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The Einstein-Podolsky-Rosen and Bell arguments revisited

(Part I)

by Nickola S. TODOROV

Georgie Nadjakov Institute of Solid State Physics,
72 boulevard Lenin
1184 Sofia, Bulgaria

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I. On the validity of certain critiques against the current interpretation of Bell's argument

Abstract : *An outburst of critiques in the literature against the interpretation of Bell's argument as a general proof of the nonlocality idea in physics represents a requiem for both one more "no-go-theorem" and, de facto, for a whole heroic era when certain restrictive considerations were treated as theorems solving foundational philosophical problems (e.g. determining the nature of any future physical theory) - cf. Part II. In Part I we show why the Einstein-Podolsky-Rosen (EPR) argument was never refuted, why (and in what sense) certain critiques -which we examine in sufficient detail- actually disprove the said "no-go-theorem" and why certain attempts at their invalidation (and sometimes -misrepresentation) are ineffective.*

Résumé : *L'afflux de publications critiquant l'interprétation du raisonnement de Bell comme une démonstration générale de l'idée de non-localité sonne le glas à la fois pour un théorème d'interdiction de plus, et pour toute une époque héroïque où certaines considérations restrictives*

étaient traitées comme des théorèmes résolvant des problèmes philosophiques fondamentaux (par exemple, déterminer la nature de n'importe quelle théorie à venir) - cf. Partie II. Dans la première partie, nous montrons pourquoi le raisonnement d'Einstein-Podolski-Rosen (EPR) n'a jamais été réfuté, pourquoi (et en quel sens) certaines critiques -que nous examinerons en détail- infirment en fait le théorème d'interdiction en question et pourquoi certaines tentatives pour les contredire (et parfois les trahir) sont sans effet.

I. INTRODUCTION

The proliferation of critiques in recent years against the idea to treat Bell's argument ([1]) (combined with certain experimental results) as a general theorem determining the possible nature of any future physical theory poses serious problems. Some of them are the following :

(a) A number of such considerations rediscover, in principle, arguments that have already been proposed in the earlier literature.

(b) Bell's basic assumptions which lead to his inequality ([1]) are not usually clearly formulated in such papers. This prevents, on its turn, the clear formulation of a general basis for the critiques and the clear realization of the fact that the "no-go-theorem" interpretation of the said argument has *already* been refuted.

(c) Some models proposed in the critical literature (e.g. ([2])) contain implications which are probably unnoticed yet. Others ([3-5]) are so nontrivial that it is easy to misunderstand or misinterprete them. An explanatory comment is thus really necessary.

(d) Natural attempts at rescuing the "no-go-theorem" interpretation by its proponents appeared in the literature. (D'Espagnat, for instance, has gone so far as to allege invalid ([6]) all the existing critiques). The explicit formulation of the basic principle of the critiques would make the answer to attempts of this kind much easier (and in most cases practically obvious).

The present paper represents an attempt at a creation of a more general basis for a discussion not only of the Bell literature and its critique but of the very idea too that "no-go-theorems" are indeed theorems giving an ultimate answer to foundational philosophical problems (cf. Part II for a more detailed discussion). In what follows we shall employ certain notions set forth in a recent paper of ours ([7]) but the exposition here is given so that the ideology be understandable without a previous study of ([7]). We are certainly aware that there may exist better variants of consideration of the problems in question and shall be glad if the present paper would stimulate them. At the same time, we should like to apologize to all authors whose critiques of Bell's argument are not considered here. This is not due to a possible underestimation of their work but is determined by the goals of our paper which is not envisaged as a review article.

2. ON THE EPR-BOHR CONTROVERSY

There exists such a strong similarity between the evolution of the EPR ([8])-Bohr ([9]) controversy in the literature in the past and the way in which the discussion of Bell's argument evolves nowadays that it is certainly worth dwelling a little on this item.

As well known, the dispute arose due to the way in which EPR treated certain correlations between sufficiently separated subsystems S' and S" of an overall system S. The EPR approach rested on the (inexplicit) postulate (which was physically preferable for them and for many others) that measurements on one of the subsystems could in no way affect the other. Bohr's thesis rested on precisely the opposite postulate : for him S', S", and the measuring apparatus represented an inseparable entity irrespective of distances. We have thus two totally different interpretations (employing *different axioms*) which, formally, cannot invalidate each other being based on different types of logic, so that one can make a free choice between them (or discard both of them) depending on one's own philosophical attitude. But the Copenhagen school brooked no alternatives : the Copenhagen interpretation would not be just *a* interpretation, it had to be *the* interpretation of quantum mechanics (some of the reasons for this are mentioned in our paper ([10])). So Bohr qualified the EPR argumentation as misleading ([9]). (His followers would qualify it later on as a "fallacy"). The way in which the logically sound (and, in my opinion, physically superior) EPR agument was usually presented to the public is well illustrated by Blokhintsev's allegation ([11]) that it is easier to "explain" (meaning in fact to refute) the argument than to formulate it.

The wearing off of the novelty of the Copenhagen interpretation and its impact on the thinking of physicists resulted in the realization by many that the said doctrine contained, in itself, no evidence of its uniqueness as an interpretation of microphenomena. This entailed the augmentation of a large number of arguments pointing out the fact that the EPR "paradox" was never really refuted. The very "mass" of these arguments made it clear that an unresolved problem is still on the addenda. But, in an "idealized physical society", all that could be restricted to the simple observation the logic of an argument (say Bohr's) cannot invalidate another argument that is logically incompatible with the former.

Bell, no doubt, was aware of this and contrived an argument in favour of nonlocality that seemed to allow, at the same time, an experimental check-up of the essential validity of Bohr's point of view. The enthusiasm which Bell's argument aroused among the adepts of the orthodox viewpoint may well be illustrated with a number of assertions in the literature (cf. as an example the one quoted in ([7])). The story was then repeated once again : a fast augmentation of critiques against the "no-go-theorem" interpretation of Bell's argument began. On their turn, counter-critiques appeared, aiming at the invalidation of the said critical arguments from Bell's positions. Some of them are frank attempts at misrepresenting the significance of the critiques. An example of this is, once again, ref. ([6]) in which d'Espagnat tried to create the impression that such critiques are even not worth reading ("an attempt at an exhaustive review of all the objections to the Bell theorem that were proposed (and disproved) would be a boring and unrewarding enterprise". -ref. ([6]), p. 259). And

once again the actual state of affairs in quite different : A Bell type of reasoning cannot invalidate arguments that go outside the framework of Bell's assumptions. This will be evident from our consideration which begins with the formulation of Bell's basic assumptions (axioms).

3. BELL'S AXIOMS

As the developments of Bell's argument have not really achieved its significant generalization ([12]), we shall examine the basic assumptions in ([1]). The consideration in ([1]) deals with a certain class (restricted, as we shall see, and the most general one, as it is usually visualized in the Bell literature) of deterministic local hidden-parameters (HPs) theories that conform to the following three axioms.

(i) A set Λ of HPs λ (completely specifying the state of motion) exists.

(ii) A normalized probability density distribution (pdd) $\rho(\lambda)$ yielding all the necessary probabilities (see below) exists on Λ.

(iii) λ-determined bivalued response functions $A(a,\lambda)$, $B(b,\lambda)$ ($= \pm 1$ in the spin-$\frac{1}{2}$ case) exist for two-particle decays of a spinzero initial "molecule".

The parameters a and b in (iii) denote the orientations of the instruments M' and M" measuring $A(a,\lambda)$ and $B(b,\lambda)$, respectively, for each member of the couple. Locality is taken into account by the dependence of A on a only, and of B on b only.

4. A REFORMULATION OF THE PROBLEM

We shall adduce here certain notations and specifications employed in ([7]).

The HPs λ in Bell's axioms (whose values are certainly determined with respect to a given coordinate system K) obviously refer to states prior to measurements. We examine two-particle correlations, so $\lambda = (\lambda_1, \lambda_2)$, λ_1 and λ_2 denoting the states of motion of particles 1 and 2, respectively, say at the moments of their impinging on M' and M". Axioms (i-iii) give then

$$(1) \qquad \int \rho(\lambda_1,\lambda_2)d\lambda_1\, d\lambda_2 = 1$$

$$(2) \qquad P(a,b) = \int \rho(\lambda_1,\lambda_2)A(a,\lambda_1)B(b,\lambda_2)d\lambda_1\, d\lambda_2,$$

where P(a,b) stands for relevant correlation value at an orientation a of M' and b - of M".

We can attach, however, individual coordinate systems K' to M' and K" -to M" in a manner determined by the physical nature of M' and M" ([7]) and give the following

General deterministic definition of locality : The display of M' must depend solely on the numerical value(s) w' of the set of HPs of subsystem 1 (call it S') plus those of M', all these HPs taken with respect to coordinate system K', and that of M" -on the value(s) w" of the state of S" and the possible HPs of M" in K".

The adding of the possible HPs of M' and M" (see below) to the HPs w' and w" of S' and S" represents a certain generalization of the definition given in ([7]).

Eqns. (1) and (2) (which take into consideration just the states w', w" of S', S" in conformity with Bell's argument) can be recast now in the form

$$(3) \qquad \int r(n',n'';w',w'')dw'dw'' = 1$$

$$(4) \qquad P(n',n'') = \int r(n',n'';w',w'')C(w')C(w'')dw'dw'',$$

where n', n" stand for the orientations of M', M" ; C(w') and C(w") are the response functions of M' and M" in terms of w' and w" (these bivalued functions, = ±1, are of an identical form due to the identical nature of M' and M") ; the new pdd r (in terms of w', w") must be n', n"-dependent now as the C-functions are, by their very sense, parameter-independent.

It is worth illustrating the above somewhat abstract assertion with a concrete simple example.

5. A SIMPLE ILLUSTRATION

Let λ_1, λ_2 vary in the interval [0,1] along the z-axis, n', n", and z lie on the same plane and the angles θ', θ'' between the orientations n', n" of the (purely hypothetical, not necessarily spin-measuring) instruments M', M", respectively, and the z-axis vary in the interval $(0,\pi/2)$ (Fig.1). Assume that the response of M' is determined by the numerical value of the orthogonal projection of λ_1 onto n' (that is, $w' = \lambda_1 \cos\theta'$, $0 \leqslant w' \leqslant \cos\theta'$), and that of M" -by the orthogonal projection of λ_2 onto n" (i.e. $w'' = \lambda_2 \cos\theta''$, $0 \leqslant w'' \leqslant \cos\theta''$) and let the normalized pdd $\rho(\lambda_1,\lambda_2)$ be equal to $\delta(\lambda_1 - \lambda_2)$ inside the ranges of variation of λ_1 and λ_2 and zero outside them. We shall have then $A(n',\lambda_1) = A(\lambda_1 \cos\theta') = C(w')$, $B(n'',\lambda_2) = B(\lambda_2 \cos\theta'') = C(w'')$, $\rho(\lambda_1,\lambda_2) = \delta\left(\frac{w'}{\cos\theta'} - \frac{w''}{\cos\theta''}\right) = \cos\theta' \cos\theta''\, \delta(w'\cos\theta'' - w''\cos\theta')$, $d\lambda_1 d\lambda_2 = dw'dw''/\cos\theta' \cos\theta''$. Consequently, $\rho(\lambda_1,\lambda_2)d\lambda_1 d\lambda_2 = \delta(w'\cos\theta'' - w''\cos\theta')dw'dw''$, so that the new pdd (in terms of w',w") will be $r(\theta',\theta'';w',w'') = \delta(w'\cos\theta'' - w''\cos\theta')$, that is, it is parameter-dependent now (together with the ranges of variation of w',w" ; in particular, eqn. (3) will now read

$$\int_0^{\cos\theta'}\int_0^{\cos\theta''} r(\theta',\theta'';w',w'')dw'dw'' = 1,$$

while the corresponding correlation result will be given by

$$\int_0^1\int_0^1 \delta(\lambda_1-\lambda_2)A(n',\lambda_1)B(n'',\lambda_2)d\lambda_1 d\lambda_2 =$$

$$\int_0^{\cos\theta'}\int_0^{\cos\theta''} \delta(w'\cos\theta'' - w''\cos\theta')C(w')C(w'')dw'dw'').$$

At a first sight, formulations and examples of the above kind do not achieve noticeable generality from the viewpoint of invalidating the "no-go-theorem" interpretation of Bell's argument. Indeed, Bell's consideration just does not apply to pdd's that depend arbitrarily on the parameters of interest (cf. the derivation in ([1]) or the brief derivation in ([7])) but the parameter-dependence of our r's up to now is such (call it *inessential*) that it allows the existence of a parameter-independent *normalized* pdd $\rho(\lambda_1,\lambda_2)$ in K, which automatically leads to Bell's inequality ([1]). However, our definition of locality shows that the way in which r's depend on n', n" has no relevance to the locality problem : the locality requirement consists in parameter-independence of only the C-functions. It can be easily shown with simple examples ([7]) that general parameter-dependence of relevant pdd's leads to a violation of Bell's inequality without introducing nonlocality. This will certainly be true in models which do not conform to at least one of the axioms (i - iii). That is, axioms (i - iii) are not synonymous with locality and local HP models may not obey Bell's inequality.

We are going to examine now several counterarguments to Bell's discussion on the basis of the above concepts.

6. LOCHAK'S COUNTER-ARGUMENT AND ITS IMPLICATIONS

In his critique of Bell's argument Lochak ([2]) accepts axioms (i - iii) as valid only prior to the impinging of S' and S" on the analysers of apparata M' and M" and points out that Bell has not taken into account the possible local effect of the very act of measurement on the pdd's and the magnitudes P(a,b) of interest. (In Lochak's terminology the magnitudes $\rho(\lambda_1,\lambda_2)$ and P(a,b) in eqns. (1) and (2) are hidden quantities ; a succint formulation of the argument together with certain ideas of de Broglie that generated it may be found in ([13])). An HP theory should explain the P(a,b) and ρ-values obtained in the process of actual measurement. Translated into the language of K' and K"-HPs this would mean that eqn. (2) should be replaced by

$$(5)\qquad P(a,b) = \int \rho_m(n',n";w'_m,w"_m)C(w'_m)C(w"_m)dw'_m dw"_m,$$

where subscript m denotes quantities taken after the passage of the particles through the analysers (and prior to detection). As $C(w'_m)$, $C(w"_m)$ are parameter-independent, formula (5)

does not imply nonlocality. Bell's axioms require *inessential* parameter-dependence (see above) of ρ_m, while Lochak's argument admits *essential* n', n"-dependence of ρ_m (i.e. violating Bell's inequality). If one accepts axioms (i) and (ii) as valid prior to measurement, then essential pdd's ρ_m may appear only when axiom (iii) is not obeyed in the process of measurement. (If axiom (iii) is satisfied in the course of measurement, then all the assumptions (i - iii) would be satisfied too and the inequality automatically obtains). The sense in which axiom (iii) should be invalid can be made somewhat more precise : a result in the literature ([12]) says that a necessary condition for an effective violation of (iii) is nonfulfilment of the factorisability requirement $p(XY;\lambda_1,\lambda_2) = p(X,\lambda_1)p(Y,\lambda_2)$, where say $p(X,\lambda_1)$ gives the probability for a + 1 value of observable X measured in state λ_1 (the deterministic response functions being replaced now by relevant probabilities), while $p(XY;\lambda_1,\lambda_2)$ is the probability that X = +1 when measured in state λ_1 and Y = +1 in λ_2. Obviously, a necessary condition for the absence of factorizability without a violation, in principle, of locality is the nonexistence of at least one of the individual probabilities $p(X,\lambda_1)$, $p(Y,\lambda_2)$. Consequently, Lochak's argument de facto requires nonexistence of probabilities of the kind $p(X_i,\lambda_i)$, i = 1,2 or, equivalently, of deterministic functions $A(a,\lambda_1)$, $B(b,\lambda_2)$ that might describe the performance of the analysers and it may therefore preserve locality if the requirement for determinism (in Bell's sense) is relaxed. This argument may not be thus invalidated within the framework of Bell's axioms since it goes beyond them.

Several questions immediately arise at this point:
(a) Does violation of determinism in the sense of Bell's axioms necessarily mean violation of determinism in general ?
(b) Does there exist, *in principle*, a physical explanation for a possible stochastic behaviour (in the above sense) of the measuring instruments ?
(c) How it might be that measuring instruments transforming the pdd's of interest in such a "totally stochastic" local fashion may permit at the same time the observation of defi-

nite (probabilistic) correlations in the motion of S' and S" ?

There may possibly be a number of different answers to the above questions. We shall offer here one such answer which will show that HP theories in Lochak's sense are, in principle, physically conceivable.

Let us point out, firstly, that an "elementary" particle should not be visualized, generally, as a pointlike structureless entity : the very presence of inherent magnetic properties (or, equivalently, spin) is already an indication of a possible complex inner structure and inner dynamics (other indications for inner structure that have led to a quark-gluonic picture of hadrons may be found in elementary particle physics ; at the same time, there are no a priori reasons to assume that leptons must be structureless). We know -or at least hope that we do- what is the force with which a given macroscopic magnetic field can act on a given magnetic moment that travels inside the field but we do not know how this field may influence the internal dynamics of the particle so that it acquires definite magnetic properties (e.g. a definite spin component or, mutatis mutandis, polarization in the case of photons). For that reason one may assume that a given macroscopic distribution of, say, the magnetic field $\vec{H}(\vec{r})$ of a measuring instrument does not give a full description of the possible properties of the field and that finer HPs may be necessary in order to describe the way in which the field may affect the inner properties of microparticles. (*A thermodynamical analogy* : the temperature T of a thermostat determines only a certain macroscopic property of this apparatus and gives no account of its finer microscopic properties).

Denote the possible "magnetic" HPs of M' (with respect to K) with h' and those of M" (in K) -with h". We see then that Bell's assumption (iii) : $A = A(a,\lambda_1)$, $B = B(b,\lambda_2)$, is restrictive in the general case. A more general assumption would be

$$(6) \qquad A = A(a,h',\lambda_1), \quad B = B(b,h'',\lambda_2)$$

This certainly means that Bell's equation (2) is restrictive too. If we assume, say, that the distributions of h' and h" are given by normalizable pdd's $R'(h')$ and $R''(h'')$, then the more general form of eqn. (2) would be

$$(7) \quad P(a,b) = \int \rho(\lambda_1,\lambda_2)R'(h')R''(h'')A(a,h',\lambda_1)B(b,h'',\lambda_2)d\lambda_1 d\lambda_2 \, dh'dh''$$

Denoting $\int R'(h')A(a,h',\lambda_1)dh'$ as $\alpha(a,\lambda_1)$ and the corresponding integral over h" -as $\beta(b,\lambda_2)$, we obtain

$$(8) \qquad P(a,b) = \int \rho(\lambda_1,\lambda_2)\alpha(a,\lambda_1)\beta(b,\lambda_2)d\lambda_1 d\lambda_2,$$

where α and β can be, generally, $\neq \pm 1$. This remark is sufficient to invalidate Bell's initial "self-obvious" consideration ([1]) as a most general one since Bell essentially employs there the ± 1 values of his spin functions $A(a,\lambda_1)$ and $B(b,\lambda_2)$, while here the role of A and B is taken by α and β which have different properties. Due to our restrictive assumption, however, that $R'(h')$ and $R''(h'')$ are normalizable and give all the necessary probabilities we have in fact a factorizable stochastic model up to now and the somewhat more general CHSH inequalities will survive ([12]). The latter possibility can be eliminated by removing the above normalizability restriction on "magnetic" pdd's in the spirit of our paper ([7]) (cf. also Section 8 here) or employing for such pdd's a consideration of the kind given by Pitowsky ([3],[5]) (cf. also Section 7 here).

In such a way Lochak's critique ([2]) (based on certain ideas of de Broglie - cf. e.g. ([13])) is effective : one can assume that HP models are possible in which the pdd's yielding the quantum predictions are in fact generated in the act of measurement and must not be identified with the initial hidden distributions. At the same time, we have answered above all the questions (a - c) : the answer to (a) is *no*, to (b) - *yes* (the HPs of the measuring instruments), while the "how" of (c) is answered by the fact that the more general deterministic functions A and B in eqn. (6) depend once again on a and b, respectively, and a relevant averaging procedure over the product AB will give an a,b-dependent result.

One more question which should be answered here (just *in principle,* as long as Bell's argument does not examine a given specific theory but claims to be applicable to a whole class of possible theories) is the following.

(d) How is it possible that one can obtain experimentally definite probabilities for the bivalued displays of M' (or M") if any one of the instruments behaves in a "totally stochastic way" (i.e. precluding the existence of definite statistics of the responses at a fixed λ_1 or λ_2, respectively) in a non-factorizable HP model ?

In order to see a possible answer, examine the case of, say, a nonnormalizable distribution R'(h') of the HPs h' of M'. Such a pdd may be regarded, in a sense, as corresponding to a "very large" HP space {h'} and the "unrestricted number of possibilities" for the values of h' at a fixed state λ_1 of S' may in principle preclude definiteness of the statistics of the M' responses for any given λ_1. (The latter requirement may possibly be relaxed to a certain extent but this is not really essential here). On the other hand, the definite probabilities of the M' responses may well be connected with the problem of finding the probabilities in question in a relevant ensemble of λ_1-values *at a fixed arbitrary* h'. We have thus two essentially different problems from the point of view of logic (statistics of the responses of say M' at a fixed λ_1 and variable h'-values and statistics at a fixed h' and variable λ_1-values, {h'} nonnormalizable, $\{\lambda_1\}$ normalizable) and this gives, in principle, an answer to question (d).

Consequently, there are no a priori reasons to expect that local deterministic HP models in Lochak's sense (i.e. violating Bell's axiom (iii)) may be impossible from the point of view of logic. There are other arguments too (3-5,7) which show that Bell's axiom (ii) for $\rho(\lambda_1,\lambda_2)$ is restrictive in itself, even if the possible HPs of M' and M" are not taken into account. We shall examine them briefly in Sections 7 and 8.

8. PITOWSKY'S COUNTER-ARGUMENT

In order to capture the essence of Pitowsky's consideration (3,5) (which is highly advanced from a purely mathematical point of view) employ the correlation requirement (1) $B(n'',\lambda_2) = -A(n'',\lambda_1)$ (in its original form : $B(b,\lambda) = -A(b,\lambda)$), n" denoting an arbitrary orientation of M", and rewrite eqn. (2) in the form

$$(9) \qquad P(n',n'') = -\int_{\Lambda_1} \rho(\lambda_1)A(n',\lambda_1)A(n'',\lambda_1)d\lambda_1,$$

where $\rho(\lambda_1) = \int_{\Lambda_2} \rho(\lambda_1,\lambda_2)d\lambda_2$ is the individual parameter-independent normalized pdd for S' and Λ_1, Λ_2 are the ranges of variation of λ_1, λ_2. Consequently, P(n',n") in eqns. (2) or (9) represents a sum of four obvious probabilities p[.], each of which is multiplied by +1 or −1 (depending on the sign of $A(n',\lambda_1)B(n'',\lambda_2)$ or $-A(n',\lambda_1)A(n'',\lambda_1)$, respectively, in relevant subsets of Λ_1 and Λ_2). Bell's argument thus requires that two-particle probabilities be equal to relevant single-particle ones. For instance, in obvious notations, it yields $p[A(n') = +1 \text{ and } B(n'') = +1] = \frac{1}{2}\sin^2(\theta_{n'n''}/2)$ (14) = $p[A(n') = +1 \text{ and } A(n'') = -1]$, the "obvious" assumption (applied subsequently) being that in a local HP theory probabilities of this sort must exist *simultaneously* for every given couple of directions n', n". We know from quantum mechanics (3,5) that the first probability in the above chain is in fact $= \frac{1}{2}p[A(n'') = -1/A(n') = +1]$, where p[./.] is the *conditional* probability for a −1 value of A(n") given A(n') = +1. Pitowsky's model aims precisely at the explanation of the $\frac{1}{2}$p[./.] single-particle probabilities. The individual HP v that determines (with the help of a spin-function $s_v(x) = \pm\frac{1}{2}$) the value of A(n") [= $s_v(n'')$] at a fixed value of A(n') in this model is given by the set {x} of all points x on the two-dimensional unit sphere $S^{(2)}$. (For the sake of clarity I have introduced here notations somewhat different from those in ref. (3)). The bivalued functions $s_v(x)$ are defined so that the QM conditional single-particle measures p[./.] be obtained with probability 1 on relevant circles ([3]) covering $S^{(2)}$ (i.e. with respect to the Lebesgue measure on circles) for each s_v. It turns out ([3],[5]) that the set of all $x \in S^{(2)}$ having the said property at a given s_v is nonmeasurable with respect to the Lebesgue measure on $S^{(2)}$ and -due to the model assumption that all v's are equally probable and the set {v} is obtained through the O(3) transformations of a given v_0- the same property will be possessed by the set {v} of total HPs (or, equivalently, the set $\{s_v\}$, $v \in \{v\}$). More precisely, no uniform probability measure (as would be required by the above assumptions) may be introduced on {v} that could give account, at the same time, of the single-particle conditional probabilities in question ([3]). (Another way to say the same is that the uniform measure in v-space that is required for the description of the equivalence of the v's from a probabilistic point of view and which turns out to be "isomorphic" to the Lebesgue $S^{(2)}$-measure cannot generate definite values for the relevant single-particle conditional probabilities). This means that no conventional probability space of the kind assumed by Bell (incorporating axiom (ii) and ensuring the violation of quantum mechanical correlations) exists in Pitowsky's model which explains the quantum correlations exactly through single-particle conditional probabilities. That is, the different couples of directions n', n", for which the model yields the quantum expectations with probability 1, must be examined *separately* (a separate probability space being associated with each such couple), as the relevant probabilities associated with the different couples of directions cannot be defined on a unique probability space ([3]) of a conventional Kolmogorovian type, contrary to the implications of Bell's eqns. (2) or (9). This is nothing else than the "language" of our parameter-dependence of pdd's here, i.e. no parameter-independent normalized "overall" $\rho(\lambda_1,\lambda_2)$ -obeying Bell's assumptions- may be introduced and a specific parameter-dependent $\rho_{n'n''}$ must be employed for any specific couple of directions n', n". This inference follows directly too from the proof of Theorem 4 in ref. ([5]), where it is demons-

trated that probabilities of the kind $\frac{1}{2}$ p[A(w) = -1/A(x) = +1] and $\frac{1}{2}$ p[A(w) = -1/A(y) = +1] exist simultaneously only on a subset {w} of $S^{(2)}$ of a zero Lebesgue $S^{(2)}$-measure, i.e. there is no chance for a simultaneous existence in Pitowsky's extraordinary model of relevant large-number limits yielding the above probabilities ([4],[5]) which gives an unexpected account of the fact that experiments can determine the mentioned probabilities for only one couple of directions at a time.

Consequently, Pitowsky's counter-argument goes definitively beyond Bell's system of axioms (i - iii) by negating axiom (ii) and is inaccessible to critiques based on a Bell type of reasoning. This argument may theoretically be invalidated only from a purely mathematical point of view if for some reasons the theorems proved in ([5]) would turn out to be wrong. But even if this were really so we would have just a refutation of a concrete model and not of Pitowsky's penetrating idea ([3]) that probability theory should be regarded in the same manner as Riemann and Einstein treated geometry (that is, different concepts of "probability" and "event" may exist in different physical theories, while Bell's argument inexplicitly postulates Kolmogorov's axiomatics as the only possibility in a stochastic HP theory).

8. OUR ARGUMENT

This argument was published recently in the Annales ([7]) and we shall be quite brief about it.

As shown in ([7]) Bell's axiom (ii) may turn out to be restrictive from a different point of view too : essential parameter-dependence of suitable normalized pdd's may in principle be obtained with the help of a *nonnormalizable* parameter-independent pdd $\rho(\lambda_1,\lambda_2)$ in K. (This argument was contrived in fact independently of the Pitowsky one, of which I learned later on). The latter approach employs too the concept of "nonmeasurability" (but not Lebesgue nonmeasurability !) and has the merit of being "constructivistic" ([7]). That is, nonmeasurable sets in the sense of ref. ([7]) can actually be constructed and they are not, in principle, "pathological". The interference, however, is the same as that in ([3]) : different concepts of "event" may be possible in different physical theories and there are no a priori reasons to restrict the probabilistic nature of a possible theory to Kolmogorov's axiomatics.

9. CONCLUSION

The explicit formulation of Bell's basic assumptions shows why and in what sense certain critical arguments in the earlier literature invalidate the idea that his argument represents a general theorem on locality and determinism in microphysics : from the point of view of a number of existing possibilities Bell's argument has relevance to only a restricted class of local deterministic HP theories, while a large class of such theories that are possible in principle remain totally out of reach of the argument. It is thus really naive to think that the experiments inspired by the said argument can say anything about the basic philosophical EPR standpoint whose falsification was the ultimate goal of the argument. A more detailed discussion of the philosophical side to the problem, together with additional arguments showing that even the seemingly innocent axiom (i) is restrictive too will be given in Part II of the present page.

In the Appendix below we demonstrate the ineffective character of certain attempts at invalidating the Lochak and Pitowsky counter-arguments.

APPENDIX

Critiques against Pitowsky's counter-argument ([3,5]) may be found in ([6,15,16]). The way in which this complex argument is dealt with in ([6]) is worth quoting in full.

"The Wigner procedure (ref. ([14]) -my note) eliminates the hypothesis considered by some authors (see e.g. ref. ([3])) that the violation of the Bell inequalities should be compatible with a local hidden variables theory provided that the hidden parameters values would range over a nonmeasurable set. The same result can be obtained (still within the strict correlation case) by using Stapp's procedure (e.g. ref. ([17]) -my note) which is then unobjectionable (see also references

([15]),([16]),([33d])". (ref. ([6]), p. 232 ; I have altered above d'Espagnat's numeration of the literature to conform with mine, the only exception being his ref. ([33d]) which is a preprint by Stapp). However :

1. The arguments due to Mermin ([15]) and Macdonald ([16]) are based precisely on the assumption for a simultaneous existence of single-particle probabilities for arbitrary couples of directions which was shown to be impossible in Pitowsky's model (cf. Section 7). This was pointed out in ([4]) (not cited in ([6])). Ref. ([5]), containing the necessary results (in the proof of Theorem 4 there) for an answer to the critiques in ref. ([15]) and ([16]) is also not cited by d'Espagnat ; at the same time, the coordinates of ref. ([3]) and the name of its author are incorrectly given by him.

2. Wigner's argument ([14]) assumes the existence of joint single-particle probabilities for arbitrary triples of directions (a necessary condition ([12]) for the validity of Bell's consideration). Such probabilities, if existing, would entail the (impossible) simultaneous existence too (as marginals) of the above two-parameter single-particle probabilities (cf. also the counter-example in ([7])).

3. Stapp's argument ([17]) de facto assumes the existence of joint single-particle probabilities for quadruples of directions (a necessary condition ([12]) for the validity of Bell's argument). The argument in point 2 literally applies here too (Stapp's probabilities would entail the Wigner ones as marginals).

This conveniently brief "invalidation" of Pitowsky's model is followed, at the end of ([6]), by a wordy attack against Lochak's argument, the final conclusion being as follows.

"Taking all the foregoing considerations into account we may assert that, contrary to Lochak's statement, the idea of using one and the same $\rho(\lambda)$ in relation with different experimental arrangements is in no way in contradiction with the factual content of quantum mechanics. This being established, we are then led to claim that a realist can hardly dissociate the said idea from his a priori expectation that separability should hold true : to operate this dissociation would be equivalent to reducing the principle of separability into its "assumption A" part and this would be utterly artificial since as soon as we set up to define the words "separability" and "locality" so that the new precise acceptations preserve a similarity with the vague, customary ones, we are forced to introduce the "assumption B" part -or some equivalent statement- into the definitions searched for". (([6]), p. 262).

The concrete interpretation of Lochak's argument in our Section 6 leads to a somewhat unexpected conclusion (from the view-point of those who are fond of determining what a realist should think and what not). Namely, d'Espagnat's "assumption A" part itself (([6]), p. 206) is dispensable for local deterministic HP theories and may not be obeyed by them. Really, assumption A contains a set of restrictions imposed on hypothetically existing conditional probabilities (e.g. on $(A|\lambda)$, where A denotes the result of given measurement and λ- a set of parameters "specifying completely the objective state of the source S at the time of emission" ([6])). We saw in Section 6, however, that such conditional probabilities may just not exist and that this should not generally be regarded as a supernatural or artificial fact. In any case, nonquantitative statements as "artificial", "natural", etc., may have a certain meaning only within the context of a *concrete* HP theory, employing a *concrete* mathematical apparatus and giving *concrete* results that may be compared with known facts and "general" qualifications of the above kind represent just a personal perception devoid of objective significance.

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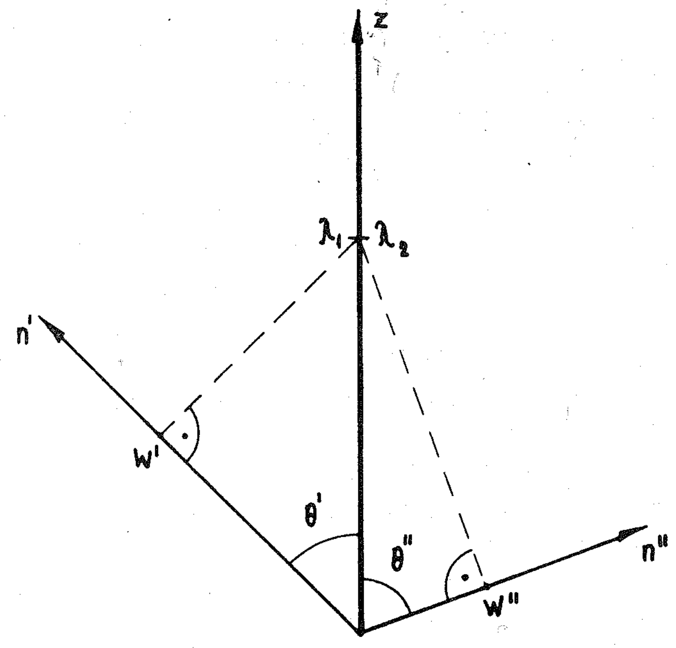

Fig. 1