On the quantum-mechanical description of the Stern-Gerlach experiment

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ABSTRACT. Although the Stern-Gerlach experiment is very important both in atomic physics and as an example of a quantum process of measurement, we do not find in the literature a convincing theoretical analysis of this phenomenon. We present here a detailed quantum description of that experiment for hydrogen atoms or alkali atoms. The conclusion is that, considering the approach where the spin-orbit coupling is disregarded, the experiment makes in fact possible to determine the component of atomic magnetic moment along the field direction. The results obtained can also prove two of the basic postulates of quantum mechanics.

1. Introduction

It is known that the importance of the Stern-Gerlach type experiments is due to the fact that they have been considered capable of determining the atomic magnetic moments [1] and more recently these experiments have been often used as the example of a typical process of measurement in quantum mechanics [2]. However, a bibliographical analysis shows that there exists in fact no satisfactory quantum description of the interaction between the Stern-Gerlach device and the atoms which travel through it ; an indirect proof of this is the fact that many excellent books on quantum mechanics and atomic physics [3] deal with the Stern-Gerlach experiment and all these approaches raise a serious degree of amazement or criticism. Therefore, it is not even certain that experiments of this type make it in fact possible to determine the atomic magnetic moments.

[1] The first mention of this experiment corresponds to Ref. [1] and [2]. Results of further experiments of the same type are given mainly in Refs [3-5].

[2] Cf., for example, Refs [6-13], merely for information.

[3] Cf. mainly Refs. [6.9.10.14 and 15].

The purpose of this paper is exactly to demonstrate that the Stern-Gerlach experiment, at least in the case of hydrogen or alkali atoms, is one of the processes of measuring a component of the atomic magnetic moment, in case it is possible to disregard the term of the hamiltonian which represents the spin-orbit coupling. If this term is also considered, the quantity determined by such an experiment becomes the component of the "effective magnetic moment", as a second paper will show. On a third paper we finally show that the theory predicts that the spots resulting from the impact of the atoms on the detectors of the actual Stern-Gerlach apparatus are spacially separated. This conclusion is obviously indispensable so that we may consider these apparatus as being measurement devices. Such are the conclusions of this theoretical analysis, in which we try to explain the approaches that will be needed along the calculations.

The structure of the Stern-Gerlach device itself led to consider, as usually, that the space between the collimator appliance C (which is located in front of the furnace F, that emits the atoms) and the detector D is divided into three regions, regions I and III having no external field at all, whereas in region II the hamiltonian H^{II} includes a supplementary term that expresses the presence of a magnetic field produced by the magnet M.

Figure 1.

Once the general solutions of the evolution equations in any of these regions are found and then connected on the planes $X = A$ and $X = B$ following the usual requirements of continuity, it is finally seen that the physical parameters of the device are such that it is possible to confer to each atom, according to its impact position in the detector, a well defined value of the component of

the magnetic moment along the field direction. Furthermore, it can be proved that the statistic frequency according to which the various quantified values of the component of the magnetic moment occur is proportional to the squared modulus of the decomposition coefficient of function ψ^I along the eigenfunctions of the operator corresponding to that physical quantity, as required by the usual postulates of the theory.

2. The state function in region I

Let us consider that the furnace emits hydrogen atoms or alkali atoms, which we may consider as made up of an electron (coordinate $\vec{r}_1$, mass m_1, charge $e_1 = -e$) interacting with a stable structure (coordinate $\vec{r}_2$, mass $m_2 \gg m_1$, charge $e_2 = +e$) which is either a proton or a core surrounded by completely filled electronic shells. In this latter case, the interaction potential is no longer Coulombic but, as we know, it is still expressed as a function of $|\vec{r}_1 - \vec{r}_2|$ and, since we decided to disregard here the spin-orbit coupling, the hamiltonian in region I will simply be

$$H^I(\vec{r}_1, \vec{r}_2, \vec{p}_1, \vec{p}_2) = \frac{\vec{p}_1^2}{2m_1} + \frac{\vec{p}_2^2}{2m_2} + V(|\vec{r}_1 - \vec{r}_2|) \tag{1}$$

where $\vec{p}_i$ represents the linear momentum operator corresponding to $\vec{r}_i$. By inserting the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$ and the center of mass coordinate $\vec{R} = (m_1\,\vec{r}_1 + m_2\,\vec{r}_2)/(m_1 + m_2)$ it is immediately seen that this hamiltonian is now written

$$H^I(\vec{r}, \vec{P}, \vec{p}) = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2m} + V(r) \tag{1b}$$

with $\vec{P} = -i\hbar\vec{\nabla}_{\vec{R}}$, $\vec{p} = -i\hbar\vec{\nabla}_{\vec{r}}$, $M = m_1 + m_2$, $m = m_1\,m_2/(m_1 + m_2)$.

Let us consider that the "particle" of mass M indicates the global movement of the atom and describes its evolution in physical space until it reaches detector D.

Since hamiltonian (1b) may be decomposed to

$$H^I(\vec{r}, \vec{P}, \vec{p}) = H_0^I(\vec{P}) + H_1^I(\vec{r}, \vec{p}) \tag{2}$$

and it is supposed that no previous coupling or connection have previously occured between both "particles", the more general solution of the evolution equation

$$H^I\,\psi^I(\vec{R}, \vec{r}, t) = i\hbar\,\frac{\partial}{\partial t}\,\psi^I(\vec{R}, \vec{r}, t) \tag{3}$$

should be written

$$\psi^I(\vec{R},\vec{r},t) = \Phi^I(\vec{R},t)\,\chi^I(\vec{r},t) \tag{4}$$

where, by mere replacement, it is seen that Φ^I and χ^I are defined by the equations

$$\frac{\vec{P}^2}{2M}\,\Phi^I(\vec{R},t) = i\hbar\,\frac{\partial}{\partial t}\Phi^I(\vec{R},t) \quad , \quad \left(\frac{\vec{p}^2}{2m} + V\right)\,\chi^I(\vec{r},t) = i\hbar\,\frac{\partial}{\partial t}\,\chi^I(\vec{r},t) \tag{5}$$

the normalization of ψ^I being assured if we consider that functions Φ^I and χ^I are both normalized.

Indicating now by $F_{nlm_l}(r,\theta,\phi) = R_{nl}(r)\,Y^l_{m_l}(\theta,\phi)$ the orthonormal eigenfunctions of hamiltonian H^I_1 and by f_σ ($\sigma = \pm 1$) the Pauli elementary spinors $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$, inserted at this stage only thinking of the subsequent transformations of the atom, we can use this basis to decompose function χ^I and, instead of (4), we have the spinorial function

$$\psi^I(\vec{R},\vec{r},t) = \Phi^I(\vec{R},t)\sum_{nlm_l\sigma}\theta^I_{nlm_l\sigma}(t)\,F_{nlm_l}(r,\theta,\phi)\,f_\sigma \tag{4b}$$

It is thus not difficult to realize that this expression can be simplified if we make it include the eigenvalues E_{nl} of H^I_1, in other words, it can be written

$$\psi^I(\vec{R},\vec{r},t) = \Phi^I(\vec{R},t)\sum_{nlm_l\sigma}a_{nlm_l\sigma}\,e^{-iE_{nl}t/\hbar}\,F_{nlm_l}(\vec{r})\,f_\sigma \tag{4c}$$

By knowing the expression of wave packet $\Phi^I(\vec{R},t)$ and the numerical values of coefficients $a_{nlm_l\sigma}$ it is possible to completely define the original state of the system.

3. The atom hamiltonian in region II

If we take the Breit hamiltonian written as in the Pauli approach [16] and eliminate its relativistic terms and, as we mentionned above, its term of spin-orbit coupling (which means that we also disregard the spin-spin coupling), we are led to confer to H^{II} the expression

$$H^{II}(\vec{r}_1,\vec{r}_2,\vec{p}_1,\vec{p}_2,\vec{\sigma}_1,\vec{\sigma}_2) = \sum_{i=1}^{2}\left\{\frac{1}{2m_i}\,\left[\vec{p}_i^{\,2} - \frac{e_i}{c}\,\vec{A}\,(\vec{r}_i)\right]^2 - \frac{e_i\hbar}{2m_ic}\,\vec{B}(\vec{r}_i)\cdot\vec{\sigma}_i\right\} + V(|\vec{r}_1-\vec{r}_2|) \tag{6}$$

this hamiltonian appearing as a natural generalization of (1) for the situation present in region II, where we have an external magnetic field $\vec{B}$, whose vector potential is represented by $\vec{A}$.

We reach then a first simplification of (6) by imposing the Gauge condition $\operatorname{div} \vec{A} = 0$ and taking into consideration that the intensity of the external field is never high enough to confer real meaning to the presence of the diamagnetic terms. If we then express, as in (1b), operators $\vec{p}_1$ and $\vec{p}_2$ in terms of $\vec{P}$ and $\vec{p}$ and operator V in terms of $\vec{r}$, we have

$$H^{II} = \frac{\vec{P}^2}{2M} + \frac{\vec{p}^2}{2m} - \sum_{i=1}^{2} \left\{ \frac{e_i}{m_i\, c}\, \vec{A}(\vec{r}_i) \cdot \vec{p}_i + \frac{e_i \hbar}{2m_i c}\, \vec{B}(\vec{r}_i) \cdot \vec{\sigma}_i \right\} + V(r) \qquad (7)$$

Furthermore, it is necessary to explicitly consider the physical features of the magnetic fields that are really used in the Stern-Gerlach type experiments. It is known that, in these experiments, the fields generated by the magnetic pole pieces near the symmetry plane OXZ are pratically directed along OZ, $\vec{B} = B\, \vec{e}_Z$, and, besides that, not only the gradient of B has that same direction but it can be considered as constant. [4] With a field intensity of 10^4 gauss and dB/dZ not higher [5] than 10^5 gauss.cm $^{-1}$, it is easily seen that B varies slowly in the same order as the atom dimensions, and it is thus possible to write in the neighbourhood of the symmetry plane,

$$\vec{A}(\vec{r}_i) = \frac{1}{2}\, B(\vec{r}_i)\, \vec{e}_Z \times \vec{r}_i \qquad (8)$$

With such numerical values, an expansion in power series of $B(\vec{r}_i)$ near the point $\vec{R}$ gives as a result

$$B(\vec{r}_i) \approx B(\vec{R}) + \left(\frac{dB}{dz_i} \right)_{z_i = Z} (z_i - Z)$$

where the numerical value of the second term is of the order of 10^{-3}. We then conclude that we should take

$$B(\vec{r}_i) = B(\vec{R}) \qquad (9)$$

[4] It is well known that the field gradient is only approximately constant and this approach is unsatisfactory mainly near the side edges of the pole pieces, where the field presents a complicated and scarcely known form (edge effect). To develop the calculation we have to disregard this effect, which is in fact the stronger restriction of this analysis of the experiment.

[5] Cf. mainly Refs. [2-4].

Due to (8) and (9) and since we have here $e_2 = -e_1 = e$, the terms under the summation sign present in (7) can now be rewritten

$$-\frac{eB(\vec{R})}{2c}\vec{e}_Z.\left\{\frac{\vec{R}\times\vec{p}}{m}+\frac{\vec{r}\times\vec{P}}{M}+\frac{m_2-m_1}{m_1m_2}(\vec{r}\times\vec{p})-\frac{\hbar}{m_1}\vec{\sigma}_1+\frac{\hbar}{m_2}\vec{\sigma}_2\right\}$$

As in the usual theory of the Zeeman effect, the two first terms can be disregarded in the presence of the third one and, since $m_2 >> m_1$, the last one has no value relatively to the last but one. As a consequence of that, if operator $\vec{r}\times\vec{p}=\vec{l}$, is expressed in $\hbar$ units (as was already the case for operators $\vec{\sigma}$) and since $(m_2-m_1)/m_1m_2 \approx 1/m_1$, we can conclude that H^{II} can be reduced to the expression

$$\begin{aligned} H^{II} &= \frac{\vec{P}^2}{2M}+\frac{\vec{p}^2}{2m}+\frac{e\hbar}{2m_1c}B(\vec{R})\,(l_Z+\sigma_Z)+V(r) \\ &= \frac{\vec{P}^2}{2M}+\frac{\vec{p}^2}{2m}+\mu_0 B(\vec{R})\,(l_Z+\sigma_Z)+V(r) \end{aligned} \tag{10}$$

where μ_0 represents the Bohr magneton.

4. The state function in region II

The presence of the term $\mu_0 B(\vec{R})(l_Z+\sigma_Z)$ in hamiltonian H^{II} prevents ψ^{II} to be expressed as a $\Phi^{II}(\vec{R},t)\chi^{II}(\vec{r},t)$ product. However, since the normalized eigenfunctions $F_{nlm_l}f_\sigma$ of H_1^I are still defining an orthonormalized basis in the $\vec{r}$ space, it is possible to go on writing the general solution of the evolution equation in region II using the expression

$$\psi^{II}(\vec{R},\vec{r},t)=\sum_{nlm_l\sigma}\theta^{II}_{nlm_l\sigma}(\vec{R},t)F_{nlm_l}(\vec{r})f_\sigma \tag{11}$$

Since functions $F_{nlm_l}f_\sigma$ are also eigenvectors of operator $\mu_0 B(\vec{R})(l_Z+\sigma_Z)$, to which correspond eigenvalues $\mu_0 B(\vec{R})(m_l+\sigma)$ and since (11) must satisfy an equation similar to (3) but where hamiltonian (10) is present, it is not difficult to conclude that $\theta^{II}_{nlm_l\sigma}$ will be governed by the equation

$$\left[H_0^I+E_{nl}+\mu_0 B(\vec{R})(m_l+\sigma)\right]\theta^{II}_{nlm_l\sigma}(\vec{R},t)=i\hbar\frac{\partial}{\partial t}\theta^{II}_{nlm_l\sigma}(\vec{R},t) \tag{12}$$

and that, therefore,

$$\theta^{II}_{nlm_l\sigma}(\vec{R},t)=b_{nlm_l\sigma}e^{-iE_{nl}t/\hbar}\Phi^{II}_{m_l+\sigma}(\vec{R},t)$$

with

$$\left[H_0^I + \mu_0 B(\vec{R})(m_l + \sigma)\right]\Phi^{II}_{m_l+\sigma}(\vec{R},t) = i\hbar\frac{\partial}{\partial t}\Phi^{II}_{m_l+\sigma}(\vec{R},t) \tag{12b}$$

and, instead of (11), we will write

$$\psi^{II}(\vec{R},\vec{r},t) = \sum_{nlm_l\sigma} b_{nlm_l\sigma} e^{-iE_{nl}t/\hbar}\Phi^{II}_{m_l+\sigma}(\vec{R},t)F_{nlm_l}(\vec{r})f_\sigma \tag{13}$$

This general expression of the system state function in region II must however be connected to a corresponding expression in region I, expressed by (4c), which mainly causes both functions to coincide on the $X = A$ plane,

$$\left[\psi^{II}(\vec{R},\vec{r},t)\right]_{X=A} = \left[\psi^{I}(\vec{R},\vec{r},t)\right]_{X=A} \tag{14}$$

and, since ψ^I and ψ^{II} are expressed in the same orthonormalized basis, it is easy to infer that (14) corresponds to the imposition of

$$b_{nlm_l\sigma}\left[\Phi^{II}_{m_l+\sigma}(\vec{R},t)\right]_{X=A} = a_{nlm_l\sigma}\left[\Phi^{I}(\vec{R},t)\right]_{X=A} \tag{14b}$$

On the other hand, the requirement of uniqueness of function $\Phi(\vec{R},t)$ in the $X = A$ plane causes

$$\left[\Phi^{II}_{m_l+\sigma}(\vec{R},t)\right]_{X=A} = \left[\Phi^{I}(\vec{R},t)\right]_{X=A} \tag{15}$$

to be a physically intuitive equality. If we insert it into (14b) , the result will be

$$b_{nlm_l\sigma} = a_{nlm_l\sigma}$$

and (13) will finally be written

$$\psi^{II}(\vec{R},\vec{r},t) = \sum_{nlm_l\sigma} a_{nlm_l\sigma} e^{-iE_{nl}t/\hbar}\Phi^{II}_{m_l+\sigma}(\vec{R},t)F_{nlm_l}(\vec{r})f_\sigma \tag{13b}$$

5. The state function in region III

Although in region III the hamiltonian has again the expression (1b) valid in region I, we can no longer write $\psi^{III}(\vec{R},\vec{r},t) = \Phi^{III}(\vec{R},t)\chi^{III}(\vec{r},t)$ because the correlation established in region II between both particles continues to exist.

Therefore, the general expression of ψ^{III} must be, as in (11), a summation of the type

$$\psi^{III}(\vec{R},\vec{r},t) = \sum_{nlm_l\sigma} \theta^{III}_{nlm_l\sigma}(\vec{R},t) F_{nlm_l}(\vec{r}) f_\sigma \tag{16}$$

Since ψ^{III} continues to be the solution of the evolution equation, now again (3), we have

$$[H_0^I + E_{nl}]\theta^{III}_{nlm_l\sigma}(\vec{R},t) = i\hbar \frac{\partial}{\partial t}\theta^{III}_{nlm_l\sigma}(\vec{R},t) \tag{17}$$

which implies

$$\theta^{III}_{nlm_l\sigma}(\vec{R},t) = c_{nlm_l\sigma} e^{-iE_{nl}t/\hbar}\Phi^{III}(\vec{R},t)$$

with

$$H_0^I \Phi^{III}(\vec{R},t) = i\hbar \frac{\partial}{\partial t}\Phi^{III}(\vec{R},t) \tag{18}$$

Since this function Φ^{III} is not defined as dependent on the quantum numbers n, l, m_l, σ, we could be led to infer that (16) is after all not formally similar to (11) but to (4c). That conclusion would however be wrong, since we did not yet consider the correlation of particles, which is only shown through the connection to be imposed on plane $X = B$. In fact, instead of (14), we will have here

$$\left[\psi^{III}(\vec{R},\vec{r},t)\right]_{X=B} = \left[\psi^{II}(\vec{R},\vec{r},t)\right]_{X=B} \tag{19}$$

that is to say

$$c_{nlm_l\sigma}\left[\Phi^{III}(\vec{R},t)\right]_{X=B} = a_{nlm_l\sigma}\left[\Phi^{II}_{m_l+\sigma}(\vec{R},t)\right]_{X=B} \tag{19b}$$

and, on the other hand, just as before,

$$\left[\Phi^{III}(\vec{R},t)\right]_{X=B} = \left[\Phi^{II}_{m_l+\sigma}(\vec{R},t)\right]_{X=B} \tag{20}$$

The equalities (19) and (20), in the same way as (14) and (15), are directional equalities, the first term of each having to equal the second one. And, in the same way we concluded in (15) that functions $\Phi^{II}_{m_l+\sigma}(\vec{R},t)$ on plane $X = A$ were after all independent of m_l and σ, we must admit here that $\Phi^{III}(\vec{R},t)$ on plane $X = B$ is to represent a whole set of $\left[\Phi^{III}_{m_l+\sigma}(\vec{R},t)\right]_{X=B}$ functions. Since, due to (19), $c_{nlm_l\sigma} = a_{nlm_l\sigma}$, the general expression of ψ in region III will be written

$$\psi^{III}(\vec{R},\vec{r},t) = \sum_{nlm_l\sigma} a_{nlm_l\sigma} e^{-iE_{nl}t/\hbar}\Phi^{III}_{m_l+\sigma}(\vec{R},t) F_{nlm_l}(\vec{r}) f_\sigma \tag{16b}$$

(16b) being different from (13b) only because it contains function $\Phi^{III}_{m_l+\sigma}$ instead of $\Phi^{II}_{m_l+\sigma}$.

6. A more adequate form of the state function

If we insert functions

$$g_{m_l\sigma}(\vec{r},t) = \sum_{nl} a_{nlm_l\sigma} e^{-iE_{nl}t/\hbar} F_{nlm_l}(\vec{r}) \tag{21}$$

expressions (4c), (13b) and (16b) will become

$$\psi^I(\vec{R},\vec{r},t) = \Phi^I(\vec{R},t) \sum_{m_l\sigma} g_{m_l\sigma}(\vec{r},t) f_\sigma \tag{22}$$

$$\psi^N(\vec{R},\vec{r},t) = \sum_{m_l\sigma} \Phi^N_{m_l+\sigma}(\vec{R},t) g_{m_l\sigma}(\vec{r},t) f_\sigma \quad (N = II, III) \tag{23}$$

or, considering the well known relations among the possible values of quantum numbers n, l and m_l, and defining $k = m_l + \sigma$ with $\sigma = \pm 1$,

$$\begin{aligned} (22b) \qquad \psi^I(\vec{R},\vec{r},t) &= \Phi^I(\vec{R},t) \sum_{k=-l-1}^{l+1} \left[g_{k-1,1}(\vec{r},t) f_1 + g_{k+1,-1}(\vec{r},t) f_{-1} \right] \\ &= \Phi^I(\vec{R},t) \sum_{k=-l-1}^{l+1} G_k(\vec{r},t) \\ (23b) \qquad \psi^N(\vec{R},\vec{r},t) &= \sum_{k=-l-1}^{l+1} \Phi^N_k(\vec{R},t) \left[g_{k-1,1}(\vec{r},t) f_1 + g_{k+1,-1}(\vec{r},t) f_{-1} \right] \\ &= \sum_{k=-l-1}^{l+1} \Phi^N_k(\vec{R},t) G_k(\vec{r},t) \end{aligned}$$

where, according to (21), the $g_{k\mp1,\pm1}$ mean

$$g_{k\mp1,\pm1}(\vec{r},t) = \sum_n \sum_{l=|k\mp1|}^{n-1} a_{nl,k\mp1,\pm1} F_{nlk\mp1}(\vec{r}) e^{-iE_{nl}t/\hbar} \tag{24}$$

It is easy to realize that functions

$$G_k(\vec{r},t) = g_{k-1,1}(\vec{r},t) f_1 + g_{k+1,-1}(\vec{r},t) f_{-1} \tag{25}$$

satisfy the eigenvalue equation of operator $-\mu_0(l_Z + \sigma_Z)$, the operator of the component of the atomic magnetic moment along OZ, the corresponding eigenvalues being $-\mu_0 k$ $(k = -n, -n+1, \ldots, n)$.

For the expressions (22b) and (23b) to be expressed in terms of normalized functions, we have to insert functions $G'_k(\vec{r}, t) = G_k(\vec{r}, t)/c_k$ in the place of $G_k(\vec{r}, t)$ with

$$c_k = \left[\sum_n \sum_{l=|k-1|}^{n-1} |a_{nl,k-1,1}|^2 + \sum_n \sum_{l=|k+1|}^{n-1} |a_{nl,k+1,-1}|^2 \right]^{1/2} \tag{26}$$

which leads us to write

$$\psi^I(\vec{R}, \vec{r}, t) = \Phi^I(\vec{R}, t) \sum_{k=-l-1}^{l+1} c_k G'_k(\vec{r}, t) \tag{22c}$$

$$\psi^N(\vec{R}, \vec{r}, t) = \sum_{k=-l-1}^{l+1} c_k \Phi^N_k(\vec{R}, t) G'_k(\vec{r}, t) \quad (N = II, III) \tag{23c}$$

Written in these terms, state function ψ expresses the essential part of the results obtained by this theoretical analysis of the Stern-Gerlach experiment. First we conferred to the elements of the atomic beam we studied any state vector, given by (4c) or by (22c), whose expression was only implicitly restricted by the experimental limitations of the device itself. Then it was possible to see that, after travelling through the inhomogeneous magnetic field, that state vector according to (23c), was expressed as a summation of correlated pairs of functions with very specific features : each pair consisted of a wave packet Φ^{III}_k which described the evolution of the system mass center in physical space, and of a specific eigenfunction of the component operator of the magnetic moment along the field direction.

7. Conclusions

In order not to interrupt abruptly this subject neither to make this paper too long, we leave for another paper, as mentioned in the Introduction, the analysis of the space-time evolution of the Φ^N_k wave packet. That analysis enables us to admit here, due to the physical parameters characteristic of the Stern-Gerlach devices, that the Φ^{III}_k wave packets will cause the appearing in the detector of spacially separated spots, each of them corresponding to a well determined value of k. Therefore, the position of a small spot appearing in the detector shows

in which of the wave packets the atom was located. Consequently, and due to the correlation between the Φ_k^{III} and the G'_k expressed in (23c), the position of the spot enables us to give the component of the atomic magnetic moment its corresponding $-\mu_0 k$ value. The explanation of the wave function collapse that corresponds to the appearing of the spot in the context of common quantum mechanics is a question out of the reach of this article. Anyway, it is certainly possible to conclude that, according to the approach considered in this paper, the Stern-Gerlach experiment is the example of a form of measuring the component of the atomic magnetic moment along the field direction.

But we can proceed further. Since the possible results of such a measurement can only be the $-\mu_0 k$ values, which are the eigenvalues of the operator corresponding to the quantity here considered, one of the postulates of quantum mechanics is here justified.

Concerning the probabilities of determining the various values of $-\mu_0 k$ they ought to be the probabilities of finding the corpuscle in the corresponding Φ_k^{III} wave packets or, in other words, the potentialities of presence of the atom in each of the wave packets. So according to (23c), the position probability density $\rho_k(\vec{R}, t)$ of the atom in one point of region $D_k(t)$, where the generic wave packet Φ_k^{III} is not null, should be written

$$\rho_k(\vec{R}, t) = \int \left|\Phi_k^{III}(\vec{R}, t) c_k G'_k(\vec{r}, t)\right|^2 d^3r \tag{27}$$

$$= |c_k|^2 \left|\Phi_k^{III}(\vec{R}, t)\right|^2 \int |G'_k(\vec{r}, t)|^2 d^3r = |c_k|^2 \left|\Phi_k^{III}(\vec{R}, t)\right|^2$$

and, therefore, the probability P_k of locating the corpuscle in $D_k(t)$ will have the value [6]

$$P_k = |c_k|^2 \int_{D_k} \left|\Phi_k^{III}(\vec{R}, t)\right|^2 d^3R = |c_k|^2 \tag{28}$$

the c_k being obviously still given by definition (26).

Now let us try, using the expression of function ψ^I, to define the probability P'_k of the result of measuring the component of the atomic magnetic moment being $-\mu_0 k$. According to another of the postulates of quantum mechanics, we must thus calculate P'_k by the squared modulus of the corresponding coefficient included in the decomposition of ψ^I along the normalized eigenfunctions G'_k of

[6] P_k is independent of t because each of the Φ_k^{III} satisfies the Schrödinger equation, which assures that its norm is kept.

the respective operator ; [7] therefore, if we use (22c), we will immediately have

$$P'_k = \int |c_k \Phi^I(\vec{R}, t)|^2 d^3R = |c_k|^2 \tag{29}$$

the agreement of the values of P_k and P'_k being in fact a justification of this basic postulate of quantum mechanics. [8] The truthfulness of this postulate depends only on the truthfulness of the hypothesis that $|\psi|^2$ determines the position probability density.

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[7] It is easy to conclude that we reach the same result if the decomposition is not made according to G'_k but to the usual normalized functions of operator $-\mu_0(l_Z + \sigma_Z)$.

[8] Note that the above reasoning leads to prove the special form of the quantum postulate in a case like this, where the eigenfunctions of the operator do not depend on all variables present in the state vector.

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RESUME. Malgré l'importance de l'expérience de Stern-Gerlach soit en physique atomique, soit comme exemple d'un processus quantique de mesure, on ne trouve dans la littérature aucune analyse théorique convaincante de ce phénomène. On présente ici une description quantique détaillée de cette expérience pour l'atome d'hydrogène ou les atomes alcalins. On montre que, en négligeant l'interaction spin-orbite, l'expérience conduit effectivement @ la détermination de la composante du moment magnétique atomique selon la direction du champ. Les résultats obtenus permettent par ailleurs de retrouver deux des postulats fondamentaux de la mécanique quantique.