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MAGNETOHYDRODYNAMIC WAVES, ELECTROHYDRODYNAMIC WAVES AND PHOTONS

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Résumé : Deux nouveaux sujets ont dernièrement attiré fortement l'attention : la Magnétohydrodynamique (m.h.d.) et la théorie du laser. Tout aussi important est le domaine de l'Electrohydrodynamique (c.h.d.), qui pourrait expliquer des phénomènes spectaculaires mais terrifiants de l'électricité atmosphérique : la foudre et la tornade. Ce dernier a été l'objet d'une étude introductive de l'auteur [1].

*Or, de toute évidence, toutes les ondes électromagnétiques portent des photons ; c'est le mérite de L. de Broglie [2] d'avoir concilié la validité des équations de Maxwell avec l'existence du photon. J'ai récemment [3] déduit les équations de de Broglie des équations C.**

Il semble naturel d'admettre que les ondes de la m.h.d. portent aussi des photons ; mais comment concilier les axiomes de la m.h.d. avec l'existence des photons ? ... c'est

*N.D.L.R. - Les "équations C" auxquelles l'auteur se réfère font en réalité partie de ses travaux originaux, mais n'ont pas jusqu'ici été publiées par lui. Aussi l'avons-nous prié de les expliciter plus amplement, ce qu'il a fait dans l'appendice I.

un problème qui, jusqu'ici, a échappé à l'attention des physiciens. Aussi bien, nous parlons de l'herbe toujours verte sans trop nous demander pourquoi elle est verte.

Dans ce qui suit, nous tentons d'incorporer de façon simple les photons dans les ondes m.h.d. (resp. dans les ondes e.h.d.).

Abstract : Two news subjects have lately attracted increased attention : the magnetohydrodynamics (m.h.d.) and the theory of lasers. Equally important is the subject of electrohydrodynamics (e.h.d.) which might explain spectacular but awesome phenomena in the atmospheric electricity : the lightning and the tornado ; an introductory study of the latter has been written by the author [1].

Now, clearly, all electromagnetic waves cary photons ; it is the merit of Louis de Broglie [2] to have had reconciled the validity of the Maxwell equations with existence of the latter. I [3] have, recently, derived L. de Broglie's equations from the equations C.*

It seems natural to assume that the m.h.d. waves carry also photons, but how to reconcile the m.h.d. axioms with the existence of photons ? ... a problem which has, so far, escaped the notice of physicists. Well, we talk about the grass as continuing to be green, without thinking too much why it is green !

In the lines which follows, an attempt is made to incorporate the photons in the m.h.d. waves, re e.h.d. waves in a rather simple fashion.

I hope that a more expanded account of this *first* investigation will be continued, and that suitable experiments will be made. The relativistic magnetohydrodynamics, superbly analyzed by Lichnerowicz [4], would get a new growth and a number of important problems, in physics, astrophysics, and biology, could be solved.

*Voir page précédente

1. Fundamental equations and some simple consequences

The equations of magnetohydrodynamics, re. electrohydrodynamics, are the electromagnetic and hydrodynamic equations, modified to take account of the interaction between the motion and the magnetic and electric fields.

Here, the electromagnetic variables are supposed to be measured in *meter-kilogram-second-coulomb* units.

We shall be concerned with the following equations:

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\mu \vec{J}_m, \qquad \text{div } \vec{B} = \mu \rho_m, \tag{1}$$

$$\text{curl } \vec{B} - \frac{1}{C^2}\frac{\partial \vec{E}}{\partial t} = \mu \vec{J}_e, \quad \text{div } \vec{E} = \frac{1}{\varepsilon}\rho_e,$$

$$\vec{J}_e - \rho_e \vec{v} = \sigma_e(\vec{E}+\vec{v}\times\vec{B}), \quad \frac{\partial \rho_e}{\partial t} + \text{div } \vec{J}_e = 0, \tag{2}$$

$$\vec{J}_m - \rho_m \vec{v} = \sigma_m(\vec{B} - \frac{1}{C^2}\vec{v}\times\vec{E}), \quad \frac{\partial \rho_m}{\partial t} + \text{div } \vec{J}_m = 0,$$

$$\vec{E} = -\frac{\partial \vec{A}_e}{\partial t} - \text{grad } \phi_e - C^2 \text{curl } \vec{A}_m, \tag{3}$$

$$\vec{B} = -\frac{\partial \vec{A}_m}{\partial t} - \text{grad } \phi_m + \text{curl } \vec{A}_e,$$

$$\rho \frac{D\vec{v}}{Dt} = -\text{grad } p + \rho\vec{g} + \rho_e\vec{E} + \vec{J}_e\times\vec{B} + \rho_m\vec{B} - \frac{1}{C^2}\vec{J}_m\times\vec{E}, \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho\vec{v}),$$

where $\vec{E}$ is the electric field, $\vec{B}$ the magnetic induction, $\vec{J}_e$ and $\vec{J}_m$ the electric and magnetic current densities, ρ_e and ρ_m the electric and magnetic charge densities, σ_e and σ_m the electric and magnetic conductivities, $\vec{A}_e$, ϕ_e and $\vec{A}_m$, ϕ_m the electromagnetic potentials ; $\vec{v}$ is the velocity of a fluid particle, p the pressure, ρ the density of the fluid, and $\vec{g}$ the extraneous mechanical force, say, gravity, per unit volume.

C *represents the velocity of propagation of an electromagnetic wave front in the material* (in vacuum C = c, the velocity of light), and ε and μ are the permitivities (dielectric constant and magnetic permeability) of the fluid corresponding to the condition : $\varepsilon\mu C^2 = 1$. D/Dt is the mobile operator

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\vec{v}.\nabla) \ . \tag{5}$$

The dimensions of various classical electromagnetic quantities are given in the Appendix I (p. 602) in the valuable book "Electromagnetic Theory" by Stratton [5]. In order to check our equations (I must say, I get lost in other units), I shall implement them by the following :

$$\begin{aligned} &[\rho_m] = L^{-2}T^{-1}Q, \quad [\phi_m] = MLT^{-1}Q^{-1}, \quad [\sigma_m] = M^{-1}L^{-1}T^{-1}Q^2, \\ &[\vec{J}_m] = L^{-1}T^{-2}Q, \quad [\vec{A}_m] = MQ^{-1}, \quad [\vec{A}_e] = MLT^{-1}Q^{-1} \ . \end{aligned} \tag{6}$$

It is worth observing that : $[\sigma_m/\sigma_e] = L^2T^{-2} = [v^2]$.

The quantities $\vec{J}_m$ and $\vec{A}_m$ represent *axial vectors* and ρ_m is a *pseudoscalar* (the divergence of a tensor of second order which is $\vec{H}$ is a vector ; hence, ρ_m is a vector of length zero !). There is not yet definite confirmation of physical existence of the quantities $\vec{J}_m$ and ρ_m ; *they are, however, useful !*

After this, a little long but "*sine qua non*" preamble, we come to the subject of magnetohydrodynamics. We rewrite the first equation (1) as follows

$$-\text{curl}(\vec{v}\times\vec{B}) + \frac{\partial\vec{B}}{\partial t} = -\text{curl}\ \vec{E}^* - \mu\vec{J}_m, \tag{7}$$

where, as customary, we put

$$\vec{E}^* = \vec{E} + \vec{v} \times \vec{B}. \tag{8}$$

Expanding the curl, we have

$$\frac{D\vec{B}}{Dt} - (\vec{B}.\nabla)\vec{v} - \mu\ \rho_m\vec{v} - \frac{1}{\rho}\frac{D\rho}{Dt}\vec{B} = -\ \text{curl}\ \vec{E}^* - \mu\ \vec{J}_m. \tag{9}$$

Thus

$$\frac{D}{Dt}\left(\frac{\vec{B}}{\rho}\right) = \left(\frac{\vec{B}}{\rho}.\nabla\right)\vec{v} = -\ \frac{1}{\rho}\ \text{curl}\ \vec{E}^* - \frac{\mu}{\rho}(\vec{J}_m - \rho_m\vec{v}), \tag{10}$$

which is of fundamental importance.

When the conditions

$$\sigma_m = 0, \quad \vec{E} + \vec{v} \times \vec{B} = 0, \tag{11}$$

are fulfilled, equation (10) reduces to

$$\frac{D}{Dt}\left(\frac{\vec{B}}{\rho}\right) = \left(\frac{\vec{B}}{\rho}.\nabla\right)\vec{v} \ , \tag{12}$$

which shows that *the quantity* $\vec{B}/\rho$ *is frozen in the fluid.*

The resulting equations

$$\vec{J}_e - \rho_e\vec{v} = 0, \quad \vec{J}_m - \rho_m\vec{v} = 0 \tag{13}$$

yield, by virtue of *equations of continuity* in (2),

$$\frac{1}{\rho_e}\frac{D\rho_e}{Dt} + \text{div}\ \vec{J}_e = 0, \quad \frac{1}{\rho_m}\frac{D\rho_m}{Dt} + \text{div}\ \vec{J}_m = 0. \tag{14}$$

which lead to :

$$\frac{D}{Dt}\left(\frac{\rho_e}{\rho_m}\right) = 0, \quad \frac{\rho_e}{\rho_m} = \text{const., } \textit{along a trajectory}. \tag{15}$$

We also have

$$\frac{\rho_e}{\rho} = \text{const.}, \quad \frac{\rho_m}{\rho} = \text{const.}, \tag{16}$$

where the *hydrodynamic equation of continuity* in (4) has been used. The first relation (16) constitues a beautiful result due to Lichnerowicz [6](p.101).

Before going further, let us stop for a moment to observe an elegant theorem, regarding the "induction equation" (12), due to Elsasser [7], namely : "*on the basis of purely electromagnetic measurements one cannot distinguish between a state of uniform rotation, say, the earth, and a state of rest*". We shall prove this theorem, in a little while, for the electric field.

We now go, *mutatis mutandis*, to the subject of electrohydrodynamics.

Equation (10) is now replaced by

$$\frac{D}{Dt}\left(\frac{\vec{E}}{\rho}\right) = \left(\frac{\vec{E}}{\rho}.\nabla\right)\vec{v} + \frac{C^2}{\rho}\,\mathrm{curl}\,\vec{B}^* - \frac{1}{\epsilon\rho}(\vec{J}_e - \rho_e\vec{v}), \tag{17}$$

where

$$\vec{B}^* = \vec{B} - \frac{1}{C^2}\,\vec{v}\times\vec{E}\ . \tag{18}$$

When the conditions

$$\sigma_e = 0,\quad \vec{B} - \frac{1}{C^2}\,\vec{v}\times\vec{E} = 0, \tag{19}$$

are fulfilled, equation (17) reduces to

$$\frac{D}{Dt}\left(\frac{\vec{E}}{\rho}\right) = \left(\frac{\vec{E}}{\rho}.\nabla\right)\vec{v}, \tag{20}$$

which shows that, this time, *the quantity* $\vec{E}/\rho$ *is frozen in the fluid*.

It is to be noticed that the *relations (13) to (16) continue to be valid*.

We now prove the invariance of equation (20) under a uniform rotation $\vec{\Omega}$ of the axes. As well known, we have

$$\frac{D}{Dt}\left(\frac{\vec{E}}{\rho}\right) = \left[\frac{D}{Dt}\left(\frac{\vec{E}}{\rho}\right)\right]_0 + \vec{\Omega}\times\frac{\vec{E}}{\rho}\ , \tag{21}$$

where the subscript 0 refers to the nonrotating system. This formula is valid for any vector whatever. On applying it to the

velocity $\vec{v}$, we have

$$\vec{v} = \vec{v}_0 + \vec{\Omega}\times\vec{r},\quad \left(\vec{v} = \frac{D\vec{r}}{Dt} = \dot{\vec{r}}\right). \tag{22}$$

The invariance of (20) follows (see Appendix II).

2. Magnetohydrodynamic waves and photons

We shall take Cowling's [8] *Magnetohydrodynamics* as a reference text. Consider an infinite mass of uniform fluid penetrated by a uniform field $\vec{B}_0$. As a result of a small disturbance, suppose that a velocity field $\vec{v}$ is produced in a certain region of the fluid, and that the magnetic induction becomes $\vec{B}_0 + \vec{b}$, we shall assume an infinite electric conductivity of the fluid, $\sigma_e \to \infty$; this leads to the condition

$$\vec{E} + \vec{v}\times\vec{B}_0 = 0\ . \tag{23}$$

We shall further assume that the charges are vanishingly small, and neglect squares and products of small quantities. Under these conditions, the equations giving the variations in $\vec{b}$ and $\vec{v}$ [see equations (10) and (4)] are

$$\frac{\partial\vec{b}}{\partial t} = (\vec{B}_0.\nabla)\vec{v} - \mu\,\vec{J}_m, \tag{24}$$

$$\rho_0\,\frac{\partial\vec{v}}{\partial t} = -\,\mathrm{grad}(p + \rho_0 U) + \vec{J}_e\times\vec{B}_0 \tag{25}$$

where it is assumed that : $\vec{g} = -\,\mathrm{grad}\,U$.

The latter equation can be rewritten as

$$\rho_0\,\frac{\partial\vec{v}}{\partial t} = -\,\mathrm{grad}\left(p + \frac{\vec{B}_0.b}{\mu} + \rho_0 U\right) + \frac{1}{\mu}(\vec{B}_0.\nabla)\vec{b}\ . \tag{26}$$

As shown by Cowling [8](p.35), the quantity $p + \frac{\vec{B}_0.\vec{b}}{\mu} + \rho_0 U$ is a constant. Thus, by taking Oz parallel to $\vec{B}_0$, equations (24) and (26) become

$$\frac{\partial\vec{b}}{\partial t} = B_0\,\frac{\partial\vec{v}}{\partial z} - \mu\,\vec{J}_m, \tag{27}$$

$$\frac{\partial \vec{v}}{\partial t} = \frac{1}{\mu \rho_0} B_0 \frac{\partial \vec{b}}{\partial z} \tag{28}$$

We now introduce the photons by writing*

$$\vec{J}_m = -\frac{1}{\mu} C^2 k_0^2 \vec{A}_m, \tag{29}$$

where $k_0 = (1/h)m_0C$ is L. de Broglie's constant ($2\pi h$ is Planck's constant), and m_0 is the proper mass of the photon in the fluid.

Thus equation (27) becomes

$$\frac{\partial \vec{b}}{\partial t} = B_0 \frac{\partial \vec{v}}{\partial z} + C^2 k_0^2 \vec{A}_m . \tag{30}$$

Cross-differentiation of the terms in equations (28) and (30) yields

$$\begin{aligned} \frac{\partial^2 \vec{b}}{\partial t^2} &= V_A^2 \frac{\partial^2 b}{\partial z^2} + C^2 k_0^2 \frac{\partial \vec{A}_m}{\partial t}, \\ \frac{\partial^2 \vec{v}}{\partial z^2} &= V_A^2 \frac{\partial^2 \vec{v}}{\partial z^2} + \frac{V_A^2}{B_0} C^2 k_0^2 \frac{\partial \vec{A}_m}{\partial z}, \end{aligned} \tag{31}$$

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*The relations

$$\text{(B)} \quad \begin{aligned} \vec{J}_e &= -\frac{1}{\mu} k_0^2 \vec{A}_e, & \rho_e &= -\varepsilon k_0^2 \phi_e, \\ \vec{J}_m &= -\frac{1}{\mu} C^2 k_0^2 \vec{A}_m, & \rho_m &= \frac{1}{\mu} k_0^2 \phi_m, \end{aligned}$$

were derived by the author [3] in a previous Article from the modified equations C. One can, backwards, find the latter equations by substitution of the values (B) in the relations (3). One has

$$\text{(C)} \quad \begin{aligned} \text{curl } \vec{J}_e - \frac{1}{C^2} \frac{\partial \vec{J}_m}{\partial t} &= \text{grad } \rho_m - \frac{1}{\mu} k_0^2 \vec{B}, \\ \text{curl } \vec{J}_m + \frac{\partial \vec{J}_e}{\partial t} &= -C^2 \text{ grad } \rho_e + \frac{1}{\mu} k_0^2 \vec{E}. \end{aligned}$$

where $V_A = B_0/\sqrt{\mu\rho}$ is the Alfvén velocity. This is the new wave *motion of our conducting fluid embedded in the field* B_0 *and carrying photons*. When m_0 is null or negligible one finds the m.h.d. waves, in the classical sense (see, Çowling [8]).

We shall improve this our result with the help of relations (31) and L. de Broglie equations. We have

$$\frac{\partial \vec{A}_m}{\partial t} = -\vec{b} + \text{curl } \vec{A}_e = -\vec{b} + \frac{1}{k_0^2} \nabla^2 \vec{b}, \tag{32}$$

for $\phi_m = 0$ ($\rho_m = 0$), and $\vec{A}_e = -\frac{1}{k_0^2} \text{ curl } \vec{b}$ by virtue of de Broglie's equations. Thus equations (31) become

$$\begin{aligned} \Box \vec{b} + \frac{V_A^2}{C^2} \frac{\partial^2 b}{\partial z^2} &= k_0^2 \vec{b}, \qquad \Box = \nabla^2 - \frac{1}{C^2} \frac{\partial^2}{\partial t^2} \\ \Box \vec{v} + \frac{V_A^2}{C^2} \frac{\partial^2 \vec{v}}{\partial z^2} &= k_0^2 \vec{v}. \end{aligned} \tag{33}$$

In the absence of the field $\vec{B}_0$, these equations (or rather the first equation) reduce to L. de Broglie's equations.

These equations admit solutions of the form (plane waves)

$$e^{i(\omega t - \vec{k}.\vec{r})}, \qquad \vec{k}.\vec{r} = k_x x + k_y y + k_z z,$$

which will be discussed in our incoming book [9]. Let us notice here that the *wave-number surface* (locus of points k_x, k_y, k_z for given ω) is

$$k_x^2 + k_y^2 + \left(1 + \frac{V_A^2}{C^2}\right)k_z^2 = \frac{\omega^2}{C^2} - k_0^2 .$$

This is an ellipsoid of revolution, with its axis the k_z axis (which represents the undisturbed magnetic field). The *reduced length* of the latter lies between 1, corresponding to $B_0 = 0$, and 2, which corresponds to $V_A = C$.

The effect of compressibility (sound wave) is omitted here. In the absence of photons, the reader is referred to the beautiful studies of Lighthill [10] and contributions of this writer [11].

3. Electrohydrodynamic k^2 axes and photons. *"Plus ça change, plus c'est la même chose"* tells a French old proverb. This is conspicuously true for electrohydrodynamic waves and photons when compared to m.h.d. waves. As we shall see, shortly, the final equations we obtain are exactly same equations (33), where only $\vec{b}$ is repladed by the variation $\vec{e}$ in the electric field, and the Alfvén velocity V_A is replaced by the velocity V_C, whose definition is given below.

But, let us start approximating the equation (17). We shall assume that there is a uniform electric field $\vec{E}_0$ which penetrates an infinite mass of uniform fluid. As a result of a small disturbance the electric field becomes $\vec{E}_0 + \vec{e}$. The condition (23) is replaced by

$$\vec{B} - \frac{1}{C^2}\,\vec{v} \times \vec{E}_0 = 0 \qquad (35)$$

assuming, this time, that the fluid under consideration has an infinite *magnetic conductivity*, $\sigma_m \to \infty$.

The equations giving the variations in $\vec{e}$ and $\vec{v}$ are

$$\frac{\partial \vec{e}}{\partial t} = (\vec{E}_0.\nabla)\vec{v} - \frac{1}{\varepsilon}\,\vec{J}_e, \qquad (36)$$

$$\rho_0\,\frac{\partial \vec{v}}{\partial t} = -\mathrm{grad}(p + \rho_0 U) - \frac{1}{C^2}\,\vec{J}_m \times \vec{E}_0. \qquad (37)$$

The latter equation can be rewritten as

$$\rho_0\,\frac{\partial \vec{v}}{\partial t} = -\mathrm{grad}(p + \varepsilon\vec{E}_0.\vec{e} + \rho_0 U) + \varepsilon(\vec{E}_0.\nabla)\vec{e}. \qquad (38)$$

Observing, as before, that the quantity

$p + \varepsilon\vec{E}_0.\vec{e} + \rho_0 U$ is a constant, and taking Oz parallel to $\vec{E}_0$, equations (36) and (38) become

$$\frac{\partial \vec{e}}{\partial t} = E_0\,\frac{\partial \vec{v}}{\partial z} - \frac{1}{\varepsilon}\,\vec{J}_e\,, \qquad (39)$$

$$\rho_0\,\frac{\partial \vec{v}}{\partial t} = \varepsilon\, E_0\,\frac{\partial \vec{e}}{\partial z}\,. \qquad (40)$$

We now introduce the photons by writing

$$\vec{J}_e = -\frac{1}{\mu}\,k_0^2\,\vec{A}_e. \qquad (41)$$

Equation (39) becomes

$$\frac{\partial \vec{e}}{\partial t} = E_0\,\frac{\partial \vec{v}}{\partial z} + C^2 k_0^2\,\vec{A}_e. \qquad (42)$$

Cross-differentiation of the terms in equations (40) and (42) gives

$$\boxed{\begin{aligned} \frac{\partial^2 \vec{e}}{\partial t^2} &= V_C^2\,\frac{\partial^2 \vec{e}}{\partial z^2} + C^2 k_0^2\,\frac{\partial \vec{A}_e}{\partial t}\,, \\ \frac{\partial^2 \vec{v}}{\partial t^2} &= V_C^2\,\frac{\partial^2 \vec{v}}{\partial z^2} + \frac{V_C^2}{E_0}\,C^2 k_0^2\,\frac{\partial \vec{A}_e}{\partial z}\,, \end{aligned}} \qquad (43)$$

where V_C is the velocity : $V_C = E_0\sqrt{\varepsilon/\rho_0}$. As we see, these equations are identical with equations (31), if we replace there B_0 by E_0, $\vec{A}_m$ by $\vec{A}_e$ and V_A by V_C. This is the *wave motion of a magnetically conducting fluid embedded in an electric field* E_0 *and carrying photons.*

In order to improve this result, we again use relation (3) and the equation : curl $\vec{e} = C^2 k_0^2\,\vec{A}_m$. We have

$$\frac{\partial \vec{A}_e}{\partial t} = -e - C^2\,\mathrm{curl}\,\vec{A}_m = -e + \frac{1}{k^2}\,\nabla^2\,\vec{e}, \qquad (44)$$

for $\phi_e = 0$ ($\rho_e = 0$).

The first equation in (43) becomes

$$\nabla^2 \vec{e} - \frac{1}{C^2}\frac{\partial^2 \vec{e}}{\partial t^2} + \frac{V_C^2}{C^2}\frac{\partial^2 \vec{e}}{\partial z^2} = k_0^2 \vec{e}, \qquad (45)$$

that is

$$\Box \vec{e} + \frac{V_C^2}{C^2}\frac{\partial^2 e}{\partial z^2} = k_0^2 \vec{e}\ , \qquad \Box = \nabla^2 - \frac{1}{C^2}\frac{\partial^2}{\partial t^2}, \qquad (46)$$

which, in the absence of the electric field $\vec{E}_0$ ($V_C = 0$), reduces to the L. de Broglie equation.

The second equation in (43) gives

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 \vec{v}}{\partial t^2} - V_C^2 \frac{\partial^2 \vec{v}}{\partial z^2}\right) = \frac{V_C^2}{E_0} C^2 k_0^2 \frac{\partial}{\partial E}\frac{\partial \vec{A}_e}{\partial t}$$

$$= C^2 k_0^2\left(-\frac{\partial \vec{v}}{\partial t} + \frac{1}{k^2}\nabla^2 \frac{\partial \vec{v}}{\partial t}\right) \qquad (47)$$

by virtue of equations (44) and (40). Thus

$$\frac{\partial}{\partial t}\left(\nabla^2 \vec{v} - \frac{1}{C^2}\frac{\partial^2 \vec{v}}{\partial t^2} + \frac{v^2}{C^2}\frac{\partial^2 \vec{v}}{\partial z^2} - k_0^2 \vec{v}\right) = 0. \qquad (48)$$

It follows from (48) that the quantity in paranthesis is constant. If ever in its past history this quantity has vanished, this constant must be zero. Thus

$$\Box \vec{v} + \frac{V_C^2}{C^2}\frac{\partial^2 v}{\partial E^2} = k_0^2 \vec{v} \quad . \qquad (49)$$

Conclusion

Townes invented masers and lasers were built by Kastler. These are of great interest to the theoretician. Now, nature makes the lightning ! It seems to the author that lightning is, say, an analogue to the man-made laser, which analogy deserves close attention. It looks to us that, from a macroscopic view point, they are somewhat related via electrohydrodynamic wave and photons. The lines above may constitute an introduction and a limited model to the phenomena in question which are, to be noted, essentially non linear. We end with the words by Louis de Broglie in the conclusion of his booklet : "... *Mais il me paraît vraisemblable que tout ce qui paraît impossible à représenter dans le cadre des théories linéaires trouvera un jour son explication dans le cadre plus vaste des théories non linéaires*". Painlevé [12] said : "*Entre le problème théorique et la réalité à laquelle on l'applique, il n'y a sans doute pas beaucoup plus de ressemblance qu'entre un cube de glaise et le visage humain qu'y modèlera le sculpteur : c'est le cube de glaise pourtant qui fournit la matière à modeler*". I foresee, for all together, a new beginning !

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APPENDIX I - The equations C

For the completeness of this article we give, below, the equations C, which appear now here else in the literature.

It seems natural to assume that the densities of electric current $\vec{J}_e$ and magnetic current $\vec{J}_m$ are not independent, but which are the equations, if any, governing them ? We are not as fortunate as Maxwell was ; he translated into a mathematical form the observations and ideas of Faraday. The quantities $\vec{J}_m$ and ρ_m (density of magnetic charge) are considered by many authors as fictitous quantities with no physical existence. Observations and ideas on the subject are controversial. All writers agree, however, that these quantities are useful. We shall not enter here into these discussions but, instead, we shall postulate the equations C, which relate the quantities $\vec{J}_e$, $\vec{J}_m$, ρ_e and ρ_m, and test them by their consequences. We shall subject the quantities in question to the following equations (*the equations* C) :

$$\text{(A1)}\quad \begin{aligned} &\operatorname{curl} \vec{J}_e - \frac{1}{C^2}\frac{\partial \vec{J}_m}{\partial t} = \operatorname{grad} \rho_m, \qquad \frac{\partial \rho_m}{\partial t} + \operatorname{div} \vec{J}_m = 0,\\ &\operatorname{curl} \vec{J}_m - \frac{\partial \vec{J}_e}{\partial t} = -C^2 \operatorname{grad} \rho_e, \qquad \frac{\partial \rho_e}{\partial t} + \operatorname{div} \vec{J}_e = 0. \end{aligned}$$

Simple manipulations of vector calculus (curl curl $= -\nabla^2 +$ grad div, where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, etc.) show that

$$\text{(A2)}\quad \begin{aligned} &\nabla^2 \vec{J}_e - \frac{1}{C^2}\frac{\partial^2 \vec{J}_e}{\partial t^2} = 0, \qquad \nabla^2 \rho_e - \frac{1}{C^2}\frac{\partial^2 \rho_e}{\partial t^2} = 0,\\ &\nabla^2 \vec{J}_m - \frac{1}{C^2}\frac{\partial^2 \vec{J}_m}{\partial t^2} = 0, \qquad \nabla^2 \rho_m - \frac{1}{C^2}\frac{\partial^2 \rho_m}{\partial t^2} = 0, \end{aligned}$$

that is the respective quantities are propagated in the material, in all directions, with the velocity C (in *vacuum* : C = c, the speed of light).

Before going farther, let us observe that the Maxwell equations, with densities of magnetic current and magnetic charge, i.e. the equations (1) in the text, together with the equations C, determine all variables involved : $\vec{E}$, $\vec{B}$, $\vec{J}_e$, $\vec{J}_m$, ρ_e and ρ_m. The equations C, just as Maxwell's equations, are invariant under the Lorentz transformation.

Combining the modified Maxwell equations (1) with the equations C, we obtain

$$\text{(A3)}\quad \nabla^2 \vec{E} - \frac{1}{C^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0, \qquad \nabla^2 \vec{B} - \frac{1}{C^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0,$$

that is, the electric field $\vec{E}$ and the magnetic induction $\vec{B}$ are propagated with the velocity C. This proposition is proved, in the literature, assuming : $\vec{J}_e = \vec{J}_m = \rho_e = \rho_m = 0$. *Our equations* C *are imperative !*

The Maxwell equations and the equations C assume that the proper mass m_0 of the photon is zero. They change their expressions if $m_0 \neq 0$, no matter how small this quantity may be ; in fact (in vacuum) $m_0 < 10^{-48}$ k. The arduous task of the physicist is to reformulate these equations when $m_0 \neq 0$.

We shall rewrite the equations C in the form :

$$\text{(A4)}\quad \begin{aligned} &\operatorname{curl} \vec{J}_e - \frac{1}{C^2}\frac{\partial \vec{J}_m}{\partial t} = \operatorname{grad} \rho_m - \frac{1}{\mu} k_0^2 \vec{B},\\ &\operatorname{curl} \vec{J}_m + \frac{\partial \vec{J}_e}{\partial t} = -C^2 \operatorname{grad} \rho_e + \frac{1}{\mu} k_0^2 \vec{E}, \end{aligned}$$

where $k_0 = (1/\hbar)m_0 C$ is L. de Broglie's constant ($2\pi\hbar$ is Planck's constant). It is an infinitesimally small quantity.

Comparaison of the latter equations with the equations for electromagnetic potentials, namely the equations (3) in the text, yields

$$\text{(A5)}\qquad \vec{J}_e = -\frac{1}{\mu}k_0^2\vec{A}_e, \qquad \rho_e = -\varepsilon k_0^2\phi_e,$$
$$\vec{J}_m = -\frac{1}{\mu}C^2 k_0^2\vec{A}_m, \qquad \rho_m = -\frac{1}{\mu}k_0^2\phi_m.$$

The field equations (1) become

$$\text{(A6)}\qquad \text{curl}\,\vec{E} + \frac{\partial\vec{B}}{\partial t} = C^2 k_0^2\vec{A}_m, \qquad \text{div}\,\vec{B} = -k_0^2\phi_m,$$
$$\text{curl}\,\vec{B} - \frac{1}{C^2}\frac{\partial\vec{E}}{\partial t} = -k_0^2\vec{A}_e, \qquad \text{div}\,\vec{E} = -k_0^2\phi_e,$$

which, in the absence of the magnetic potentials $\vec{A}_m$ and ϕ_m, and in *vacuum* ($C = c$, $\vec{B} = \mu_0\vec{H}$) are precisely the *wave equations for the particle photon written by the genius of L. de Broglie*[2]. Actually, de Broglie uses a different sets of units, and complex quantities (as required by the formalism of quantum mechanics).

The *continuity equations* in the equations C yield

$$\text{(A7)}\qquad \frac{1}{C^2}\frac{\partial\phi_m}{\partial t} + \text{div}\,\vec{A}_m = 0, \qquad \frac{1}{C^2}\frac{\partial\phi_e}{\partial t} + \text{div}\,\vec{A}_e = 0,$$

where the latter equation is the relation of Lorentz between the potentials $\vec{A}_e$ and ϕ_e.

Inserting the values of $\vec{E}$ and $\vec{B}$ [formulas (3)] into the equations (A6), we obtain at once the following Klein-Gordon equations :

$$\text{(A8)}\qquad \Box\vec{A}_e = k_0^2\vec{A}_e,\ \Box\vec{A}_m = k_0^2\vec{A}_m,\ \Box\phi_e = k_0^2\phi_e;\ \Box\phi_m = k_0^2\phi_m,$$

where : $\Box = \nabla^2 - (1/C^2)(\partial^2/\partial t^2)$.

Using again the relations (3), we obtain

$$\text{(A9)}\qquad \Box\vec{E} = k_0^2\vec{E}, \qquad \Box\vec{B} = k_0^2\vec{B}.$$

The equations C and (A6) have shown, in a simple and elegant fashion, their utility. But, much remains to be done.

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APPENDIX II

We give here the computation. On substituting (21) and (22) into (20) and using the identity

$$(\frac{\vec{E}}{\rho}.\nabla)(\vec{\Omega}\times\vec{r}) = \frac{1}{\rho}(E_x\vec{\Omega}\times\frac{\partial\vec{r}}{\partial x} + E_y\vec{\Omega}\times\frac{\partial\vec{r}}{\partial y} + E_z\vec{\Omega}\times\frac{\partial\vec{r}}{\partial z})$$
$$= \frac{1}{\rho}\vec{\Omega}\times(E_x\vec{i} + E_y\vec{j} + E_z\vec{k}) = \vec{\Omega}\times\frac{\vec{E}}{\rho},$$
$$(\vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}),$$

we obtain

$$\text{(20)'}\qquad \left[\frac{D}{Dt}(\frac{\vec{E}}{\rho})\right]_0 = (\frac{\vec{E}}{\rho}.\nabla)\vec{v}_0.$$

Using tensor notations, we have

$$\text{(21)'}\qquad (\vec{E}.\nabla)\vec{v}_0 = E_i\frac{\partial\vec{v}_0}{\partial x_i} = E_i\frac{\partial v_0}{\partial x_{ok}}\frac{\partial x_{ok}}{\partial x_i}, \qquad i,k = 1,2,3.$$

where, again, the subscript 0 refers to the nonrotating frame.

Now, in a rotation, we have

$$x_i = \alpha_{ik}x_{ok}, \qquad x_{ok} = \alpha_{jk}x_j,$$
$$\alpha_{ij}\alpha_{kj} = \alpha_{ji}\alpha_{jk} = \delta_{ik} = \begin{cases} 0, & k \neq i, \\ 1, & k = i. \end{cases}$$

Thus

$$\frac{\partial x_{ok}}{\partial x_i} = \alpha_{jk} \frac{\partial x_j}{\partial x_i} = \alpha_{ik} \quad ,$$

and the formula (21)' becomes

$$(22)' \quad (\vec{E}.\nabla)\vec{v}_0 = \alpha_{ik} \; E_i \frac{\partial \vec{v}_0}{\partial x_{ok}} = E_{ok} \frac{\partial \vec{v}_0}{\partial x_{ok}} = (\vec{E}_0.\nabla_0)\vec{v}_0 ,$$

which concludes our proof. We note that Elsasser[7] stops short his proof for the magnetic induction B (p.143) without showing that

$$(\vec{B}.\nabla)\vec{v}_0 = (\vec{B}_0.\nabla_0)\vec{v}_0 \quad ,$$

which does not seem obvious !