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Contributions to the theories of electromagnetism

and gravitation

by John CARSTOIU

International Consultant Scientists Corporation
44 Winslow Road, Brookline, Massachusetts 02146

Abstract : *The variety and originality of the subjects discussed may surprise the reader. There is a complete system of equations for an electromagnetic theory, whereby the new "equations C" play a decisive role. The L. de Broglie equations for the particle photon are easily derived from the equations C. The oscillating electric dipole of L. de Broglie leads, mutatis mutandis, to the oscillations of the Earth magnetic field and photon displays, of importance in a theory of the Aurora Borealis. The change in color of the aurorae is attributed to the L. de Broglie collisions of photons. A theory of rotating bodies (rotating frames of reference), which is conspicuously missing in relativity and electromagnetism appears as a contributor to planetary magnetism, as shown by the examples investigated here. One finds relativistic formulations for the addition of rotations (whose upper limit Ω was computed by Poincaré) and a relation energy-moment of inertia* : $E = I\Omega^2$, *which replaces Einstein's energy-mass relation* : $E = Mc^2$. *There is no time dilatation in rotating frames of reference. Correcting an error of Sommerfeld, the author shows unidirectional propagation of the electromagnetic field apparent to an observer at rest with respect to the uniform motion of an electron;*

the velocity of propagation is the velocity of the electron. The gravitational analog of this effect concerns and comple- tes the first principle of dynamics ; the uniform and rec- tilinear motion of a massive particle in the absence of any outside force involves a second gravitational field in addi- tion to the intrinsec field of gravity generated by the par- ticle. The two fields in question propagate in the direction of the motion which the velocity of the particle (or massive body).

Reconsidering his equations for the two gravita- tional fields $\vec{E}$ and $\vec{\Omega}$ (the latter is called the "gravitational vortex") the author introduces a second gravitational mass density ρ_ω in addition tho the mass density ρ_g. The ratio ρ_ω/ρ_g equals the ratio ρ_m/ρ_e between the densities of magnetic and electric charges. Thus, a subtle coupling between gravita- tion and electromagnetism is apparent. This coupling is actual- ly realized by simple relations between the electric field $\vec{E}$ and gravity field $\vec{G}$, and between the magnetic field $\vec{B}$ and the gravitational vortex $\vec{\Omega}$. It is verified that the fine-weather electric field at the surface of the Earth : $|\vec{E}| \simeq 100$ volt/m corresponds to the acceleration of gravity $|\vec{g}| \simeq 9,81$ m/sec².

The gravitational analog of the Larmor precession in magnetism shows an extremely slow rotation of the planetary orbits in their own plane due to the gravitational vortex $\vec{\Omega}$. It explains reasonably well the advance observed in the perihe- lion of Mercury.

The author reformulation of Bode's law predicts the semi-major axis of the "missing planet" in the solar sys- tem : a ≈58.5 astronomical units ; the distance derived is ba- sically the same as that computed from the observed perturba- tions in the orbits of Uranus and Neptune, corroborated by as- tronomer R.S. Harrington of the "U.S. Naval Observatory".

In appendix, the author shows how the equations C, in the presence of photons, lead to the exponential decay with the time of the fields and charges in a conductor. One may no- tice the new relation : $C^2 = \sigma_m/\sigma_e$, between the velocity of propagation C and the magnetic and electric conductivities σ_m and σ_e.

A thorough study of planetary motion forms the subject of appendix II.

Résumé : La variété et l'originalité de sujets traités peuvent étonner le lecteur. On y trouve un système complet d'équations pour une théorie électromagnétique où les nouvelles "équations C" jouent un rôle décisif. En partant de ces équations on dérive facilement les équations de L. de Broglie pour la particule photon. Le dipôle oscillant de L. de Broglie conduit, mutatis mutandis, aux oscillations du champ magnétique terrestre et le déploiement des photons dans ce champ, qui sont importants dans une théorie des aurores boréa- les. Le changement de couleur dans les aurores est attribué aux collisions de L. de Broglie entre photons. Une théorie de corps en rotation (systèmes de référence en rotation), qui est singulièrement absente en relativité et l'électromagnétisme, semble jouer un rôle important dans le magnétisme planétaire, comme le montrent les exemples étudiés ici. On donne des for- mules relativistes pour l'addition des rotations (la rotation limite étant donnée par Poincaré), et une relation énergie- moment d'inertie : $E = I\Omega^2$, qui remplace $E = Mc^2$ de Einstein. Il n'y a pas de dilatation du temps dans un système de référen- ce en rotation. En corrigeant une erreur de Sommerfeld, l'au- teur montre que le mouvement uniforme et rectiligne d'un élec- tron produit des ondes électromagnétiques qui se propagent dans la direction du mouvement de l'électron avec la vitesse de ce- lui-ci. L'analogue gravitationnel de cet effet implique l'exis- tence d'un second champ de gravitation, en outre de celui pro- duit par la particule massive. Cela concerne et complète le premier principe de la dynamique. Un observateur, au repos par rapport au mouvement de la particule libre, va détecter des ondes gravitationnelles se propageant dans la direction du mou- vement avec la vitesse de la particule (ou corps massif).

En reconduisant ses équations pour les deux champs de gravitation $\vec{G}$ et $\vec{\Omega}$ (le dernier, appelé "tourbillon gravita- tionnel"), l'auteur introduit une seconde densité de masse ρ_ω, en plus de la densité ρ_g. Le quotient : ρ_ω/ρ_g est égal à celui des densités de charges magnétique et électrique. Il apparaît ainsi un accouplement subtil de la gravitation et l'électroma- gnétisme. Cet accouplement est effectivement réalisé par des

relations simples entre le champ électrique $\vec{E}$ et le champ de gravitation $\vec{G}$, et entre le champ magnétique $\vec{B}$ et le tourbillon gravitationnel $\vec{\Omega}$. On vérifie que le champ électrique, par beau temps, à la surface de la Terre, à savoir $|\vec{E}| \simeq 100$ volt/ m correspond à la pesanteur $g \simeq 9,81$ m/sec².

L'analogue gravifique de la rotation de Larmor en magnétisme met en évidence une rotation extrêmement lente des orbites planétaires dans leur propre plan causée par le tourbillon gravitationnel $\vec{\Omega}$. Cela explique raisonnablement bien l'avance observée du périhélie de Mercure.

La reformulation de la loi de Bode prédit le demi-grand axe de la "planète manquante" dans le système solaire : $a \simeq 58,5$ unités astronomiques, ce qui est en bon accord avec la valeur calculée par la méthode des perturbations -celles observées dans les orbites de Uranus et Neptune, comme nous confirme l'astronome R.S. Harrington de "U.S. Naval Observatory".

Dans l'appendice, l'auteur montre comment les équations C, en présence de photons, conduisent à l'amortissement exponentiel des champs et charges dans un milieu conducteur. Il y a à remarquer la formule nouvelle : $C^2 = \sigma_m/\sigma_e$, entre la vitesse de propagation C et les conductivités magnétique et électrique σ_m et σ_e.

Une étude approfondie du mouvement planétaire fait l'objet de l'appendice II.

I. Introduction

Electromagnetic phenomena are the best understood of all nature's manifestations. It is hoped that our contributions may clarify the mysteries that still persist in this and related areas. Much less is known in gravitation. In his last book, Brillouin [1] speaks of the mystery of gravitation. Bridgman [2] wrote : "Einstein did not carry over into his general relativity the lessons and insights which he himself has taught us in his special relativity". The voluminous treaty entitled "Gravitation" by Misner, Thorne and Wheeler [3] is a

highly organized textbook. It goes far into geometrodynamics and cosmology. About the latter, Brillouin [1] said : "...we still are very far from understanding cosmogony. It remains a dream, a wonderful and evading dream". The great book [3] ends with the vers (sur l'Air de "Auprès de ma bonde") by M.A. Tonnelat et all. I quote only partially :

> De ravissants modèles
> Pour la cosmologie
> Pour moi ne m'en faut guère
> Car j'en ai un joli
> ....
> Il est dans ma cervelle
> Voici mon manuscrit.

Yes, it is good to dream "auprès de nos ondes". This brings us to very serious problems in *biophysics*, but this is another story !

Now, the lasers built by Kastler change our perspective of the world. The *theory of the double solution of L. de Broglie* [4] *may be the basis for lasers' development*, fact which is not well known in the United States. The reader is refered to the "preface" of a very remarkable booklet by L. de Broglie [5] for an illuminating discussion of the subject. The whole booklet is a marvel, very helpful to many readers in all these modern developments of lasers. L. de Broglie does not give there his non-Maxwelian system of equations, which, in my opinion, is equally important. I study in this mémoire a singular electromagnetic field : $\vec{E} = c\vec{B}$, and our equations C lead to a system of equations very similar to N.M. equations of L. de Broglie. In fact, by a suitable transformation, our equations in question become identical. This might lead to a new type of laser !

Brillouin [1] concluded his book looking forward for a *graser* a powerful amplifying device for gravity waves. He said : "Such a discovery would spark a big new chapter in physics and engineers might even build gravity transmitters and receivers competing with radio !" I may add that *with a graser we could communicate with submerged submarines*, a problem which preoccupied Brillouin and this writer.

With this in mind, I review and reexamine in this mémoire some old and new problems, bringing in new contributions. I search for simplicity ; it may be awarding. As Brillouin [1] said : "traveling along high roads is no fun, but wondering on forgotten tracks may lead to some wild summit from which you suddenly discover the whole landscape with an uncommon beauty". It is with this perspective that I record my findings here. It is too evident that I am greatly influenced by Brillouin's teaching and work. We went together to Washington D.C., in the spring of 1969, to talk to science officials about a program of research on gravitation initiated by Brillouin. One could imagine my emotion when, in a famous restaurant, in the presence of our wives, Brillouin told me : "nous nous complétons très bien". It was our last diner. A few months later Brillouin faded away. Brillouin died in the rising sun of the day of October 4, 1969. Caracteristic to a genius, he left to us his "Relativity Reexamined" [1], the manuscript of which was handled to his faithful "Academic Press" a few weeks before his untimely death. He answers there the objections of those "officials" that declined our proposal a few days before his death. Let me quote here these words by Heaviside [6, p. XXVII] ; "No matter how eminent they may be in their departments, officials need not be scientific men. It is not expected of them. But should they profess to be, and lay down the law outside their knowledge, and obstruct the spreading of views they cannot understand, their official weight imparts a fictious importance to their views, and are most deleteriously in propagating error, especially when their official position is held up as a screen to protect them from criticism".

Magnetic charge, current and conductivity were conceived by the singular genious of Heaviside [6]. There is a remarkable article on "gravitation and magnetic charge" by Kursunoglu [7]. Their existence and use could drastically change our nowadays technology and satisfy our wildest dreams. This is why I examined them in great detail. After all, science is a game peculiar to us as human beings. But "fortune favours the prepared mind" said Pasteur. A scientific law is an interpretation of nature, an actual *creation of negentropy* by human thought. Theories come and go, as we do, but facts remain.

I appreciate the comments by Dr. E.J. Post communicated to Professor John A. Wheeler, who kindly transmitted them to me. I thank them both. These comments concern a statement which I made in one of my unpublished papers, and which appears (generalized) in this mémoire, namely : "*An observer in the rotating frame (x',y',z') and his colleague in the fixed frame (x,y,z) will measure exactly same magnetic field*". Due to their importance, I take the liberty to quote here the essence of these comments : "... In fact, the Oppenheimer paradox (see L.I. Schiff, Proc. Natl. Ac. Sci. U.S. 25 (1939) 391) when seen in the light of that statement could well be considered as a potential contributor to planetary magnetism ... The author should know and mention that there is a measure of experimental support for his statement. The statement is in fact a dual of the Kennard (Phil. Mag. 33 (1917) 179) & Pegram (Phys. Rev. 10 (1917) 591) observations..."

Monsieur William P. Allis, Professor Emeritus of Physics at M.I.T., and a former consultant of "International Consultant Scientists Corporation" has encouraged the author, morally and materially, to publish his research. I have discussed with him many a problem, and his advice and help have been a light in my path wicked sometimes by vicissitudes of life. May he find here a token of respect and gratitude.

II. The Maxwell equations and the need to implement them. A new system

1. Fundamental equations. The Maxwell equations are (*see*, for reference Stratton [8]) :

$$(1) \quad \begin{aligned} \text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0, & \text{div } \vec{B} &= 0, \\ \text{curl } \vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{J}_e, & \text{div } \vec{D} &= \rho_e, \end{aligned}$$

where the electromagnetic variables are measured in *meter-kilogram-second-coulomb* units. Between the five vectors $\vec{E}$, $\vec{B}$, $\vec{D}$, $\vec{H}$, $\vec{J}_e$ and ρ_e there are but two vector and two scalar relations and one is, therefore, obliged to inquire further conditions if the system is to be made determinate.

If we consider the densities of magnetic current $\vec{J}_m$ and magnetic charge ρ_m, we shall write

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = -\mu \, \vec{J}_m, \qquad \text{div } \vec{B} = \mu \, \rho_m,$$

(2)

$$\text{curl } \vec{H} - \frac{\partial \vec{D}}{\partial t} = \vec{J}_e, \qquad \text{div } \vec{D} = \rho_e,$$

and the situation becomes more complicated.

It seems natural to implement the equations (2) by the equations

$$C^2 \text{ curl } \vec{D} + \frac{\partial \vec{H}}{\partial t} = -\vec{J}_m, \qquad \text{div } \vec{H} = \rho_m,$$

(3)

$$\text{curl } \vec{B} - \frac{1}{C^2} \frac{\partial \vec{E}}{\partial t} = \mu \, \vec{J}_e, \qquad \text{div } \vec{E} = \frac{1}{\varepsilon} \rho_e,$$

where C represents the velocity of propagation of a wave front in a medium of inductive capacities ε and μ.

We now subject the densities of currents and charges to the following equations

$$\text{curl } \vec{J}_e - \frac{1}{C^2} \frac{\partial \vec{J}_m}{\partial t} = \text{grad } \rho_m, \qquad \frac{\partial \rho_m}{\partial t} + \text{div } \vec{J}_m = 0,$$

(4)

$$\text{curl } \vec{J}_m + \frac{\partial \vec{J}_e}{\partial t} = -C^2 \text{ grad } \rho_e, \qquad \frac{\partial \rho_e}{\partial t} + \text{div } \vec{J}_e = 0.$$

These are "the equations C" ; *see*, Carstoiu [9, Appendix I].

Equations (2), (3) and (4) determine *all* the electromagnetic variables, subject to initial and boundary conditions. These verify the equation

(5) $\nabla^2 - \frac{1}{C^2} \frac{\partial^2}{\partial t^2} = 0, \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2},$

which shows that *the quantities in question propagate in the*

medium with the velocity C.

Our equations above are invariant under the following transformation essentially due to Schwinger [10,11]. (He omites the quantities $\vec{B}$ and $\vec{D}$, and uses other units) :

$$\vec{E}' = \vec{E} \cos\theta + C \, \vec{B} \sin\theta,$$

$$\vec{B}' = \vec{B} \cos\theta - \frac{1}{C} \vec{E} \sin\theta,$$

(6) $\quad \vec{D}' = \vec{D} \cos\theta + \frac{1}{C} \vec{H} \sin\theta,$

$$\vec{H}' = \vec{H} \cos\theta - C \, \vec{D} \sin\theta,$$

$$\vec{J}_e' = \vec{J}_e \cos\theta + \frac{1}{C} \vec{J}_m \sin\theta,$$

$$\vec{J}_m' = \vec{J}_m \cos\theta - C \, \vec{J}_e \sin\theta,$$

with the electric and magnetic charge densities ρ_e and ρ_m following the pattern of electric and magnetic current densities $\vec{J}_e$ and $\vec{J}_m$. The transformation (6) expresses a rotation through the arbitrary angle θ.

If we set $\rho_m' = 0$ (absence of magnetic charge in purely electromangetic considerations), we have

(7) $\qquad \rho_m = C \, \rho_e \tan \theta,$

which represents a straight line in the charge plane (C ρ_e, ρ_m). On the bisector $\theta = 45°$, we have

(8) $\qquad \rho_m = C \, \rho_e,$

an important relation which we are going to study in a little while. We also note, with Schwinger, the two invariants of this rotation :

(9) $\qquad C^2 \rho_{e_1} \rho_{e_2} + \rho_{m_1} \rho_{m_2}, \quad \rho_{e_1} \rho_{m_2} - \rho_{e_2} \rho_{m_1},$

corresponding to lenghts (C$^2 \rho_e^2 + \rho_m^2$) and angles between two-dimensional vectors.

The transformation (6) shows that when $\vec{D} = \varepsilon\vec{E}$, $\vec{H} = (1/\mu)\vec{B}$, we necessarily have : $\varepsilon\mu C^2 = 1$. Assuming ε and μ to be constant, equations (2) and (3) become identical.

Let us assume that ε and μ are variable. We then have

$$(10) \qquad \text{div } \vec{B} = \mu \text{ div } \vec{H} + \vec{H}.\text{grad } \mu = \mu \, \rho_m,$$

or, because of the relation : $\text{div } \vec{H} = \rho_m$,

$$(11) \qquad \vec{H}.\text{grad } \frac{\mu}{\rho_m} = 0,$$

that is

$$(12) \qquad \frac{\partial \mu}{\partial x} = \frac{\partial \mu}{\partial y} = \frac{\partial \mu}{\partial z} = 0.$$

Similarly, we have

$$(13) \qquad \frac{\partial \varepsilon}{\partial x} = \frac{\partial \varepsilon}{\partial y} = \frac{\partial \varepsilon}{\partial z} = 0.$$

Hence, our ε and μ represent maximum or minimum values (saddle points). *These depend on frequency.*

Equations (10) and (11) differ from those given by Sommerfeld [12, pp. 40-41], who assumes : $\text{div } \vec{B} = 0$.

In *vacuum*, we write : $\vec{D} = \varepsilon_0\vec{E}$, $\vec{H} = (1/\mu_0)\vec{B}$, $\varepsilon_0\mu_0 C^2 = 1$, where C is the velocity of light.

2. <u>A theorem of L. de Broglie</u>. We insert here a very important remarque due to L. de Broglie [13, pp. 2-4]. Let us assume that there are neither charges nor currents in the medium considered and ε, μ are constant. We shall further assume that the field vectors contain the time only as a factor $\exp(i\omega t)$. Writing $\vec{D} = \varepsilon\vec{E}$, and $\vec{B} = \mu\vec{H}$, the field equations are

$$(14) \quad
\begin{aligned}
\text{curl } \vec{E} + i\omega\mu\vec{H} &= 0, & \text{div } \vec{H} &= 0, \\
\text{curl } \vec{H} - i\omega\varepsilon\vec{E} &= 0, & \text{div } \vec{E} &= 0.
\end{aligned}$$

In our units, we shall put

$$(15) \qquad \vec{E}' = \sqrt{\frac{\varepsilon}{\varepsilon_0}}\,\vec{E}, \quad \vec{H}' = \sqrt{\frac{\mu}{\mu_0}}\,\vec{H}, \quad \omega' = \omega\sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}},$$

where $\sqrt{\varepsilon\mu/\varepsilon_0\mu_0} = c/C = n$ is the index of refraction with respect to vacuum, say the air.

Equations (14) become

$$(16) \quad
\begin{aligned}
\text{curl } \vec{E}' + i\omega'\mu_0\vec{H}' &= 0, & \text{div } \vec{H}' &= 0, \\
\text{curl } \vec{H}' - i\omega'\varepsilon_0\vec{E}' &= 0, & \text{div } \vec{E}' &= 0.
\end{aligned}$$

These equations have the same form as the Maxwell equations in *vacuum* for a harmonic electromagnetic field of frequency $\omega'/2\pi$. Hence, this elegant result of L. de Broglie, transposed in *meter-kilogram-second-coulomb* units :

If, *for a certain enclosure empty of matter, one has found a harmonic solution of the Maxwell equations corresponding to a certain value of ω', one will obtain a solution valid for same enclosure filled with matter of dielectric constant ε and magnetic permeability μ by replacing in the former solution ω', $\vec{E}'$ and $\vec{H}'$ by the values given by (15).*

Verification of these formulas is simple. Equations (14) and (16) yield propagation equations in matter and in vacuum

$$(17) \quad
\begin{aligned}
\nabla^2\vec{E} + \frac{\omega^2}{C^2}\,\vec{E} &= 0, & \nabla^2\vec{H} + \frac{\omega^2}{C^2}\,\vec{H} = 0 & \quad (C = \frac{1}{\sqrt{\varepsilon\mu}}), \\
\nabla^2\vec{E}' + \frac{\omega'^2}{C^2}\,\vec{E}' &= 0, & \nabla^2\vec{H}' + \frac{\omega'^2}{C^2}\,\vec{H}' = 0 & \quad (c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}).
\end{aligned}$$

These equations are identical if we take the values (15).

3. <u>Retarded potentials</u>. Concentrating our attention on the field vectors $\vec{E}$ and $\vec{B}$, that one eventually measures, we introduce the electromagnetic potentials by writing

$$\vec{E} = - \frac{\partial \vec{A}_e}{\partial t} - \text{grad } \phi_e - C^2 \text{ curl } \vec{A}_m,$$

(18)

$$\vec{B} = - \frac{\partial \vec{A}_m}{\partial t} - \text{grad } \phi_m + \text{curl } \vec{A}_e.$$

We shall subject these potentials to the conditions

(19) $\quad \frac{1}{C^2} \frac{\partial \phi_e}{\partial t} + \text{div } \vec{A}_e = 0, \quad \frac{1}{C^2} \frac{\partial \phi_m}{\partial t} + \text{div } \vec{A}_m = 0.$

Our equations (2) and (3) yield

$$\nabla^2 \vec{A}_e - \frac{1}{C^2} \frac{\partial^2 \vec{A}_e}{\partial t^2} = -\mu \vec{J}_e, \quad \nabla^2 \vec{A}_m - \frac{1}{C^2} \frac{\partial^2 \vec{A}_m}{\partial t^2} = - \frac{\mu}{C^2} \vec{J}_m,$$

(20)

$$\nabla^2 \phi_e - \frac{1}{C^2} \frac{\partial^2 \phi_e}{\partial t^2} = - \frac{1}{\varepsilon} \rho_e, \quad \nabla^2 \phi_m - \frac{1}{C^2} \frac{\partial^2 \phi_m}{\partial t^2} = -\mu \rho_m.$$

Equations (20) give

$$4\pi \frac{1}{\mu} \vec{A}_e = \int \frac{[\vec{J}_e]}{r} d\tau, \quad 4\pi \frac{C^2}{\mu} \vec{A}_m = \int \frac{[\vec{J}_m]}{r} d\tau,$$

(21)

$$4\pi\varepsilon \phi_e = \int \frac{[\rho_e]}{r} d\tau, \quad 4\pi \frac{1}{\mu} \phi_m = \int \frac{[\rho_m]}{r} d\tau,$$

where the quantities in brackets [ ] are the values of intensities of currents and charges not at the time t of observation but at the earlier time : $t' = t - r/C$, $r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$ and $d\tau = d\xi d\eta d\zeta$; the integration is carried out over all infinite space. *Formulas (21) define our retarded potentials $\vec{A}_e$, $\vec{A}_m$, ϕ_e and ϕ_m.*

By using the equations C, we obtain

$$\vec{E} = \frac{\mu}{4\pi} \int \frac{1}{r^3} \{C^2 [\rho_e] \vec{r} - [\vec{J}_m] \times \vec{r}\} d\tau,$$

(22)

$$\vec{B} = \frac{\mu}{4\pi} \int \frac{1}{r^3} \{[\rho_m] \vec{r} + [\vec{J}_e] \times \vec{r}\} d\tau,$$

which generalize, in a remarkable fashion, the *Coulomb field* in electromagnetism.

Anticipating our formulas in gravitation, we have the following expressions for *the two gravitational fields $\vec{G}$ and $\vec{\Omega}$* :

$$\vec{G} = - \frac{\gamma}{C^2} \int \frac{1}{r^3} \{C^2 [\rho_g] \vec{r} - [\vec{J}_\omega] \times \vec{r}\} d\tau,$$

(23)

$$\vec{\Omega} = - \frac{\gamma}{C^2} \int \frac{1}{r^3} \{[\rho_\omega] \vec{r} + [\vec{J}_g] \times \vec{r}\} d\tau,$$

where γ is Newton's constant and $\vec{J}_g$, $\vec{J}_\omega$, ρ_g and ρ_ω denote intensities of gravitational currents and masses. The first formula in (23) *generalizes Newton's field*. Clearly, $\vec{G}$ and $\vec{\Omega}$ propagate with the velocity $C = c$ in vacuum, where $\gamma = \gamma_0$.

III. The L. de Broglie electromagnetic waves and photons, and the equations C. Various consequences

1. Klein-Gordon equation of propagation. Waves mechanics discovered in 1923 by L. de Broglie marks a new era in physics. L. de Broglie has published numerous papers and books on the subject. We refer here to his booklet [5] where he discuses his *theory of the double solution and the theory of lasers.*

Written in *meter-kilogram-second-coulomb* units, L. de Broglie equations for the particle photon, in vacuum, are

$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0, \qquad \text{div } \vec{B} = 0,$$

(24)

$$\text{curl } \vec{B} - \frac{1}{C^2} \frac{\partial \vec{E}}{\partial t} = -k_0^2 \vec{A}_e, \qquad \text{div } \vec{E} = -k_0^2 \phi_e,$$

where k_0 is L. de Broglie constant defined by : $k_0 = (1/\hbar) m_0 c$, m_0 being the proper mass of photon and $2\pi\hbar$ the Planck constant. The value of m_0 is extraordinary small (certainly inferior to 10^{-45} kg) but L. de Broglie did not want to consider it rigo-

rously null. As required by the formalism of quantum mechanics, the quantities in equations (24) are essentially complex. F being one of them, we have $F = F_1 + iF_2$, where F_1 and F_2 are real.

If we consider the magnetic potentials $\vec{A}_m$ and ϕ_m, we shall write

(25)
$$\text{curl } \vec{E} + \frac{\partial \vec{B}}{\partial t} = c^2 k_0^2 \vec{A}_m, \qquad \text{div } \vec{B} = -k_0^2 \phi_m,$$

$$\text{curl } \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = -k_0^2 \vec{A}_e, \qquad \text{div } \vec{E} = -k_0^2 \phi_e.$$

Equations (25) immediately give the relations (19) between the potentials.

L. de Broglie admits *a priori* that the components F of the field vectors and potentials obey the equation of Klein-Gordon

(26)
$$\Box F = k_0^2 F \qquad \Box = \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2},$$

and then verifies that the equations (24) lead to the definition of potentials. We prefer to admit *a priori* the existence of relations (18) and then verify that the quantities F obey the equation of Klein-Gordon (26). Let us remark, that our *relations (18) are true equations which together with the equations (25) and (19) determine the field vectors $\vec{E}$ and $\vec{B}$, and the potentials $\vec{A}_e$, $\vec{A}_m$, ϕ_e and ϕ_m, for which we have four vector equations and two scalar equations. Our potentials $\vec{A}_e$, $\vec{A}_m$, ϕ_e and ϕ_m are as real as our field vectors $\vec{E}$ and $\vec{B}$ are.*

Now, in order to take into account L. de Broglie's constant k_0 in our equations C, we rewrite the latter as follows

(27)
$$\text{curl } \vec{J}_e - \frac{1}{c^2} \frac{\partial \vec{J}_m}{\partial t} = \text{grad } \rho_m - \frac{1}{\mu_0} k_0^2 \vec{B},$$

$$\text{curl } \vec{J}_m + \frac{\partial \vec{J}_e}{\partial t} = -c^2 \text{ grad } \rho_e + \frac{1}{\mu_0} k_0^2 \vec{E}.$$

Comparison of the equations (27) with the equations for the potentials (18) yields the important relations

(28)
$$\vec{J}_e = -\frac{1}{\mu_0} k_0^2 \vec{A}_e, \qquad \rho_e = -\epsilon_0 k_0^2 \phi_e,$$

$$\vec{J}_m = -\frac{1}{\mu_0} c^2 k_0^2 \vec{A}_m, \qquad \rho_m = -\frac{1}{\mu_0} k_0^2 \phi_m.$$

Our equations (2) and (3) (written for vacuum) and the equations (25) become identical. The continuity equations in our equations C (4) yield the relations (19) between the potentials.

Thus, we answer, at least partially, the problem put by L. de Broglie [5, Preface] : *"donner une image claire et précise dont peuvent se concilier la validité des équations de Maxwell et l'existence des photons".*

Simple computation shows that all electromagnetic variables are subject to the Klein-Gordon equation (26).

2. <u>Plane monochromatic waves. Relativistic relations. Doppler principle</u>. The Klein-Gordon equation admits solutions of the form

(29)
$$e^{i(\omega t - \vec{k}.\vec{r})}, \qquad \vec{k}.\vec{r} = k_x x + k_y y + k_z z,$$

that is the quantities above propagate as plane monochromatic waves, provided that

(30)
$$|\vec{k}|^2 - \frac{\omega^2}{c^2} + k_0^2 = 0.$$

In a Lorentzian frame of reference, we shall have solutions of the form

(31)
$$e^{i(\omega' t' - \vec{k}'.\vec{r}')},$$

under the condition

(32)
$$|\vec{k}'|^2 - \frac{\omega'^2}{c^2} + k_0^2 = 0.$$

We require that

$$(33) \quad \omega' t' - (k'_{x'}\, x' + k'_{y'}\, y' + k'_{z'}\, z') \equiv \omega t - (k_x x + k_y y + k_z z).$$

Now, the Lorentz transformation is

$$(34) \quad x' = x, \quad y' = y, \quad z' = \frac{1}{\sqrt{1-\beta^2}}(z - vt), \quad t' = \frac{1}{\sqrt{1-\beta^2}}\left(t - \frac{v}{c^2}z\right),$$

$$\beta = \frac{v}{c}$$

Hence

$$(35) \quad k'_{x'} = k_x, \quad k'_{y'} = k_y, \quad k'_{z'} = \frac{1}{\sqrt{1-\beta^2}}\left(k_z - \frac{v}{c^2}\omega\right),$$

$$\omega' = \frac{1}{\sqrt{1-\beta^2}}(\omega - v k_z).$$

We have

$$(36) \quad \omega = \frac{1}{\hbar}E, \quad \vec{k} = \frac{1}{\hbar}\vec{p}, \quad E = Mc^2, \quad \vec{p} = M\vec{u}.$$

The formulas (35) give

$$(37) \quad p'_{x'} = p_x, \quad p'_{y'} = p_y, \quad p'_{z'} = \frac{1}{\sqrt{1-\beta^2}}\left(p_z - \frac{v}{c^2}E\right),$$

$$E' = \frac{1}{\sqrt{1-\beta^2}}(E - v p_z),$$

known in the theory of relativity ; see W. Pauli [14, p. 87].

The latter formula yields

$$(38) \quad M' = \frac{M}{\sqrt{1-\beta^2}}\left(1 - \frac{v u_z}{c^2}\right),$$

which generalizes Einstein's relation, and reduces to the latter when $u_z = v$.

When $u_z = 0$, we have

$$(39) \quad M' = \frac{M}{\sqrt{1-\beta^2}},$$

that is

$$(40) \quad M = M_0 \sqrt{1-\beta^2}, \quad M' = M_0,$$

which is our result. We note that the formula of slowing down of clocks : $\nu = \nu_0 \sqrt{1-\beta^2}$ is an immediate consequence of (40).

Finally, the first three relations in (37) and (38) give

$$(41) \quad u'_x = \frac{M}{M'}u_x = \frac{\sqrt{1-\beta^2}}{1 - \frac{v u_z}{c^2}}u_x, \quad u'_y = \frac{\sqrt{1-\beta^2}}{1 - \frac{v u_z}{c^2}}, \quad u'_z = \frac{u_z - v}{1 - \frac{v u_z}{c^2}}$$

due to Poincaré ; see Pauli (14, p. 16).

We now rapidly reformulate the Doppler principle taking into account the L. de Broglie constant k_0. Relations (30) and (31) show that

$$(42) \quad |\vec{k}'|^2 - \frac{\omega'^2}{c^2} = |\vec{k}'|^2 - \frac{\omega^2}{c^2} = -k_0^2.$$

Assuming that the wave propagates in the (y,z) plane, we write

$$(43) \quad k_x = 0, \quad k_y = \frac{2\sigma}{K}\sin\alpha, \quad k_z = \frac{2\sigma}{K}\cos\alpha, \quad \frac{\omega}{c} = \frac{2\pi}{\lambda}, \quad k_0 = \frac{2\pi}{L_0}.$$

Formulas (35) give

$$(44) \quad \frac{\sin\alpha'}{K'} = \frac{\sin\alpha}{K}, \quad \frac{\cos\alpha'}{K'} = \frac{1}{\lambda\sqrt{1-\beta^2}}\left(\frac{\lambda}{K}\cos\alpha - \beta\right),$$

$$(45) \quad \frac{1}{\lambda'} = \frac{1}{\lambda\sqrt{1-\beta^2}}\left(1 - \beta\frac{\lambda}{K}\cos\alpha\right),$$

where, by virtue of (42),

$$(46) \quad \frac{\lambda}{K} = \sqrt{1 - \frac{\lambda^2}{L_0^2}} = 1 - \frac{1}{2}\frac{\lambda^2}{L^2} + \cdots$$

The relation (45) represents our *reformulation of the Doppler principle which takes into account the L. de Broglie wave length L_0*. The latter is a very large quantity : $L_0 > 2.18 \times 10^6$ m. Our correction is, therefore, quite negligible unless λ is large.

Forming the ratio of equations (44), we have

$$(47) \qquad \tan \alpha' = \frac{\frac{\lambda}{K} \sin\alpha \sqrt{1-\beta^2}}{\frac{\lambda}{K} \cos\alpha - \beta},$$

which is to be compared to the formula for the aberration of light in optics ; *see*, Sommerfeld [15, p. 68].

3. Generalization of the L. de Broglie flux-density. We go on generalizing L. de Broglie's [5, p. 17] *flux-density* by writing

$$\rho = \frac{i}{\hbar c^2}[\vec{A}_e^* \cdot \vec{E} - \vec{E}^* \cdot \vec{A}_e - c^2(\vec{A}_m^* \cdot \vec{B} - \vec{B}^* \cdot \vec{A}_m)],$$

$$(48)$$

$$\vec{j} = \rho\vec{v} = \frac{i}{\hbar}[\vec{A}_e^* \times \vec{B} + \vec{B}^* \times \vec{A}_e + \vec{A}_m^* \times \vec{E} + \vec{E}^* \times \vec{A}_m$$

$$+ \frac{1}{c^2}(\phi_e^* \vec{E} - \phi_e \vec{E}^*) - (\phi_m^* \vec{B} - \phi_m \vec{B}^*)],$$

where the quantities considered are complex fields and potentials and the star indicate conjugate quantities. We note with L. de Broglie that ρ and $\vec{j}$ are *real* quantities.

Using the relations (18), (25) and formulas of vector analysis we check the continuity equation of fluid mechanics, namely

$$(49) \qquad \frac{\partial \rho}{\partial t} + \text{div}(\rho\vec{v}) = 0.$$

Our definition (48) reduces to the definition of L. de Broglie of the *flux-density* if we take $\vec{A}_m = 0$, $\phi_m = 0$.

The density $\rho(x,y,z,t)$ *defines the probability of presence of a photon at the point* x, y, z *at the time* t, *whereas the velocity* $\vec{v} = \vec{j}/\rho$ *defines the guiding of the photon by*

the electromagnetic field and potentials.

4. Oscillations of the Earth magnetic field and photon displays. Toward a new theory of Aurora Borealis. As an important application of our relations (48), we consider an oscillating magnetic dipole, say the Earth's magnetic field. This complements a study by L. de Broglie [5, pp. 44-46] of an electric dipole. Taking the dipole axis as the Oz-axis, O being the origin of the coordinates where the dipole is located, we write

$$(50) \quad A_{e_x} = A_{e_y} = A_{e_z} = \phi_e = 0 \; ; \; A_{m_x} = A_{m_y} = 0, \; A_{m_z} = -\frac{M}{r} e^{i\alpha},$$

where M is the magnetic moment of the dipole and $\alpha = \omega t - |\vec{k}|r$, with $\omega^2/c^2 = |\vec{k}| + k_0^2$.

In spherical coordinates (r, θ, ϕ) used for the Earth's magnetic field, we have

$$(51) \quad A_{m_r} = -\frac{M\cos\theta}{r} e^{i\alpha}, \; A_{m_\theta} = -\frac{M\sin\theta}{r} e^{i\alpha}, \; A_{m_\phi} = 0.$$

Thus

$$(52) \; \frac{1}{c^2}\frac{\partial \phi_m}{\partial t} = -\text{div}\,\vec{A}_m = -\frac{\partial A_{m_z}}{\partial z} = -\frac{\partial A_{m_z}}{\partial r}\cos\theta$$

$$= -M\cos\theta(\frac{1}{r^2} + \frac{i|\vec{k}|}{r})e^{i\alpha}.$$

Hence, integrating on the time

$$(53) \qquad \phi_m = \frac{ic^2}{\omega} M \cos\theta(\frac{1}{r^2} + \frac{i|\vec{k}|}{r})e^{i\alpha}.$$

Computing the non-vanishing components of the field, we obtain

$$(54)\begin{cases} B_r = -\dfrac{\partial A_{m_r}}{\partial t} - \dfrac{\partial \phi_m}{\partial r} = \dfrac{ic^2 k_0^2}{\omega}\dfrac{M\cos\theta}{r}e^{i\alpha} + \dfrac{ic^2}{\omega}\dfrac{2M\cos\theta}{r^2}\left(\dfrac{1}{r}+i|\vec{k}|\right)e^{i\alpha}, \\[2mm] B_\theta = -\dfrac{\partial A_{m_\theta}}{\partial t} - \dfrac{1}{r}\dfrac{\partial \phi_m}{\partial\theta} = \dfrac{i\omega M\sin\theta}{r}e^{i\alpha} + \dfrac{ic^2}{\omega}\dfrac{M\sin\theta}{r}\left(\dfrac{1}{r}+i|\vec{k}|\right)e^{i\alpha}, \\[2mm] E_\phi = \dfrac{c^2}{r}\left[\dfrac{\partial}{\partial r}(rA_{m_\theta}) - \dfrac{\partial A_{m_r}}{\partial\theta}\right] = -i|\vec{k}|c^2\dfrac{M\sin\theta}{r}e^{i\alpha} + c^2\dfrac{M\sin\theta}{r^2}e^{i\alpha}. \end{cases}$$

Our equations (54) record the oscillations of the Earth's magnetic field. By the way of digression, we note that the electric field E_ϕ is quite strong in equatorial zones where hurricanes originate, and might influence the development of the latter, an important problem open for investigation.

Equations (54) are new and may surprize the geophysicist. However, they seem to be of great importance in the physics of aurora and airglow. It is for this reason that we now shall compute the probability of the presence of photons in the geomagnetic field and their guidance by the latter.

Our first formula in (48) gives

$$(55) \qquad \rho = -\frac{i}{\hbar}(A_{m_r}^* B_r + A_{m_\theta}^* B_\theta - \text{conj.}).$$

Using the relations (54), where we assume r large enough as to keep only the terms in 1/R, we obtain

$$(56) \qquad \rho = \frac{2}{\hbar}\frac{c^2}{\omega}\frac{M^2}{r^2}(k_0^2\cos^2\theta + \frac{\omega^2}{c^2}\sin^2\theta).$$

Similarly, we have

$$(57)\begin{cases} \rho v_r = \frac{i}{\hbar}(A_{m_\theta}^* E_\phi - \phi_m^* B_r - \text{conj.}) \\[1mm] \qquad = -\frac{2}{\hbar}\frac{c^4|\vec{k}|}{\omega^2}\frac{1}{r^2}(k_0^2\cos^2\theta + \frac{\omega^2}{c^2}\sin^2\theta), \\[1mm] \rho v_\theta = \frac{i}{\hbar}(-A_r^* E_\phi - \phi_m^* B_\theta - \text{conj.}) = 0 \\[1mm] \rho v_\phi = 0. \end{cases}$$

Inspection of formulas (56) and (57) shows that

$$(58) \qquad v_r = -\frac{|\vec{k}|}{\omega}c^2.$$

When $k_0 = 0$ ($m_0 = 0$), $|\vec{k}| = \omega/C$ and, therefore, $|v_r| = C$ as we expected.

In order to check these results, we compute

$$(59) \qquad E = \frac{m_0 c^2}{\sqrt{1-\dfrac{v_r^2}{c^2}}} = \frac{m_0 c\omega}{k_0} = \hbar\omega = h\nu \qquad \nu = \frac{\omega}{2\pi},$$

which is correct.

The density vanishes if

$$(60) \qquad \cot^2\theta = -\frac{1}{k_0^2}\frac{\omega^2}{c^2} = -\frac{1}{k_0^2}(|\vec{k}|^2 + k_0^2) = -\frac{|\vec{k}|^2}{k_0^2} - 1,$$

which is impossible if the wave number $|\vec{k}|$ is real. We, therefore, must assume that $|\vec{k}|$ becomes a purely imaginary quantity : $|\vec{k}| = -i\gamma$; then

$$(61) \qquad \cot^2\theta = \frac{\gamma^2}{k_0^2} - 1.$$

If we take $(\gamma/k_0)^2 = 2$, then $\cot^2\theta = 1$, $\theta = \pm45°$; it follows that our massive photons can exist only inside the volume of the cone with vertex at 0 and semi-angle 45°. On this cone and beyond, the exponentials $e^{-i|\vec{k}|r}$ become $e^{-\gamma r}$ and the oscillations of the Earth's magnetic field decay exponentially.

Returning to the relation : $\omega^2/c^2 = |\vec{k}| + k_0^2$, we shall have

$$(62) \qquad \frac{\omega^2}{c^2} = -\gamma^2 + k_0^2.$$

Hence, for ω real

$$(63) \qquad \gamma^2 - k_0^2 \leqslant 0.$$

Therefore, if $\gamma = k_0$, then $\omega = 0$ and the oscilla-

tions become stationnary. This can happen at the poles or close to them. Finally, if $\gamma > k_0$, ω becomes a purely imaginary quantity and our oscillations decay exponentially with the time.

To sum up, *we associate the aurorae with the presence of the L. de Broglie photons. The latter seem to be produced by the "fusion" of the electrons abundant in polar regions.* For the *"theory of fusion"*, we refer the reader to the remarkable book by L. de Broglie [16].

The change in color of aurorae is undoubledly caused by collisions of the photons. L. de Broglie [5, pp. 82-84] shows, by a simple and elegant computation, that two red photons produce by collision a violet photon, etc. There is, of course, much to be told about the subject. As Störmer [17] said : "This fascinating phenomenon, the aurora, guards its secrets well and it may be far in the future before they are completely yielded up to man".

We have made here a beginning, a *new beginning* toward this future. For what has been done so far, we refer the reader to the pioneering work of Störmer [17], the classical treatise by Chapman and Bartels [18] and the excellent books by Chamberlain [19] and Omholt [20].

5. A singular electromagnetic field. The non-Maxwellian equations of L. de Broglie. The relation (8) leads us to consider the field

$$(64) \qquad \vec{E} = C\vec{B},$$

of importance in electro-optics. Taking the divergence of both sides we have

$$(65) \qquad \frac{\rho_e}{\varepsilon} = C\mu\rho_m,$$

that is

$$(66) \qquad \frac{\rho_m}{\rho_e} = \frac{1}{\varepsilon\mu C} = C,$$

which is precisely our relation (8).

We now put

$$(67) \qquad \vec{J}_e = \rho_e\vec{v}, \qquad \vec{J}_m = \rho_m\vec{v},$$

$\vec{v}$ being the velocity of a particle. We have

$$(68) \qquad \begin{aligned} &\operatorname{curl} \vec{E} + \frac{\partial\vec{B}}{\partial t} = -\mu\rho_m\vec{v}, \\ &\operatorname{curl} \vec{B} - \frac{1}{C^2}\frac{\partial\vec{E}}{\partial t} = \mu\rho_e\vec{v}. \end{aligned}$$

Hence, by virtue of relations (64) and (66) we have

$$(69) \qquad \begin{aligned} &\operatorname{curl} \vec{B} = 0, \qquad \frac{\partial\vec{B}}{\partial t} = -\mu C\rho_e\vec{v} \\ &\operatorname{curl} \vec{E} = 0, \qquad \frac{\partial\vec{E}}{\partial t} = -\mu C^2\rho_e\vec{v} = -\frac{1}{\varepsilon}\rho_e\vec{v}. \end{aligned}$$

On the other hand, the modified equations C, for a medium of inductive capacities ε and μ, yield

$$(70) \qquad \begin{aligned} &\operatorname{curl}(\rho_e\vec{v}) - \frac{1}{C^2}\frac{\partial(\rho_m\vec{v})}{\partial t} = \operatorname{grad} \rho_m - \frac{1}{\mu}k^2\vec{B}, \\ &\operatorname{curl}(\rho_m\vec{v}) + \frac{\partial(\rho_e\vec{v})}{\partial t} = -C^2\operatorname{grad} \rho_e + \frac{1}{\mu}k^2\vec{E}, \end{aligned}$$

where the L. de Broglie constant k corresponds to the proper mass of photon in the medium.

It follows that

$$(71) \qquad \operatorname{curl}(\rho_e\vec{v}) = 0, \qquad \frac{\partial(\rho_e\vec{v})}{\partial t} + C^2\operatorname{grad} \rho_e - \frac{1}{\mu}k^2\vec{E} = 0.$$

Thus

$$(72) \qquad \vec{E} = \operatorname{grad} I, \qquad \rho_e\vec{v} = \operatorname{grad} I_2.$$

Equations (68) and (71) give

(73) $\operatorname{grad}(\frac{\partial I}{\partial t} + \frac{1}{\varepsilon} I_2) = 0$, $\operatorname{grad}(\frac{\partial I_2}{\partial t} + C^2 \rho_e - \frac{1}{\mu} k^2 I) = 0.$

We shall take

(74) $\frac{\partial I}{\partial t} + \frac{1}{\varepsilon} I_2 = 0$, $\frac{\partial I_2}{\partial t} + C^2 \rho_e - \frac{1}{\mu} k^2 I = 0.$

Equations (74) together with the equation of continuity :

(75) $\frac{\partial \rho_e}{\partial t} + \operatorname{div}(\rho_e \vec{v}) = \frac{\partial \rho_e}{\partial t} + \nabla^2 I = 0,$

will determine the quantities I, I_2, ρ_e and $\vec{E}$ when $\vec{v}$ is known.

There is a curious similarity between our equations (72)-(75) and the non-Maxwellian equations of L. de Broglie [16, p. 109]. In fact, if we write

(76) $\rho_e \vec{v} = i k \vec{\sigma}$, $i k \sigma_4 = C^2 \rho_e - \frac{1}{\mu} k^2 I,$

equations (72-75), in vacuum (C=c, k=k₀), become identical with those of L. de Broglie (rewritten in our units), namely :

(77) $\begin{cases} -\dfrac{\partial I_2}{\partial t} = i k_0 \sigma_4, \quad \operatorname{grad} I_2 = i k_0 \vec{\sigma}, \quad \dfrac{1}{c^2} \dfrac{\partial \sigma_4}{\partial t} + \operatorname{div} \vec{\sigma} = -i k_0 I_2, \\[2mm] \qquad\qquad \operatorname{curl} \vec{\sigma} = 0, \quad \dfrac{\partial \vec{\sigma}}{\partial t} + \operatorname{grad} \sigma_4 = 0. \end{cases}$

Just as the L. de Broglie variables, our quantities obey to the Klein-Gordon equation.

III. Rotating bodies and new equations for the field vectors

1. Fundamental mapping. We begin by considering the helicoidal motion of a rigid body. Both rotation and translation, are limiting cases of the latter. Now, a helicoidal motion of a rigid body is described by the helicoidal motion of a frame of reference (x',y',z') tied to the body with respect to a fixed frame (x,y,z). We have the mapping

(78) $x' = x \cos\omega t + y \sin\omega t, \quad y' = -x \sin\omega t + y \cos\omega t,$
$z' = z - \ell\omega t, \qquad\qquad t' = t$

where ω is the angular velocity of the body and ℓ is the helicoidal parameter which a constant. We shall assume a uniform rotation, that is $\omega = $ const.

Let us write

(79) $\ell \omega = v, \qquad \ell = \frac{v}{\omega}.$

when $\ell = 0$, $v = 0$ and we have a pure rotation. Now, when $\omega \to 0$, we shall assume that $\ell \to \infty$ in such a way that v exists and is finite. Formulas (78) define then a pure translation (the "Galilean transformation").

We generalize the mapping (78) by writing

(80) $x' = x \cos\omega t + y \sin\omega t, \quad y' = -x \sin\omega t + y \cos\omega t,$
$z' = \frac{1}{\sqrt{1-\beta^2}}(z - vt), \qquad t' = \frac{1}{\sqrt{1-\beta^2}}(t - \frac{v}{C^2} z), \quad \beta = \frac{v}{C}.$

Clearly, our transformation (80) reduces to the Lorentz transformation when $\omega \to 0$, by virtue of our relation (79).

On the other hand, when $\ell = 0$, $v = 0$ and the mapping (80) becomes

(81) $x' = x \cos\omega t + y \sin\omega t, \quad y' = -x \sin\omega t + y \cos\omega t,$
$z' = z, \qquad\qquad t' = t,$

which represents the rotation of a rigid body.

Thus, contrary to the Lorentzian physicist, *an observer in the rotating frame (x',y',z') will measure exactly same time t as his colleague in the fixed frame (x,y,z).*

Corresponding to our transformation (80), we have

$$B'_{x'} = \frac{1}{\sqrt{1-\beta^2}}[(B_x + \frac{v}{C^2} E_y)\cos\omega t + (B_y - \frac{v}{C^2} E_x)\sin\omega t],$$

(82)
$$B'_{y'} = \frac{1}{\sqrt{1-\beta^2}}[(B_y - \frac{v}{C^2} E_x)\cos\omega t - (B_x + \frac{v}{C^2} E_y)\sin\omega t],$$

$$B'_{z'} = B_z,$$

$$E'_{x'} = \frac{1}{\sqrt{1-\beta^2}}[(E_x - vB_y)\cos\omega t + (E_y + vB_x)\sin\omega t],$$

(83)
$$E'_{y'} = \frac{1}{\sqrt{1-\beta^2}}[(E_y + vB_x)\cos\omega t - (E_x - vB_y)\sin\omega t],$$

$$E'_{z'} = E_z,$$

$$J'_{e_{x'}} = J_{e_x}\cos\omega t + J_{e_y}\sin\omega t, \quad J'_{m_{x'}} = J_{m_x}\cos\omega t + J_{m_y}\sin\omega t,$$

$$J'_{e_{y'}} = J_{e_y}\cos\omega t - J_{e_x}\sin\omega t, \quad J'_{m_{y'}} = J_{m_y}\cos\omega t - J_{m_x}\sin\omega t,$$

(84)
$$J'_{e_{z'}} = \frac{1}{\sqrt{1-\beta^2}}(J_{e_z} - v\rho_e), \quad J'_{m_{z'}} = \frac{1}{\sqrt{1-\beta^2}}(J_{m_z} - v\rho_m),$$

$$\rho'_e = \frac{1}{\sqrt{1-\beta^2}}(\rho_e - \frac{v}{C^2} J_{e_z}), \quad \rho'_m = \frac{1}{\sqrt{1-\beta^2}}(\rho_m - \frac{v}{C^2} J_{m_z}).$$

When $\omega=0$, we obtain the Lorentzian transformation (C=c) :

$$B'_{x'} = \frac{1}{\sqrt{1-\beta^2}}(B_x + \frac{v}{C^2} E_y), \quad E'_{x'} = \frac{1}{\sqrt{1-\beta^2}}(E_x - vB_y),$$

(85)
$$B'_{y'} = \frac{1}{\sqrt{1-\beta^2}}(B_y - \frac{v}{C^2} E_x), \quad E'_{y'} = \frac{1}{\sqrt{1-\beta^2}}(E_y + vB_x),$$

$$B'_{z'} = B_z, \quad E'_{z'} = E_z,$$

$$J'_{e_{x'}} = J_{e_x}, \quad J'_{m_{x'}} = J_{m_x},$$

$$J'_{e_{y'}} = J_{e_y}, \quad J'_{m_{y'}} = J_{m_y},$$

(86)
$$J'_{e_{z'}} = \frac{1}{\sqrt{1-\beta^2}}(J_{e_z} - v\rho_e), \quad J'_{m_{z'}} = \frac{1}{\sqrt{1-\beta^2}}(J_{m_z} - v\rho_m),$$

$$\rho'_e = \frac{1}{\sqrt{1-\beta^2}}(\rho_e - \frac{v}{C^2} J_{e_z}), \quad \rho'_m = \frac{1}{\sqrt{1-\beta^2}}(\rho_m - \frac{v}{C^2} J_{m_z}).$$

Inversion of these formulas are obtained by replacing v by −v, and ω by −ω.

2. Rotating frames of reference. Invariance of the Maxwell equations and new equations for the field vectors. For a pure rotation, v = 0, and the mapping (82)-(84) reduces to

$$B'_{x'} = B_x\cos\omega t + B_y\sin\omega t, \quad E'_{x'} = E_x\cos\omega t + E_y\sin\omega t,$$

(87)
$$B'_{y'} = B_y\cos\omega t - B_x\sin\omega t, \quad E'_{y'} = E_y\cos\omega t - E_x\sin\omega t,$$

$$B'_{z'} = B_z, \quad E'_{z'} = E_z,$$

$$J'_{e_{x'}} = J_{e_x}\cos\omega t + J_{e_y}\sin\omega t, \quad J'_{m_{x'}} = J_{m_x}\cos\omega t + J_{m_y}\sin\omega t,$$

(88)
$$J'_{e_{y'}} = J_{e_y}\cos\omega t - J_{e_x}\sin\omega t, \quad J'_{m_{y'}} = J_{m_y}\cos\omega t - J_{m_x}\sin\omega t,$$

$$J'_{e_{z'}} = J_{e_z}, \quad J'_{m_{z'}} = J_{m_z},$$

$$\rho'_e = \rho_e, \quad \rho'_m = \rho_m.$$

Let us study the equation

$$(89) \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \frac{\partial B_x}{\partial t} = -\mu J_{m_x}.$$

We have

$$\frac{\partial}{\partial x} = \cos\omega t \frac{\partial}{\partial x'} - \sin\omega t \frac{\partial}{\partial y'},$$

$$\frac{\partial}{\partial y} = \sin\omega t \frac{\partial}{\partial x'} + \cos\omega t \frac{\partial}{\partial y'},$$

$$(90) \quad \frac{\partial}{\partial z} = \frac{\partial}{\partial z'},$$

$$\frac{\partial}{\partial t} = \omega(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'}) + \frac{\partial}{\partial t}.$$

Using the formulas (86)-(87), the equation (88) yields

$$(91) \quad \frac{\partial E'_z}{\partial y'} - \frac{\partial E'_y}{\partial z'} + \frac{\partial B'_x}{\partial t} + \omega(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})B'_{x'} - \omega B'_{y'} = \mu J'_{m_{x'}},$$

$$\frac{\partial E'_{x'}}{\partial z'} - \frac{\partial E'_{z'}}{\partial x'} + \frac{\partial B'_{y'}}{\partial t} + \omega(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})B'_{y'} + \omega B'_{x'} = -\mu J'_{m_{y'}}.$$

Finally, the equation

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{\partial B_z}{\partial t} = -\mu J_{m_z}$$

yields

$$(92) \quad \frac{\partial E'_{y'}}{\partial x'} - \frac{\partial E'_{x'}}{\partial y'} + \frac{\partial B'_{z'}}{\partial t'} + \omega(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})B'_{z'} = -\mu J'_{m_{z'}}.$$

We, therefore, have

$$(93) \quad \text{curl}'\vec{E}' + \frac{\partial \vec{B}'}{\partial t} = -\mu \vec{J}'_m,$$

provided that

$$(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})B'_{x'} - B'_{y'} = 0,$$

$$(94) \quad (y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})B'_{y'} + B'_{x'} = 0,$$

$$(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})B'_{z'} = 0.$$

Similarly, we shall have : $(95) \quad \text{curl } \vec{B}' - \frac{1}{C^2} \frac{\partial \vec{E}'}{\partial t} = \mu \vec{J}'_e,$

provided that

$$(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})E'_{x'} - E'_{y'} = 0,$$

$$(96) \quad (y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})E'_{y'} + E'_{x'} = 0,$$

$$(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'})E'_{z'} = 0.$$

One easily verifies the divergence equations

$$(97) \quad \text{div } \vec{B}' = \mu \rho'_m, \quad \text{div } \vec{E}' = \frac{1}{\varepsilon} \rho'_e.$$

Therefore, *subject to the conditions (94) and (96), our Maxwell equations are invariant.*

Now, the equations (94) and (96) admit the following solutions :

$$(98) \quad B'_{x'} = -M_m \frac{3x'z'}{r'^5}, \quad B'_{y'} = -M_m \frac{3y'z'}{r'^5}, \quad B'_{z'} = M_m \frac{r'^2 - 3z'^2}{r'^5},$$

$$E'_{x'} = -M_e \frac{3x'z'}{r'^5}, \quad E'_{y'} = -M_e \frac{3y'z'}{r'^5}, \quad E'_{z'} = M_e \frac{r'^2 - 3z'^2}{r'^5},$$

where $r'^2 = x'^2 + y'^2 + z'^2$ and M_m and M_e are constants.

The solutions (98) represent magnetic and electric dipoles, of moments M_m and M_e, located at 0, with axes along Oz-axis.

Using the relations (81) and (87), we have

(99) $$B_x \cos\omega t + B_y \cos\omega t = -M \frac{3(x\cos\omega t + y\sin\omega t)z}{r^5},$$

that is

(100) $$B_x = -M_m \frac{3xz}{r^5}, \quad B_y = -M_m \frac{3yz}{r^5}, \quad B_z = M_m \frac{r^2-3z^2}{r^5},$$

and, similarly

(101) $$E_x = -M_e \frac{3xz}{r^5}, \quad E_y = -M_e \frac{3yz}{r^5}, \quad E_z = M_e \frac{r^2-3z^2}{r^5}.$$

Thus, *an observer in the rotating frame* (x',y',z') *and his colleague in the fixed frame* (x,y,z) *will measure exactly same magnetic and electric dipoles.*

We can write

(102) $$\vec{B}' = M_m \operatorname{grad} \phi', \quad \vec{E}' = M_e \operatorname{grad} \phi'$$

where

(103) $$\phi' = \frac{z'}{r'^3}.$$

We now see that putting : $\partial/\partial t = 0$, $\vec{J}'_e = \vec{J}'_m = 0$ *in the equations* (93) *and* (95), *these are also satisfied by the solutions* (98).

The magnetic dipole in (98) and (100) gives an excellent representation of the Earth magnetic field, *whose north pole is presently located close to the geographic north.* It is known that the field has reversed its polarity several times in the past 300 million years. Let us note that equations (94) allow a reversed polarity, for if $\vec{B}$ is a solution, $-\vec{B}$ is also a solution of these equations.

3. Invariance of the equations C and consequences. The rotation of a conducting body is more complicated. We interrogate on the subject our equations C (4). They tell us that, in absence of charges and in stationary fields ($\partial/\partial t = 0$), we have

(104)
$$J'_{e_{x'}} = -C_e \frac{3x'z'}{r'^5}, \quad J'_{e_{y'}} = -C_e \frac{3y'z'}{r'^5}, \quad J'_{e_{z'}} = C_e \frac{r'^2-3z'^2}{r'^5},$$

$$J'_{m_{x'}} = -C_m \frac{3x'z'}{r'^5}, \quad J'_{m_{y'}} = -C_m \frac{3y'z'}{r'^5}, \quad J'_{m_{z'}} = C_m \frac{r'^2-3z'^2}{r'^5},$$

where C_e and C_m are constants. It is apparent, in this case, that the equations C are invariant.

We, therefore, have to integrate the equations

(105) $$\operatorname{curl} \vec{E}' = -\mu\vec{J}'_m, \quad \operatorname{curl} \vec{B}' = \mu J_e,$$

where $\vec{J}'_e$ and $\vec{J}'_m$ are given by (104). We obtain

(106)
$$E'_{x'} = -\mu C_m \frac{y'}{r'^3} - M_e \frac{3x'z'}{r'^5},$$

$$E'_{y'} = \mu C_m \frac{x'}{r'^3} - M_e \frac{3y'z'}{r'^5},$$

$$E'_{z'} = M_e \frac{r'^2-3z'^2}{r'^5},$$

(107)
$$B'_{x'} = \mu C_e \frac{y'}{r'^3} - M_m \frac{3x'z'}{r'^5},$$

$$B'_{y'} = -\mu C_e \frac{x'}{r'^3} - M_m \frac{3y'z'}{r'^5},$$

$$B'_{z'} = M_m \frac{r'^2-3z'^2}{r'^5}.$$

If charges are present, they are subject to the equation

(108) $$\left(y' \frac{\partial}{\partial x'} - x' \frac{\partial}{\partial y'}\right) \begin{array}{c} \rho'_e \\ \rho'_m \end{array} = 0.$$

Hence, we can take

(109) $$\rho'_e = \alpha_e \frac{z'}{r'^3}, \quad \rho'_m = \alpha_m \frac{z'}{r'^3},$$

where α_e and α_m are constants.

We, therefore, have

$$(110) \quad \begin{aligned} J'_{e_{x'}} &= \alpha_m \frac{y'}{r'^3} - C_e \frac{3x'z'}{r'^5}, \\ J'_{e_{y'}} &= -\alpha_m \frac{x'}{r'^3} - C_e \frac{3y'z'}{r'^5}, \\ J'_{e_{z'}} &= C_e \frac{r'^2 - 3z'^2}{r'^5} \end{aligned}$$

$$(111) \quad \begin{aligned} J'_{m_{x'}} &= -\alpha_e C^2 \frac{y'}{r'^3} - C_m \frac{3x'z'}{r'^5}, \\ J'_{m_{y'}} &= \alpha_e C^2 \frac{x'}{r'^3} - C_m \frac{3y'z'}{r'^5}, \\ J'_{m_{z'}} &= C_m \frac{r'^2 - 3z'^2}{r'^5}. \end{aligned}$$

Finally, here is a solution of the equations (105) when $\vec{J}'_e$ and $\vec{J}'_m$ are given by (110) and (111) :

$$(112) \quad \begin{aligned} E'_{x'} &= -\mu C_m \frac{y'}{r'^3} - M_e \frac{3x'z'}{r'^5}, \\ E'_{y'} &= \mu C_m \frac{x'}{r'^3} - M_e \frac{3y'z'}{r'^5}, \\ E'_{z'} &= -\frac{1}{\varepsilon}\alpha_e \frac{1}{r'} + M_e \frac{r'^2 - 3z'^2}{r'^5}, \end{aligned}$$

$$(113) \quad \begin{aligned} B'_{x'} &= \mu C_e \frac{y'}{r'^3} - M_m \frac{3x'z'}{r'^5}, \\ B'_{y'} &= -\mu C_e \frac{x'}{r'^3} - M_m \frac{3y'z'}{r'^5}, \\ B'_{z'} &= -\alpha_m \mu \frac{1}{r'} + M \frac{r'^2 - 3z'^2}{r'^5}. \end{aligned}$$

Using again the relations (81) and (83), we have

$$(114) \quad E_x \cos\omega t + E_y \sin\omega t =$$

$$-\mu C_m \frac{-x\sin\omega t + y\cos\omega t}{r^3} - M_e \frac{3(x\cos\omega t + y\sin\omega t)z}{r^5}$$

that is

$$(115) \quad \begin{aligned} E_x &= -\mu C_m \frac{y}{r^3} - M_e \frac{3xz}{r^5}, \\ E_y &= \mu C_m \frac{x}{r^3} - M_e \frac{3yz}{r^5}, \\ E_z &= -\frac{1}{\varepsilon}\alpha_e \frac{1}{r} + M_e \frac{r^2 - 3z^2}{r^5}, \end{aligned}$$

etc. *Our statement above persists : the two observers measure, in their respective frames of reference, same electric and magnetic fields, densities of currents and charges.*

4. Return to the Lorentz transformation. The intrinsic field of an electron in uniform motion and the first principle of dynamics. In the Lorentz transformation (34), we have

$$(116) \quad \begin{aligned} \frac{\partial}{\partial x} &= \frac{\partial}{\partial x'}, \qquad \frac{\partial}{\partial y} = \frac{\partial}{\partial y'}, \\ \frac{\partial}{\partial z} &= \frac{1}{\sqrt{1-\beta^2}}\left(\frac{\partial}{\partial z'} - \frac{v}{C^2}\frac{\partial}{\partial t'}\right), \quad \frac{\partial}{\partial t} = \frac{1}{\sqrt{1-\beta^2}}\left(-v\frac{\partial}{\partial z'} + \frac{\partial}{\partial t'}\right). \end{aligned}$$

In *stationary fields in the fixed frame of reference we put* $\partial/\partial t = 0$. Hence

$$(117) \quad v\frac{\partial}{\partial z'} - \frac{\partial}{\partial t'} = 0,$$

that is

$$(118) \quad v^2 \frac{\partial^2}{\partial z'^2} - \frac{\partial^2}{\partial t'^2} = 0.$$

An observer on the moving body senses propagation of the fields in the direction of the motion with the velocity v.

We also have

(119) $$\frac{\partial}{\partial z} = \sqrt{1-\beta^2}\ \frac{\partial}{\partial z'}.$$

The equation

(120) $$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu\ J_{m_x},$$

changes into

(121) $$\frac{\partial E_{z'}'}{\partial y'} - \frac{\partial}{\partial z'}(E_{y'}' - vB_{x'}') = -\mu\ J_{m_{x'}}',$$

that is

(122) $$\frac{\partial E_{z'}'}{\partial y'} - \frac{\partial E_{y'}'}{\partial z'} + \frac{\partial B_{x'}'}{\partial t'} = -\mu\ J_{m_{x'}}'\ ;$$

the relations (85), (86), (117) and (119) have been used.

We conclude that to the system

(123)
$$\operatorname{curl}\ \vec{E} = -\mu\ \vec{J}_m,$$
$$\operatorname{curl}\ \vec{B} = \mu\ \vec{J}_e,$$

in the fixed frame of reference, corresponds the system

(124)
$$\operatorname{curl}'\ \vec{E}' + \frac{\partial \vec{B}'}{\partial t'} = -\mu\ \vec{J}_m',$$
$$\operatorname{curl}'\ \vec{B}' - \frac{1}{C^2}\frac{\partial \vec{E}'}{\partial t'} = \mu\ \vec{J}_e',$$

in the Lorentzian frame.

We now note with Sommerfeld [12, pp. 239-241] that in a frame of reference (x,y,z) which moves with an electron in uniform motion, the intrinsic field of the electron is electrostatic in character. Thus, for $r = \sqrt{x^2+y^2+z^2}$:

(125) $$\vec{E} = -\frac{e}{4\pi\epsilon_0}\ \operatorname{grad}\ \frac{1}{r}, \qquad \vec{B} = 0.$$

For an observer at rest, with respect to whom the electron moves in the direction of the negative z-axis, we then have in view of (85)

(126)
$$E_{x'}' = \frac{1}{\sqrt{1-\beta^2}}\ E_x, \quad E_{y'}' = \frac{1}{\sqrt{1-\beta^2}}\ E_y, \quad E_{z'}' = E_z,$$

$$B_{x'}' = \frac{v}{C^2\sqrt{1-\beta^2}}\ E_y, \quad B_{y'}' = -\frac{v}{C^2\sqrt{1-\beta^2}}\ E_x, \quad B_{z'}' = 0.$$

Now (*see*, (34)) :

(127)
$$x = x', \quad y = y', \quad z = \frac{1}{\sqrt{1-\beta^2}}(z'-vt')$$

$$r = r' = \sqrt{x'^2 + y'^2 + \frac{(z'-vt')^2}{1-\beta^2}}.$$

We then obtain

(128)
$$E_{x'}',E_{y'}',E_{z'}' = \frac{e}{4\pi\epsilon_0\sqrt{1-\beta^2}}\ \frac{x',y',z'-vt'}{r'^3},$$

$$B_{x'}',B_{y'}',B_{z'}' = \frac{\mu_0 ev}{4\pi\ \sqrt{1-\beta^2}}\ \frac{y',-x',0}{r'^3}.$$

Thus, our primed observer, unlike one moving with the electron, is aware of a *magnetic* field in addition to the electric field. Further more, *he senses electromagnetic waves propagating in the* z'-*direction with the velocity* v *of the electron* ; for he senses the equation

(129) $$\left(v^2\ \frac{\partial^2}{\partial z'^2} - \frac{\partial^2}{\partial t'^2}\right)\begin{matrix}E_{x'}',E_{y'}',E_{z'}'\\B_{x'}',B_{y'}'\end{matrix} = 0.$$

We verify this important result by substitution of (128) into the equation (129) or, simply, by observing that *our quantities are functions of* z'-vt'. It was predicted by our equations

(117)-(118). We have corrected here the error of Sommerfeld : his z' is replaced by z'-vt'. Sommerfeld missed, therefore, in his beautiful study the waves demonstrated above !

The Sommerfeld theorem, so corrected, has an interesting analog in gravitation. *The first principle of dynamics states uniform and rectilinear motion in the absence of any outside force.* Let M_0 be the mass of the physical body in motion with a constant velocity v. This mass must generate an intrinsic gravitational field

$$(130) \qquad \vec{G} = \gamma_0 M_0 \, grad \, \frac{1}{r},$$

measured in a frame of reference (x,y,z) tied to the body in uniform motion say in the direction of the negative z-axis ; γ_0 is Newton's constant in vacuum. *An observer at rest in the frame (x',y',z') is aware of a second gravitational field $\vec{\Omega}'$ in addition to the gravitational field $\vec{G}'$*, and will record the relations

$$
(131) \quad
\begin{aligned}
G'_{x'} &= \frac{1}{\sqrt{1-\beta^2}} G_x, \quad & G'_{y'} &= \frac{1}{\sqrt{1-\beta^2}} G_y, \quad & G'_{z'} &= G_z, \\
\Omega'_{x'} &= \frac{v}{c^2\sqrt{1-\beta^2}} G_y, \quad & \Omega'_{y'} &= -\frac{v}{c^2\sqrt{1-\beta^2}} G_x, \quad & \Omega'_{z'} &= 0.
\end{aligned}
$$

We then have in view of (130) and (127)

$$
(132) \quad
\begin{aligned}
G'_{x'}, G'_{y'}, G'_{z'} &= -\frac{\gamma_0 M_0}{\sqrt{1-\beta^2}} \frac{x', y', z'-vt'}{r'^3} \\
\Omega'_{x'}, \Omega'_{y'}, \Omega'_{z} &= -\frac{\gamma_0 M_0 v}{c^2\sqrt{1-\beta^2}} \frac{y', -x', 0}{r'^3}.
\end{aligned}
$$

Obviously, $\vec{\Omega}$ represents a rotation about the z-axis and indeed its dimensions $[\vec{\Omega}] = T^{-1}$. We also notice Einstein's mass relation : $M_0/\sqrt{1-\beta^2} = M$.

We, clearly, have the correspondence

$$(133) \quad M_0 \rightarrow e, \quad \gamma_0 \rightarrow -\frac{1}{4\pi\varepsilon_0}, \quad \vec{G} \rightarrow \vec{E}, \quad \vec{\Omega} \rightarrow \vec{B},$$

which will be studied in detail in the next section.

We have

$$(134) \quad \left(v^2 \frac{\partial^2}{\partial z'^2} - \frac{\partial^2}{\partial t'^2}\right) \begin{matrix} G'_{x'}, G'_{y'}, G'_{z'} \\ \Omega'_{x'}, \Omega'_{y'} \end{matrix} = 0,$$

which shows that *the two gravitational fields $\vec{G}'$, $\vec{\Omega}'$ propagate in the z'-direction with the velocity v.*

It is strange that such an important property had been ignored since Newton's time, and does not figure in relativity where it belongs ! *It complements the first principle of Newton.* L. de Broglie [4, p. 1] said : "... il est naturel de supposer que, pour la matière aussi, il existe un aspect corpusculaire et un aspect ondulatoire, ce dernier jusque là méconnu". He refers to his work of 1923-1924 which opened the way to his wave mechanics.

5. Relativistic formulas for rotations. We conclude this section with some relativistic formulas which we do not find in literature.

Two rotations ω_1 and ω_2 having the same direction combine according to the formula

$$(135) \qquad \omega = \frac{\omega_1 + \omega_2}{1 + \frac{\omega_1 \omega_2}{\Omega^2}}.$$

The upper limit Ω has been computed by Poincaré : $\Omega^2 = 2\pi\gamma_0\rho_g$; *see* Lamb [21, pp. 687-689].

We have

$$(136) \quad M = \frac{M_0}{\sqrt{1 - \frac{\omega^2}{\Omega^2}}}, \quad E = I\Omega^2, \quad I = \frac{I_0}{\sqrt{1 - \frac{\omega^2}{\Omega^2}}},$$

where I is the moment of inertia about the axis of rotation.

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APPENDIX I – <u>Electromagnetic field and photons in a conducting</u>
<u>medium</u>

We call σ_e and σ_m the electric and magnetic con-
ductivities of the medium, and write

(AI.1) $\quad \vec{J}_e = \sigma_e \vec{E}$ (Ohm's law), $\quad \vec{J}_m = \sigma_m \vec{B}$.

Using Schwinger's transformation (formulas (6) in
the text), we have

(AI.2) $\quad \vec{J}_m \cos\theta + C \vec{J}_e' \sin\theta = \sigma_m(-\vec{B}' \cos\theta + \frac{1}{C} \vec{E}' \sin\theta)$.

Thus

(AI.3) $\quad \vec{J}_m' = \sigma_m B'$, $\quad J_e' = \frac{m}{C} E' = {}_e E'$.

We, therefore, obtain the following important re-
sult :

(AI.4) $\qquad \frac{\sigma_m}{\sigma_e} = C^2$,

which can be rewritten as

(AI.5) $\qquad \mu\sigma_m = \frac{\sigma_e}{\varepsilon}$, $\qquad (\varepsilon\mu C^2 = 1)$.

(AI.4) give Assuming σ_e and σ_m to be constant, the equations

(AI.6) $\qquad \text{curl } \vec{E} - \frac{\partial \vec{B}}{\partial t} = \frac{1}{\sigma_e} \text{ grad } \rho_m$,

$\qquad \text{curl } \vec{B} + \frac{1}{C^2} \frac{\partial \vec{E}}{\partial t} = - \frac{1}{\sigma_e} \text{ grad } \rho_e$,

$\qquad \frac{\partial \rho_e}{\partial t} + \frac{\sigma_e}{\varepsilon} = 0$, $\qquad \frac{\partial \rho_m}{\partial t} + \mu\sigma_m \rho_m = 0$,

where relations (AI.4) and (AI.5) have been used.

Comparing now the equations (AI.6) with the respec-
tive equations in (2)–(3), we obtain

(AI.7)
$$2 \frac{\partial \vec{E}}{\partial t} + \mu\sigma_m \vec{E} = - \frac{\sigma_m}{\sigma_e^2} \text{ grad } \rho_e,$$

$$2 \frac{\partial \vec{B}}{\partial t} + \mu\sigma_m \vec{B} = - \frac{1}{\sigma_e} \text{ grad } \rho_m,$$

(AI.8)
$$2 \text{ curl } \vec{E} + \mu\sigma_m \vec{B} = \frac{1}{\sigma_e} \text{ grad } \rho_m,$$

$$2 \text{ curl } \vec{B} - \mu\sigma_e \vec{E} = - \frac{1}{\sigma_e} \text{ grad } \rho_e.$$

The solution of these equations is

(AI.9)
$$\rho_e = \rho_{e_0} e^{-\frac{\sigma_e}{\varepsilon} t}, \quad \rho_m = \rho_{m_0} e^{-\mu\sigma_m t},$$

$$\vec{E} = \frac{1}{\mu\sigma_e^2} \text{ grad } \rho_{e_0} \cdot e^{-\frac{\sigma_e}{\varepsilon} t}, \quad \vec{B} = \frac{1}{\mu\sigma_e \sigma_m} \text{ grad } \rho_{m_0} \cdot e^{-\mu\sigma_m t},$$

where $\rho_{e_0} = (\rho_e)_{t=0}$, $\rho_{m_0} = (\rho_m)_{t=0}$ are functions of (x,y,z).

We have

(AI.10) $(\vec{E})_{t=0} = \vec{E}_0 = \frac{1}{\mu\sigma_e^2} \text{ grad } \rho_{e_0}$, $(\vec{B})_{t=0} = \vec{B}_0 = \frac{1}{\mu\sigma_e \sigma_m} \text{grad } \rho_{m_0}$,

where ρ_{e_0} and ρ_{m_0} are subject to the conditions :

(AI.11) $\quad \nabla^2 \rho_{e_0} = \frac{\mu}{\varepsilon} \sigma_e^2 \rho_{e_0}$, $\quad \nabla^2 \rho_{m_0} = \mu^2 \sigma_e \sigma_m \rho_{m_0}$.

To take into account the L. de Broglie photons, we use the equations (27), where we replace the constant k_0 by k of the medium. These equations give

$$\text{curl } \vec{E} - \frac{\partial \vec{B}}{\partial t} + \frac{1}{\mu \sigma_e} k^2 \vec{B} = \frac{1}{\sigma_e} \text{ grad } \rho_m,$$

(AI.12)

$$\text{curl } \vec{B} + \frac{1}{C^2} \frac{\partial \vec{E}}{\partial t} - \frac{1}{\mu \sigma_m} k^2 \vec{E} = - \frac{1}{\sigma_e} \text{ grad } \rho_e.$$

Before going further, it is convenient to introduce the following quantities

(AI.13) $\sigma_e^+ = \sigma_e(1 + \frac{1}{\mu^2 \sigma_e \sigma_m} k^2), \quad \sigma_m^+ = \sigma_m(1 - \frac{1}{\mu^2 \sigma_e \sigma_m} k^2).$

Comparison of equations (AI.12) with the respective equations in (2)-(3) yields

$$2 \frac{\partial \vec{E}}{\partial t} + \mu \sigma_m^+ \vec{E} = - \frac{\sigma_m}{\sigma_e^2} \text{ grad } \rho_e,$$

(AI.14)

$$2 \frac{\partial \vec{B}}{\partial t} + \mu \sigma_m^+ \vec{B} = - \frac{1}{\sigma_e} \text{ grad } \rho_m,$$

$$2 \text{ curl } \vec{E} + \mu \frac{\sigma_e^+ \sigma_m}{\sigma_e} \vec{B} = \frac{1}{\sigma_e} \text{ grad } \rho_m,$$

(AI.15)

$$2 \text{ curl } \vec{B} - \mu \sigma_e^+ \vec{E} = - \frac{1}{\sigma_e} \text{ grad } \rho_e.$$

The solution of these equations is

(AI.16) $\vec{E} = \frac{1}{\mu \sigma_e \sigma_e^+} \text{ grad } \rho_{e_0} . e^{-\frac{\sigma_e}{\varepsilon} t}, \quad \vec{B} = \frac{1}{\mu \sigma_e \sigma_m^+} \text{ grad } \rho_{m_0} . e^{-\mu \sigma_m t},$

where the charge densities $\rho_{e_0}(x,y,z)$ and $\rho_{m_0}(x,y,z)$ are subject to the equations

(AI.17) $\nabla^2 \rho_{e_0} = \frac{\mu}{\varepsilon} \sigma_e \sigma_e^+ \rho_{e_0}, \quad \nabla^2 \rho_{m_0} = \mu^2 \sigma_e^+ \sigma_m \rho_{m_0}.$

The densities of currents $\vec{J}_e = \sigma_e \vec{E}$, and $\vec{J}_m = \sigma_m \vec{B}$ are immediately obtained and so are the electromagnetic potentials (see, our formulas (28) in text) :

$$\vec{A}_e = - \frac{\mu}{k^2} \vec{J}_e = - \frac{\mu \sigma_e}{k^2} \vec{E}, \quad \vec{A}_m = - \frac{\mu}{C^2 k^2} \vec{J}_m = - \frac{\mu \sigma_e}{k^2} \vec{B},$$

(AI.18)

$$\phi_e = - \frac{1}{\varepsilon k^2} \rho_e, \quad \phi_m = - \frac{\mu}{k^2} \rho_m.$$

Our theory applies to a medium at rest. For moving media, one may use the books by Sommerfeld (12, pp. 280-322) and Penfield, Jr. and Haws [33].

We hope that the analysis above may stimulate experiments to elucidate the mystery of magnetic matter and its interaction with light. This may have far-reaching results in physics and biology. We may build magnetic semiconductors and edify a theory of photosynthesis !