform the other experiment. Sometimes however it is possible to perform e and f together. This means that there is a new experiment which we shall denote $e \times f$ with outcome set $E \times F$, such that the performance of $e \times f$ is the performance of e and f together. This means that if we perform the experiment $e \times f$, and we find one of the outcomes (x,y) for a certain $x \in E$ and an arbitrary $y \in F$, then we interpret this as the outcome x for the experiment e. If we perform the experiment $e \times f$ and we find one of the outcomes (x,y) for certain $y \in F$ and an arbitrary $x \in E$, then we interpret this as the outcome y for the experiment f.

Example 1 : Consider a system S of two spin 1/2 particles in the singlet spin state. We perform an experiment e that consists of measuring the spin of one of the particles in a certain direction in one region of space, and a measurement f of the spin of the other particle in the same direction in an opposite region of space. The outcome sets of e and f are {0,1} where 0 means that the electron is absorbed and 1 means that the electron has passed the Stern Gerlach. What we mean is the well known experiment proposed by Bohm [6] and carried out meanwhile several times to test Bell inequalities.



$e \times f$ consists of performing e and f at once. The outcome set of $e \times f$ is {(0,0), (0,1), (1,0), (1,1)}. As is shown by the experiments [7] and as is also predicted by quantum mechanics, for $e \times f$ we always find one of the outcomes (0,1) or (1,0). For e however we can find the outcome 0 and 1 both with probability 1/2 and also for f we can find the outcome 0 and 1 both with probability 1/2.

Example 2 : Consider a system S consisting of two vessels containing each 10 l. of water connected by a tube as is shown on the figure. The experiment e consists of testing whether the volume of the water contained in the first vessel is more than 10 l. We perform this experiment by emptying the vessel by means of a siphon and collecting the water in a reference vessel. We give the outcome 1 if the water stops flowing after it depasses 10 l. in the reference vessel and we give the outcome 0 if the water stops flowing before it depasses 10 l. The experiment f consists of testing whether the volume contained in the second vessel is more or equal to 10 l.



$e \times f$ consists of performing e and f at once. Again we see that for $e \times f$ we always find the outcomes (0,1) or (1,0). For e however we always find the outcome 1 and for f we always find the outcome 1.

In both examples we see that some of the combinations of outcomes of e and f are not possible for the experiment $e \times f$. Indeed in both cases 1 is a possible outcome for e and 1 is a possible outcome for f but (1,1) is not possible for $e \times f$.

This indicates that e × f is really a new experiment. It is not just the performance of e and f together. In both examples this is due to the fact that the system consists of two systems that are not separated. These examples inspire us the following intuitively clear definition of separated experiments.

Definition 1 : Two experiments e and f are separated iff

i) they can be performed together
ii) if x is a possible outcome for e and y is a possible outcome for f, then (x,y) is a possible outcome for e × f
iii) if (x,y) is a possible outcome for e × f, then x is a possible outcome for e and y is a possible outcome for f.

Clearly in both examples this definition is not satisfied such that in both examples e and f are non separated experiments.

Definition 2 : Two physical systems S_1 and S_2 are separated iff all experiments of S_1 are separated from all experiments of S_2.

4. The missing elements of reality

We shall now proof that an experiment of the type e × f where e and f are separated experiments cannot be described by quantum mechanics.

Theorem 1 : If e and f are separated experiments on a physical system S, then the experiment e × f cannot be described by quantum mechanics.

Proof : Suppose we can describe e × f by quantum mechanics. Then there exist a Hilbert space H and a selfadjoint operator O corresponding to this experiment. Since also e and f are experiments, there exists selfadjoint operators R and S corresponding to the experiments e and f. If E is the outcome set of e and F is the outcome set of f, the E × F is the outcome set of e × f. If $x \in E$ and $y \in F$ we will denote by P_x, P_y and $P_{(x,y)}$ the projection operators of the spectral decompositions

of the operators R, S and O. Consider now an arbitrary (x,y) $\in E \times F$ and a vector $v \in \mathcal{H}$ such that $v \not\perp P_{(x,y)}$. If the physical system is in the state v, then (x,y) is a possible outcome for e × f. As a consequence x is a possible outcome for e and y is a possible outcome for f. Hence $v \not\perp P_x$ and $v \not\perp P_y$. So if $w \in H$ and $w \perp P_x$ or $w \perp P_y$, then $w \perp P_{(x,y)}$. This means that $1 - P_x \subset 1 - P_{(x,y)}$ and $1 - P_y \subset 1 - P_{(x,y)}$. From this follows that $P_{(x,y)} \subset P_x$ and $P_{(x,y)} \subset P_y$. Or $P_{(x,y)} \subset P_x \cap P_y$. Consider now a vector $v \in H$ such that $P_x v = v$ and $P_y v = v$. Then if the physical system is in the state v, x is a certain outcome for e and y is a certain outcome for f. As a consequence (x,y) is a certain outcome for e × f.

This means that $P_{(x,y)} v = v$. From this follows that $P_x \cap P_y \subset P_{(x,y)}$ and as a consequence $P_{(x,y)} = P_x \cap P_y$. But then $\oplus_{(x,y) \in E \times F} P_x \cap P_y = 1$. From this follows that $[P_x, P_y] = 0$. As a consequence $[R,S] = 0$, which shows that e and f have to be compatible experiments. If R and S commute we also have $P_x \cap P_y = P_x . P_y$ and from the foregoing then follows that $P_{(x,y)} = P_x . P_y$.

Consider now two outcomes x, z of e and two outcomes y, t of f such that $P_x . P_y \neq 0$ and $P_z . P_t \neq 0$. Consider two vectors v, w such that $P_x . P_y \, v = v$ and $P_z . P_t \, w = w$ and consider then the vector u = v + w.

Then
$$P_x(u) = P_x(v) + P_x(w) = v$$
$$P_z(u) = P_z(v) + P_z(w) = w$$
$$P_y(u) = P_y(v) + P_y(w) = v$$
$$P_t(u) = P_t(v) + P_t(w) = w$$

Hence u is not an eigenvector of P_x, P_z, P_y, P_t such that for e the outcomes x and z are possible and for f the outcomes y and t are possible.

We can even choose very easily u in such a way that we have probability 1/2 to have the outcome x for e and probability 1/2 to have the outcome z for e and that we have probability 1/2 to have the outcome y for f and probability 1/2 to have the outcome t for f.

But $P_x.P_y(u) = v$ and $P_z.P_t(u) = w$ while

$P_x.P_t(u) = 0$ and $P_z.P_y(u) = 0$.

This means that for e × f we only have possible outcomes (x,y) and (z,t) if the system is in the state u, and (x,t) and (z,y) are not possible.

This shows that e and f are not separated experiments since ii) of the definition 1 is not satisfied.

Suppose now that we have two separated systems S_1 and S_2. If e is an experiment on S_1 and f is an experiment on S_2 then e × f is an experiment of the type that cannot be described by quantum mechanics. It are the elements of reality of the joint system S consisting of the two separated systems S_1 and S_2 defined by experiments of the type e × f that cannot be described by quantum mechanics. This incapacity of quantum mechanics leads to the contradiction in the E.P.R. reasoning.

Completing quantum mechanics cannot be achieved by changing the description of the subsystems by adding additional variables (often called hidden variables) to this description because it is the description of the joint system which is wrong and has to be changed. In [2] and [3] we give such a description of separated physical systems in a more general theory as quantum mechanics, and we see that the mathematical structure of the set of states of the joint system is indeed not a vectorspace structure. Such that the superposition principle shall not be valid in the description of this joint system.

5. The E.P.R. reasoning

Since we showed that separated systems cannot be described by quantum mechanics, the E.P.R. reasoning is still valid but becomes a reasoning ex absurdum. Indeed E.P.R. suppose that quantum mechanics *can* describe separated systems. Let us analyse again their reasoning knowing this result. E.P.R. consider the following two sentences (1) quantum mechanics is not complete (2) physical quantities that are not compatible cannot have simultaneous reality. Obviously these two sentences cannot both be wrong. Indeed if two non compatible quantities can have simultaneous reality then quantum mechanics is not complete, because the wave function cannot describe these elements of reality. So we have one of the three cases

A (1) false (2) true   B(1) true (2) false
C (1) true (2) true

Once E.P.R. come to this conclusion they consider the situation of two separated systems S_1 and S_2. *By applying quantum mechanics* to describe these two separated systems, and using the Schrödinger effect they can show that it is possible to attach simultaneously elements of reality to non compatible quantities. Hence (1) has to be true. So what E.P.R. show is the following :

Quantum mechanics describes correctly separated systems
Quantities that are not compatible can have simultaneous reality
Quantum mechanics is not complete.

From this they can conclude that quantum mechanics does not describe correctly separated systems or quantum mechanics is not complete. E.P.R. mention in the beginning of their paper that they suppose quantum mechanics to be correct, and then they can indeed conclude that quantum mechanics is not complete. If one supposes quantum mechanics to give a correct description of separated systems, this reasoning of E.P.R. indicates which are the missing elements of reality in quantum mechanics. These are elements of reality corresponding to non commuting observables. This made a lot of people think that it should be possible to solve the problem by introducing classical hidden variables that take into account these missing elements of reality, and in this way complete quantum mechanics. If we take into account that quantum mechanics is wrong in the description of separated systems, then the E.P.R. reasoning is still valid and the conclusion is correct. But because the premise is false it does not indicate missing elements of reality. Indeed the statement "Quantum mechanics is not complete because quantities that are not compatible can have simultaneous reality" is not a true statement, because it is only a statement used in the reasoning ex absurdum. I want to put the attention to this fact because often people are not interested in the conclusion of E.P.R., namely that quantum mechanics is incomplete if it is correct, but in the conclusion that quantities that are not compatible can have simultaneous reality if quantum mechanics is correct. And it is this conclusion that leads directly to the thought that to avoid the E.P.R. problem, we have to build a theory that takes into account the fact that non compatibles physical quantities can have simultaneous reality. Let us sum up all this and see that there is no paradox left. There are two possible situations for two systems.

First situation :

The two systems are separated. Then quantum mechanics gives not a correct description of this situation. Correcting quantum mechanics does not happen by adding states to the subsystems but by taking states away of the compound system.

Second situation :

The two systems are not separated. In this case, as we explained in section 2, it is not possible to make the E.P.R. reasoning. Indeed, it is not possible to give an element of reality to one system by performing a measurement on the other system.

So as we see, the E.P.R. paper touches at a major shortcoming of quantum mechanics, namely its incapacity to describe separated systems. Since the E.P.R. reasoning is a reasoning ex absurdum, it however does not indicate the way to solve the problem.

6. Conclusion

We can conclude and say that the shortcoming of quantum mechanics of not being able to describe separated systems is due to the fact that the mathematical structure of quantum mechanics always forces by means of the superposition principle the existence of states that should not be there in the case of separated systems. One could think in the following way : why not just drop these states and go on with quantum mechanics. This is indeed the solution that has been taken for the problem in connection with superselection rules.

For the case of separated systems this solution is not possible because the wrong superpositions are not superpositions between states of different orthogonal subspaces. If one drops the states that should not be there, nothing of the rest of the mathematical structure of quantum mechanics can be applied anymore. Let us give an example to show this.

In general the evolution of a system from one instant to another one is described by an automorphism of the set of states. In quantum mechanics such automorphisms are represented by unitary (or antiunitary) operators of the Hilbert space describing the system. In classical mechanics they are permutations of the state space describing the system. If we have in classical mechanics a system S described in a state space Γ consisting of two systems S_1 and S_2 described in state spaces Γ_1 and Γ_2, then $\Gamma = \Gamma_1 \times \Gamma_2$. It is easy to see that a permutation of Γ cannot always be decomposed in the product of a permutation of Γ_1 and a permutation of Γ_2. Moreover it are these non product permutations of Γ that make it possible to describe an evolution with interactions between S_1 and S_2. On the other hand, if we have in quantum mechanics a system S described in a Hilbert space H composed of two systems S_1 and S_2 described in Hilbert spaces H_1 and H_2, then $H = H_1 \otimes H_2$. If we would now try to give a description of the evolution of two separated systems in this tensorproduct Hilbert space, we must consider these unitary transformations that conserve product states. But a unitary transformation U that conserves product states is always of the form $U_1 \otimes U_2$. We shall then never be able to describe interactions between the two systems.

We must make one other remark. A logical possible way out is to say that separated systems do not exist, and therefore we do not have to try to describe them by quantum mechanics. Often people seem to consider this solution as the correct one. This is however because they have a wrong intuitive image of what separated systems are. When two systems are separated this does not mean that there is no interaction between the two systems. No, in general there is an interaction and by means of this interaction the dynamical change of the state of one system is influenced by the dynamical change of the state of the other system. In classical mechanics for example, almost all two body problems are problems of separated bodies (e.g. the Keplerproblem). It are the analogies of these problems for quantum systems that can not be treated in quantum mechanics. Two systems are non separated, when a measurement on one system disturbs the other system. Or we would even say, two systems are non separated, when it is possible to define a property (element of reality) of one system that can be tested by means of a measurement on the other system. For two classical bodies this is for example the case when they are connected by a rigid rod. But we know that in this case the two bodies connected by a rigid rod are treated as a one body problem. To give still a better intuitive feeling of what are separated systems we could say, that one system is separated from the rest of the universe but one system is not separated from the measuring apparatus during a measurement.

Si it is this whole range of interesting situations of two separated systems with interaction between them that can not be treated in quantum mechanics.

REFERENCES

[1] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. 47, 777 (1935)

[2] D. Aerts, "The one and the many, Towards a unification of the quantum and the classical description of one and many physical entities"
Doctoral thesis, Vrije Universiteit Brussel, TENA, (1981)

[3] D. Aerts, "Description of many separated physical entities without the paradoxes encountered in quantum mechanics"
Foundations of Physics 12, 1131 (1982)

[4] D. Aerts, "How do we have to change quantum mechanics in order to describe separated systems"
in "The wave-particle dualism" edited by S. Diner, D. Fargue, G. Lochak, F. Selleri ; D. Reidel (1984).

[5] D. Aerts, "The physical origin of the Einstein Podolsky Rosen paradox"
Submitted to Foundations of Physics.

[6] D. Bohm, Quantum Theory (Prentice-Hall, 1951).