Examples of explicit position-velocity coexistence and their physical implications in a 'minimal' stochastic interpretation of quantum mechanics - Part III Generalizations and comparisons

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ABSTRACT. Comparison of our results in Part II with those of certain earlier considerations is carried out. Our results give evidence for the validity of the locality viewpoint in physics and the additional considerations carried out in this Part lead to the same conclusion. The said viewpoint was extensively criticised in the past and a number of arguments (invariably tautological and restrictive) known as no-go-"theorems" and aiming at its invalidation were contrived. These arguments disregard possibilities as the one discussed in detail in Appendix B. Namely, it is shown there that the strange 'phantom' in microphysics may turn out to be particle statistics and not the particle itself.

VI. Generalizations and conclusions

The preceding sections of this paper contain a number of inferences within the frame of the 'minimal' SI. We shall not reiterate them here but shall rather point out a major conclusion following from our considerations. Namely, the CI does not offer an unquestionable outlook on both the nature of the uncertainty relations of the kind (1.1) and the process of measurement: Our discussion of a number of exactly solvable QM problems provides counter-examples showing that the said relations are, generally, ensemble concepts while measurement is, generally, a noninstantaneous process of a natural evolution (which just begins at a given moment t) of the QM states of motion in suitably chosen physical conditions. As a consequence of this neither von Neumann's projection postulate (stipulating instantaneous state-reduction in measurement at moment t) nor the idea for the existence of a specific QM inseparability (based on the concept of QM nonlocality) of our physical world can be attributed universal validity. Indeed, these concepts are not indispensable for obtaining the well known QM probability distributions: In our ensemble picture *exactly the same* probabilities (in the problems examined) arose from a de facto negation –resulting from quantum dynamics itself– of the mentioned concepts. (The direct corroboration of the possibility for coexistence of seemingly incompatible local magnitudes as positions and velocities of microparticles is already an indication that nonlocality cannot be a universally valid notion; cf. also our subsequent discussion).

These conclusions, however, give no grounds to regard the 'minimal' SI as an attempt at restoring a purely Newtonian outlook on the nature of the physical world. As it was already mentioned, this interpretation is completely open for the introduction of new physical mechanisms that may possibly explain typical QM phenomena as interference, diffraction, tunnelling, etc. (In such a case the SI would certainly no longer be just 'minimal'; de Broglie's theory sketched in the Introduction and Bohm's outlook discussed in Appendix A represent attempts at 'nonminimal' interpretations aiming at the explanation of such phenomena). But the 'minimal' SI does show, in consonance with Nelson's opinion [8], that there were no reasons for so radical a departure from the pre-quantum physical notions as the one present in the CI.

A result in Sec. IV implies a possible outlook on measurement and related probabilities which may turn out to be more physical than the one offered by the CI. Namely, we saw that macrovelocity cannot be defined both theoretically and –as a consequence– experimentally for too short time-intervals $(0, t)$, $t > 0$. This fact may be generalized as follows: The macroscopic physical magnitudes that may be attributed to microparticles do not exist generally *in themselves* but should be discussed from the viewpoint of the proper objective physical conditions ('measurement devices') under which these magnitudes may be exhibited in relevant q-ensembles. At a first sight this seems to be no more than a simple reiteration of the CI statement of the quantum wholeness of the Universe (of which the presumable nonseparability of the system measured object-measuring apparatus is a private case). In the 'minimal' SI, however, the said assertion is based on a different physical idea. Really, in an ensemble picture of particle motion the macroscopic magnitudes of interest (say, macrovelocities) may not be theoretically definable even in a purely classical context (for instance, in a realistic c-ensemble with an initial spread of positions $\sim d_0$ and too small $t > 0$). Therefore, there are no reasons to assert that our general statement is necessarily underlied by nonlocality. As it should be clear from the entire line of reasoning in this paper, the said statement means that in a *statistical ensemble picture* of particle dynamics (which picture must be adopted in a stochastic theory) one is forced to employ in the general case (even in c-ensembles) magnitudes which acquire a definite physical sense only in the process of evolution of the ensemble state in certain definite physical conditions. These should be regarded as an inseparable part of the appropriate measuring device (even when its 'analysing' part consists of just empty space). The probability distributions observed in QM refer to magnitudes of such a kind.

In this way we arrive once again, within the frame of the 'minimal' SI, to de Broglie's idea that "predicted probabilities" (observable in actual experiments) acquire a definite physical sense only taking into consideration the specific physics of the process of measurement. In this sense the said probabilities may be regarded as a result of measurement itself and may have little in common with the eventual "hidden probabilities" for "hidden parameters" in a possible more detailed theory of microphenomena. (It may even turn out that certain events in the space of hidden local variables may not have definite probabilities –cf. Appendix B).

The above discussion leads to the idea that the attaching of a general locality postulate to the 'minimal' SI would not be an unwarranted step. This postulate is understood here exactly in the EPR sense. According to the said conception, a physical system S' which is separated by a sufficiently large distance from another physical system S'' cannot in any way or sense exert a noticeable influence on the state of motion of the latter system and vice versa. (The possibility of position-velocity coexistence for individual particles, as demonstrated first by EPR, may be regarded as a consequence of such a standpoint. In general, it would be hopeless to achieve an objective local description of individual systems, say a given particle, if its state of motion "here" might somehow be influenced by that of another particle in a far-off galaxy).

The locality postulate is so alien to the CI conceptions and seems to be so easy to refute with the aid of simple arguments (as, say, the one of Bell [23]) that its natural character certainly deserves special consideration. In the simple argument that we are going to adduce here locality will turn out to be a consequence of the logic of the 'minimal' SI.

Assume that a physical system S' is enclosed in an impenetrable volume V' (so that its wave functions have zero values at the walls of V' and outside it) and, analogously, another system S'' is enclosed in an impenetrable volume V''. Let the distance between V' and V'' be sufficiently large to cancel any noticeable direct interactions between S' and S'' and let $\{\psi_n(q')\}$ $(n = 1, ..., \infty)$, $\{\varphi_k(q'')\}$ $(k = 1, ..., \infty)$ be arbitrary complete orthonormal sets of wave functions, q' and q'' denoting the sets of all 'co-ordinates' (including spin variables) of S' and S'', respectively. According to current notions, any function

$$\xi(q', q'') = \sum_{n,k=1}^{\infty} a_{nk}\ \psi_n(q')\varphi_k(q'') \tag{6.1}$$

represents a possible state of motion of the overall system $S' + S''$ (under the only requirement $\sum_{n,k=1}^{\infty} \mid a_{nk} \mid^2 = 1$ on the co-ordinate-independent parameters a_{nk}). This state can be recast in the form

$$\xi(q', q'') = \sum_{k=1}^{\infty} \sqrt{p_k}\ \phi_k(q')\varphi_k(q'') \tag{6.2}$$

where

$$\phi_k(q') = \frac{1}{\sqrt{p_k}} \sum_{n=1}^{\infty} a_{nk}\psi_n(q') \tag{6.3}$$

and

$$p_k = \sum_{n=1}^{\infty} \mid a_{nk} \mid^2 \leq 1 \tag{6.4}$$

(obviously, $\sum_{k=1}^{\infty} p_k = 1$). At a first sight, eq. (6.2) or (6.1) gives evidence for the existence of a strong mutual influence of the motions of S' and S'' in state $\xi(q', q'')$ as, say, the probability density $\mid \xi(q', q'') \mid^2$ cannot be represented in the form $\mid \xi \mid^2 = \mid \psi(q') \mid^2 \mid \phi(q'') \mid^2$, $\psi(q')$ and $\phi(q'')$ being arbitrary states of S' and S'', respectively. On its turn, this fact leads to the CI idea of a specific QM interconnectedness of subsystems (or quantum wholeness of the overall system), the intensity of which is independent of the distance between S' and S'' (equivalently, V' and V'').

[As a further step, one may arrive at the more concrete conjecture that the said interconnectedness of S' and S'' is due to the presence of specific quantum interaction forces whose independence of distance has no analogue in the ordinary forces (gravitational, electromagnetic) of the classical world; this is the case, for instance, with Bohm's ideology examined in Appendix A]. The 'minimal' SI, however, gives a different explanation of phenomena of this sort.

It is well known [13],[20] that wave functions of the kind (6.1-2) yield density operators (density matrices) for the states of motion of individual subsystems S' and S''. For instance, the state of motion of S' represents a mixture of normalized states $\phi_k(q')$ (not necessarily mutually orthogonal) in which each state ϕ_k participates with a weight (probability) $p_k \leq 1$ ($\sum_{k=1}^{\infty} p_k = 1$). We have to point out immediately that for an observer O' who can carry out experiments just on S' this mixture is incoherent, so that the statistical ensemble E' which O' can assign to S' will consist of subensembles E'_k corresponding to the 'disjoint' states $\phi_k(q')$ $k = 1, ..., \infty$, and taken with weights p_k in the overall symbolic union $E' = \bigcup_{k=1}^{\infty} p_k E'_k$. Really, according to current axiomatics, the average value of any physical magnitude (operator) $\hat{R}'$ of subsystem S' will be given by

$$< \hat{R}' >=< \xi(q', q'') \mid \hat{R}' \mid \xi(q', q'') >= \sum_{k=1}^{\infty} p_k < \phi_k(q') \mid \hat{R}' \mid \phi_k(q') > \tag{6.5}$$

having in mind that $\hat{R}'$ acts only on functions of variable q' and that $< \varphi_m \mid \varphi_l >= \delta_{ml}$ due to the requirement for orthonormality of $\{\varphi_k\}$, $k = 1, ..., \infty$. Besides, the density distribution $\rho(q')$ of variable q' will be equal to

$$\rho(q') = \sum_{k=1}^{\infty} p_k \mid \phi_k(q') \mid^2 \tag{6.6}$$

In such a way the ensemble state of motion of the individual subsystem S' represents indeed just a mixture of states $\phi_k(q')$ of the mentioned kind. (A totally analogous consideration will evince a similar fact for S''). One would be able to say that S'' may exert a specific quantum long-range influence on S' only if a certain perturbation of the (ensemble) state of motion of S'' would somehow affect the state of S' (once again in the absence of direct potential-dependent interactions between S' and the rest of the

Universe). Assume then that, at moment t_0, S'' begins to interact with a third system s whose co-ordinates are denoted by q and which cannot interact directly with S', that is, the interaction energy between s and S' is (practically) equal to zero in the QM evolution equation. Let the state of s at moment t_0 be $\eta(q)$, so that the state of the overall system $S' + S'' + s$ be given by

$$\xi(q', q'', q)_{t_0} = \sum_{k=1}^{\infty} \sqrt{p_k}\phi_k(q')\varphi_k(q'')\eta(q) \tag{6.7}$$

at that moment. Every initial product $\varphi_k(q'')\eta(q)$ will be transformed into a function $f_k(q'', q)$ at $t > t_0$ as a result of the direct interaction between S'' and s and the unitarity of the evolution operator of $S'' + s$ will preserve orthonormality at any $t > t_0$, so that

$$< f_1(q'', q) \mid f_m(q'', q) >= \delta_{lm} \tag{6.8}$$

at an arbitrary $t > t_0$. Consequently, the overall state

$$\xi(q', q'', q)_{t>t_0} = \sum_{k=1}^{\infty} \sqrt{p_k}\phi_k(q') f_k(q'', q) \tag{6.9}$$

will give once again exactly the same density operator for S' as the one in the absence of s. That is, the ensemble state of motion of S' is not affected at all by the presence of s and by the (arbitrary) modifications of the state of S''. (Only the correlations between S' and S'' will be modified by the perturbing effect of s on S'').

A reasonable statistical interpretation gives thus no grounds to assert the existence of specific long (and even infinite)-range quantum interactions between practically isolated subsystems of an overall system. This result, achieved in a natural setting, makes it necessary to find a different explanation for the correlations in the motion of S' and S'', without resorting to nonlocality arguments. But the explanation may be found in fact in the very sense of the word "correlation". As it was pointed out earlier by us [24], long (or even infinite)-range correlations in the motion of isolated systems may exist in a stochastic variant of classical mechanics too. [For instance, one can predict exactly the position of a classical oscillator S' by registering the position of an oscillator S'' located arbitrarily far from S' if it is known, say, that the inherent parameters and the amplitudes of S' and S''

are equal and the phase difference of their oscillations is a fixed number for all pairs (S', S'') in the ensemble]. The existence of correlations means that the motion in the ensemble of pairs (S', S'') is not statistically independent (probably due to past interactions, conservation laws, etc.), without any influence "now" of any subsystem on its partner in the ensemble.

The fact that the mentioned correlations can influence the future evolution of the system $S' + S''$ if S' and S'' are brought so close to each other that their direct interaction can no logner be neglected is also a property that is not typical for QM only: The same would apply to the evolution of the states of motion of c-ensembles too.

The above consideration shows that it would be just a matter of time to find counterarguments to any concrete argument (however intricate) aiming at presenting a proof of the quantum-nonseparability idea in *present-day* theory. A possibility (representing a counterargument to Bell's nonlocality argument) that was recently proposed in the literature is examined in detail in Appendix B. (In ref. 24 the reader may find additional locality arguments asserting the possibility for restricted validity of the time-dependent Schrödinger equation in the case of potentials swiftly varying with time. This idea may be employed in a different approach to the problem of entropy increase with time [25] compared to well known ones).

To summarize, we encounter two kinds of nonlocality effects in QM : (i) wave-like properties of individual particles, and (ii) long-range correlations in the motion of spatially separated subsystems. 'Nonminimal' models as de Broglie's [1-3] or Bohm's [4,5] show that the effects of case (i) are compatible with local concepts as position-velocity coexistence for individual microparticles. [We have to point out here, however, that there exists in principle another –practically unexplored– kind of a possible 'nonminimal' theory for case (i), namely, a theory taking into consideration the logical admissibility of pseudo-wave properties arising due to a possible specific complex interaction of a pointlike particle (which may additionally be endowed with an inherent structure of its own) with an extended body, say a crystal, that may either respond *as a whole* to the perturbation introduced by the incident particle or generate resonant inner motions in the particle, or both]. As for case (ii), the 'minimal' SI is sufficient, as we saw, to demonstrate that the introduction of concepts as specific quantum long (infinite)-range interactions entailing, in particular, infinitely-fast state reductions in peculiar perturbations called measurements is unnecessary (at least for the time being, in the absence of fundamental reasons that might *force* us to resort to such radical conceptions).

The 'minimal' SI thus makes physics more understandable indeed. In addition to all previous considerations, we shall examine one more typical experimental situation illustrating this. Examine a set of excited atoms emitting electromagnetic radiation. A spectrometer, located arbitrarily far from the volume containing the set of atoms, will register some of the emitted photons. According to the CI idea of nonseparability of the system measured object-measuring apparatus (in which conception measurement always plays an active role in one sense or the other) one will have to regard the process of emission of light by the atoms –at least for the registered events– as intimately and inseparably linked with the fact that a spectrometer is present in some part of the Universe. According to the 'minimal' SI, however, the spectrometer and the set of emitting atoms represent two de facto isolated subsystems S' and S'' of the system $S = S' + S''$. Consequently, the latter interpretation will insist on an objective description of the process of light emission on the basis of inherent physical properties of the individual atom, the spectrometer itself being of no interest to this part of the theory. Explanations of such a kind may turn out to be incompatible with the idea of unrestricted validity of certain QM evolution equations [24]. But we do adhere to the opinion that a really good interpretation of a theory is one which, in addition to the numerical explanation of a set of experimental facts, gives evidence about the possibility for modifications of the theory since practice shows that theories are always subject to modifications. Consequently, chances exist that the 'minimal' SI may turn out to be, in the above sense, a good interpretation of quantum theory.

APPENDIX A

COMPARISONS WITH EARLIER CONSIDERATIONS

(1) Schrödinger's example

Interesting enough, Schrödinger himself discovered first [26] certain classical features in the motion of wave packets that evolve from a very special case of our initial wave function (4.6) in the harmonic potential field. (There exists a recent discussion of this work [27]; cf. also ref. 22 and §23, Problem 3, in ref. 20). Namely, for $\psi(x,0) \sim exp[-(\sqrt{m\omega/\hbar}x - A)^2/Z]$, $A = \text{const}$, one obtains position densities that have a time-invariable form around point $x(t)$ obeying the classical law of motion $x(t) = x_0 \cos\omega t$ (valid for $v_0 = 0$), where $x_0 = \sqrt{\hbar/m\omega}A$. One may easily be tempted here to conclude that QM wave packets are capable of giving a detailed account of the motion of individual particles (even imitations of classical type motions – but with a position spread $\sim \sqrt{\hbar/m\omega}$ of the individual particle). However, the application of such a viewpoint to the more general initial condition (4.6) would immediately lead to an incorrect inference. Indeed, as shown in ref. 27 (for purposes quite different from ours) for initial packets of the form $\psi(x,0) = (\alpha/\pi)^{1/4}\ exp[-\alpha(\sqrt{m\omega/\hbar}x - A)^2/2]$ [once again a special case of (4.6)] combined with the Newtonian requirement $x(t) = x_0 \cos\omega t$ one obtains position densities that undergo periodical distorsions about point $x(t)$, the deformations tending to infinity in the limit $\alpha \to \infty$ (which corresponds to our limit $\sigma \downarrow 0$). One would have to declare then that in this limit the behaviour of the individual particle is highly nonclassical. Such an inference, however, would be at complete variance with that following from the ensemble picture, where the behaviour of the relevant ensemble explicitly tends to be totally classical as $\sigma \downarrow 0$. As the primary concepts in statistical theories are those of adequate statistical ensembles, one must abandon the single-particle interpretation and accept the ensemble picture, the latter being the antipode of the former for the case in question in treating the alternative: classical/nonclassical (motion).

(2) Comparison with Bohm's theory

Bohm's ideology [4,5] has been of a considerable conceptual importance as it clearly demonstrated that even typically quantal effects (say, tunnelling or the behaviour of systems of identical particles) may be 'classically understandable' in the terms of a Hamilton-Jacobi-type theory. Briefly, this theory is the following.

Examine an N-body QM system and write its wave function in the form

$$\psi(\vec{x}_1, ..., \vec{x}_N, t) = R(\vec{x}_1, ..., \vec{x}_N, t) exp[iS(\vec{x}_1, ..., \vec{x}_N, t)/\hbar] \tag{A.1}$$

where R and S are real-valued functions, S playing the role of an action function. Defining, respectively, the classical-type momentum of the n-th particle as

$$\vec{p}_n = \nabla_n S \tag{A.2}$$

one arrives at the continuity equation

$$\frac{\partial P}{\partial t} + \sum_{n=1}^{N} \nabla_n \frac{(P\nabla_n S)}{m} = 0 \tag{A.3}$$

($P = R^2$ being the probability density in configuration space) and a Hamilton-Jacobi-type equation

$$\frac{\partial S}{\partial t} + \sum_{n=1}^{N} \frac{(\nabla_n S)^2}{2m} + U(\vec{x}_1, ..., \vec{x}_N) + Q(\vec{x}_1, ..., \vec{x}_N, t) = 0, \tag{A.4}$$

where U is the usual classical interaction potential of the N particles while

$$Q = -\frac{\hbar^2}{2m} \sum_{n=1}^{N} \frac{(\nabla_n)^2 R}{R} \tag{A.5}$$

is an additional potential, absent in the classical case. (For that reason it is called a *quantum potential*).

In such a way Bohm's theory is seen to represent an ensemble theory [cf., in particular, eq. (A.3)] for an arbitrary N-body system, the n-th member of the system ($n = 1, ..., N$) having a velocity $\vec{v}_n = \nabla_n S/m$ [cf. (A.2)] at each moment t. The difference from a purely Newtonian ensemble picture is that an additional *quantum force*

$$\vec{F}_{Qn} = -\nabla_n Q \tag{A.6}$$

acts on the n-th particle. This force has the unusual property to act intensely at arbitrarily large distances (see also the subsequent consideration). One thus comes, in this model, to the idea that there exists a specific quantum wholeness of the world, the motion of each particle in the Universe being strongly influenced by the behaviour of each other particle irrespective of distances. (More details of this view-point may be found in the cited references).

Eqs. (A.1-6) and their interpretation show that the wave function ψ plays a specific dual role in Bohm's theory. On the one hand, $\mid \psi \mid^2$ gives the position distribution function in the statistical ensemble representing the QM state of motion ψ. On the other hand, ψ is envisioned as a real physical field generating actual forces described by the above formulae.

The results in the previous sections of this paper can be obtained in the terms of Bohm's theory too. For instance, in the case of certain nonormalizable states of motion we obtained total identity of the behaviour of appropriate q- and c-ensembles. Bohm's theory contains a ready explanation of this fact: The quantum potentials in these problems are easily seen to be exactly equal to zero at all $t > 0$ [with the exception of an inessential denumerable set of isolated moments of time (which are irregular for the classical picture too –cf. Sec. V) in the harmonic oscillator case]. We shall examine in some more detail the results following from the form of the Q-potential for the normalized state $\psi(x,t)$ which evolves from an initial wave function (4.7) in the absence of external fields (free motion). The exact expression for this $\psi(x,t)$ is given by (4.8) and its representation in the form (A.1) leads to a "quantum force"

$$F_Q(x,t) = -\frac{\partial Q(x,t)}{\partial x} = \frac{m^3\sigma^4 x}{\hbar^2 t^4 \left(1+\frac{m^2\sigma^4}{\hbar^2 t^2}\right)^2} \tag{A.7}$$

At a fixed point x we have a 'normal' behaviour of $F_Q(x,t)$ with the course of time: (A.7) shows that $F_Q \to 0$ at $t \to \infty$ for an arbitrary given x. But the spatial behaviour of F_Q for an arbitrary fixed moment t is totally unusual: $\mid F_Q(x,t) \mid$ increases linearly with the increase of $\mid x \mid$ and tends to infinity for $\mid x \mid \to \infty$. In other words, F_Q becomes (practically) infinite in the region of space where $\psi(x,t)$ is (practically) equal to zero.

Let us solve, in this picture, the equation of motion for an individual particle with an initial position x_0 in the q-ensemble. From (A.2) it follows that, in our case, all particles in the ensemble have the same initial velocity $v_0 = 0$. The positions and velocities of a given particle will therefore be functions $x(x_0,t)$ and $v(x_0,t)$ of x_0 and t. Employing once again (A.2), we obtain

$$v(x_0,t) = \partial x(x_0,t)/\partial t = \frac{x(x_0,t)}{t}\,\frac{1}{1+\frac{m^2\sigma^4}{\hbar^2 t^2}} \tag{A.8}$$

[From (A.8) it follows in a straightforward manner that for the particle in question expression $m\partial v(x_0,t)/\partial t$ is equal to

$$m\partial v(x_0,t)/\partial t = \frac{m^3\sigma^4 x(x_0,t)}{\hbar^2 t^4\left(1+\frac{m^2\sigma^4}{\hbar^2 t^2}\right)^2} \tag{A.9}$$

in agreement with (A.7)].

The integration of the second eq. (A.8) yields

$$x(x_0,t) = x_0\left(1+\frac{\hbar^2 t^2}{m^2\sigma^4}\right)^{1/2} \tag{A.10}$$

so that

$$v(x_0,\infty) = x_0\hbar/m\sigma^2 \tag{A.11}$$

Having in mind the initial zero-velocity condition [satisfied by eq. (A.10)] for all particles, we see that expression (A.11) agrees with the unusual behaviour of $\mid F_Q(x,t)\mid$ at large x. Namely, the larger forces acting on particles with larger $\mid x_0\mid$ will lead, in the limit $t\to\infty$, to larger final velocities $\mid v(x_0,\infty)\mid$ of these particles when $v_0=0$. On the other hand, the finite value (A.11) of $v(x_0,\infty)$ for any given x_0 is a consequence of the specific form (A.7) of $F_Q(x,t)$ which leads to the result that the quantum force acting on any *given* particle in the ensemble tends to zero as $t\to\infty$ after a law preventing infinite final v's. And, in the end, the one-one correspondence between v_∞ and x_0, following from (A.11), entails

$$\mid\psi(x_0)\mid^2\ dx_0 = R(v_\infty)dv_\infty \tag{A.12}$$

where $R(v_\infty)$ is the density distribution of the final velocities and $dv_\infty = (\hbar/m\sigma^2)dx_0$. The initial state $\psi(x_0)$ is given by eq. (4.7) in which x must be replaced with x_0 in agreement with our present notations. Replacing further x_0 with $m\sigma^2 v_\infty/\hbar$ [cf. (A.11)] and taking into consideration (A.12), we arrive at

$$R(v_\infty) = \frac{m\sigma}{\sqrt{\pi}\hbar}\ exp(-m^2\sigma^2 v_\infty^2/\hbar^2) \tag{A.13}$$

which is exactly the result (4.10), in which v is replaced by v_∞.

In such a way, both the 'minimal' *SI* and Bohm's theory lead to the same final ensemble pictures in the case just examined too. We nevertheless prefer the SI picture due to the patnological behaviour of F_Q at large x that contradicts physical intuition. This behaviour leads to an additional difficulty as well. Namely, assume that our particle is enclosed in a 'wave guide' with impenetrable walls and of a constant cross section in the (y,z)-plane, the motion along the x-axis being once again free for all $x \in (-\infty,\infty)$. For three-dimensional states of the form $\chi(\vec{r},t) = \varphi(y,z,t)\psi(x,t)$ with the above $\psi(x,t)$ [eq. (4.8)] we shall have a nonzero quantum-force field in the entire wave guide [that is, for every $x \in (-\infty,\infty)$], the magnitude of the force tending to infinity with the increase of $\mid x \mid$. From the viewpoint of our usual concepts we may infer that the total energy associated with such a force field in the wave guide must be infinite. That is, the unrestricted application of Bohm's model may turn out to lead to divergencies for individual particles similar to those in quantum field theory. Consequently, a more developed theory of this kind should probably contain certain "cut-offs" (or other restrictions based on adequate physical principles) that would remove the difficulty by making the theory more local. The consideration of well-separated subsystems S' and S'' of an overall system $S = S' + S''$ in Sec. VI also agrees with the locality viewpoint. Indeed, we saw there that in a reasonable interpretation there is no direct influence *of any kind* of any subsystem on the state of motion of the other one. On the other hand, in the quantum-potential model such an influence exists as the relevant potentials (and forces) are essentially nonzero irrespective of distances. [In this model the said influence is always envisioned as an *active factor*, even in subtle interpretations as that in the last ref. 5, where the authors state that the multidimensional wave function cannot be treated "as a field producing a force or pressure that would transfer energy to the particles". They suggest, instead, "that the particles move under their own energies but that the form of this motion is *fundamentally affected* (my italics) by a multidimensionally ordered kind of information that is represented by the wave function". But there should be no other criterion for fundamental active influence than the ability of relevant physical factor to introduce changes in the states of motion of the (sub)systems in question (even if this factor plays a role similar to that of a classical magnetic field which may modify the state of motion of a particle without changing its energy). From this viewpoint the 'minimal' SI gives no evidence in favour of the active interpretation].

To summarize, the above discussion leads the present author to the following conclusion. If one would try to explain the quantum behaviour of microparticles exactly in Hamilton-Jacobi terms, then one would certainly have to adhere to the concept of quantum potential, etc. (probably, with appropriate “cut-offs” that may result, say, from possible modifications of the basic QM evolution equation). But there may also exist entirely local models formulated in suitable different local terms as implied by the ‘minimal’ SI, in which models the introduction of active factors for the explanation of the correlations in the motion of (practically) isolated systems would be unnecessary. [A well known analogous alternative: Ancient physics admitted the necessity of forces for the maintaining of inertial motion. Galileo’s postulate, however, preserves inertia exactly in the absence of any forces].

It is interesting to point out, in the end, that Bohm’s model, giving one-one $x-v$ correspondence at any $t>0$ in the case examined, possesses the same feature as the one discussed in Sec. IV. Namely, the “quantum force” [cf. (A.7)] would act for a longer time for smaller values of $\hbar$ (for a thought transition $\hbar \downarrow 0$), the relevant variable factor ensuring a correct transition to the classical picture in the zero limit being once again σ.

(3) Comparison with Blokhintsev’s ensemble approach

Blokhintsev’s approach (cf. ref. 7 and the references therein) represents an attempt at introducing statistical ensembles as primary concepts of QM and developing on this basis a theory of quantum measurements. In this theory the measured system is treated as a small open physical system interacting with a large system (the measuring apparatus), the latter being in an unstable macroscopic state of motion that makes it possible to enhance to a macroscopic scale the effect of its interaction with the small system. Once again, analysers are assumed to play an *active* role in measurement and this was, possibly, one of the reasons that prevented the said author from detaching his theory from the CI viewpoint on incompatible measurements (in spite of the fact that one may encounter in ref. 7 the statement that Heisenberg’s uncertainty relations refer to statistical dispersions in the ensemble of interest). As a consequence of this the coexistence of physical magnitudes connected via ineq. (1.1) is considered impossible. An additional argument in favour of this in the position-velocity case is the impossibility of introducing a positive-valued density distribution in phase space.

We shall illustrate the essence of the latter viewpoint with the simple case of one-dimensional free motion examined in Sec. IV. As we know, the state of motion of the particle will be described, generally, by a density matrix [written as $\rho(x, x', t)$ in the co-ordinate representation]. In the case of a normalizable pure state of motion $\psi(x, t)$ (that will be examined here for the sake of simplicity) the density operator reduces to

$$\rho(x, x', t) = \psi(x, t)\psi^*(x', t) \tag{A.14}$$

In order to arrive at a suitable expression for ρ in phase space (x, p), Blokhintsev employs, essentially, Kirkwood's variant [28] of the Wigner consideration [29]. Kirkwood's $\rho(p, x, t) = P(p, x, t)$ is given by

$$P(p, x, t) = \frac{e^{-ipx/\hbar}}{2\pi\hbar}\ \psi(x, t) \int_{-\infty}^{\infty} \psi^*(x', t) e^{ipx'/\hbar} dx' \tag{A.15}$$

As demonstrated by Kirkwood [28] and Blokhintsev [7], the average values of physical magnitudes that are functions of p or x can be obtained with the aid of $P(p, x, t)$ by employing an algorithm that has the same appearance as the classical one for such cases. But $P(p, x, t)$ is an essentially nonclassical magnitude as it satisfies an equation that differs from the Liouville equation for the classical phase density $\rho(p, x, t)$. In our specific case the equation for P is

$$\frac{\partial P(p, x, t)}{\partial t} + \frac{p}{m}\frac{\partial P(p, x, t)}{\partial x} = \frac{i\hbar}{2m}\frac{\partial^2 P(p, x, t)}{\partial x^2}, \tag{A.16}$$

whereas the relevant Liouville equation will be

$$\frac{\partial \rho(p, x, t)}{\partial t} + \frac{p}{m}\frac{\partial \rho(p, x, t)}{\partial x} = 0 \tag{A.17}$$

Obviously, the nonzero complex term in the right-hand side of (A.16) does not permit the definition of a real (nonnegative)-valued $P(p, x, t)$. Besides, the second term in the left-hand side of (A.16) will, generally, be of the same order of magnitude as $i\hbar(\partial^2 P/\partial x^2)/2m$ at $t \geq 0$, so that one cannot discard the latter term. Consequently, one may be led to the inference that the motion of a microparticle is strongly non-classical at $t \geq 0$ even in the absence of force fields. In a more general setting, this is the inference made by Blokhintsev himself.

From the discussion in Sec. IV we know that a general inference of the above kind is invalid and one can immediately point out why: Eq. (A.15) can be rewritten in the form

$$P(p,x,t) = \frac{e^{-ipx/\hbar}}{\sqrt{2\pi\hbar}}\ \psi(x,t)a_p^*(t) \tag{A.18}$$

[cf. eq. (3.2)], in which the phase variables x and p are totally 'disjoint'. [That is, this P gives no link between positions and velocities at a given t]. The consideration in Sec. IV showed, however, that in the appropriate q-ensemble there exists, practically, a one-one $x - v$ correspondence in the limit of large t.

Consequently, despite the fact that certain algorithms for average-value calculations have a (misleading) classical appearance with respect to the way in which $P(p,x,t)$ is made use of, the said magnitude is not suitable for determining whether, say, position-velocity coexistence in q-ensembles is admissible or not.

APPENDIX B

'Phantoms' of statistics

There exists a number of arguments in the literature, known as "no-go theorems", that seem to rule out, in particular, the validity of a locality viewpoint in microphysics. The most popular among these are probably the arguments of von Neumann [13], Kochen and Specker [30], and Bell [23]. Any argument must certainly rest on definite (explicit or inexplicit) assumptions and the numerous critiques against all arguments as those mentioned above show that the said assumptions are not treated by everybody as ubiquitous postulates capable of determining the general nature of future physical theories. (Paradixically, some of these critiques are based on CI arguments [31]). From a more general viewpoint, the common unacceptable feature of all "no-go theorems" is the seemingly natural inexplicit axiom that every physical theory aiming at the explanation of microphenomena must contain in an *unmodified form* certain basic mathematical features of present-day quantum theory. [For instance, von Neumann's argument requires preservation of all basic rules of operator calculus exactly in its present form, while the argument of Kochen and Specker requires preservation in an unmodified form of the QM functional dependences between "compatible" physical magnitudes (and, certainly, the present-day notion of compatibility of these magnitudes). History of science itself gives evidence against such a viewpoint. Really, a new theory containing the older one as a limiting case may have an altogether different mathematical apparatus. (Compare, say, the mathematical apparatuses of general relativity or quantum theory with that of Newtonian mechanics, the latter representing a limiting case –in different senses– of the former theories). Besides, a new theory may introduce new physical magnitudes and concepts and/or drastically modify the functional form of the connexion between certain magnitudes: compare, e.g., the expressions $p = mv$ and $p = mv/\sqrt{1 - v^2/c^2}$ connecting momenta with velocities in Newtonian mechanics and special relativity; recall, besides, that the 'minimal' SI gives no evidence for incompatibility, in the CI sense, of certain magnitudes]. Bell's argument is, in a sense, a special case of a "no-go theorem".

It has drawn fire from possibly the largest number of critical papers (the assessing of which would need a special review paper) but its discussion should be really careful as this argument seems to employ only basic axioms of ordinary probability theory, e.g. the conception that an "event" must always have a definite probability. Critique against this argument along the lines of de Broglie's view on probabilities may be found in papers by Lochak (cf. e.g. ref. 3; cf. also our papers [15] in which a detailed discussion of Lochak's argument is given from the viewpoint of our approach to the problem and where it is also pointed out that all the basic assumptions of Bell are restrictive). We shall examine here only the mentioned conception of "event" in order to show that even apparently innocent assumptions – which are in fact perceived as self-obvious truths and not as explicit or inexplicit assumptions– turn into unnecessary restrictions when one regards them as an imminent feature of *every* possible theory.

It was demonstrated by Pitowsky [16] and independently by the present author [17] that there may exist events in physics that cannot be assigned definite probabilities and on this basis the former writer contrived a nontrivial local theory capable of explaining the QM theoretical correlation results. The probability-free sets of local variables were obtained by Pitowsky via the application of nonconstructivistic axioms of abstract set theory, so that he proved existence theorems but did not adduce explicit constructions of the said sets (cf. also ref. 15). In order to demonstrate the unusual properties of probability-free sets we shall examine in sufficient detail an explicit example of such sets, briefly mentioned in ref. 17.

We find the following assumption (cf. also ref. 17) in Bell's argument: If local variables λ really exist then probabilities (for correlated spin-1/2 pairs of particles) as, say, $p(a'+;c''+)$ are representable in the form

$$\begin{aligned} p(a'+;c''+) = p(a'+,b'+,c'-;a''-,b''-,c''+) \\ + p(a'+,b'-,c'-;a''-,b''+,c''+) \end{aligned} \tag{B.1}$$

Here $p(a'+;c''+)$ is the probability measure of the set T of local states λ that will cause a "+" display of a measuring instrument M' oriented along direction a and a "+" display of a measuring instrument M'' oriented along direction c, whereas $p(a'+,b'+,c'-;a''-,b''-,c''+) = p(T_1)$ is the probability of the set T_1 of states λ that will trigger a "+" display of an a- or b-oriented M'' and a "+" display of a c-oriented M'' [analogously for $p(a'+,b'-,c'-;a''-,b''+,c''+) = p(T_2)$, corresponding to a λ-set T_2, $T_1 \cap T_2 = \emptyset$, $\emptyset$ being the empty set]; the requirement $\hat{n}'\pm = \hat{n}''\mp$, $\hat{n} = a, b, c$, follows from the specific spin-correlations considered.

According to the CI, the probabilities in the right-hand side of (B.1) do not exist in QM due to the impossibility of defining spin projections for individual particles along triples of directions, as implied by the conventional theory of measurement. Measurement was seen to play a different role in the 'minimal' SI but it may turn out that eq. (B.1) is impossible in local-variable theories too (contrary to Bell's assumption) for reasons that are not directly related to those discussed above. Namely, the relevant space of local variables λ may be such that probability measures for sets as T_1 and T_2 may just not exist. At a first sight the last statement sounds wildly. Really, if local variables λ describing every possible experimental outcome exist, then it would be sufficient to count correctly the elementary acts of appearing of every concrete λ, then take the number of λ's that belong to T_1 and divide it by the (sufficiently large) total number of events, and we automatically obtain $p(a'+,b'+,c'-;a''-,b''-,c''+)$ [analogously for the other probability in the right-hand side of (B.1)]. The striking fact, however, is that this simple prescription may be ineffective. More exactly, there may exist mutually exclusive, equally correct (or –which is the same– incorrect) ways of counting elementary events in the same probabilistic experiment and it is in these cases precisely that one encounters nonmeasurable (probability-free) sets of events. Let us illustrate this with the simplest possible example (mentioned in ref. 17).

Assume that set θ consists of the points belonging to the semi-interval $[0, \infty)$ along the x-axis and that the probabilistic experiment consists in the random choice of a point in θ, no point in θ being 'privileged' (i.e. all points being equally probable –a thing usually described by the introduction of a nonnormalizable probability density $\rho(x) = \text{const}$, $0 \leq x < \infty$). Assume also that θ (which obviously must be assigned probability one in our experiment) is subdivided into consecutive nonoverlapping semi-intervals of length uq^0, uq^1, uq^2, uq^3, ..., where u and q are arbitrary given positive numbers, the only restriction being $q > 1$. [More exactly, the first semi-interval (of length $uq^0 = u$) consists of points $x \in [0, u)$, the second semi-interval (of length $uq^1 = uq$) consists of points $x \in [u, uq)$, and so on]. Call all semi-intervals corresponding to even degrees of q "red" semi-intervals and those corresponding to uneven degrees of q –"blue" semi-intervals. (One can make the above random experiment more vivid by supplying the red and blue semi-intervals with red and blue lamps, so that the random choice of a point belonging to a red semi-interval be accompanied by a flash of the red lamp attached to this semi-interval and the choice of a "blue" point –by a blue flash). Let us find now the probability for a red (or blue) flash in our stochastic experiment. Theoretically, this probability can be defined as follows.

Examine an arbitrary semi-interval of the kind $[0, w), w > 0$, and compute the ratio

$$\{\text{total length of red-point sets belonging to}[0, w)\}/w \tag{B.2}$$

If (B.2) tends to a definite limit l, $0 \leq l \leq 1$, as $w \to \infty$, then l will be the probability for a red flash.

It is not difficult to see that, for a fixed integer $n > 0$, ratio (B.2) will have a largest value L_n when $[0, w)$ contains exactly the first n red semi-intervals and the first $n-1$ blue semi-intervals and a smallest value l_n when $[0, w)$ contains exactly the first n red semi-intervals and the first n blue semi-intervals. In the former case we shall have

$$w = w_1 = u(1 + q + q^2 + ... + q^{2n-2}), \tag{B.3}$$

while in the latter case

$$w = w_2 = u(1 + q + q^2 + ... + q^{2n-1}) \tag{B.4}$$

Employing the formula

$$\sum_{m=0}^{p} z^m = (1 - z^{p+1})/(1 - z) \tag{B.5}$$

(provable by trivial induction and valid for an arbitrary number z, p being a nonnegative integer), we obtain

$$L_n = u(1 + q^2 + q^4 + ... + q^{2n-2})/w_1 = \frac{1}{1+q} \frac{1 - q^{2n}}{1 - q^{2n-1}} \tag{B.6}$$

$$l_n = u(1 + q^2 + q^4 + ... + q^{2n-2})/w_2 = 1/(1 + q) \tag{B.7}$$

In the limit $n \to \infty$ (in which we have $w \to \infty$ as well) we obtain

$$\begin{aligned} l_n &= \text{const} = 1/(1 + q) \\ lim_{n\to\infty}\ L_n &= q/(1 + q) \end{aligned} \tag{B.8}$$

Consequently, ratio (B.2) varies in the entire interval $(1/1 + q, q/1 + q)$ as $w \to \infty$ and does not tend to any definite value l in this limit. This means that no definite probability for a red or blue flash exists in our problem. Denoting by θ_1 and θ_2 the union of all red and all blue semi-intervals, correspondingly, one can therefore assert that no definite probabilities can be assigned to the disjoint point sets θ_1 and θ_2 in spite of the fact that their union $\theta = \theta_1 \cup \theta_2$ has probability one in the stochastic experiment under consideration. [Note the identical character of the purely set-theoretical properties of θ, θ_1, and θ_2 (namely, $\theta = \theta_1 \cup \theta_2$, $\theta_1 \cap \theta_2 = \emptyset$) and those of the above-mentioned T, T_1, and T_2 which satisfy analogous relations. Note, besides, that $p(t)$ and $p(\theta)$ have definite values in both cases].

The essential difference between the cases of measurable and nonmeasurable red –(equivalently blue–) point sets, consisting in the presence or absence of a limit $p(\theta_1)$ (equivalently $p(\theta_2) = 1 - p(\theta_1)$) of the ratio (B.2) as $w \to \infty$, entails a surprising 'experimental' consequence. In order to arrive at it, examine first an arbitrary case in which $p(\theta_1)$ exists, all the other assumptions about the stochastic experiment being the same as before, and take a sufficiently large w' for which the ratio (B.2) will be practically equal to $p(\theta_1)$. [More precisely, we take a value of $w = w'$ for which $(B.2) = p(\theta_1) + \sigma'$, where σ' belongs to the arbitrarily small interval $(-\epsilon, \epsilon)$ and never leaves this interval with the further increase of w']. Let us subdivide the large 'block' $B' = [0, w')$ into smaller nonoverlapping 'blocks' B_i, $i = 1, 2, ..., n'$, $n' \gg 1$, in the following manner: B_1 contains all (and only) the points of the first red and the first blue semi-intervals of B', B_2 contains the points of the second red and second blue semi-intervals in B', and so on for all $1 \leq i \leq n'$. (We have chosen a value of w' for which B' will contain exactly n pairs of red and blue semi-intervals). The above assumption about w' obviously means that if the random point has hit B', then with probability practically equal to $p(\theta_1)$ we shall have a red flash. Now, we may certainly construct another large block $B'' = [w', w'')$, containing a large number of red/blue couples and having the analogous property that the relevant σ'' belongs to the same small interval $(-\epsilon, \epsilon)$. The sub-blocks of B'' will be numerated as $n' + 1$, $n' + 2, ..., n''$, $n'' - n' \gg 1$, and once again the flash induced by them will be of a red colour with probability practically equal to $p(\theta_1)$. Continuing the same manner, we may cover the entire semi-infinite line $\theta = [0, \infty)$ with large *continuous* blocks of the above kind, each of which emits a red signal with a probability practically equal to $p(\theta_1)$ and exactly one of which will inevitably be hit by the choice of the random point. A construction of this kind may be carried out for an arbitrary value of ϵ and setting $\epsilon \to 0$ we see that the counting of events on the entire semi-interval $[0, \infty)$ can be done in fact by subdividing it into red/blue couples B_i, $i = 1, 2, ...\infty$, and registering, on the same screen, first the number i of B_i and then the colour of the flash emitted by the couple hit by the random choice. (The large blocks disappear from the picture in the limit $\epsilon \downarrow 0$ in the sense that the registering of their numbers plays no role for the statistics of events in this limit). Clearly, the probability for, say, a red flash in this process will be exactly equal to $p(\theta_1)$, so the just described way of counting of events is correct.

Until now we examined red/blue couples (i.e. couples consisting of a red semi-interval and its right-hand blue neighbour). We can examine analogously blue/red couples and will arrive at exactly the same inference for the case of a definite $p(\theta_1)$. [As the first (leftmost) semi-interval is red, it cannot be included in a blue/red couple and must be examined separately. Its length, however, is negligible compared to the total (infinite) length of θ, so that this solitary finite semi-interval cannot in any way disturb the overall statistics and may not be paid any attention].

Consequently, in every case of existence of a definite (arbitrary) probability $0 \leq p(\theta_1) \leq 1$ under the assumption of a homogeneous nonnormalizable density distribution $\rho(x)$ in $[0, \infty)$ we can also count correctly the elementary events by registering the number of the red/blue (or blue/red) couple hit by the random point and then the colour of the flash emitted by the couple. That is, counting via couples is equivalent to 'normal' counting in which only the colour of the flash is registered, the definite value $p(\theta_1)$ emerging, in any case, in the large number limit. This result is not so trivial as it might seem as there are incorrect ways [that is, ways not agreeing with the definition of probability via the limit of ratio (B.2)] of subdividing $[0, \infty)$ into a set of nonoverlapping 'mini-counters' that will lead to a wrong counting of events even in the measurable case examined here. (Examples of the mentioned kind can be easily constructed and we shall not dwell on this item).

The above-said surprising 'experimental' fact for the nonmeasurable case is the following. The couple counter approach which works correctly in any measurable case [determining a definite objective $p(\theta_1)$] is of no avail at all in the non-measurable case [an indefinite $p(\theta_1)$] examined in eqs. (B.3-8). Indeed, consider the nonmeasurable case and subdivide first $[0, \infty)$ into red/blue couples. Any such couple consists of a red semi-interval of a given length d (that varies with the number of the couple) and a blue interval of a length qd, $q > 1$. Assume that a number on the screen shows which such couple has been hit in the random experiment (in full analogy with the measurable case). Due to the constancy of $\rho(x)$ in $[0, \infty)$ one can immediately say that the probability to see a red flash after the appearing of the number on the screen is equal to $d/(d + qd) = 1/(1 + q)$ and the probability for a blue flash is $q/(1 + q) > 1/(1 + q)$. Consequently, this way of counting the elementary random events will give here $100/(1 + q)$ per cent red flashes and $100q/(1 + q)$ per cent blue flashes on the screen in an arbitrarily large finite total number of events. Subdivide now $[0, \infty)$ into blue/red couples and carry out the same random choice experiment. A totally analogous argument will give, in this case, $100q/(1 + q)$ per cent red flashes and $100/(1 + q)$ per cent blue flashes on the screen. That is, in the first case the screen was more times blue than red while now it is more times red than blue in exactly the same proportion. Which is then the correct way of counting in the nonmeasurable case ?

The answer is that in the nonmeasurable case the correct way of counting is just a 'phantom': All thinkable ways of counting are equally correct (or incorrect) as one has no definite probabilities that may serve as a criterion of correctness in this case. Thus the phantom which is 'more blue than red' can turn, with the same success, into one that is 'more red than blue', like we this chameleon behaviour or not. A nice feature of these two phantoms is that they are tame: They appear when invoked and each one of them readily offers a definite statistics of its own –which, unfortunately, is of no use for solving the 'correctness problem'. But one may encounter in the same random-choice experiment wild phantoms too. Such will be the case, e.g., when the random choice is made without any subdivision of $[0, \infty)$ into 'mini-counters', that is, when one employs 'normal' counting in which only the colour of the flash is registered. The absence of a definite $p(\theta_1)$ will mean in this case that no set of elementary events will be sufficiently large to reveal a definite statistics. That is, even if for a certain large number of experiments one may imagine to have approached a definite limit l of (B.2), wild subsequent fluctuations will inevitably vitiate the result since any limit is known to be impossible in this problem.

We examined above a very simple case indeed of nonmeasurable sets. It may turn out that local-variable theories are possible in which the λ-space is much more intricate and its nonmeasurable sets have much richer and surprising properties (as it was already mentioned, an illustrative 'nonconstructivistic' complete model for such a theory already exists [16]). But our consideration is sufficient to demonstrate the possibility for a nontrivial generalization of de Broglie's view on measurement statistics. Namely, there may exist cases of apparent simultaneous nonmeasurability of certain physical magnitudes in which certain measurements may play the role of our tame phantoms with respect to statistics. That is, a specific type of ensemble measurement may introduce a specific statistics of its own on λ-sets that are in fact nonmeasurable, the different statistics on the same λ-space corresponding to "incompatible" ensemble measurements (or, equivalently, to different QM representations). In such a way statistics of this sort would depend on the type or the parameters of the measuring instrument (say, its orientation) and, once again, there would be no necessity of introducing any incomprehensible instantaneous nonlocality effects (cf. also Sec. VI; this is the essence of the conclusions made in refs. 15-17 too). On the other hand, wild phantoms may be the prototypes of other concepts, say "uncontrollability" and "unpredictability".

We thus arrive at a seemingly strange but logically admissible (hence understandable) and de facto *objective* possible role of the observer: He may not be a demiurge that can instantaneously turn, e.g., an infinitely thin cloud into an infinitely dense material point and vice versa but may be just a harmless phantom that may arrange, at a will, the (objective) experimental physical conditions and obtain instrument-dependent statistics, at that – sometimes– out of a statistics-free set of states. In any case, it is not really necessary to evolve further these speculations in the absence, for the time being, of logically complete local-parameter theories resting on axiomatics of their own. The purpose of the above consideration was to demonstrate that profound depths may underlie the smooth surface of even the apparently most natural and innocent assumptions which, however, are endowed in "no-go theorems" with the not quite modest metaphysical role of being valid in every possible physical theory.

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