Examples of explicit position-velocity coexistence and their physical implications in a "minimal" stochastic interpretation of quantum mechanics

NICKOLA S. TODOROV

Georgi Nadjakov Institute of Solid State Physics,
72 bld. Lenin, 1784 Sofia, Bulgaria

Part I. General considerations and formulation of the problem

ABSTRACT. A brief outline of several approaches to the stochastic interpretation of quantum mechanics is given. It is pointed out that the statistical theory of quantum mechanics should be compared with a pertinent statistical variant of classical mechanics when fundamental interpretational problems of the former (as, say, the possibility of position-velocity coexistence) are considered. From this viewpoint the conventional interpretation of quantum theory, which too readily attributes ensemble properties to individual ensemble members, cannot claim to be logically consistent (to be explicitly demonstrated in the next Parts). The strategy necessary for the demonstration of the admissibility of position-velocity coexistence for micro-particles in the ensemble interpretation is formulated.

I. INTRODUCTION

It is well known that there exist two basic outlooks on the nature of quantum mechanical (QM-standing for "quantum mechanics" too) motion of microparticles :

(i) The outlook expressed by the conventional interpretation (CI) of QM. The CI rests on Bohr's complementarity thesis and uncertainty relations of the kind

$$\delta F_1 \delta F_2 \geq < \hat{M} > /2 \tag{I.1}$$

where $< ... >$ stands for the QM average value (in QM states that are *normalizable to unity* : $\int |\psi|^2 dV = 1$) of the dynamical variable inside the brackets,

$$\delta F_i = \sqrt{<(\Delta\hat{F}_i)^2>} \quad , \quad \Delta\hat{F}_i = \hat{F}_i - <\hat{F}_i>$$

and $\hat{F}_i$, $(i = 1, 2)$ are self-adjoint operators satisfying the commutation relation

$$[\hat{F}_1, \hat{F}_2] = i\hat{M}, \tag{I.2}$$

$\hat{M}$ being, by assumption, a self-adjoint operator too. In the case of

$$\hat{F}_i = \hat{x} = x \quad , \quad \hat{F}_2 = \hat{p}_x = -i\hbar\partial/\partial x$$

we have $\hat{M} = \hbar$ and one comes to the fundamental uncertainty relation of Heisenberg

$$\delta x \delta p_x \geq \hbar/2 \tag{I.3}$$

which shows that in a normalized QM state of motion positions and velocities ($\hat{v}_x = \hat{p}_x/m$, m standing for the mass of the particle) cannot have simultaneously zero statistical dispersions. The CI relates facts of this kind to individual physical systems and presumable properties of measuring instruments. Namely, it asserts that measurement of one of the *complementary* magnitudes in a given couple (e.g. x or v_x) affects in an unpredictable and uncontrollable way the other (the measuring apparatus and the measured system forming an inseparable nonlocal entity), so one can never determine them simultaneously for an individual physical system. The inference is that it is, strictly speaking, senseless to employ for microparticles classical concepts as, say, positions and velocities at a given moment t : no realizable experiment can give a simultaneous account of them, so pairs of complementary magnitudes are *in principle* simultaneously nonexisting.

(ii) The outlook expressed by the stochastic interpretation (SI) of QM. More precisely, there exist a number of such interpretations at different levels of nonorthodoxy. [For instance, in de Broglie's approach [1-3] (cf. also the earlier references cited therein) neither the conventional outlook on measurement nor the conventional outlook on the nature of the uncertainty relations (I.1) is preserved. Bohm's approach [4-6] (ref. 6 offering in fact a somewhat different variant of this ideology) preserves the idea of a quantum non-separability of the entity measuring apparatus-measured system and the conception of unpredictable and uncontrollable effects of measurement on the measured system in *actual practice*. In this consideration, however, positions and velocities of members of the relevant statistical ensemble may be thought of as coexisting all the time. Bloknintsev's ideology [7] represents an example of a practically orthodox way of reasoning, with the only exception that the preeminence of the statistical ensemble concept is recognized from the very beginning. Still different versions of the SI may be found in Nelson's work [8], resting on the conception of a Markovian character of the behaviour of micro-particles in a subquantum stochastic medium, and in stochastic electrodynamics [9], where a non-Markovian picture of the motion of charged particles is set forth. Reviews and detailed considerations of certain aspects of the SI are given in the literature [10],[11].]

Most versions of the SI (we shall have in mind these variants in what follows) support the idea that any 'pointlike' particle must have, say, a reasonably defined position and velocity at every moment t. [Velocities that exist, by hypothesis, at any moment t will be called here *microvelocities* while velocities which can be well defined only in a certain limit (e.g. at $t \to \infty$) will be called *macrovelocities* and it will be demonstrated in Part II that both macro- and microvelocities can coexist with position in the sense that experiment does not rule out, generally, the possibility of a simultaneous measurement of individual positions and velocities with an *arbitrary* degree of precision]. According to this different interpretation, however, the said magnitudes do not play the same role in QM and classical mechanics. For instance, one of the possibilities indicated by the SI is that QM in fact reveals the insufficiency of positions and velocities for a complete determination of the actual state of motion of microparticles (certain additional parameters might be necessary to this end).

The SI essentially employs the concept of a statistical ensemble representing a given physical situation. (The fact that statistical ensembles must be employed in comparing QM and classical results due to the correspondence of a QM ensemble in the quasiclassical case to a classical ensemble of particles with indefinite initial conditions was pointed out, e.g., by Einstein himself [12]. Other arguments on the importance of statistical ensembles in QM may be found in ref. 10. The content of the present paper is determined by the ensemble concept too). The possibility that an ensemble of classical particles may be characterised by some spread, say, of positions is certainly 'classically conceivable'. The 'classically inconceivable' point (depriving, according to the CI, positions and velocities of a simultaneous reality) is the inevitable spread δp_x of momenta accompanying, as required by relation (I.3), a spread δx of positions in the ensemble since in classical mechanics it is conceptually possible to build statistical ensembles with nonzero δx and zero δp_x or vice versa. As it was mentioned, the SI assumes that the interconnectedness of δx and δp_x may be due to additional factors which determine the state of motion of microparticles and which must be taken into account as well in a statistical picture of a given situation. For instance, in a stochastic electrodynamical picture such factors are the interaction of an electrically charged particle with the presumable vacuum electromagnetic fluctuations and with its own electromagnetic field, the latter interaction promoting e.g. acceleration to the rank of a state-determining magnitude (cf. ref. 9 for a consideration of this problem and a list of earlier references ; see also our brief discussion of de Broglie's more general ideas below). On the other hand, magnitudes such as spin may be (and most probably really are) a direct consequence of a non-pointlike nature of the microparticles, that is, a result of an inner structure and inner dynamicas. Therefore, complex structure presents another possibility for explaining certain features of the behaviour of microparticles. These remarks make it necessary to reconsider here briefly the possible role of the different dynamical variables in microphysics.

Conventional theory has a well known feature : it treats all complete sets of compatible dynamical magnitudes as equivalent, in a sense (though mutually exclusive from the view-point of Bohr's complementarity principle for such sets). Namely, the different representations -as determined by "complementary" sets of variables, e.g. co-ordinates or momenta- of the states of motion describe equally well any given state. It seems natural to assert that measurements within different complete sets of compatible magnitudes have certain universal common features too, so the conventional theory of measurement [10],[13],[14] rests on the concept of an instantaneous reduction of the initial wave function to an eigenfunction of the measured magnitude(s) at the moment of impinging of the physical system under investigation on the measuring apparatus. This instantaneous process is envisioned as taking place, generally, in the act of measurement of every dynamical magnitude and the corresponding experimental eigennumbers are not regarded as actually preexisting in the initial wave packet. (Uncontrollability and unpredictability of measurement perturbations of the corresponding complementary magnitudes agrees well with the postulated instantaneous nature of measurement since such an ideology rules out any causal dynamical evolution that might somehow be associated with predictability or controllability of measurement results).

Certainly, alike the CI, the different variants of the SI also offer general outlooks on the role of measurement in microphysics. But in the more detailed picture of quantum phenomena given by them the above-mentioned equivalence of state descriptions may turn out to be absent in the general case. It has been repeatedly stressed by de Broglie, for instance, that position appears to be a preeminent quantity compared to all other physical magnitudes since all measurements are performed in fact through position localizations and, besides, position localizations do not require any specific state modifications before detection by means of convenient analysers (cf. e.g. refs. 2,3). The idea of a permanent localization of a microparticle helps to evade certain difficulties in the CI [2]. Besides, this idea is so natural from the viewpoint of our usual image of particles that a tendency of preserving it is understandable indeed. The apparent contradiction of permanent localization with inequality (I.3). [a permanently localized pointlike particle must obviously have a well defined trajectory -hence velocity and momentum- at any moment t while (I.3) seems to forbid this] is overcome, in principle, in de Broglie's variant of the SI by assuming that the actual velocity at a given moment t is a hidden magnitude with respect to the usual experiments exhibiting velocity distributions. Really, in such (always macroscopic) experiments one needs sufficiently large distances (cf. e.g. refs. 2, 10) or intervals of time (see below) in order to be able to determine with a necessary precision certain momentum values. It is quite possible, then, that what one actually measures in this way is just the time-averaged result of the motion of the particle under the influence of the hypothetical additional factors and not microvelocity itself.

Let us examine briefly de Broglie's ideology on measurements and the role of permanent localization of microparticles in it. We have the following general assumption in this theory : Each measuring apparatus consists of a spectral analyser and a detector, with the exception of position-measuring devices which consist of detectors only. The essential part of the measuring device is the analyser which decomposes the initial wave packet into a suitable set of practically non-overlapping wave packets. The particle (which has a well defined position at any moment of time) may be found in only one of these wave packets and the registering of its *actual* position by the detector gives information about the value of the physical magnitude of interest. The very act of registering the position by the detector is physically uninteresting as it informs us about an already existing fact (the presence of the particle in a given wave packet at a suitable distance behind the analyser). The important point in measurement is the concrete *state preparation* by the concrete analyser (call it A') since the measuring of another physical magnitude (m'') that is incompatible with the first one (m') would require a different analyser (A'') which is incompatible with A' in the sense that it gives a preparation of states that is incompatible with the one produced by A'. In other words, the set of wave packets produced by A'' (the incident wave packet being the same for both A' and A'') carry a totally different information compared to that given by A', A'' being able to produce wave packets determining only m'', while A' emits wave packets determining just m'. (Therefore, if we use simultaneously both analysers, A'' being placed behind A', then A'' will obliterate the effect of A', thus destroying the information about the value of m').

Consequently, the SI needs, generally, no concepts as instantaneous and uncontrollable effects of measurements in order to explain the incompatibility of certain QM measurement procedures : In the SI, incompatibility is of a purely dynamical nature (that is, it is describable in terms of properties of the solutions of certain dynamical equations) and comes as a result of the possible fundamental difference of the physical nature of certain magnitudes. This difference entails as a direct consequence the above-mentioned difference in the types (or logic) of experiments determining the "complementary" quantities.

[It would be logical to expect that in a consistent physical theory the conception of fundamental difference of the nature of certain physical magnitudes of a given physical system should have a counterpart in the statistical apparatus of the theory. This is the case indeed with de Broglie's theory in which we encounter concepts as *present probabilities* (determining actual co-ordinate distributions at a moment t of interest) and *predicted probabilities* that give, say, macroscopic velocity distributions, without being able to reveal the microscopic, *hidden velocity distributions*. It is worth recalling, though, that observables of the kind of spin-component values, being possibly determined by an inner particle structure and motion, may not have microscopic analogues that actually exist at any t. Other possible aspects of the role of measurement that were recently considered in the literature [15] are discussed in Appendix B, where one may find also a more detailed physical consideration of the essence of a new mathematical proposal [16],[17]].

In spite of the fact that the different variants of the SI exhibit a number of conceptual advantages [1-6],[8,10] the more popular interpretation continues to be the CI. One of the possible reasons for this (besides obvious reasons of a historic nature) may be the following. In the proposed variants of the SI measurement continues to play a specific role that has no analogue in classical theory. Indeed, in classical mechanics, measurements were in fact of no interest to the theory as the relevant devices there were 'self-obviously' treated as registering actually existing magnitudes, that is, these devices were envisioned as uninteresting *detectors* of actual numbers. As we saw, certain variants of the SI employ the concepts of *analyser* and *incompatible analysers* for "complementary" pairs of dynamical magnitudes (with the only exception of position x but even here the "complementary" magnitude p_x requires, by assumption, an analyser for its determination). One may be led then to the inference that the SI does not represent an essential step in a forward direction from a conceptual point of view since the important role of the concepts of analyser and incompatible analysers there may be interpreted to imply a de facto return to the conventional outlook on "complementarity".

The goal of this paper is to demonstrate that the SI represents no disguised return to the CI. On the contrary, it gives a self-consistent physical picture for cases in which the CI does not work. We shall concentrate our attention mainly on the position-momentum couple (x, p_x) [or, equivalently, $(x, v_x), v_x = p_x/m$]. Our consideration will be based on the detailed analysis of the objective properties of certain explicit solutions to the nonstationary Schrödinger equation for the one-dimensional case and their discussion from the viewpoint of a 'minimal' variant of the SI. This variant employs basic statistical concepts of stochastic theories, evading as much as possible specific physical assumptions whose introduction may be as yet premature. In spite of its minimal character the discussion will reveal useful physical implications that can be generalized later on and will give good possibilities for comparisons with earlier ideas. In particular, we shall show that the use of analysers will be unnecessary also for the determining of certain physical magnitudes that differ from position (in our case - momentum). This leads to the important inference that the conception of compatibility or incompatibility of physical magnitudes needs revision. (Indeed x and v_x will be compatible magnitudes in the problems examined, so the EPR notion [18] on the admissibility of position-velocity coexistence in QM will obtain a direct corroboration).

Let us specify now the basic concepts of the 'minimal' SI.

II. ON THE STATISTICAL CONCEPTS OF QUANTUM THEORY

It should be an obvious fact that any physical theory of a statistical nature must essentially employ the concept of a relevant *statistical ensemble* for a given physical situation to which the statistical predictions of the theory apply (cf. also the Introduction). This is certainly necessary from the viewpoint of coherence of logic as statistical concepts (in QM : wave functions, probabilities, dispersions, correlations, etc.) always apply to a given ensemble of events and are checked on this ensemble. Consequently, in order to evade inferences that are not sound enough, one must attribute, generally, the statistical theoretical properties to the entire ensemble itself and not necessarily to inherent features of its individual members (say, physical systems in the same QM state). In the CI the latter distinction is not clearly delineated [10] (in fact, certain ensemble properties are too readily attributed by it to individual systems), so a number of its assertions are unacceptable to those who would like to have a more rigorous physical approach to the interpretation of QM.

An immediate objection to the above-said seems to be existing. Namely, one is interested in the physical properties of individual systems (say microparticles) and not in the properties of abstract (at that infinite - in order to be able to define precisely certain probabilities) ensembles of identical physical systems. Why should one attribute then experimental observations to imaginary formations ?

The above-said gives an answer to the question but it is worth dwelling on this item a little longer. The replacing of individual systems by ensembles of such -mutually noninteracting- systems gives certain advantages, which in fact explains why one resorts sometimes to the introduction of statistical concepts in physics. For instance, in classical statistical mechanics the practically impossible computation of time-averages of physical magnitudes of an individual system is replaced, by postulation, by the much easier computation of ensemble averages of the same magnitudes in relevant statistical ensembles (microcanonical, canonical, etc.) that are assumed to correspond in one way or the other to a given physical situation. In QM one can also formulate rules for the computation of probabilities and averages of physical magnitudes in relevant ensembles. But in both cases one has to pay a serious price for such a convenience : The introduction of statistics automatically requires a modification of our language which must now contain statistical ensemble concepts only, so that, say, the concept of a state of motion of a system should now be no other than statistical. [For the case of statistical mechanics this point has been clearly understood and the ensemble concept of state of motion (in the classical case - the density distribution $\rho(p, q, t)$ in $6N$-dimensional phase space, N being the number of particles in the physical system) enters in the axioms of this theory [19]]. This fact entails difficulties which come when one tries to answer the natural question how precisely the ensemble concepts should be translated in the terms of physical properties of individual systems and these difficulties may be severe. [For instance, the absence of entropy increase in the ensemble picture of statistical mechanics is a well known difficulty that has not found a final resolution until the present moment. The same applies to the interpretation of the physical sense of ineq. (I.1) and the related problem of wave-particle dualism in QM]. In other words, great interpretational difficulties arise when one tries to extract physical properties of individual systems from the postulated statistical picture which -although suitable for certain computational purposes- turned out to be the source of sharp theoretical controversies. These controversies in QM would probably not appear or at least be much milder if it were clearly realized from the very beginning that for some reasons (which should better not be prematurely specified), we are forced to employ for the time being a statistical description of the microworld and must bear in mind all consequences of this fact for the proper language that should be used in the theory. Therefore, in order to decide whether the coexistence of essentially classical magnitudes as, say, positions and velocities is actually ruled out or not by the uncertainty relations of QM, one has no other logically consistent choice than employ suitable quantum and classical statistical ensembles for given physical situations and compare the corresponding results for position and momentum distributions. (This is the most that can be done as long as statistics is concerned). If the QM and the classical pictures would give coinciding statistical results in certain concrete situations, then there would be no grounds for rejecting the possibility for position-velocity coexistence (which possibility is asserted invalid by the CI for all thinkable cases). The ensemble approach outlined above will be employed by us below.

To summarize, we treat all QM states and relations [including ineq. (I.1)] as purely statistical concepts the physical sense of which may be clarified by a careful analysis in each specific case. [For instance, in the (x, v_x) case -by a comparison of the conduct of suitable QM and classical statistical ensembles of identically prepared systems]. Strictly speaking, this makes it necessary to introduce two different concepts of measurement, namely, (an act of) *measurement* and *ensemble measurement.* By *measurement* of a dynamical magnitude we mean a relevant experiment determining this magnitude on an individual member of the ensemble. *Ensemble measurement* of the same magnitude represents the set of all individual acts of measurement carried out on every member of the overall ensemble. (Statistical physical magnitudes, e.g. probabilities, can be determined only via ensemble measurement). Consequently, uncertainty relations of kind (I.1), for instance, being ensemble concepts, can be exhibited or verified only with the help of ensemble measurement while measurement may, in principle, be capable of giving an arbitrarily precise information about "complementary" dynamical quantities of an individual system. This fact was, essentially, pointed out in the earlier literature (cf. e.g. ref. 10). It must be mentioned too that the celebrated EPR argument envisions precisely individual measurements as giving indirect evidence about position-velocity coexistence in specific correlation experiments. (As promised above, the *direct* demonstration of the same will be one of the main concerns of the 'minimal' SI examined in this paper).

The 'minimal' SI, some basic features of which were outlined in this section, does not differ essentially from the one examined in ref. 10. In combination with the no-analyser requirement and the consideration of explicit time-dependent solutions of the one-dimensional Schrödinger equation, however, it makes certain additional insights possible. In order to arrive at them in the remaining sections and appendices of the present work we shall apply the following strategy.

III. THE CONCRETE ENSEMBLE APPROACH

We want to demonstrate the possibility for objective position-velocity coexistence in the case of individual members (here - microparticles) of the QM ensemble by examining the unperturbed (by analysers and so on) evolution with time of the wave function in the co-ordinate representation. The properly chosen QM ensemble must correspond to the given physical situation in a manner that should be obvious and unique. Objective (x, v)-coexistence at a given moment t will mean that at this t one can objectively attribute a well defined velocity $v = v_x(t) = v(x, t)$ to a given position x of an individual particle in the QM ensemble, the product of the individual *objective* uncertainties (determined by the specific physical conditions) being much smaller than the one in ineq. (I.3). As we recall, positions and velocities are classical concepts. (In particular, velocities are determined in practice in macroscopic experiments with the aid of a classical terminology). Our way of reasoning simply means then that we shall be trying in fact to check whether we may find a suitable classical ensemble of particles with the same inherent characteristics (mass, etc.) that evolves with time in the same physical conditions (fields) in such a way that -perhaps after the elapsing of a sufficiently large stretch of time $T \geq 0$- definite velocities can be attributed to given positions at any t of interest $(t \geq T)$ in this ensemble after a law that may unequivocally be ascribed to the QM case. This will be possible, for instance, in the following case. Assume that, for the sake of simplicity, we employ a classical ensemble in which positions define velocities and vice versa with an absolute precision (that is, after a one-one $x - v$ correspondence) at every $t \geq 0$, the concrete form of the said correspondence being determined with the aid of Newton's laws. (Such will be the case when, say, all members of the classical ensemble have the same initial position at $t = 0$. The more realistic case will also be considered when necessary). Obviously, there will exist a unique link between position distribution $r(x, t)$ and velocity distribution $R(v, t)$ of the particles in this ensemble, so that $R(v, t)$ will be a consequence of $r(x, t)$ (for special fields of course) and vice versa.

Assume now that $r(x, t)$ coincides at all $t \geq 0$ or in the limit $t \to \infty$ with the position distribution $|\psi(x, t)|^2$ in the QM ensemble. This would certainly mean that we would have at our disposal an unique algorithm for assigning definite velocities to given positions at the above-said moments t in the QM case too, namely, the mentioned Newtonian algorithm. Really, the (practical) coincidence of the evolution laws for $|\psi(x, t)|^2$ and $r(x, t)$ at *all* moments of interest [$R(v, t)$ being a direct consequence of $r(x, t)$] would permit us to explain this evolution in both cases with the same physical notions, that is, with the classical concepts of positions and velocities that are (practically) one-one functions of each other at $t \geq T$, the functional dependence being determined in the above-mentioned way. Consequently, in problems for which our assumption is valid the (practically or totally) coinciding QM and classical ensemble pictures will give (at all $t \geq 0$ or, at least, at sufficiently large $t \geq T$) evidence for the admissibility of objective position-velocity coexistence in QM too. And, in the end, the validity of the above assumption would make it possible to check in our problems the validity of the QM postulate on velocity distribution $R_{QM}(v,t)$, namely,

$$R_{QM}(v,t) = m|a_p(t)|^2, \tag{3.1}$$

where

$$a_p(t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x,t) \exp(-ipx/\hbar) dx \tag{3.2}$$

and m is the mass of the particles. Indeed, as we know, the coincidence of $r(x,t)$ with $|\psi(x,t)|^2$ at *all* t of interest would directly lead to a velocity distribution that is the same ($= R(v,t)$) in both cases, so from the viewpoint of logic the QM postulate (3.1,2) for the (macro)velocity density turns into an assertion that needs a purely mathematical check-up : It will be valid in the case of coincidence of $R_{QM}(v,t)$ with the primary ensemble magnitude $R(v,t)$.

We pass now to the equations and formulae that will be necessary for us.

In our case of classical ensembles with a one-one $x - v$ correspondence we have the following relation between r and R (valid for distributions which can be both normalizable or nonnormalizable to unity) :

$$r(x,t)dx = R(v,t)dv, \tag{3.3}$$

where either x is a free variable and v is a definite function $v(x,t)$ of x and t or the converse [v is independent and $x = x(v,t)$], both viewpoints being equivalent. Consequently, the relevant phase space classical distribution $\rho(x,v,t)$ can be represented in any one of the equivalent forms

$$\rho(x,v,t) = r(x,t)\delta(x - x(v,t))|\frac{\partial x(v,t)}{\partial v}| \tag{3.4}$$

$$\rho(x,v,t) = R(v,t)\delta(v - v(x,t))|\frac{\partial v(x,t)}{\partial x}| \tag{3.5}$$

Indeed, integration of eq. (3.4) over v and eq. (3.5) over x gives, correspondingly,

$$r(x,t) = \int_{-\infty}^{\infty} \rho(x,v,t)dv \tag{3.6}$$

and

$$R(v,t) = \int_{-\infty}^{\infty} \rho(x,v,t)dx \tag{3.7}$$

[This fact is a consequence of the well known general formula $\delta[f(y)] = \delta(y-y_0)/|df/dy|$ for Dirac's δ-function, where $f(y_0) = 0$ and, alternatively, $f = f_1(v) = x - x(v,t)$ or $f = f_2(x) = v - v(x,t)$, x being a constant parameter in f_1 and v being a constant parameter in f_2]. On the other hand, integration of (3.4) over x and of (3.5) over v yields

$$R(v,t) = r[x(v,t)]\Big|\frac{\partial x(v,t)}{\partial v}\Big| \tag{3.8}$$

and

$$r(x,t) = R[v(x,t)]\Big|\frac{\partial v(x,t)}{\partial x}\Big|, \tag{3.9}$$

which relations represent, in agreement with eq. (3.3), the exact mathematical expression of the fact that under our assumptions only one of the probability distributions r and R may be regarded as independent, the form of the other distribution following directly from the form of the 'independent' one via Newton's dynamics of individual particles in the specific physical conditions. (Certainly, it is just a matter of convenience which distribution will be treated as independent). The relevant initial conditions for the solution of Newton's problem will be chosen at $t = 0$. Correspondingly, the QM wave function will also be given at $t = 0$.

From the above consideration it follows that there exist different equivalent ways of checking whether the conduct of the QM ensemble in given conditions coincides (probably as $t \to \infty$) with that of a relevant classical ensemble with a one-one $x - v$ correspondence. One of them was already described here. Namely, find $r(x,t)$ in the classical ensemble and $|\psi(x,t)|^2$ in the QM ensemble, compare them and assign, in the case of coincidence, the classical $R(v,t)$ [eq.(3.8)] to the QM ensemble. (We shall compare in fact r and $|\psi|^2$ at suitable time-variable positions). Another way consists in determining $|\psi(x,t)|^2$, replacing x with the relevant Newtonian $x(v,t)$

and multiplying the expression so obtained by $|\partial x(v,t)/\partial v|$. If this would give (probably as a limit) a picture of a velocity distribution that can be attributed to a classical ensemble with a one-one $x-v$ correspondence in the given physical conditions, then we would automatically have a classical ensemble whose (x,v)-evolution is equivalent to the QM one. We shall make use of both these ways of action below.

Let us adduce, in the end, several formulae that will be employed in the next sections. The QM states $\psi(x,t)$, $t>0$, participating in expression (3.2), can be obtained via formula

$$\psi(x,t)=\int_{-\infty}^{\infty}K(x,t;x',0)\psi(x',0)dx', \tag{3.10}$$

where $K(x,t;x',0)$ is the propagator corresponding to the given physical conditions. We shall examine cases with time-independent basic potentials $U(x,t)=U(x)$ at $t\geq 0$, for which the explicit expressions for K are known. Namely, for $U(x)=0$ we have

$$K(x,t;x',0)=\Big(\frac{m}{2i\pi\hbar t}\Big)^{1/2}\exp[im(x-x')^2/2\hbar t] \tag{3.11}$$

The expression for K in the case $U(x)=-Fx$, $F=$const , is

$$K(x,t;x',0)=\Big(\frac{m}{2\pi i\hbar t}\Big)^{1/2}\exp\left\{\frac{i}{\hbar}\left[\frac{m(x-x')^2}{2t}+\frac{Ft}{2}(x+x')-\frac{F^2t^3}{24m}\right]\right\} \tag{3.12}$$

and for the case of harmonic motion $U(x)=m\omega^2x^2/2$ we have

$$K(x,t;x',0)=\Big(\frac{m\omega}{2\pi i\hbar\sin\omega t}\Big)^{1/2}\exp\left\{\frac{im\omega}{2\hbar\sin\omega t}[(x^2+x'^2)\cos\omega t-2xx']\right\}. \tag{3.13}$$

Références

[1] L. de Broglie, Found. Phys. **1** (1970) 5.

[2] L. de Broglie, G. Lochak, J.A. Beswick, and J. Vassalo-Pereira, Found. Phys. **6** (1976) 3.

[3] G. Lochak, De Broglie's Initial Conception of De Broglie Waves in "The Wave-Particle Dualism" (S. DIner et al, Eds.) D. Reidel Publ. Co. 1984.

[4] D. Bohm, Phys. Rev. **85** (1952) 166, 180.

[5] D. Bohm and B.J. Hiley, Found. Phys. **5** (1975) 93 ; ibid. **12** (1982) 1001 ; Phys. Rev. Lett. **55** (1986) 2511.
[6] D. Bohm and J.P. Vigier, Phys. Rev. **96** (1954) 208.
[7] D.I. Blokhintsev, *Foundational Problems of Quantum Mechanics*, Nauka, Moscow, 1966 ; *Quantum Mechanics*, Atomizdat, 1981.
[8] E. Nelson, Phys. Rev. **150** (1966) 1079.
[9] L. de la Peña-Auerbach and A.M. Cetto, J. Math. Phys. **18** (1977) 1612.
[10] L.E. Ballentine, Rev. Mod. Phys. **42** (1970) 358.
[11] G.C. Ghirardi, C. Omero, A. Rimini, and T. Weber, Rivista del Nuovo Cimento **1** (1978) 1.
[12] A. Einstein in *Scientific Papers Presented to Max Born*, Oliver & Boyd, Edinburgh, page 33, 1953.
[13] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Princeton University Press, Princeton, 1955.
[14] E.P. Wigner, Am. J. Phys. **31** (1963) 6.
[15] N.S. Todorov, Ann. Fond. L. de Broglie **10** (1985) 207 ; ibid., page 273.
[16] I. Pitowsky, Phys. Rev. Lett. **48** (1982) 1299 ; ibid. **49** (1982) 1216 ; Phys. Rev. **D27** (1983) 2316.
[17] N.S. Todorov, Ann. Fond. L. de Broglie **10** (1985) 49.
[18] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**.
[19] R. Balescu, *Equilibrium and Nonequilibrium Statistical Mechanics*, Chap. 2, John Wiley & Sons, New York-London-Sydney-Toronto, 1975.

(Manuscrit reçu le 20 juin 1987, révisé le 10 septembre 1987)