Heisenberg uncertainty principle*

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ABSTRACT. The Heisenberg uncertainty principle is a theoretical postulate which has been validated by the agreement of quantum electrodynamics with experimentation. The postulate is consistent with the wave-packet definition of a particle and with the wave-particle duality concept. An analysis of the Heisenberg principle reveals, however, that the principle can be derived from actual experiments, rather than being postulated, provided an interaction force is assumed to exist between particles. Such novel interaction is universal in nature, and different from any known particle-particle interaction. Once the law of interaction is inferred from the analysis of the experiments which led to the postulation of the Heisenberg principle and is applied to well-known experimental situations, the wave-packet definition of a particle does not need to be called upon in order to determine the behaviour of the particle. Interestingly, in the case of photons, the application of the interaction law yields the same equations, namely Kirchhoff's and Helmholtz', which are obtained when the classical wave interpretation of the nature of light is adopted to explain diffraction and interference phenomena. An experiment of diffraction with statistically independent laser photons is reported which is capable of discriminating between the two models of light, the wave-packet model and the interacting

*Accounts of this work have appeared in : Annales Fondation Louis de Broglie, **6** , 197 (1981) ; **10** , 1 (1985) ; Spec. Sc. Tech. **8** , 35 (1985) ; Proc. N.A.T.O. Advanced Research Workshop "Quantum Violations : Recent and Future Experiments and Interpretations" (eds. W.M. Honig, D.W. Kraft, E. Panarella), Bridgeport, Conn., U.S.A., 23-27 June 1986 (Plenum Press, New York).

particles model. The results are in agreement with the latter model and at variance with the former.

1. Introduction

One of the fundamental principles of modern physics is the Heisenberg principle. In essence, it is a statement of the unavoidable interaction between measuring apparatus and measured system, which limits the precision with which certain pairs of physical variables can be simultaneously measured. Although the interaction is essential for the observation and measurement, its central role in the Heisenberg principle has not been emphasized because the principle can be incorporated "in toto", i.e., without explicit attention to the underlying cause of it, in the definition of a particle as a wave-packet. We shall prove that this definition is tantamount to a postulation of the Heisenberg principle. Although such postulation is theoretically legitimate, when the role of the interaction is re-examined, a derivation (rather than a postulation) of the principle is obtained.

The interaction between system and apparatus, which makes the measurement possible, is not an instantaneous process. As we shall prove, the interaction takes place continuously, thus continuously affecting the system, be it an atom, an electron, a neutron or a photon, and such influence goes on irrespective of whether some knowledge of the system is required or not. If a photon, for instance, is brought close to an electron for the purpose of observation, the two particles do not experience a collision type of interaction, but rather they influence one another continuously, and such continuous interaction takes place anyway, whether or not the observation or measurement of some parameters regarding the electron (position, for instance) is required at all.

The present study attempts to determine the role of this interaction and to establish the interaction law. One finds that such "Heisenberg interaction" is different from, and in addition to, any other known particle-particle interaction. As to the law of interaction, it is obtained from an analysis of the experiments upon which the derivation of the Heisenberg principle is based. It will be shown that, as a first approximation, the law can be expressed as

$$p(r) = h/r$$

where r is the separation between the interacting particles (i.e., the target or observed particle and the observing particle) and $p(r)$ is the amount of momentum transferred from one particle to the other along the r-direction (h = *constant*). It is then shown that, when this law is applied to a collection of photons, for instance, it leads to the general form of Kirchhoff's equation (Sec. 7), of fundamental importance in optics for understanding diffraction phenomena of light, or to Helmholtz' formula, in case of monochromatic photons (Sec. 10). Subsequent discussion shows that from this result one may infer that photons, as particles, arrange themselves on a diffraction pattern, with maxima and minima, not because they are guided by waves, but because they are constantly under the influence of such mutual "Heisenberg interaction". Moreover, this interaction model of photons is also shown to be capable of explaining some novel experimental results of diffraction of statistically independent photons, results which are reported here and which cannot be explained on the basis of the classical wave-particle model of light.

In summary, the purpose of this paper is two-fold : on the one hand it proposes a new physical interpretation for Heisenberg's uncertainty principle from which the properties of light can be derived without resorting to the wave-particle duality of photons, and on the other hand it reports the results from an experiment in which the observed behaviour of statistically independent photons is not as predicted by the wave-particle duality, but in agreement with the proposed interacting photons model.

The article is structured along the following steps. In Sec. 2 we shall briefly review the Heisenberg principle, as presently postulated on the basis of the accepted wave-particle duality. In Sec. 3 it will be shown that the Heisenberg principle, rather than being postulated, can be derived in a natural fashion once an interaction force is assumed to exist between particles. Some arguments will then be put forward in Sec. 4 in favour of the reality of such an interaction force. In the same section it will be shown that even the case of particles which are supposed to be isolated, and therefore that cannot interact with other particles, can be successfully dealt with because the existence of these isolated particles is essentially a hypothesis which has never been experimentally verified. In Sec. 5 we shall report the results of an experiment which, in fact, confirm that statistically independent photons, namely photons that are supposed to be isolated within an interferometer, in reality are not isolated, otherwise they would interfere as predicted by the wave-particle duality hypothesis. In Sec. 6 we shall derive from plausible arguments the interaction law and in Sec. 8 the interaction force. The legitimacy of the interaction law for photons will be demonstrated in Secs. 7 to 10, where the Kirchhoff's theorem and the Helmholtz equation governing the distribution of photons in polychromatic and monochromatic light, respectively, are derived from the interaction law. Finally, Sec. 11 will be dedicated to a discussion of the possibility that the interacting particles theory expounded here might be classified as a "hidden-variables theory". It will be demonstrated that, if this is the case, it should be considered as one of a "nonlocal" type. Any such theory is, in general, indistinguishable from quantum theory in terms of its predictive ability. However, for the particular experimental results reported in this article, it will be shown that the former theory is in better agreement with the experiment than the latter.

2. Analysis

The analysis begins with a study of the Heisenberg principle and the way it was derived. One of the derivations, and indeed the most famous one, was given by Heisenberg himself [1]. He considered an electron moving along an axis x. In order to observe the electron and to determine its position, a microscope is used. Any photon used to observe the electron transmits a momentum to the electron which is uncertain by

$$\Delta p_x = \frac{h}{\lambda}\sin\theta$$

where θ is the microscope angular aperture. On the other hand, since the resolving power of the microscope is

$$\Delta x \approx \frac{\lambda}{\sin\theta}$$

one gets

$$\Delta p_x \,.\, \Delta x \approx h \tag{1}$$

It is clear therefore that Heisenberg's principle can be formulated only because, in this case, a photon interacts with an electron and momentum is transferred from the former to the latter. If the two particles ignored one another, the photon would have proceeded undisturbed by the electron and the indeterminacy principle could not have been established. That an observer, in turn, picks up the information carried by the scattered photon

and makes what is called a "measurement" does not change the physical reality that the photon interacted and transferred momentum to the electron. The question of observability vs. physical reality is therefore irrelevant here.

The Heisenberg principle does not need to be derived from experiments. It can be postulated as true, by hypothesizing that it is intrinsically associated with any particle, which must then have an inherent undefined position and an undefined momentum. This postulate alone, however, is not sufficient. Additional assumptions are required in order to : 1. make the particle position and momentum uncertainties Δx and Δp, respectively, satisfy relation (1) ; 2. make the particle uncertainties conceptually acceptable (a single particle cannot, intuitively, have a spread Δx of position and Δp of momentum - a collection of particles can).

One finds that, by ascribing to the particle another nature -a wave nature- and prescribing a functional relation

$$p = h/\lambda \tag{2}$$

between particle's momentum p and wavelength λ, the foregoing requirements are satisfied. In fact, the spread of particle momentum Δp becomes now a spread of particle wavelength :

$$\Delta p = h\,\Delta(1/\lambda) \tag{3}$$

which is conceptually acceptable because waves can have a distribution of frequencies around a central frequency. Moreover, since the waves associated with the particle, in order to be able to represent the position of the particle, must be localized within an interval Δx, they must interfere destructively outside this interval. The interval Δx, therefore, must contain at least one wavelength more of the wave λ_1 than of λ_2, where λ_1 and λ_2 are the shortest and the longest of all possible waves associated with the particle, respectively [2]. In other words :

$$\left(\frac{\Delta x}{\lambda_1} - \frac{\Delta x}{\lambda_2}\right) \geq 1 \tag{4}$$

Multiplying both sides of inequality (4) by h, one gets :

$$\Delta x \,.\, h\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \geq h$$

which, on account of assumptions (2) and (3), leads to

$$\Delta x \,.\, \Delta p \geq h$$

i.e., to the Heisenberg principle. This shows that the association of a group of waves (a wave-packet) with a particle, together with assumption (2), leads to, or is consistent with, the Heisenberg principle. Conversely, the postulation of the latter leads to the wave-particle duality.

A principle is believed to be true as long as its validity is verified by experiments. No doubt can be cast upon the validity of the Heisenberg principle because of the demonstrated agreement of quantum electrodynamics with experimentation. And the authority of the proponents of the intimate connection between the wave and the particle nature of matter, namely Bohr, Born and Heisenberg, reinforced the argument in favour of the postulation of the Heisenberg principle, despite the conceptual difficulties and objections raised by the wave-particle duality even in minds of no less authority, namely Einstein, De Broglie, and Schroedinger.

We believe that the most successful of the postulates, capable of predicting and verifying a wealth of experimental results, is not satisfactory as long as it is not assisted by intuition. To exemplify, the principle of conservation of energy is an intuitively acceptable principle. Likewise, the principle of conservation of momentum, that of mass, etc. are principles justified by intuition.

The Heisenberg uncertainty principle, by hinging so directly upon the wave-particle duality, lacks that degree of persuasiveness to make it acceptable 'toto corde' or without reservation. We believe that the association of a wave packet with a particle represents a useful mathematical model that lacks physical reality. The purpose of our study is to attempt to find the physical reality behind that mathematical model.

3. Derivation of the Heisenberg Principle. The Case of Photons

The derivation of the Heisenberg principle, even from a simple experiment like the " γ-ray microscope" experiment described earlier, conveys two important concepts : a) the principle does not need to be postulated, but it can be experimentally derived, and b) there is physical reality in the interaction of particles necessary for their observation. Therefore, if we are aiming at an understanding of the Heisenberg principle in terms of physical reality, and not at mathematical modeling, we have to base our reasoning on

experiments. In doing so we should stay away from "gedanken" experiments, which are fraught with danger about their ability to represent a possible, real experiment.

The requirement that the principle be derived from real experiments is satisfied by all known particles. These, in fact, are able to interact with other particles, thus leading to the experimental derivation of the Heisenberg principle. Photons are the only exception. When one deals with photons, for them no real experiment seems to exist from which the principle can be derived. The reason is that, in order to establish an uncertainty principle for photons from the generalization of the outcome of real experiments, one should try to proceed in the same way as we previously did for electrons and have the photon acting as a target for other particles. If one considers a possible experiment in which the photon acts as a target for a beam of electrons, for instance, this would imply the detection of an "inverse-Compton effect", involving the scattering of electrons by photons, and such an effect has not been experimentally verified as yet*. Moreover, even if such "inverse-Compton effect" were found, the determination of the photon position from the scattering of the electrons, which is necessary for the formulation of the uncertainty principle, would just be impossible, because no experiment can ever provide an image of a photon. It seems, therefore, that the Heisenberg principle for photons cannot be derived but only postulated.

We would like to offer here the following derivation of the Heisenberg principle for photons, based on the concepts previously outlined and on an experiment routinely performed. We refer to Fig. 1 which shows the classical experiment of a beam of photons crossing a narrow slit. After crossing the slit, the x-momentum of each and every photon is changed from zero to anywhere between $-p_x$ and $+p_x$. If we disregard, as proposed, the wave-particle model of interpretation of this phenomenon and if we reinstate the role of the interaction, the change of photon momentum can occur only if the photons interact, some of their momentum being transferred to the

*Since we have excluded "gedanken" experiments from our considerations, we need real experiments in order to derive the Heisenberg principle for photons. Unfortunately the "inverse-Compton effect" is still awaiting an experimental verification, although such an effect certainly exists. The effect might be observed using an experimental set up proposed by Kapitza and Dirac [3] with standing light waves. Despite several attempts to observe the Kapitza-Dirac effect, none of them has provided convincing evidence of its detection (see the review paper by Eberly [4]).

surrounding photons. Since the x-component of the momentum acquired by each and every photon, after crossing the slit, ranges between zero and $\pm p_x$, this means that such momentum is not precisely known and therefore is uncertain by :

$$\Delta p_x = p \sin\theta \tag{5}$$

where p is the original photon momentum and θ is the deflection angle. On the other hand, in order to establish another relation between the various parameters, one notices that, by gradually decreasing the slit width, the following relation holds :

$$\Delta x \, . \sin\theta = k \tag{6}$$

where k is known because Δx and θ are measurable quantities. In other words, by reducing the slit width Δx, we experimentally find that the maximum deflection angle θ increases. But Δx is the uncertainty of the photon position. Hence, inserting sin θ from (6) into (5), we find

$$\Delta p_x = p \cdot \frac{k}{\Delta x} \tag{7}$$

and

$$\Delta p_x \, . \, \Delta x = p \, . \, k = \text{const} \tag{8}$$

We find experimentally therefore that the product of the uncertainties of position and momentum is a constant, as previously found. In other words, without postulating the wave-particle duality for photons and analyzing the single-slit experiment in terms of interacting photons, one derives the same uncertainty principle as previously found when the momentum of a photon was postulated to be h/λ in terms of its wave nature. This interpretation of interacting photons, therefore, provides the required experimental proof that photons obey the uncertainty principle.

4. Analysis of the Single-Slit Experiment When the Photons Cross the Slit One at a Time

Apparently, the hypothesis of interacting photons cannot be accepted because it does not explain diffraction effects obtained when isolated photons cross the slit one at a time. In this case, in fact, the photons cannot interact with other photons and the diffraction pattern can be explained only with the

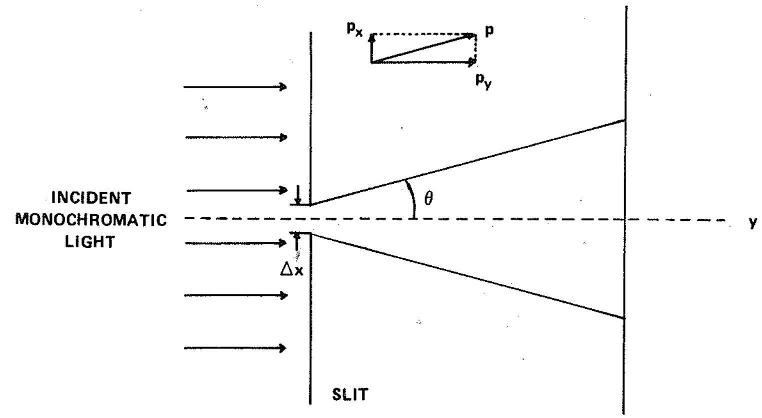

Fig. 1. Diffraction of light through a slit. The photons crossing the slit are deflected and the maximum angular deflection θ is related to the photon momentum p according to: $\Delta p_x = p \sin\theta$. Another relation that applies here is: $\Delta x . \sin\theta = k$, where Δx is the slit width and k is a constant.

wave-particle duality of the photons. Likewise, the formulation of the uncertainty principle in terms of interacting photons, based as it is on diffraction effects caused by interaction, is impossible.

If one considers carefully this objection, however, one finds that it would have validity only if the existence of isolated photons was a proven fact, and not just on assumption. In other words, only if a clear demonstration of the existence of isolated photons were provided by experiments, which is not the case thus far, then the objection would be valid. Even when one analyzes the experiments of detection of single photons, in this instance too one cannot be sure that these single photons are isolated (by isolation we mean, of course, at a distance from one another that they cannot reasonably interact). This is because the limited quantum efficiency of any detector precludes the detection of all photons. In addition, the limited time resolution of even the fastest of all possible detectors is another factor that precludes the possibility of verifying whether the photons are isolated or not. On the other hand, if one examines the available literature, one finds that photons cannot be radiated as isolated particles from any source. Rather, one always deals with sources of light whose individual atoms cannot emit photons independently of each other, because they are constantly interacting with a common radiation field [5]. This implies that photons are not emitted at random but have certain characteristic bunching properties [6]. This "clumping" effect for photons, on the other hand, has been successfully demonstrated by Hanbury-Brown and Twiss [7] who detected a correlation between light beams of narrow spectral width. In summary, the evidence in favour of collection of photons, as opposed to isolated photons, is by far preponderant.

However, since the model of interacting photons is crucially dependent upon the notion that the photons are not isolated, whereas the wave-packet model is crucially dependent on the notion that they can be isolated, it is important to perform an experiment capable of discriminating between the two models of light. We have designed and performed such an experiment. The experiment rests on the notion that, if statistically independent photons (namely photons that, on average, are widely separated) obey the wave-particle model, the interference pattern created by them after crossing an interferometer is built by one photon at a time independently of the others. Therefore, two interference or diffraction patterns produced at different light intensities should be rigorously the same when the total number of photons used to create the patterns at the two light intensities is the same. In other words, the phenomenon should be linear with light intensity, because it is essentially a single particle phenomenon. On the other hand, if this does not happen, i.e. the phenomenon is nonlinear, this would imply that several photons are involved in the production of the interference or diffraction patterns, and this result would be compatible with the interacting photons model.

In the next section the experimental apparatus is described and the results are reported.

5. Experiment of Diffraction with Statistically Independent Photons at Two Light Intensities

Consider the experimental apparatus of Fig. 2 which has been used to produce statistically independent photons. A 5 mW cw TEM_{00} mode Spectra-Physics Mod. 135 He-Ne laser was the source of light. The laser emitted a Gaussian beam of radius $a = 0.35$ mm at $1/e^2$ points. The peak light

intensity in the central part of the beam was :

$$I_p = \frac{2P_0}{\pi a^2} = 2.59 \text{ W.cm}^{-2} \qquad (P_0 = 5 \times 10^{-3} \text{ W})$$

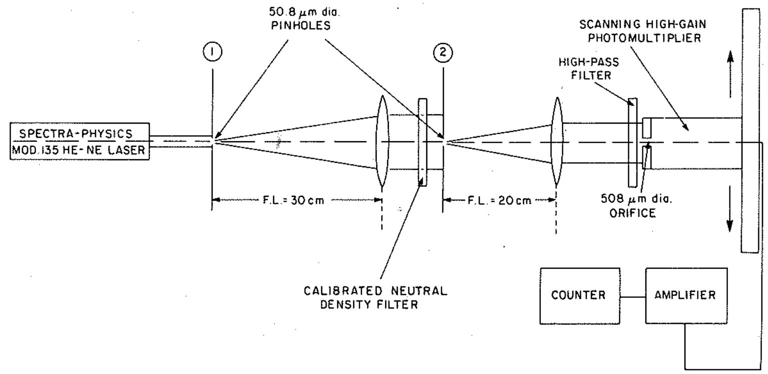

Fig. 2. Experimental apparatus used to reveal the effect of the degree of statistical independence on the photon distribution on a diffraction pattern. With a photon flux $\lambda = 1.90\ 10^{10}$ photons . sec^{-1} within the interferometer (i.e. between pinholes 1 and 2), a clear diffraction pattern is revealed with a counting time of $2\ 10^{-3}$ sec. After reducing the light intensity 769-fold with a neutral density filter inserted in the light path, the same diffraction pattern does not appear, despite having increased the counting time 1000-fold to 2 sec.

The light intensity profile was smoothed out by means of a pinhole of diameter $d = 5.08 \times 10^{-3}$ cm positioned at the center of the beam, at the point of maximum light intensity. The resultant emerging bright central disc of the Airy pattern was collimated by means of a simple double-convex lens located at a distance from the pinhole equal to the lens focal length $f = 30$ cm. The intensity of light at the center of the Airy pattern resulted in [8] :

$$I_0 = AP_1 / \lambda^2 f^2 = 2.95 \times 10^{-4}\ \mathrm{W/cm^2}$$

where $A = \pi d^2/4$ is the pinhole area and $P_1 = I_p A$. A second pinhole of diameter $d = 5.08 \times 10^{-3}$ cm was positioned at the center of the Airy disc. Since the light intensity across this pinhole was essentially constant, the photon flux entering the pinhole resulted in being 1.90×10^{10} photons/sec .

The diffracted light out of this second pinhole was then recollimated by means of a simple double-convex lens located at a distance from the pinhole equal to the lens focal length $f = 20$ cm and the light detection occurred along the vertical diameter of the diffraction pattern by means of a high gain photomultiplier. For good space resolution, the photomultiplier was provided with a small orifice of 5.08×10^{-2} cm diameter drilled on its front cover. The signal from the photomultiplier was then amplified and sent to a counter.

More in detail, the detection system consisted of a fourteen-stage, flat-faceplate RCA photomultiplier, type 7265, having a multialkali photodiode $([Cs])Na_2KSb)$ with S-20 response. The photomultiplier current amplification was 2×10^7. The tube was operated at $2050V$, i.e. below the maximum permissible voltage of $2400V$, in order to reduce the dark current count from thermionic emission and to increase the signal-to-noise ratio [9]. In order to further reduce the dark current, the photomultiplier was cooled with a blanket of dry ice to -15 deg C. Light uniformity over the photocathode area was achieved by inserting a diffuser within the photomultiplier case, right behind the entrance orifice. Great care was also taken to reduce any stray light entering the photomultiplier and this was achieved by enclosing the entire apparatus containing the laser and related optics within a black box, so that only a small opening was available for the laser beam to get out of pinhole No. 2. As to the residual light from the laser discharge tube going through the pinhole, it was cut out almost completely by placing in front of the photomultiplier a high-pass filter having transmission 84 % at the laser wavelength of 6,328A and rapidly falling down to 0.03 % at 5.540A . Finally, the entire experiment was carried out in a small, windowless, dark room completely shielded from any external light.

The experiment consisted in moving the photomultiplier by equidistant steps of 5/1000 of an inch ($= 1.27 \times 10^{-2}$ cm) and arresting it at each step just for the time required for pulse counting. The counting was done with a Tennelec 546P Scaler and 541A Timer, the signal from the photomultiplier having been amplified by a factor of 10 through an amplifier having input

resistance 1000 ohms.

The counting time was chosen rather short, 2×10^{-3} sec and 2 sec for the two experiments that we ran, respectively, because this offered some distinct advantages over long counting times. This avoids problems of photomultiplier fatigue and decrease of sensitivity [9], and the dark count can be greatly reduced with an appropriate choice of short counting time.

The experimental results are reported in Fig. 3. The solid circles represent the counts obtained when the photon flux within the interferometer*, i.e. between pinholes 1 and 2, was 1.90×10^{10} photons/sec (the average photon separation is 1.57 cm, i.e. much less than the length of the interferometer 42 cm) and the counting time 2×10^{-3} sec. The open circles are the counts obtained when the photon flux was decreased 769-fold to 2.47×10^7 photons/sec by inserting a calibrated neutral density filter along the light path (the average photon separation is now 1214 cm, much greater than the interferometer length) (see Figure 2) and the counting time increased 1000-fold to 2 sec.

As one can see, the relative amplitudes of the central peaks of the diffraction patterns at the two light intensities are not what one would expect. Were the phenomenon linear, in fact, by reducing the light intensity by a factor of 769 and increasing the counting time by a factor of 1000, the central peak of the second diffraction pattern (open circles in Figure 3) would have amplitude :

$$\frac{1000}{769} \cdot 335 + 52 = 487 \text{ counts}$$

instead of 163 counts (335 counts is the peak amplitude of the high light intensity case and 52 counts is the average noise count in the low intensity case).

In order to have a measure of the detection nonlinearity we subtracted the noise-free signal amplitude of the low light intensity case from the expected noise-free signal, and divided this difference by the former amplitude:

$$\frac{(487-52)-(163-52)}{(163-52)} = \frac{435(\text{expected}) - 111(\text{found})}{111(\text{found})} = 2.91 = 291\ \%$$

*The interferometer, in our case, is essentially a Young apparatus, as explained in Ref. 10, and therefore its length is the distance between pinholes 1 and 2.

a quite large nonlinearity.

In conclusion, the nonlinearity found* proves that we are not dealing with a single-particle, but with a many-particle phenomenon and this result is compatible with the interacting photons model of light previously proposed.

In the following section we shall derive, from plausible arguments, the interaction law.

6. Derivation of the Interaction Law

We have proven thus far that the Heisenberg principle refers to interacting particles. The principle has never been contradicted. It has the status of a physical law experimentally found. The interpretation, however, of the terms Δp and Δx has always been given in terms of uncertainties of the outcome of the measurement or observation. Since we have already recognized that the observation is not necessary and have admitted that the Heisenberg's relation is a consequence of an interaction between particles, the interpretation of the principle must of necessity change. Let us refer again to Fig. 1. Assume that the intensity of light is such that only two photons, at a particular instant of time, cross the slit. Because of the mutual interaction that tends to push the two photons away one from the other, and because of the absence of other photons from the immediate surrounding, the two photons position themselves at a distance equal to the slit width Δx just before emerging from the slit. Immediately after, when they are unrestricted by any physical bound, the component of each photon's momentum in the x-direction changes by an amount $\Delta p_x = p \sin\theta$. We have to assume the maximum amount of momentum change in order not to violate the Heisenberg principle. In fact, assuming the deflection angle θ' to be less than θ, we would have :

$$\Delta p_x{}' = p \sin\theta' \tag{9}$$

On the other hand, experimentally one finds :

$$\Delta x = \frac{k}{\sin\theta} \tag{10}$$

where Δx is the slit width, and this relation is independent of the intensity of

*The experiment reported here is the third of a set of experiments that we have carried out (Spec. Sc. Tech. **5** , 501, 1982; **5** , 509, 1982; **6** , 383, 1983). All three experiments confirm that the phenomenon is nonlinear at very low light intensities.

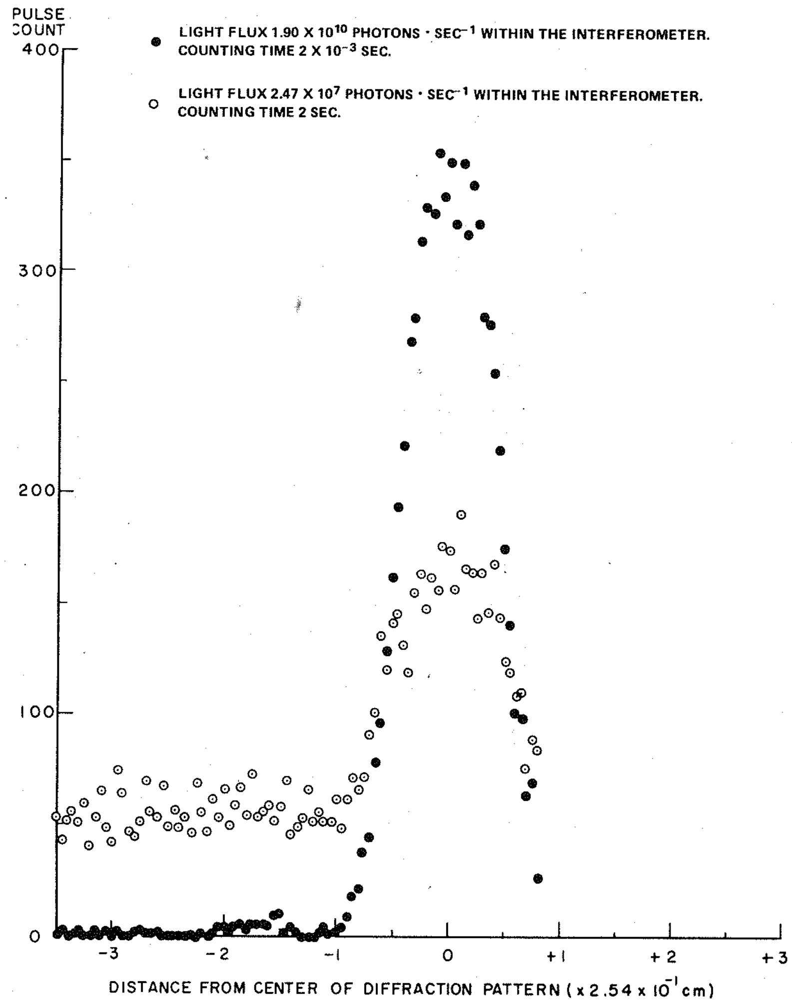

Fig. 3. *Solid circles:* regular diffraction pattern obtained with a photon flux $\lambda = 1.90\times10^{10}$ photons $.sec^{-1}$ within the interferometer and counting time 2×10^{-3} sec. *Open circles:* the diffraction

pattern does not have the same amplitude as before when the light flux is reduced 769-fold, despite having increased the counting time 1000-fold to 2 sec.

light, i.e. it is valid even with two photons crossing the slit. Multiplying (9) by (10), one gets :

$$\Delta p_x{}' \ . \ \Delta x = pk\frac{\sin\theta'}{\sin\theta} < pk \quad \text{because} \quad \theta' < \theta$$

Since this would violate the Heisenberg principle, θ' must be equal to θ.

From the foregoing, it can be said that each photon receives an amount of momentum from the other in the x-direction equal to Δp_x and the product of $\Delta x = x$ (slit width or photon separation) and Δp_x brings back the known formula $\Delta x \ . \ \Delta p_x =$ const. Based on the new interpretation of its constituent terms, this formula can be written in a more concise way :

$$x \ . \ p_x = \text{const.}$$

or

$$p_x = \frac{\text{const}}{x} \tag{11}$$

where p_x is now the momentum in the x-direction (measured in the laboratory frame of reference, see Sec. 9) transferred from one photon to another photon located at a distance x from the first.

Equation (11) is then the interaction law between these two particular photons just emerging from the slit, x being the photon separation. We might generalize Eq. (11) by writing it in the following form :

$$p_r = \frac{\text{const}}{r} \tag{11a}$$

This means that, rather than considering a slit of width extending only in the x-direction (Fig. 1), we consider a circular opening of diameter r and find the photon momentum transfer along the r-direction. We further generalize Eq. (11a) by writing it in vectorial form, which will point out that the momentum transfer occurs only in the r-direction, and not in any other direction. A vectorial form of the equation is the following :

$$\vec{p}_r = \frac{\text{const}}{r}\,\vec{r}_u \tag{12}$$

where $\vec{r}_u$ is a unit vector in the r-direction emanating from the position P of one of the two photons. Finally, we postulate that Eq. (12) is the interaction law between *any two photons* separated by a distance r *. In other words, we assume that Eq. (12) applies to any two photons, no matter where they are. Equation (12) is therefore a formula of general validity representing a law of interaction or of momentum transfer between any two photons separated by a distance r. Since the motion of a photon is perturbed in this way by the presence of all surrounding photons, we are interested in determining the amount of the perturbation, i.e. the displacement of the position of that photon from the one it would have if all other photons were absent.

7. Derivation of Kirchhoff's Formula

We would like to inquire whether Eq. (12) is capable of explaining diffraction phenomena, i.e. the arrangement of photons on a wave pattern, successfully explained so far only in terms of the wave nature of light. Because each photon is now subject to a field (12) of interaction or field of momentum transfer generated by each and every surrounding photon, the perturbation of its motion can be derived by making use of standard potential theory [11] and applying Green's theorem to a region R containing the position $P(x,y,z)$ of a test photon :

$$\int_R (U\,\nabla^2 V - V\,\nabla^2 U)\,dv = \int_S \left(U\,\frac{\partial V}{\partial n} - V\,\frac{\partial U}{\partial n}\right) ds \tag{13}$$

In this identity we take as a function V one of the three cartesian components of the momentum vector $\vec{p}_r$ (Eq. 12) :

$$p_x = \frac{\text{const}}{r}\sin\theta\cos\phi \ ; \ p_y = \frac{\text{const}}{r}\sin\theta\sin\phi \ ; \ p_z = \frac{\text{const}}{r}\cos\theta \tag{12a}$$

where r, θ and ϕ are spherical polar coordinates with origin at the test photon. Hence

$$V = \frac{K}{r} \tag{12b}$$

*Equation (12) is a universal law applicable to all particles, and not just photons, because no reference exists to mass, size, density, initial momentum, etc. of the interacting particles. Clearly, this law of interaction is in addition to any other known particle-particle interaction which may also lead to momentum-energy transfer and may in fact be predominant. In Appendix A we shall compare the interaction force derived from Eq. (12) with the electrostatic and gravitational forces.

where the value of the constant K depends on the particular cartesian component (12a) we have chosen. The analysis that follows therefore refers to a cartesian component of the momentum vector $\vec{p}_r$. Since V has a singularity for $r = 0$, the identity (13) cannot be applied to the whole region R. Therefore, we surround P with a small sphere σ with P as a centre and remove from R the interior of the sphere. For the resulting region R' we have, since V is harmonic in R' :

$$-\int_{R'} \frac{1}{r} \nabla^2 U \, dv = \int_S \left(U \frac{\partial}{\partial n} \frac{1}{r} - \frac{1}{r} \frac{\partial U}{\partial n}\right) ds + \int_\sigma \left(U \frac{\partial}{\partial n} \frac{1}{r} - \frac{1}{r} \frac{\partial U}{\partial n}\right) ds \qquad (14)$$

where n denotes the normal to the boundary of R, pointing outward from R', so that on σ it has the direction opposite to the radius r. Hence, the last integral may be written

$$\int_\Omega \left(U \frac{1}{r^2} + \frac{1}{r} \frac{\partial U}{\partial r}\right) r^2 \, d\Omega = \bar{U} \, . \, 4\pi + \int_\Omega r \frac{\partial U}{\partial r} \, d\Omega \qquad (15)$$

Here $\bar{U}$ is a value of U at some point of σ, and the integration is with respect to the solid angle subtended at P by the element of σ. As the radius of σ approaches zero, the limit of the integral over σ in (14) is $4\pi \, U(P)$ and the volume integral on the left converges to the integral over R. We thus arrive at :

$$U(P) = -\frac{1}{4\pi} \int_R \frac{\nabla^2 U}{r} \, dv + \frac{1}{4\pi} \int_S \frac{\partial U}{\partial n} \frac{1}{r} \, ds - \frac{1}{4\pi} \int_S U \frac{\partial}{\partial n} \frac{1}{r} \, ds \qquad (16)$$

This is the expression to be satisfied by the second function U in Green's theorem if the first function V is equal to $\frac{K}{r}$.

We shall take the function U to represent the total momentum transferred to the test photon by all other photons. We are interested in knowing the form to be taken by the function U in order to satisfy our physical conditions and equation (16). Since we are dealing with a stream of photons moving with velocity c, whose position is changing with time t, the total momentum U applied to the test photon at point P has to depend on time t also :

$$U = U(P, t) \qquad (17)$$

Moreover, since simultaneity of cause and effect (action at a distance) is excluded here, any signal emitted by the moving photons will be transmitted with finite velocity c and will be received at the point P after a time r/c.* Hence, the integration in (16) must be performed not at time t, but at the retarded time $t - r/c$. The function U to be inserted in (16) is then :

$$U = U(x, y, z, t - \frac{r}{c}) = [U] \qquad (18)$$

where the square brackets indicate the retarded value of the function.

The perturbation or "optical disturbance" produced at a point P by a collection of streaming photons is given by $U(x, y, z, t)$. In order to find this function, we consider initially only one photon travelling with velocity $\vec{c}$. In polar coordinates, the field of interaction (or field of momentum transfer) of such a moving source is expressed as [12] :

$$p(\vec{r}, t) = \frac{\text{const}}{\rho - \frac{\vec{c} . \vec{\rho}}{c}} \qquad (19)$$

where $\rho = |\vec{r} - \vec{c}\, t_0| = c(t - t_0)$ and t_0 represents a time such that a signal emitted by the photon at t_0 will arrive at $\vec{r}$ at time t.

In a cartesian coordinate system, in which the x-axis coincides with the velocity vector $\vec{c}$, expression (19) becomes :

$$p(x, y, z, t) = \frac{\text{const}}{c(t - t_0) - \frac{c(x - ct_0)}{c}} = \frac{\text{const}}{ct - x} \qquad (20)$$

Equation (20) for the field of interaction (or momentum transfer) of the moving photon is a function that satisfies the wave equation :

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \qquad (21)$$

The field of a collection of photons, because of the linearity of the wave equation (21), is the sum of the fields of each photon. Hence the function U also satisfies the wave equation :

*Here, we are treating this matter in the same way as the field of a moving source [12].

$$\nabla^2 U = \frac{1}{c^2} \frac{\partial^2 U}{\partial t^2} \tag{22}$$

In summary, it has been verified that the application of Green's theorem to a region R containing the position of the test photon leads to Eq. (16), when the function V has been chosen to be the one expressed by (12b), a cartesian component of the momentum vector $\vec{p}_r$. For the physical conditions provided by a stream of photons moving with velocity $\vec{c}$, the second function to be inserted into Green's formula is $U(x,y,z,t)$, which represents the momentum transferred from all the streaming photons to the test photon, and which must be calculated at the retarded time $t - r/c$ in the integration of (16). It was also proven that the field of interaction (or momentum transfer) (20) for a moving photon satisfies the wave equation (21). Therefore, the function $U(x,y,z,t)$ also satisfies the wave equation (22). This analysis is sufficient to lead, by straightforward but tedious calculation, [13] to :

$$U(x,y,z,t) = -\frac{1}{4\pi}\int_S \left[[U]\frac{\partial}{\partial n}\frac{1}{r} - \frac{1}{r}\left[\frac{\partial U}{\partial n}\right] - \frac{1}{cr}\frac{\partial r}{\partial n}\left[\frac{\partial U}{\partial t}\right] \right] ds \tag{23}$$

This is the well-known general form of Kirchhoff's theorem, as found in any textbook on classical optics [14]. At variance from the classical case, however, it expresses now a type of "optical disturbance" which is not related to light intensity or amplitude but to the "total interaction or total momentum transfer" produced by a collection of photons, randomly distributed in space and time, to a test photon positioned at a point (P,t). Such momentum transfer displaces the test photon by an amount given by Eq. (23). We shall be seeing more clearly in Sec. 10, when we will be dealing with the derivation of the Helmholtz' formula, that such displacement makes the particles reposition themselves on a distribution of maxima and minima of number density, much like a wave distribution. Moreover, since Eq. (23) does not depend on such physical properties of the photons as energy and momentum, and therefore it is valid for any photon, we shall see that, in the particular case of monoenergetic (or monochromatic) photons a characteristic length appears in the formula, which is a parameter in all respects equivalent to the wavelength λ in the classical wave theory of light. The next sections will deal at length with this particular case, and it will be proven that diffraction phenomena can be explained as a geometrical arrangement of interacting photons.

8. Derivation of the Interaction Force

We previously stated that the interaction law between two photons is given by Eq. (12) :

$$\vec{p}_r = \frac{\text{const}}{r}\,\vec{r}_u \tag{12}$$

where r is the separation between the two photons and $\vec{r}_u$ is a unit vector in the r-direction emanating from the position of one of the two photons.

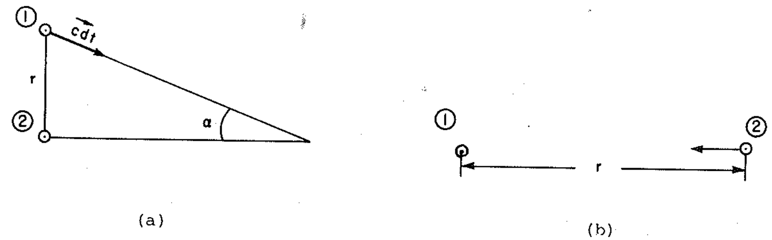

Fig. 4. Derivation of the relative velocity of photons converging towards a common focal area (case a), or belonging to counter propagating beams (case b).

We shall now derive the interaction force F. From

$$\vec{F} = \frac{d\vec{p}_r}{dt} \tag{24}$$

one gets :

$$\vec{F} = \frac{d\vec{p}_r}{dt} = \frac{d}{dt}\left(\frac{h}{r}\right)\vec{r}_u + \frac{h}{r}\frac{d\vec{r}_u}{dt} = -\frac{h}{r^2}\frac{dr}{dt}\vec{r}_u + \frac{h}{r}\frac{d\vec{r}_u}{dt} \tag{25}$$

where the constant appearing in (12) has now been designated as h. The physical meaning of expression (25) can be illustrated from the examination of two simple cases, namely the case of two photons converging towards a focal point (Fig. 5a) and the case of two counterpropagating photons (Fig. 5b). In both cases $\vec{r}_u = const$ and $\frac{d\vec{r}_u}{dt} = 0$. Hence :

$$\vec{F} = -\frac{h}{r^2}\frac{dr}{dt}\vec{r}_u \tag{26}$$

Since $\vec{p}_r$ in (12) is measured in the fixed laboratory frame of reference, $\frac{dr}{dt}$ in (26) is also measured in such frame. It is then advantageous to consider one photon fixed and the other moving against it in the r-direction in these two cases. Expression (26) states that the force is inversely proportional to the square of the distance r between the interacting photons and directly proportional to their relative velocity $\frac{dr}{dt}$. In the case of two photons directed towards a focal point (Fig. 5*a*) the relative velocity is

$$\frac{dr}{dt} = -c\sin\frac{\alpha}{2} = \text{const} \tag{27}$$

and the interaction force increases as the photons converge towards the focal area, because $F \; \alpha \; 1/r^2$. The force is obviously repulsive because it is impossible to focus to a point a beam of light. If the same two photons belong to counterpropagating beams (Fig. 5*b*), their relative velocity is

$$\frac{dr}{dt} = -c \tag{28}$$

The force is still repulsive. However, immediately after the two photons cross each other, the force becomes attractive because $\frac{dr}{dt}$ changes sign and becomes positive.

In summary, the force acting between two photons can be either attractive or repulsive, depending on whether the photons move away one from the other or approach one another, respectively. To obtain a simple mechanical representation of such an interaction force, it is as if the photons were connected by ideal tiny springs (Fig. 5*a*), the compression of the spring, which takes place when the photons approach one another, yielding a repulsive force, and the stretching of the spring, which takes place when the photons move away one from the other, yielding an attractive force. This is a very crude model, and perhaps a better one would be given by an ideal damper element (Fig. 5*b*), in which the force is proportional to the relative velocity of the damper's terminals. At any rate, we shall not attempt here to conjecture on the nature of such a force, but rather assume its existence and verify its ability to explain well known experimental results.

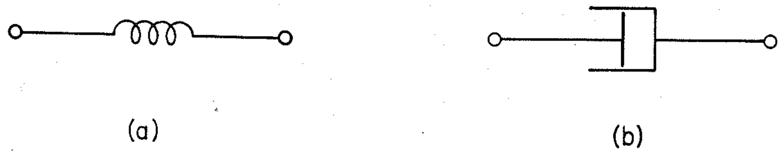

Fig. 5. a) Model of photons connected by a tiny spring. The stretching of the spring yields an attractive force between the two photons, whereas the compression of the spring yields a repulsive force. b) Model of photons connected by a damper element. In this case, the force acting between the two photons is proportional to their relative velocity.

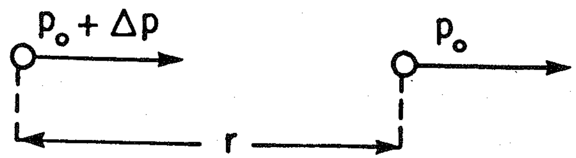

Fig. 6. The case of two photons having a small difference of momentum Δp.

9. The Equilibrium Case ($F = 0$) in a Collection of Streaming Monochromatic Photons

Let us now consider the case of a source of light which continously emits photons within a spatially narrow beam. Assume that the light is quasi-monochromatic, i.e., the photons initially have a slight spread of momentum around a central momentum p_0. Because no two photons have exactly the same velocity, they either get closer during the motion, or they separate further. In the former case a repulsive force will act on them ; in the latter, the force will be attractive. We are interested in knowing if they attain and, if so, what is the separation between the photons at the time of equilibrium when $F = 0$. To this end, let us consider the simple case of two photons of momentum p_0 and $p_0 + \Delta p$, respectively, emitted by a common source (Fig. 7). At a particular instant of time, the photons are separated by a distance equal to r. The interaction law states that the amount of momentum transferred from one photon to the other is :

$$p_r = m\ v_r = \frac{h}{r} \tag{12a}$$

where v_r is the velocity, measured in the laboratory frame of reference in the r-direction, acquired by one photon because of the presence of the other photon (it might be useful to re-examine Fig. 1 in order to be convinced that v_r is indeed the velocity in the laboratory frame of reference). If the velocity v_p of the latter photon in the r-direction were zero, v_r would be given by :

$$v_r = \frac{dr}{dt}$$

When $v_p \approx 0$:

$$v_r = \frac{dr}{dt} + v_p$$

where $\frac{dr}{dt}$ is the velocity of one photon relative to the other. Hence, eq. (12a) can be written in this way :

$$m\left(\frac{dr}{dt} + v_p\right) = \frac{h}{r}$$

or

$$\frac{dr}{dt} + v_p = \frac{h}{m}\frac{1}{r} \tag{29}$$

Let us assume that $\frac{dr}{dt}$ is the velocity of the forward photon relative to the rear one and v_p is the velocity of the latter. Equation (29) can be solved for t as follows :

$$dt = \frac{mr\ .\ dr}{h - mv_p\ .\ r} \tag{30}$$

$$t = \int \frac{mr\ .\ dr}{h - mv_p\ .r} \tag{31}$$

The integration of (31) now yields :

$$t = \int \left(\frac{\frac{h}{mv_p}}{\frac{h}{m} - v_p\ .\ r} - \frac{1}{v_p} \right) dr = \frac{1}{v_p} \int \left(\frac{h}{h - mv_p\ .\ r} - 1 \right) dr$$

$$= \frac{1}{v_p} \left[-\frac{h}{mv_p} \ln\left(h - mv_p\ .\ r\right) - r \right] + t_0 \tag{32}$$

Equation (32) shows that, as $t - t_0 \to \infty$ (t_0 is an arbitrary initial time) :

$$-\ln(h - mv_p\ .\ r) = \ln \frac{1}{(h - mv_p\ .\ r)} \to \infty \tag{33}$$

and therefore

$$(h - mv_p\ .\ r) \to 0 \tag{34}$$

If Eq. (34) is solved for r :

$$r_0 \to \frac{h}{mv_p} \quad \text{when } t \to \infty \tag{35}$$

where r_0 is the minimum (or equilibrium) distance between the two photons. But mv_p is the momentum p_0 of the rear photon. Hence

$$r_0 \to \frac{h}{p_0} = \text{const} \tag{36}$$

i.e., the two photons position themselves at the equilibrium distance $r_0 = \lambda = const$, having designated as λ such distance. Equation (36) tells us that the two particles, in the elementary one-dimensional analysis provided here, tend to be locked together at an equilibrium distance equal to λ. Clearly, in a collection of particles, this would induce a conglomeration or clustering of the particles, i.e. a lattice structure. In other words, isolated photons do not exist in this model. It is interesting to remark that this finding is consistent with the arguments of Sec. 4 and with the results of the experiment of Sec. 5.

10. Derivation of the Helmholtz Equation

Though lengthy, it is easy to prove that one can derive, from expression (20) of the field of interaction carried by each individual photon, another fundamental formula of Optics, namely the Helmholtz formula, extensively used to explain diffraction phenomena with monochromatic light.

We shall not proceed with the details of this derivation, because they can be found in Ref. 15.

11. Discussion

The analysis of the Heisenberg principle provided here has shown that two approaches can be adopted in viewing the principle itself. One is to consider the principle as a theoretical hypothesis ; the other is to admit that the principle is derivable from experiments. The first approach leads to a model of particles as wave-packets ; the second leads to a model of interacting particles. The two models do not differ in their prediction of the outcome of some experiments and the choice between the two models is then simply a matter of taste. The wave-packet representation of particles might be preferred by some, thus interpreting the wave as the probability of finding a particle in a given space at a given time. Others might instead prefer the model of interacting particles, which excludes the existence of probability waves but postulates an interaction as a physical property associated with all particles.

In the case of photons, assuming that the classical wave aspect of light stands for the probability of locating a photon in space and time, it has been shown that both models lead to the same equations, namely Kirchhoff's and Helmholtz'. Although the meaning of these equations is different in the two models, they agree on the fundamental prediction that photons arrange themselves on maxima and minima of distribution, much like a wave distribution. The case of statistically independent photons, however, is an exception and the two models yield conflicting predictions. The wave-packet model predicts that the diffraction of photons from a slit or a pinhole is a phenomenon independent of light intensity because diffraction effects arise from the linear superposition of the diffraction of each independent wave-packet or photon. In other words, each photon diffracts only with itself in this model. By contrast, the interacting photons model predicts that diffraction is a collective, or many-particle phenomenon, rather than a single-particle phenomenon and therefore nonlinear with light intensity. An experiment reported in this article

of diffraction of statistically independent laser photons at very low light intensities, yield results which are consistent with the prediction of the interacting photons model, in that the amplitude of the diffraction patterns were indeed nonlinearly related to the light intensity.

In essence, the present study has therefore provided an interaction relation which is a dispersion-free version of the Heisenberg principle, in which equality replaces inequality, and Δ-quantities become sharp. If this study is to be considered as part of a general study that goes under the name of "hidden-variables theories" [16], we would like to point out that it should then be considered as a "nonlocal" hidden-variables theory. In fact, as mentioned in the Introduction, locality is excluded in our study because "the interaction between system and apparatus, which makes the measurement possible, is not an instantaneous process" ... and "the interaction ... takes place continuously ...".

Although the interaction photon theory proposed here may be considered as a hidden-variables theory, we would like to state that its aim is not to go against the existing mathematical formulation of quantum theory. In fact, we just postulate a mechanism, an interaction force between particles which, if operative, yields the existing mathematical formulation of results, such as the uncertainty relations, Kirchhoff and Helmholz equations, etc. In other words, quantum theory prefers the route of postulating that Hermitean operators represent physical quantities, etc., whereas we prefer to postulate a physical mechanism of particle interaction.

In conclusion, that our study might be considered as a "nonlocal hidden-variables theory" is possible. That it might be distinguishable from quantum mechanics is hard. Therefore, as Rohrlich [17] has recently pointed out, for any "nonlocal hidden-variables theory" to become a viable scientific alternative to quantum mechanics it would have to account for at least one experiment not accounted for by quantum mechanics. The experiment reported in this article of diffraction of statistically independent photons under conditions of different light intensity is, we believe, such an experiment.

Acknowledgement

We are indebted to Drs. P. Savic, J. Lau, M. Vijay and E. Hanff. Their invaluable support in clarifying mathematical and physical models is greatly appreciated. In particular, Dr. Savic is the author of the analysis reported in

Section 9 for the derivation of the equilibrium distance for monochromatic photons.

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(Manuscrit reçu le 15 juillet 1986)