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Space-time structure of bradyons, luxons and tachyons*

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Abstract : *We investigate the space-time structure of regular solutions of nonlinear equations of physical fields, which may represent bradyons, luxons, strings and tachyons.*

Résumé : *On étudie la structure spatio-temporelle de solutions régulières de certaines équations de champs physiques non linéaires qui peuvent représenter des bradyons, des luxons, des cordes et des tachyons.*

In nonlinear field theory the elementary particles are considered as regular solutions of some nonlinear equation of physical fields. As far back as Einstein, some elementary bradyon-particles were considered as some regular-particle-like solutions of nonlinear field equations, i.e. as three-dimensional solitons. Such a theory allows to compute the fundamental parameters of particles -mass, spin, charge and so on- and also peculiarities of their structure if the field equations are correctly chosen. These features of nonlinear field theory constitute for them an advantage over the ordinary quantum theory of elementary particles in which each kind of particle is associated with its own linear field and its solutions ;

*Let us recall that bradyons, luxons and tachyons are respectively particles which move with velocities *less* than the velocity of light, *equal* to it, and *greater* than this velocity.

on the basis of the complementarity principle, any space-time representation of the particle structure is expeled from the theory.

Although the problem of quantization of nonlinear field theory comes up against certain difficulties, undoubtebly many advantages of this theory are already recognized and widely investigated.

Nonlinear field theory allows also to exhibit and investigate other elementary objects, for example strings and such exotic particles as tachyons. In the orthodox quantum field theory such objects, just as new elementary particles are introduced by means of a priori given new fields and hypotheses about their interaction.

Let us consider on the basis of a nonlinear field theory, i.e. from the viewpoint of possible regular solutions of nonlinear field equations, the main structural peculiarities of the different elementary objects : bradyons, luxons, tachyons, strings and so on. Let $\psi_{ik\ell...}(x,y,z,t)$ describe a set of fields, satisfying a nonlinear system of differential equations. In the most simple case it may be a single scalar function, for example, a complex function ψ solution of a nonlinear Klein-Gordon equation,

$$(1) \qquad \Box\psi + \mu[1 + F(\psi^*\psi)]\psi = 0$$

where $F(\psi^*\psi)$ is a nonlinear term which eventually may be taken in the simple form : $\alpha\psi^*\psi$. This equation has spherically-symmetric particle-like solutions of the form [1]

$$(2) \qquad \psi = U(r)e^{i\omega t}, \qquad (r^2 = x^2 + y^2 + z^2)$$

if the equation for the function U

$$(3) \qquad \Delta U + \{\omega^2 + \mu[1 + F(U^2)]\}U = 0$$

which is obtained introducing (2) in (1), has regular solutions which are finite in the domain $r = 0$ and decreases quickly to

zero when $r >> r_0 = \ell$.

In the reference frame moving along the axis x with the velocity β according to the Lorentz transformations, the function (2) obviously becomes

$$(4) \qquad \psi' = U\left(\sqrt{y'^2 + z'^2 + \frac{(x' + \beta t')^2}{1 - \beta^2}}\right)e^{i\omega\frac{t' + \beta x'}{\sqrt{1 - \beta^2}}}$$

Thus, if ℓ is considered as a caracteristic space dimension of a particle then dimensions of the moving particle in the directions x,y,z are

$$(5) \qquad \ell'_x = \ell\sqrt{1 - \beta^2}, \qquad \ell'_y = \ell, \qquad \ell'_z = \ell,$$

we have an effect of contraction of scales or flattening of the particle along the axis of motion.

Equation (3) can have also cylindrical symmetric or string-like solutions of the form [2]

$$(6) \qquad \psi = V(\rho)e^{i(\omega t - Kx)}, \qquad (\rho^2 = y^2 + z^2)$$

if the equation for

$$(7) \quad \Delta_\rho V + \{\omega^2 - K^2 + \mu^2[1 + F(V^2)]\}V = 0, \; (\Delta_\rho = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}))$$

has regular solutions similar to the solutions for U. The length of the string,discribed by the solution (6) along the axis x tends to infinity, and its width along axies y and z are limited and equal in all frame of reference moving along x. Note that the ratio $\omega/K = U$ may take values $U > 1$, $U = 1$ and $U < 1$. Let us call them respectively string-like solutions of bradyon, luxon or tachyon type.

Consider now the general form of the solution for the function $\psi_{ik\ell...}(x,y,z,t)$ in the case of tachyons and luxons. For the sake of simplicity as well as in the case of bradyons and strings we will limit ourselves with only one function $\phi(x,y,z,t)$, and even without exactly determining for the time being the form of the nonlinear equation.

A tachyon is a hypothetical particle moving with the velocity greater than light. It cannot be at rest in any real system of reference. But in the system of reference where the point-like tachyon moves with an infinite velocity along the x axis it may be represented by an infinite string instantaneously emerging and vanishing at the moment $t = 0$, i.e. as

(8) $$\phi(x,y,z,t) = \delta(y)\delta(z)\delta(t).$$

Obviously a real, instead of a point-like tachyon in this particular frame of reference, may be represented by a function :

(9) $$\phi = v(\frac{\rho}{\ell})f(\frac{t}{\tau}), \qquad (\rho^2 = y^2 + z^2)$$

where $v(\frac{\rho}{\ell})$ and $f(\frac{t}{\tau})$ are regular functions with an effective "width" ℓ and τ respectively. The space dimension of a tachyon, described by a function like (9) is infinite along the x axis but the dimensions in the perpendicular direction may be evaluated as equal ℓ. In the reference frame moving along the x axis with the velocity β, the function (9) according to the Lorentz transformation may be written as :

(10) $$\phi = v(\rho)f(\frac{t' + \beta x'}{\tau\sqrt{1 - \beta^2}}).$$

Therefore, at the moment $t' = 0$, the function ϕ has the spacial effective "width"

(11) $$\ell'_x = \frac{\tau\sqrt{1 - \beta^2}}{\beta}, \qquad \ell'_y = \ell, \qquad \ell'_z = \ell$$

so that when $\beta \to 0$, $\ell'_x \to \infty$ and, when $\beta \to 1$,

$$\ell'_x = \frac{\tau}{\beta}\sqrt{(1 - \beta)(1 + \beta)} \sim \tau\sqrt{2(1 - \beta)}$$

It is not difficult to note that string-like solutions (6) differ from tachyon-like (9) in that the function $f(\frac{t}{\tau})$ is not regular and has unrestricted "width".

The particles moving with the velocity of light, or luxons, may be described by a function of the form :

(12) $$\phi = f\{\frac{\rho}{\ell}, \frac{x - t}{\ell_x}\}, \qquad (\rho^2 = y^2 + z^2)$$

with function ϕ having regular character with respect to coordinate ρ and variable $(x - t)$. The point-like luxon obviously can be pictured by the function

(13) $$\phi = \delta(y)\delta(z)\delta(x - t)$$

In the reference frame moving along the x axis with the velocity β, according to the Lorentz transformation the function (12) becomes

(14) $$\phi = f\{\frac{\rho}{\ell}, \frac{1}{\ell_x}\sqrt{\frac{1 - \beta}{1 + \beta}}\,(x' - t')\}$$

in other words in the moving frame at the fixed time $t' = 0$, the effective "width" of the luxon is equal to :

(15) $$\ell'_x = \ell_x\sqrt{\frac{1 - \beta}{1 + \beta}}$$

With $\beta = 0$ in the initial frame, $\ell'_x = \ell_x$ as it must be. In a frame overcoming the luxon, i.e. when $\beta > 0$, the luxon stretches in the direction of motion, and in a frame moving towards the luxon i.e. when $\beta < 0$ the luxon is contracting.

In this work we don't consider concrete nonlinear equations which have the solutions of tachyon (10) or luxon (12) type. This is a problem for a future investigation. Let us only point out that the simplest equation (1) has not any solutions of tachyon type (9) or (10). It has spacial plane solutions of tachyon type

(16) $$\phi = f(\frac{t}{\tau}) \qquad \text{or} \qquad \phi = f(\frac{t' + \beta x'}{\tau\sqrt{1 - \beta^2}})$$

where (16) is a function obeying to the equation :

(17) $$\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial t^2} + \mu[1 + F(f^2)]f = 0$$

or with the variable $\xi = x - Ut$:

$$\frac{\partial^2 f}{\partial \xi^2}(1 - U^2) + \mu[1 + F(f^2)]f = 0, \tag{18}$$

this function f may be regular when $U > 1$ and with a convenient choice of the coefficient μ. But such a function f, solution of (16), is spacially unrestricted in the directions y and z and therefore cannot be considered as a tachyon-particle.

Luxon solutions (12) of a particular type are possible in the equation (1) in the form of string-like solutions

$$\psi = v(\rho)e^{iS(x-t)} \tag{19}$$

where $v(\rho)$ obeys the equation :

$$\Delta_\rho v + \mu[1 + F(v^2)]v = 0 \tag{20}$$

and $S(\xi)$ is an arbitrary function of the variable x-t.

In the particular case, if $S = \delta(\xi - \frac{\pi}{4})$,

$$e^{iS(\xi)} \to \frac{1}{\sqrt{2}}(1 + i)\delta(\xi) = e^{i\ \pi/4}\ \delta(\xi)$$

The real photon may be discribed as a luxon solution of a system of nonlinear electromagnetic field equations.

REFERENCES

[1] V.B. Glasko, F. Leruste, Ya.P. Terletskii, S.F. Shushurin, J.E.T.P. (1958), 35, p. 452-457.

[2] Ya.P. Terletskii - DAN, USSR (1977), 236, p. 828-829.

[3] Ya.P. Terletskii - DAN, USSR (1960), 133, p. 329-332.