On the distinction between fields and their metric:

the fundamental difference between
specifications concerning medium-independent fields
and constitutive specifications
concerning relations to the medium in which they exist

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ABSTRACT. A new conception of the magnetic monopole is presented based on (1) a distinction between descriptions of medium-independent fields and the constitutive relations to the medium in which they exist, and (2) the magnetic monopole defined as the excited state of a neutrino (Lochak). Using twistor formalism, it is shown that the interactive exchange between electromagnetic fields and the space-time metric (gravitational metric or aether) is a second-order differential mapping of the A-vector potential onto the metric (aether) provided by the neutrino-antineutrino pair concept (Lochak).

The excited state of the neutrino, i.e., the magnetic monopole, is proportional to the rate of change of the real part of the dielectric and magnetic susceptibilities and also of the rate of change of the dielectric dispersion and magnetic induction. A second influence on the neutrino, is a phase change which is proportional to the rate of change of the imaginary part of the dielectric and magnetic susceptibilities and also of the electrical and magnetic conductance. Thus, the electromagnetic field and the space-time metric (neutrino network) have an independent or *inherent* existence, but the excited states (magnetic monopole) and phase changes of the neutrino have a dependent existence *derived* from fluctuations in the electromagnetic field aforementioned. Justification for field-metric exchanges is found is the requirement for entropy-energy balance conservation between fields and metric.

The twistor formalism is only exactly applicable to the electromagnetic field conditioned by polarization modulation (an angular momentum twistor). The electromagnetic field without polarization modulation is well-known to be of $U(1)$ symmetry. After polarization modulation conditioning, it is of $SU(2)$ symmetry and thus of non-Abelian Yang-Mills form. Conditioning the $U(1)$ electromagnetic field into $SU(2)$ form, in effect, adds a degree of freedom to the field.

The consequences of this new picture are both an understanding of the ubiquitous nature of the magnetic monopole leading to a reformulation of Maxwell's theory, as well as an approach to the unification of electromagnetism and gravitational theory.

The major conclusion is that the relation of local fields and their metric is governed by an energy-entropy conservation condition modeled by an adiabatic polarization modulation waveguide. Experimental testing of this theory can procede at radar, infrared and visible frequencies. While the necessary speed of polarization modulation at optical frequencies (in the picosecond range) is quite difficult to obtain technically, the necessary speeds required for polarization modulation at infrared and radar frequencies are easily obtained.

INTRODUCTION

According to Cartan [1], the basic laws of electrodynamics permit a functional separation between medium-independent law statements and the constitutive specification of the medium. To include free-space, the medium independent law statements should be independent of the space-time metric. There have been early attempts to separate electromagnetism from its metric [2,3], and early and recent attempts to provide electromagnetic theories of gravitation [4-6]. Recently, the demonstration that the polarization of light is affected by wave-guide bending indicates that Maxwell's equations are independent of the affine connections of space-time [7-18].

The successfully engineered ring laser gyro, based on the Sagnac effect [19,20], implicates this independence [21-23]. The transit time around the ring contour is determined by c, the speed of light, for all observers. A photon nonetheless takes a different length of time to traverse the contour in a rotating frame.

Post [23-28], for example, functionally separates field and constitutive equations and a distinction is drawn between physical frames and the family of permissible coordinate neighborhoods associated with such physical frames. Traditionally, the phenomenological description of electromagnetism has been in two parts: firstly, the electromagnetic field equations or properties which all systems share, and, secondly, the constitutive equations of the medium, or a systematic characterization of specific systems. However, as Post has shown, free-space was not treated as belonging to the second category. Therefore, Lorentz invariance is presently incorrectly treated as a symmetry of the free-space field equations, rather than as that of the free-space constitutive equations as it should be. It is an aim of the present paper to define the magnetic monopole as an entity dependent on both changes in the electromagnetic field equations for origins and the constitutive relations for form.

In order to accomplish this aim, the paper is in three parts. The first part is a summary of Lochak's demonstration that the magnetic monopole is an excited state of the neutrino and that the Majorana equation for the neutrino/monopole is a special case of the Dirac equation for the electron. This work provides the dynamic link between the electromagnetic field (described by the Dirac equation) and the space-time metric (described by the Majorana equation) in that gauge invariance is demonstrated for the two and the latter is a special (constrained) case of the former.

The second part addresses precisely in what dynamic form the electromagnetic field must be to perturb the aether represented by the neutrino system. This part depends on twistor formalism and shows that an aether-perturbing electromagnetic field is in $SU(2)$ symmetry form, not $U(1)$.

With the field-metric relations established in part one, and the general method for perturbing the metric stated in part two, part three addresses specific ways of obtaining both excited states (monopoles) and phase changes in the metric of the neutrino system and interactive relations between field and metric based on entropy-energy balance conservation. Thus part three addresses the ways in which field and metric can interact.

I. SUMMARY OF LOCHAK'S ANALYSIS

Lochak [29] demonstrated that the Dirac equation [30,31] admits two, and only two, possible invariant gauges. One is the phase invariance $e^{i\theta}$, which gives the electromagnetic coupling with an electric charge; the other is the chiral transformation $e^{i\gamma_5\theta}$, which is only valid for a massless particle such as a neutrino. (The first of these corresponds to γ_r and ξ_r and the second to γ_i and ξ_i in Figure 18 below).

A free Dirac particle is first considered:

$$\gamma_\mu\partial_\mu\psi + (m_0c/h)\psi = 0, \tag{1.1}$$

in the relativistic coordinates $x_\mu = \{x_k, ict\}$, (where the γ-matrices are in terms of the Pauli matrices s_k):

$$\gamma_k = i\begin{pmatrix} 0 & s_k \\ -s_k & 0 \end{pmatrix} \quad , \quad k = 1,2,3 \quad , \quad \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{1.2}$$

$$\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Lochak proves that only two gauge transformations of the form:

$$\psi \mapsto e^{i\Gamma\theta}\psi \tag{1.3}$$

are admitted where Γ is a constant Hermitian matrix and θ is a constant parameter. It is then shown that there are only two possible forms of Γ [29]: (I) $\Gamma = I$, which corresponds to the classical phase invariance:

$$\psi \mapsto e^{i\theta}\psi, \tag{1.4}$$

and (II) $\Gamma = \gamma_5$, which gives the chiral gauge:

$$\psi \mapsto e^{i\gamma_5\theta}\psi. \tag{1.5}$$

Furthermore, (II) is only valid for $m_0 = 0$.

(I) defines a local gauge transformation:

$$\psi \mapsto \exp[i(e/hc)\phi]\psi \quad , \quad A_\mu \mapsto A_\mu + \partial_\mu\phi, \tag{1.6}$$

and (II) defines another local gauge transformation:

$$\psi \mapsto \exp[i(g/hc)\gamma_5]\psi \quad , \quad B_\mu \mapsto B_\mu + \partial_\mu\phi, \tag{1.7}$$

where A_μ is a polar vector (the Lorentz quadripotential), B_μ is an axial vector (an electromagnetic pseudo-potential), ϕ a scalar function, e an electric charge and g, a magnetic charge.

Whereas gauge (I) leaves the equation for an electron,

$$\gamma_\mu(\partial_\mu + (ie/hc)A_\mu)\psi - (m_0c/h)\psi = 0, \tag{1.8}$$

invariant, gauge (II) leaves the equation for a magnetic monopole,

$$\gamma_\mu(\partial_\mu + (g/hc)\gamma_5 B_\mu)\psi = 0, \tag{1.9}$$

invariant. If the monopole is in interaction with a Coulomb field, then the relation of Dirac is obtained:

$$eg/hc = n/2 \quad , \quad n = 0, 1, 2, \ldots \tag{1.10}$$

The wave equation for a massless spin 1/2 monopole can be split into two equations in terms of 2-component spinors ξ and η. Introduction of a nonlinear term introduces a mass term. The dispersion relations for this equation set have then two especially interesting solutions: (1) when ξ and η have the same phase and frequency and the proper mass > 0, then the bradyon case is obtained (a particle state with speed less than that of light); (2) when ξ and η have opposite phases and frequencies and the proper mass > 0, then the tachyon case is obtained (a particle state with speed greater than that of light). If the proper mass is zero, then the limiting luxon case is obtained.

The hypothesis is made [32] that the monopole of zero magnetic charge is the neutrino and that magnetic charge is a multiple of the fundamental charge g_0:

$$g = ng_0 \quad , \quad g_0 = hc/2e. \tag{1.11}$$

Due to a chiral invariance condition, the two component spinors in the linear (zero mass) case are related by:

$$\xi = \exp[2i(e/hc)\theta] is_2 \eta^* \quad , \quad \eta = -\exp[2i(e/hc)\theta] is_2 \xi^*, \tag{1.12}$$

where $\theta(r, t)$ is an arbitrary phase. Thus ξ and η correspond to a monopole-antimonopole couplet for which the total magnetic current is zero.

The chiral invariance condition aforementioned is:

$$\rho^2 = \Omega_1^2 + \Omega_2^2 = 0, \tag{1.13}$$

where $\Omega_1 = \psi^+\psi$ and $\Omega_2 = -i\psi^+\gamma_5\psi$. This is also the condition for a Majorana field [33]. When $\theta = 0$, the abridged Majorana equation is obtained, which is identical to the Dirac equation for the electron. Thus, the Majorana field is a constrained form of the Dirac field for $\psi = \psi_2\psi^* = \psi_c$. However, with θ nonzero, gauge invariance is obtained. Furthermore, the electrical current density is shown to be the sum of two chiral currents, but the density of magnetic current, Σ_μ, is shown to be the difference between the chiral currents.

The nonlinear generalization of the Majorana/monopole equation which gives gauge invariance is [32]:

$$\gamma_\mu(\partial_\mu + (ie/hc)A_\mu)\psi - (m_0c/h)\exp[2i(e/hc)\theta]\psi_c = 0, \tag{1.14}$$

and the relation of Uhlenbeck and Laporte can be derived from the above definitions:

$$\partial_\mu\Sigma_\mu + 2[(m_0c/h)]\Omega_2 = 0, \tag{1.15}$$

indicating that conservation of magnetic current occurs only under two conditions: (1) when $m_0 = 0$, i.e., under the condition for a magnetic monopole but in its ground state; and (2) when $\Omega_2 = 0$, i.e., in the case of a Majorana field, i.e., a field which is not invariant to gauge. The Majorana/monopole field condition can be categorized as a hybrid state of the Dirac electron which does not possess bound states, but which possesses two possible types of trajectories different according to sign, i.e., either attractive or repulsive. In a Coulombic field a Majorana electron behaves as a negatively/positively charged particle. In later sections of this paper, the following concepts will be used due to Lochak [29,32]: (i) the Dirac equation can effectively describe electron behavior. In fact, Oudet & Lochak [34] have shown its success in describing atomic magnetic moments and its uniqueness in having as a premiss that the angular momentum operator is the sum of the angular momentum and the spin, which are not separately constants of the motion; (ii) the Majorana equation describes the neutrino in its ground state and the nonlinear Majorana equation describes the excited neutrino (magnetic monopole) and the neutrino with phase change; and (iii) the Majorana equations are special instances of the Dirac equation, with state changes dependent on, or derived from, changes originating in fields described by the Dirac equation.

In the next section the exact description is considered of the dynamic form in which the electromagnetic field must be in order to perturb the metric (aether) represented by the neutrino system. To achieve that aim of exact description, twistor formalism is introduced.

II. TWISTOR FORMALISM OF THE PERTURBING FIELD

The conservation of energy and momentum law for the electromagnetic field is contained in Poynting's theorem (1884; cf. [35]). Poynting's theorem addresses the balance of the power representing a conversion of electromagnetic energy into mechanical and thermal energy with the rate of decrease of energy in the electromagnetic field within a set volume. In the usual formulation the magnetic field does no work as the magnetic force is perpendicular to that field's velocity. Other assumptions are that the macroscopic medium involved is linear in its electric and magnetic properties and that the energy is the sum:

$$W = (1/8\pi) \int \mathbf{E}.\mathbf{D}\ d^3x + (1/8\pi) \int \mathbf{H}.\mathbf{B}\ d^3x,$$

even for time-varying fields. With the total energy density given by:

$$u = (1/8\pi)(\mathbf{E}.\mathbf{D} + \mathbf{B}.\mathbf{H}),$$

a differential continuity or conservation law is obtained:

$$\partial u/\partial t + \nabla.\mathbf{S} = -\mathbf{J}.\mathbf{E}, \tag{2.1}$$

where

$$\mathbf{S} = (c/4\pi)(\mathbf{E}\times\mathbf{H})$$

represents (macroscopic) energy flow and is called Poynting's vector with the dimensions of (energy/area × time). Now the divergence of $\mathbf{S}$ appears in the conservation law (equ. (1)), so the curl of any vector can be added to it, i.e., Poynting's vector, as defined by the conservation law, *is arbitrary.* In fact, the divergence of Poynting's vector only defines the energy flowing out through the boundary surfaces of a volume per unit time. The $\partial u/\partial t$ term in equ. (1) defines the time rate of change of electromagnetic energy within that certain volume, but without specifics concerning the form that energy takes. This is also true for Poynting's theorem for microscopic fields:

$$(c/4\pi) \int_{\mathbf{S}} (\mathbf{E}\times\mathbf{H}).\mathbf{n}da + (1/4\pi) \int_{\mathbf{V}} (\mathbf{E}.\partial\mathbf{D}/\partial t + \mathbf{H}.\partial\mathbf{B}/\partial t) d^3x = -\int_{\mathbf{V}} \mathbf{E}.\mathbf{J} d^3x.$$

Thus, although, given the assumptions stated above, Poynting's theorem provides an estimate of the debits and credits of energy conservation at the surfaces of volumes, it does not provide a workable definition of a propagating wave.

This is not to deny the validity of the theorem in defining the conditions for propagation without dissipation. In fact, Crisp [36] used the theorem to derive equations for distortionless propagation of light through an optical medium. Crisp used the following form:

$$\partial u(x,t)/\partial t + \nabla.\mathbf{S}(x,t) = -E(x,t).\partial \mathbf{P}(x,t)/\partial t, \tag{2.2}$$

where

$$\begin{aligned}
\mathbf{E}(x,t) &= e_1 \; E(z,t)\cos(t-kz),\\
u(x,t) &= (\epsilon_0/4\pi)E^2(z,t)\cos^2(t-kz),\\
\mathbf{S}(x,t) &= (c/4\pi)(\epsilon_0/\mu_0)^{1/2}e_3E^2(z,t)\cos^2(t-kz),
\end{aligned}$$

for light polarized in the e_1 direction and propagating in the e_3 direction, and $\mathbf{P}(x,t)$ is the dipole moment per unit volume. Solutions are then found for $\mathbf{E}(x,t)$ under assumptions for $\mathbf{P}(x,t)$ when substituted into equ. (2.2). The solutions found are analogous to the three possible types of motion of a simple pendulum.

This approach is valid but again provides no specific form for the crafted wave. Furthermore, in requiring a polarization which is locked in phase with respect to the applied field, the approach is insufficiently general to provide guidelines for wave generation.

The intention in this section, therefore, is to define directed energy waves which propagate without dissipation in two ways. The first way is a control theory system description of wave generation; the second way is a mathematical account of that system. A fundamental assumption underlying this description is that the medium through which the energy propagates consists of interactive dipole moments which sample or interact with the energy wave according to rules of polarization and relaxation time.

The mathematical description is based on the work of Penrose and Rindler [37,38]. In this section we provide an explanation of a system's polarization modulated output in terms of twistor form. This is absolutely necessary, as the twistor form is the only precise definition of the output of the adiabatic polarization modulated waveguide system considered. To my knowledge, this is the first time that twistor theory has been applied to a system's design or that twistor theory has been shown to be necessary for system description.

Following Penrose [39], we do not view space-time as constituting, fundamentally, a mathematical continuum, but rather space-time is viewed as constituted of singularities, the interrelations of which constitute a continuum. In other words,the spinor structure of space-time is taken as more basic than its speudo-Riemannian structure. Three-dimensional space then arises according to the combinatorial rules of spin-networks [40].

The aim of the following subsection is to demonstrate that with the electromagnetic field restricted to a polarization modulated form, then it is in twistor form, and therefore with an affine relation to the twistor form of the neutrino/monopole or the basic building block of the space-time continuum. With this form field-metric interactions are possible, without it, they are not.

2.1. Control Theory Picture

The waveguide system considered here is completely general in that the output can be phase, frequency and amplitude modulated. It is, however, an *adiabatic system* and only three of the lines are waveguides –the input, the periodically delayed line, and the output. Other lines shown are energy-expending, phase-modulating lines. The basic design is shown in Figure 1. In this Figure, the input is $\mathbf{E} = E\exp(i\omega t)$. The output is:

$$\mathbf{E} = (E/4)\exp(i\omega t) + (E/4)\exp\{i[\omega + \exp(i\phi t)]t\},$$

where $\phi = \mathbf{F}(E/2)$ and $\partial\phi/\partial t = \dot{\mathbf{F}}(E/2)$.

The waveguide consists of two arms –the upper $(E/4)$ and the second $(E/4)$ with which the upper is recombined. The lower, or third, arm, merely expends energy in achieving the phase modulation of the second arm with respect to the first. This can be achieved by merely making the length of the second arm change in a sinusoidal fashion (i.e. by $\partial\phi/\partial t$), or it can be achieved electro-optically. Whichever way is used, one half the total energy of the system $(E/2)$ is spent on achieving the phase modulation in the particular example shown in Figure 1. This adiabatic system exhibits an energy-entropy conservation law discussed in Part III. It is, therefore, important to realize that the entropy change from input to output of the waveguide is compensated by energy expenditure in achieving the phase modulation to which the entropy change is due.

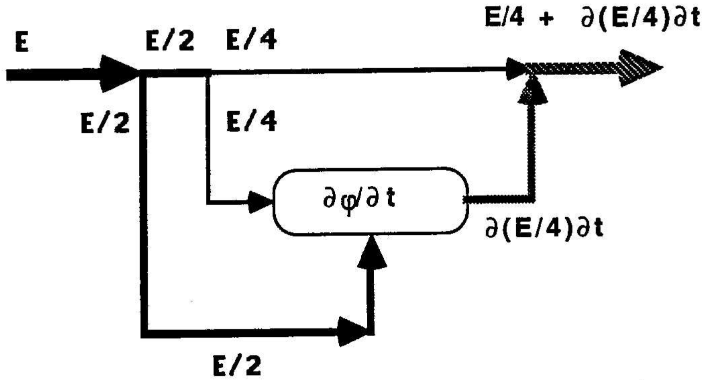


Figure 1. Waveguide system paradigm for polarization modulated ($\partial\phi/\partial t$) wave emission. This is a completely adiabatic system in which oscillating energy enters from the left and exists from the right. On entering from the left, the energy is divided into two parts equally. One part, of amplitude $E/2$, is used in providing phase modulation; $\partial\phi/\partial t$ –this energy is spent (absorbed) by the system in achieving the phase modulation; the other part, of amplitude $E/2$, is divided into two parts of the waveguide equally, so that two oscillating waveforms of amplitude $E/4$ are formed for later superposition at the output. Due to the phase modulation of one of them with respect to the other, $0 < \phi < 360°$, the output is of continuously varying polarization. The choice of wave division into two parts equally is arbitrary.

One can nest phase modulations. The next order nesting is shown in Figure 2, and other, high order nesting of order n, for the cases, $\partial\phi^n/\partial t^n$, $n = 1, 2, 3\cdots$ follow the same procedure.

The input is again: $\mathbf{E} = E\exp(i\omega t)$. The output is:

$$\mathbf{E} = (E/4)\exp(i\omega t) + (E/4)\exp\{i[\omega + \exp\{i(\phi_1 + \exp(i\phi_2 t))t\}]t\},$$

where $\phi_1 = \mathbf{F}_1(E/4)$; $\phi_2 = \mathbf{F}_2(E/4)$ and $\partial\phi^2/\partial t^2 = \dot{\mathbf{F}}_1.\dot{\mathbf{F}}_2$.

Again, the waveguide consists of two arms –the upper $(E/4)$ and the second $(E/4)$ with which the upper is recombined. The lower two arms, merely expend energy in achieving the phase modulation of the second arm with respect to the first. This again can be achieved by merely making the length of the second arm change in a sinusoidal fashion (i.e., by $\partial\phi^2/\partial t^2$), or it can be achieved electro-optically. Whichever way is used, one half the total energy of the system ($E/4+E/4 = E/2$) is spent on achieving the phase modulation in the particular example shown in Figure 2.

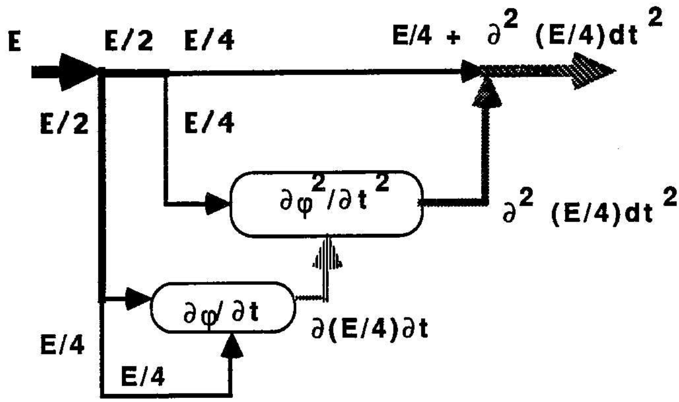
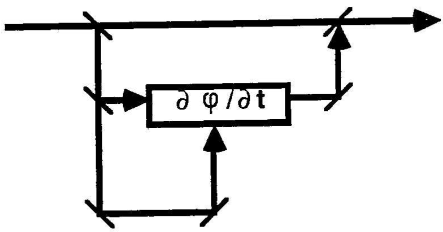


Figure 2. Waveguide system paradigm for polarization modulated ($\partial\phi^2/\partial t^2$) wave emission. A completely adiabatic system in which oscillating energy enters from the left and exists from the right. On entering from the left, the energy is divided into two parts equally. One part, of amplitude $E/2$, is used in providing phase modulation, $\partial\phi^2/\partial t^2$ –this energy is spent (absorbed) by the system in obtaining the phase modulation; the other part, of amplitude $E/2$, is divided into two parts equally, so that two oscillating waveforms of amplitude $E/4$ are formed for later superposition at the output. Unlike the system shown in Figure 1, the energy expended on phase modulating one of these waves is divided into two parts equally, of amplitude $E/4$, one of which is phase modulated, $\partial\phi/\partial t$, with respect to the other as in Figure 1. The energy of the superposition of these two waves is then expended to provide a second phase modulated $\partial\phi^2/\partial t^2$ wave which is superposed with the only nondelayed wave. Due to the phase modulation of one of them with respect to the other $0 < \phi < 360°$, the output is of continuously varying polarization. The choice of wave division into two parts equally is arbitrary.

Both the systems shown in Figures 1 and 2, and all higher order such systems, $\partial\phi^n/\partial t^n$, $n = 1, 2, 3 \cdots$, are adiabatic with respect to the field and Poynting's theorem applies to them all. However, the Poynting description, or rather limiting condition, is insufficient to describe these fields either exactly, or in their diversity and we seek a more precise analysis.

A simplification of Figure 1 is shown in Figure 3, below:

A.



B.

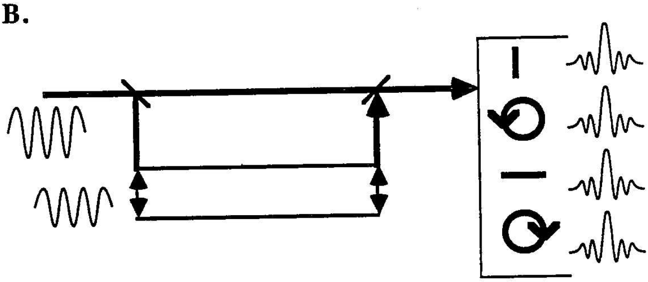


Figure 3. Phase Modulation by Three Wave Mixing. A is the generic form for Figure 1. The waveguide consists of two arms which separate and recombine. The energy in the lower arm is spent (absorbed) in achieving the phase modulation of the second arm. The combined output is the third wave. B indicates how a continuous wave of periodically varying polarization would appear as discrete pulses to a discriminator of set polarization. An adiabatic system is represented. In order to provide an output composed of two vectorial components, one of which is phase modulated with respect to the other, energy is expended to provide the delay of one vectorial component with respect to the other, so that, when superposed at the output, the vectorial components exhibit a phase modulation, $\partial\phi/\partial t$, with respect to each other. In B this phase modulation is achieved by varying in an oscillatory fashion, the path traversed by one wave with respect to another prior to superposition at the output. The output is thus a wave of continuous phase modulation, $0 < \phi < 360°$. Although a continuous wave, such a wave of periodically varying polarization would, nonetheless appear as discrete pulses to a discriminator of set polarization, and this is indicated to the right of B.

The Figure 1 system is shown in Figure 3 as a (classical) three-wave mixer, the energy of one wave being spent (absorbed) on achieving the phase modulation of one of the other two waves. The output wave, although a continuous wave when *all* polarizations are considered, exhibits discrete pulses (right side) when set polarizations are sampled (Figure 3B). The polarization of the output at some representative phase lags is shown in Figure 4 below.

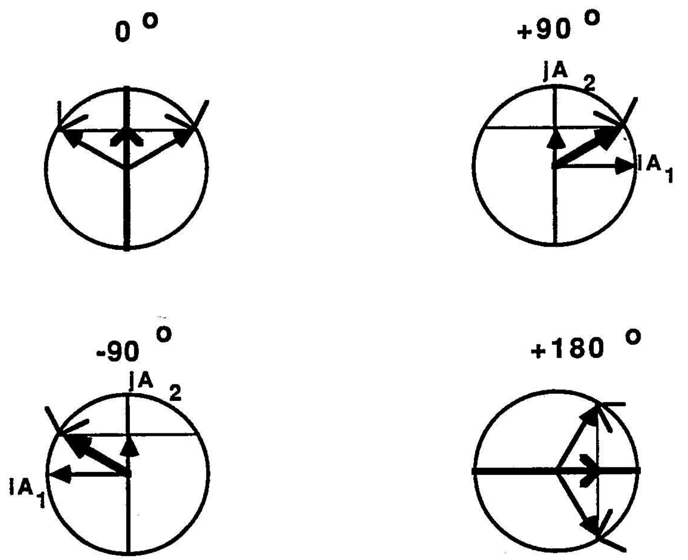


Figure 4. Modulo-evolution of phase-modulation: The Poincaré sphere: $E(x,y,z) = [A_1E_1(x,y) + A_2(z)E_2(x,y)]expik_1z$; Total Power $= (\mid A_1 \mid^2 + \mid A_2 \mid^2)$; $C = A_1(z)/A_2(z)$; angular coordinates of $C : 2\chi = \arctan(\mid A_1/A_2 \mid -1)(\mid A_1/A_2 \mid +1]$; $2\xi = \arg(A_1/A_2)$. Four depictions of polarized waves in terms of two vectorial components with phase lag. The four waves are: linearly polarized vertically at phase lag 0°; linearly polarized horizontally at phase lag 180°; right circularly polarized at phase lag +90°; and left circularly polarized at phase lag −90°. The polarized waveforms are shown as thick arrows and the vectorial components as thin arrows. These four examples illustrate waves of set polarization and vectorial phase lag, $\phi = a$ constant, but do not illustrate a polarization modulated, $\partial\phi^n/\partial t^n$, wave – *these illustrations are merely "snap shots"*.

Now, it is well known that all polarizations can be represented on a Poincaré sphere. A representative periodic phase modulation is shown as an example in Figure 5, in which a vector C is rotating through linear and circular polarizations at a set frequency of rotation. This period of rotation is shown as Δt in Figure 6 where eight representative polarizations are indicated which occur during a periodic sweep-through. It may again be indicated (Figure 7) that to an analyzer or discriminator of set polarization, the continuous wave appears as pulses of set frequency of repetition.

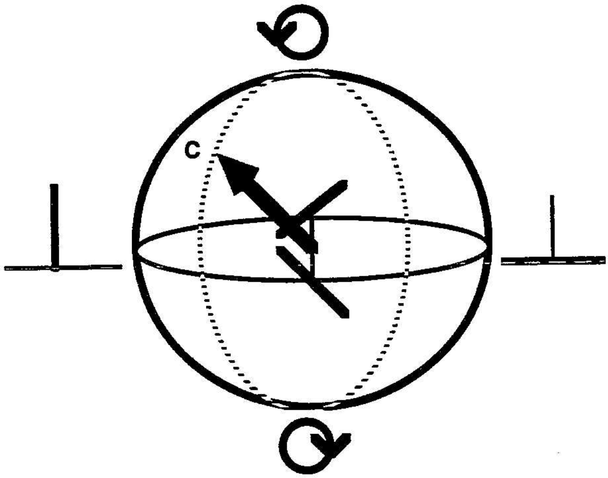


Figure 5. A wave of any polarization can be represented on a Poincaré sphere, in this case as an arrow centered at the origin. More importantly, waves of various polarization modulations, $\partial\phi^n/\partial t^n$, can be represented as trajectories on the sphere. In this case a circular trajectory, $\partial\phi/\partial t = a$ constant, is arbitrarily shown.

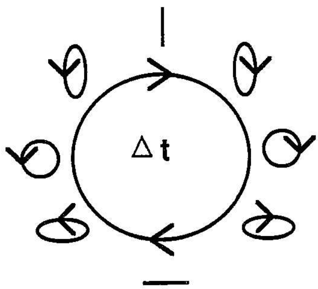


Figure 6. A representation of phase modulation which traverses continuously through an arbitrary number (8) of set polarizations in time Δt. $\partial\phi/\partial t = a$ constant $0 < \phi < 360°$.

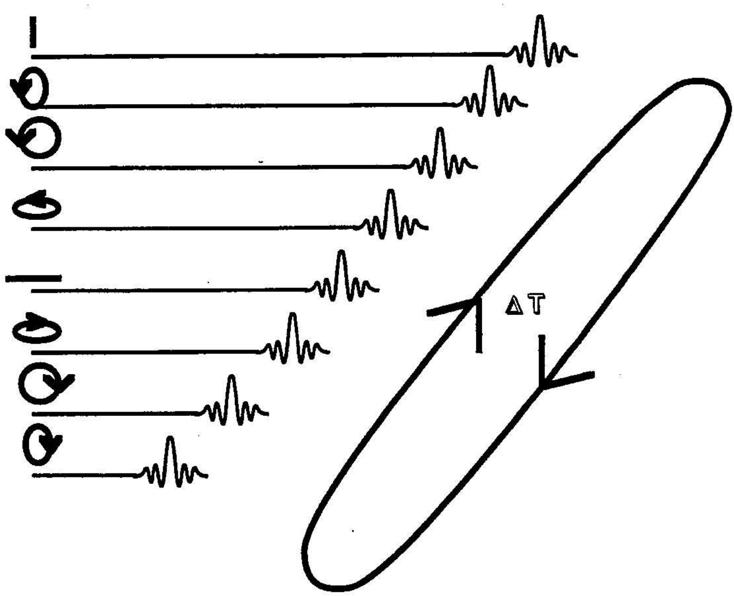


Figure 7. A representation of continuous phase modulation in period Δt; $\partial\phi/\partial t = a$ constant $0 < \phi < 360°$; with eight arbitrary representative polarization samplings. To an analyzer or material dipole moment of set polarization, the continuous wave would appear, i.e., would be sampled as a *pulse* train.

The particular polarization modulation chosen as an example is, of course, arbitrary; the vector, C, can traverse any periodic trajectory on the Poincaré sphere (Figure 5). Thus the period of modulation can be faster than the atomic or molecular relaxation time of the medium through which the energy propagates. In other words, the pulses of set polarization, constituting the continuous wave of regularly varying polarization, can have a duration shorter than the relaxation time of atomic and molecular dipole moments of the same set polarization. Alternatively, the polarization modulation rate can be faster than the relaxation rate of some dipole elements, but slower than other elements. In such cases, there will be dissipationless penetration of some media and reflectance and/or absorption by other media.

So far, we have considered a completely adiabatic system in which we have crafted the initial energy into a different form. This might be considered "energy diversification". If, however, in the three-wave mixing paradigm of Figures 1 and 3, the sources of energy from the modulated and the modulating waves are different, then the modulated wave may be said to have undergone a form of parametric amplification, but not of the phase-matched or Manley-Rowe kind. In the familiar forms of parametric amplification, a phase matching is required and the frequencies of the three waves are related by the Manley-Rowe relations. In the present paradigm, however, the phase modulation is concomitant with a frequency modulation (chirping) and the amplification appears as a rate of change dE^n/dt^n increase, where $n = 1$ (Figure 1), $n = 2$ (Figure 2), $n = 3 \cdots$ etc., or, in other words, an amplification of angular momentum and/or the differentials of the angular momentum. Therefore, by increasing the angular momentum and/or its differentials, it is possible to increase the total energy of the wave *without directly increasing its amplitude*. It is thus even possible to generate a field (wave) of relatively low amplitude but relatively high angular momentum.

In this subsection the *system's paradigm* of Figures 1-3 was explored and shown to be physically different from those paradigm's presently used in directed energy tasks. It was also indicated that merely applying the conservation law of Poynting's theorem results in limited insight into the type of signal which can be crafted. In the following subsection a remedy is found for this need and a remarkable fit is shown between the paradigm under discussion and a framework developed for the discussion of other topics in space-time. The aim of subsection 2 is to provide the appropriate mathematical tool to achieve the further aim of describing the energy conditioning according to the output of Figure 1.

2.2. Mathematical Physics Picture

The picture we shall present, pointing out its advantages over other pictures and frameworks, such as tensor analysis, is that of the theory of spinors. Spinors were first used in physics in the field of quantum mechanics by Dirac in his equation for the electron [30,31], but in their most general mathematical form, spinors were discovered by Cartan [41,42]. More recently, Penrose has extended the application of these concepts to classical subjects [37,38]. Only 2-spinors will be discussed in the following.

Before introducing the spinor concept it is first necessary to establish the physical correspondence. The correspondence will always be with the system discussed in the previous section in Figures 1 and 3, namely, a system for producing polarization modulation. That generic system is shown again in Figure 8, but with the waves labeled w, x, y and z. This will always be the system we shall have in mind, although the following synthesis can be generalized to more complicated systems such as the one shown in Figure 2.

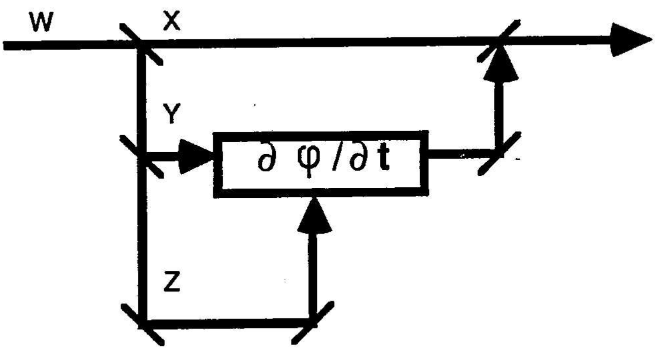


Figure 8. Waveguide system paradigm for polarization modulated $(\partial\phi/\partial t)$ wave emission as in Figure 1.

With the Figure 8 system in mind, we can define a vector V:

$$V = Ww + Xx + Yy + Zz, \tag{2.1}$$

and also what is known as a *null vector*, the coordinates of which satisfy:

$$W^2 - X^2 - Y^2 - Z^2 = 0, \tag{2.2}$$

which is clearly true for the system, it being understood that W, X, Y and Z are functions of time: $W(t)$, $X(t)$, $Y(t)$ and $Z(t)$.

Suppose, now, we wish to consider null directions at any particular time. The reason we would want to so, is that if a periodic polarization modulation is occuring, e.g., as represented on the Poincaré sphere (Figure 5), then knowing what the polarization of the output is at time t, does not provide a basis for knowing what the polarization will be at time $t+$, nor what the polarization has been at time $t-$. The abstract space whose elements are the future [past] null directions is called S^+, $[S^-]$. Thus, in analogy to a world-vector description [37], the two spaces, S^+, S^-, can be represented in any coordinate system (W, X, Y, Z) by the intersection S^+ $[S^-]$ of a future [past] null cone (Equ. (2)) with the hyperplanes $W = 1$ $[W = -1]$. In the Euclidean (X, Y, Z), i.e., output space, S^+ $[S^-]$ is a sphere with the equation:

$$x^2 + y^2 + z^2 = 1, \tag{2.3}$$

and shown in Figure 9.

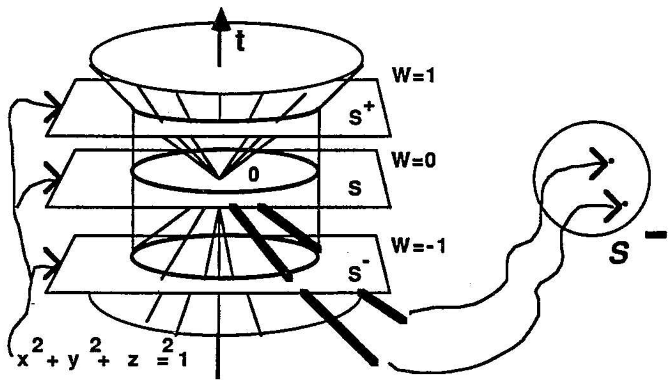


Figure 9. The relation of the abstract space whose elements are the future and past null directions, represented by the intersections, S^+ and S^-, with the W hyperplanes, to the Euclidean output space spheres $\mathbf{S}^+$ and $\mathbf{S}^-$ –in the instance highlighted– of S^- to $\mathbf{S}^-$. After Penrose & Rindler [37].

The sphere $\mathbf{S}^+$ $[\mathbf{S}^-]$ is really the well known sphere of the representation of complex numbers of which the Poincaré sphere of Figure 5 is an example. In moving toward a definition of a spinor, we may replace the coordinates x, y, z on $\mathbf{S}^+$ by a single complex number which can be represented by the stereographic correspondence between the sphere and the Argand plane (Figure 10):

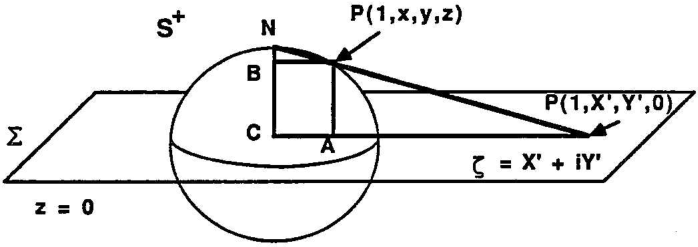


Figure 10. Correspondence between the output space sphere and an Argand plane. After Penrose & Rindler [37].

While performing this projection, it is understood that we have in mind the projection of the Poincaré sphere to the Argand plane. The plane Σ is drawn with $z = 0$ for $W = 1$ and the corresponding point $P(1, x, y, z)$ on $\mathbf{S}^+$ is mapped to $P'(1, X', Y', 0)$ on Σ. The point P' is then labeled by a single complex parameter:

$$\zeta = X' + iY'. \tag{2.4}$$

If:

$$z = 1 - CA/CP' = 1 - NP/NP' = 1 - NB/NC,$$

then:

$$\zeta = [x + iy]/[1 - z]. \tag{2.5}$$

Defining by a pair (ξ, η) of complex numbers:

$$\zeta = \xi/\eta, \tag{2.6}$$

we may represent the waves of the paradigm in Figure 8 by:

$$W = (1/\sqrt{2})[\xi\xi^* + \eta\eta^*], \tag{2.7}$$

$$X = (1/\sqrt{2})[\xi\eta^* + \eta\xi^*],$$

$$Y = (1/i\sqrt{2})[\xi\eta^* - \eta\xi^*],$$

$$Z = (1/\sqrt{2})[\xi\xi^* - \eta\eta^*],$$

so that any complex linear transformation of ξ and η results in a real linear transformation of (W, X, Y, Z). Thus, a complex linear transformation of ξ and η can be defined:

$$\xi \mapsto \xi' = \alpha\xi + \beta\eta, \tag{2.8}$$

$$\eta \mapsto \eta' = \gamma\xi + \delta\eta,$$

or

$$\zeta \mapsto \zeta' = [\alpha\zeta + \beta]/[\gamma\zeta + \delta],$$

where α, β, γ and δ are arbitrary nonsingular complex numbers.

The transformations (2.8) are *spin transformations* and imply

$$\zeta = [X + iY]/[W - Z] = [W + Z]/[X - iY], \tag{2.9}$$

and if the *spin matrix* $\mathbf{A}$ is defined:

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \det \mathbf{A} = 1, \tag{2.10}$$

then the first two transformations of (2.8) are:

$$\begin{pmatrix} \xi' & \xi \\ \eta' & \eta \end{pmatrix} = \mathbf{A} \tag{2.11}$$

Thus the spin matrix of a composition is given by the product of the spin matrices of the factors. Furthermore, any transformation of the (2.11) form is linear and real and leaves the form $W^2 - X^2 - Y^2 - Z^2$ invariant. For the present purposes this means that the physical paradigm of Figure 8 remains valid. Stated differently, the system of Figure 8 can perform, physically, a spin transformation.

Due to the unimodular condition:

$$\alpha\delta - \beta\gamma = 1, \tag{2.12}$$

the spin matrix $\mathbf{A}$ and its inverse $\mathbf{A}^{-1}$:

$$\mathbf{A}^{-1} = \begin{pmatrix} \delta & -\beta \\ -\gamma & \alpha \end{pmatrix} \tag{2.13}$$

give rise to the same transformation of ζ even although they define different spin transformations. Due to the unimodular condition (Equ. (2.12)), the $\mathbf{A}$ spin-matrix is unitary or:

$$\mathbf{A}^{-1} = \mathbf{A}^*, \tag{2.14}$$

where $\mathbf{A}^*$ is the conjugate transpose of $\mathbf{A}$. These relations have an interesting consequence.

The consequence is that every proper rotation of $\mathbf{S}^+$ corresponds to precisely two unitary spin rotations, one being the negative of the other. Now we have justified identifying the Poincaré sphere (Figure 5) representation with $\mathbf{S}^+$. The vector C in Figure 5 corresponds to two vectorial components (Figure 4), one being the negative of the other. As every unitary spin transformation corresponds to a unique proper rotation of $\mathbf{S}^+$, then any representation of $\mathbf{S}^+$ corresponds to a tri-sphere representation (Figure 11). Therefore, $\mathbf{A}^{-1}\mathbf{A} = \pm\mathbf{I}$, where $\mathbf{I}$ is the identity matrix, i.e., a spin transformation is defined *uniquely up to sign* by its effect on the $\mathbf{S}^+$ sphere.

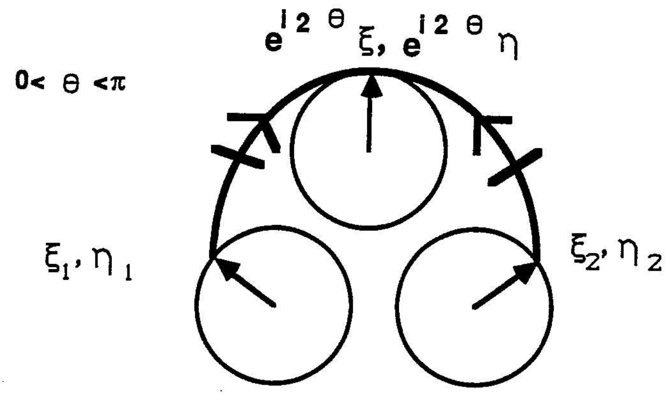


Figure 11. Tri-sphere representation of polarization mapping: $\xi_1, \eta_1; \xi_2, \eta_2 \mapsto e^{i2\theta}\xi, e^{i2\theta}\eta$; $0 < \theta < \pi$. Note that to a 360° excursion of ξ_1, η_1 and ξ_2, η_2 corresponds a 360° excursion of $e^{i2\theta}\xi, e^{i2\theta}\eta$, i.e., this is a mapping for linear polarization. Compare this with Figure 13 below.

However, the tri-sphere mapping shown in Figure 11 (or ξ_1, η_1 ; $\xi_2, \eta_2 \mapsto e^{i2\theta}\xi, e^{i2\theta}\eta$; $0 < \theta < \pi$) does not describe the output from the waveguide system shown in Figure 8 at all times, but is merely an instantaneous "snapshot". The tri-sphere mapping is a mapping $\mathbf{A}^{-1}\mathbf{A} = \pm\mathbf{I}$

and for the continuous rotation of $\xi_1, \eta_1; \xi_2, \eta_2$ through 2θ, there is a rotation of the resultant ξ, η through 2θ. Rather than the Figure 8 paradigm, this mapping is described by the system in Figure 12, below.

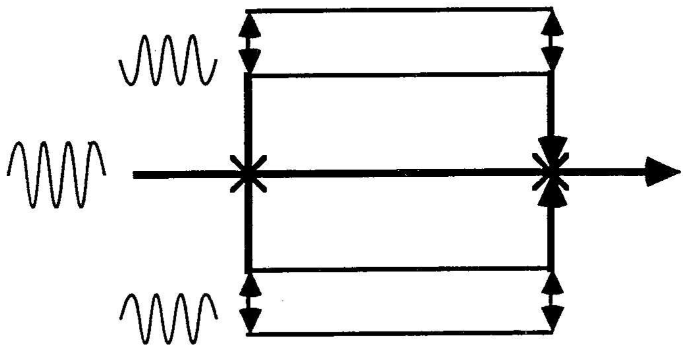


Figure 12. Waveguide system paradigm for wave emission in the absence of polarization modulation. A system in which the energy entering from the left is partitioned into three waveguide components. Two components progress along equal paths top and bottom which are modulated equally and in phase according to lengths. These two components are recombined with the third at the output. This is a system described by the mapping shown in Figure 11 and is linearly polarized. Again, corresponding to a 360° change in the top and bottom lines, is a 360° change in the middle resultant line. Note that this is *not* the case with the system described by Figure 8, the mapping of which is described in Figure 13.

It is also a consequence that if $\mathbf{A}^{-1}\mathbf{A} = \pm\mathbf{I}$, then the unitary transformation applied separately of either $\mathbf{A}$ or $\mathbf{A}^{-1}$ will not result in the identity matrix. But if the unitary transformation is applied twice, then the identity matrix *is* obtained. From this follows the curious properties of spinors that corresponding to *two* unitary transformations of, e.g., 2π, i.e., 4π, *one* null vector rotation of 2π is obtained. Such a bi-sphere correspondence is exhibited by the system paradigm of Figure 8 and is shown in Figure 13.

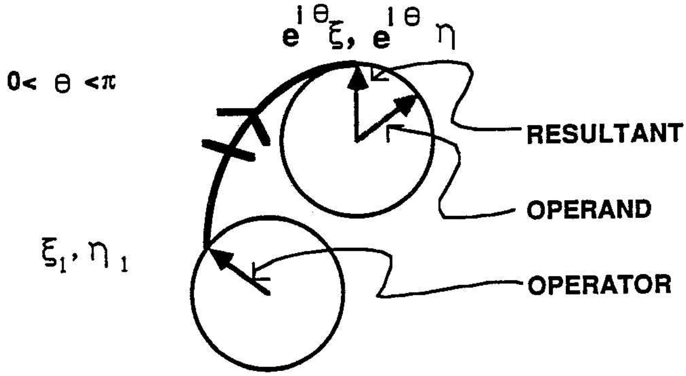


Figure 13. Bi-sphere representation of polarization modulation mapping and of the system paradigm of Figure 8 (or $\xi_1, \eta_1 \mapsto e^{i\theta}\xi, e^{i\theta}\eta$; $0 < \theta < \pi$) exhibiting the property of spinors that corresponding to two unitary transformations of e.g., 2π, i.e., 4π, a null rotation of 2π is obtained. Notice that for a 360° rotation of the resultant (i.e., the final output waveguide of the Figure 8 system), and with a stationary operand (i.e., the top throughput waveguide line of the Figure 8 system), the operator (i.e., the bottom waveguide throughput line of the Figure 8 system) must be rotated through 720°.

So far we have seen how a pair, (ξ, η), serve as coordinate for a null vector (Equ. (2.2)) which we shall call **K**. The W, X, Y, Z coordinates of **K** in Equ.s (2.7) are, however, redundant in that phase transformations: $\xi \mapsto e^{i\theta}\xi, \eta \mapsto e^{i\theta}\eta$, leave **K** unchanged. Thus the null vector **K** represents ξ and η only up to phase. What this means in terms of the system paradigm of Figure 8 can be demonstrated by reference to the Poincaré sphere of Figure 5. The ξ, η representation of the vector **C**, which is a **K** vector, gives no indication of the future position of **C**, i.e., the representation does not address the indicated hatched trajectory of the vector **C** around the Poincaré sphere. But it is precisely this trajectory which defines the particular polarization modulation for a specific wave. Stated differently: a particular position of the vector **C** on the Poincaré sphere gives no indication of its next position at a later time, as it can depart in any direction from that position when only ξ, η coordinates are given.

The geometrical structure associated with ξ, η which reduces this ambiguity up to a sign ambiguity is called a null flag [37]. A spinor, κ, can be represented, then, by not only a null direction indicated by

ξ, η or ζ, but also a real tangent vector $\mathbf{L}$ indicated in Figure 14.

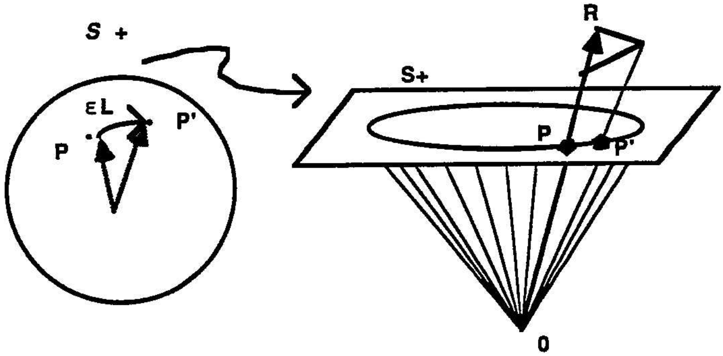


Figure 14. Relation of a trajectory in a specific direction on an output sphere $\mathbf{S}^+$ and a null flag representation on the hyperplane, W, intersection with S^+. After Penrose & Rindler [37].

In effect this represents the Poincaré vector *and its direction of change* (up to a sign ambiguity). A real tangent $\mathbf{L}$ of $\mathbf{S}^+$ at P is defined:

$$\mathbf{L} = \lambda\partial/\partial\zeta + \lambda^*\partial/\partial\zeta^*, \tag{2.15}$$

where λ is some definite expression in ξ, η. Making the choice $\lambda = -(1/\sqrt{2})\eta^{-2}$, gives:

$$\mathbf{L} = -(1/\sqrt{2})[\eta^{-2}(\partial/\partial\zeta) + \eta^{*-2}(\partial/\partial\zeta^*)], \tag{2.16}$$

whence we see that that knowing $\mathbf{L}$ at P (as an operator) means that the pair ξ, η is known completely up to sign, or, for any $f(\zeta, \zeta^*)$:

$$\lim_{\epsilon \to 0}[(1/\epsilon)(fp' - fp)] = \mathbf{L}f, \tag{2.17}$$

which *more accurately* (if not yet completely) describes the output of the system paradigm we are considering (Figure 8), or the periodic trajectory (polarization) modulation represented on the Poincaré sphere (Figure 5), *because the direction of change (in polarization modulation) is accounted for.*

More succinctly: the tangent vector $\mathbf{L}$ in the abstract space S^+ corresponds to a tangent vector L in the coordinate-dependent representation S^+ of $\mathbf{S}^+$ (cf. Figure 9). L is a unit vector if and only if K, the null vector corresponding to ξ, η, defines a point actually on $\mathbf{S}^+$. Therefore a plane Π' of K and L can be defined by:

$$aK + bL. \tag{2.18}$$

If it is required that $b > 0$, then (2.18) is a half-plane, Π, bounded by K. K and L are both spacelike and orthogonal to each other. Π and K are referred to as a *null flag* or *a flag*. The vector K is called the *flagpole*, its direction is the *flagpole direction* and the half-plane, Π, is the *flag plane*.

We have thus made some progress in defining the polarization modulation system of Figure 8 represented as a periodic trajectory of polarization modulation on a Poincaré sphere, i.e., the output wave from the system of Figure 8 is a *spinorial object*. A defining characteristic of a spinorial object is that it is not returned to its original state when rotated through an angle 2π about some axis, but only when rotated through 4π. Referring to Figure 13: *we see that for the resultant to be rotated through* 2π *and returned to its original polarization state, the operator must be rotated through* 4π. Thus, the waveguide system of Figure 8 produces spinorial objects as an output emission, and *that spinorial object exists in a different topological space from the input to the system, namely the covering space, due to the additional degree of freedom provided by the polarization bandwith which does not exist prior to modulation.* (A similar statement cannot be made for the operations shown in Figure 11 based on the system shown in Figure 12, which emits waves which are *not* polarization modulated).

For example, let us consider the constituent polarization vectors, Q^i, and let C be the space of orientations of Q^i. A spinorized version of Q^i can be constructed provided the space is such that it possesses a twofold universal covering space C^*, and provided the two different images, Q_1 and Q_2 existing in C^* of an element Q^i existing in C are interchanged after a continuous rotation through 2π is applied to a Q^i. In the case we are considering, C has the topology of the $SO(3)$ group, but C^* of the $SU(2)$ group (which is the same as that of the space of unit quaternions). There is thus a $1-2$ relation between the $SO(3)$ objects and the $SU(2)$ objects (Figure 15).

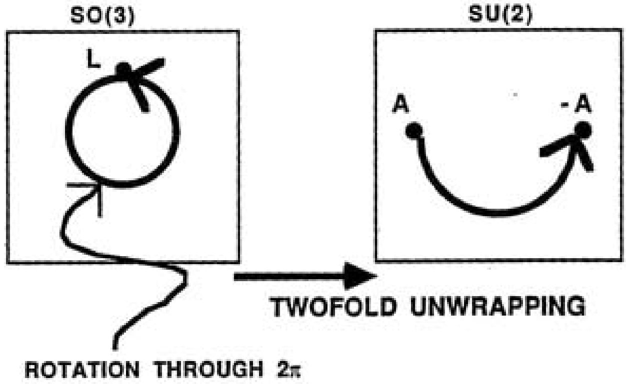


Figure 15. The left side ($SO(3)$) describes the symmetry of the top and bottom throughput waveguide lines in Figure 8; the left side describes the symmetry of the final waveguide output from the Figure 8 waveguide system. The additional degree of freedom of the system output (with respect to input) is provided by the polarization modulation bandwidth. After Penrose & Rindler [37].

We may take the Q^i to be polarization vectors (null flags) and C the space of null flags. The spinorized null flags, i.e., the output objects (waves) of the Figure 8 system, are elements of the space C^*, i.e., they are spin-vectors. Referring to Figures 13 and 15, we see that each null flag Q^i defines two associated spin vectors, which we label κ and $-\kappa$. A continuous rotation through 2π will carry κ into $-\kappa$ by acting on (ξ, η). On repeating the process, $-\kappa$ is carried back into κ, so:

$$-(-\kappa) = \kappa, \tag{2.19}$$

Any arbitrary spin-vector τ can be expressed as a linear combination of two spin vectors κ and ω:

$$\{\kappa, \omega\}\tau + \{\omega, \tau\}\kappa + \{\tau, \kappa\}\omega = 0, \tag{2.20}$$

where the brackets $\{\}$ indicates the antisymmetrical inner product. Thus any arbitrary polarization can be represented as a linear combination of spin vectors as shown in Figure 4.

A general representation of spin vectors in terms of components is obtained by adoption of the normalized pair, o, ι, as a spin frame:

$$\{o, \iota\} = 1 = -\{o, \iota\}. \tag{2.21}$$

As the components of a spin-vector in the spin frame are:

$$\kappa^0 = \{\kappa, \iota\}, \kappa^1 = -\{\kappa, o\} \tag{2.22}$$

Thus:

$$\kappa = \kappa^0 o + \kappa^1 \iota. \tag{2.23}$$

The flagpole of o is $(t+z)/\sqrt{2}$, with flag plane extending from this line in the direction of x; the flagpole of ι is $(t-z)/\sqrt{2}$ with flag plane extending from this line in the direction of $-x$. Therefore, both the output from the Figure 8 system and the Poincaré sphere exhibiting polarization modulation of Figure 5 (which is actually in Minkowski tetrad t, x, y, z form) finds a rigorous representation in the spin frame of Figure 16:

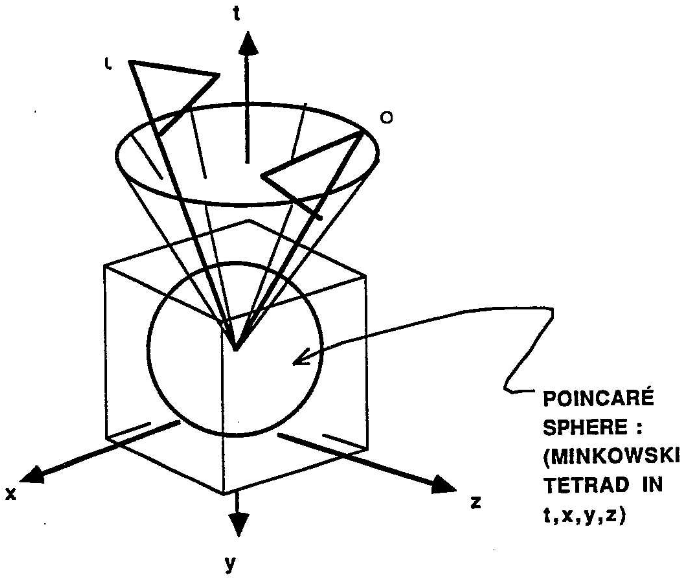


Figure 16. Spin frame representation of spin-vector by flag pole normalized pair representation of o, ι. After Penrose & Rindler [37].

The (antisymmetrical) inner product of two spin vectors can also be represented in the following way:

$$\{\kappa, \omega\} = \epsilon_{AB}\kappa^A\omega^B = -\{\omega, \kappa\}, \tag{2.24}$$

where the ϵ (or fundamental numerical metric spinors of second rank) are antisymmetrical:

$$\epsilon_{AB}\epsilon^{CB} = -\epsilon_{AB}\epsilon^{BC} = \epsilon_{AB}\epsilon^{BC} = -\epsilon_{BA}\epsilon^{CB} = \epsilon_{A}^{C} = -\epsilon_{A}^{C}, \tag{2.25}$$

and establish a canonical mapping (or isomorphism) between κ^B and κ_B:

$$\kappa^B \leftrightarrow \kappa_B = \kappa^A \epsilon_{AB}. \tag{2.26}$$

To recover the electromagnetic field a potential can be defined:

$$\Phi_a = i(\epsilon\alpha)^{-1}\nabla_a \alpha, \tag{2.27}$$

where α is a gauge:

$$\alpha\alpha* = 1 \tag{2.28}$$

and ∇_a is a covariant derivative $\partial/\partial x^a$ but without the commutation property.

Then the electromagnetic field in terms of the vector potential is:

$$F_{ab} = \nabla_a \Phi_b - \nabla_b \Phi_a, \tag{2.29}$$

and in terms of the spinor, ϕ_{AB}:

$$F_{ab} = \phi_{AB}\epsilon_{A'B'} + \epsilon_{AB}\phi *_{A'B'} . \tag{2.30}$$

The spinor, ϕ_{AB}, can also be defined in terms of the vector potential as:

$$\phi_{AB} = \nabla^{A'}_{A'(B^{\Phi}B)}, \tag{2.31}$$

where the brackets indicate the symmetry operation.

In terms of the three vector fields, E and B, F_{ab} is:

$$F_{ab} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix} \tag{2.32}$$

However, defining the field emitted by the Figure 8 waveguide system paradigm as a spin object still leaves some incompleteness in the description. An exact description is obtained by defining a spinor, say

ω^A, *which is constant under contravariant and covariant differentiation*, because an adiabatic polarization modulation system (Figure 8) with no energy dissipation implies such constancy. This required constancy is described by:

$$\nabla_{A'}^{(A}\omega^{B)} = 0, \tag{2.33}$$

which is called *the twistor equation* [38]. A solution for the twistor equation is:

$$\omega^A = \omega^{0A} - ix^{AA'}\pi^0_{A'}, \tag{2.34}$$
$$\pi_{A'} = \pi^0_{A'},$$

where ω^{0A} and $\pi^0_{A'}$ are constant spinor fields whose values coincide with those of ω^A and $\pi_{A'}$, respectively at the origin, i.e., ω^{0A} and $\pi^0_{A'}$ are the vectorial components of the output of the system of Figure 8 without polarization modulation.

Continuing the search for an exact description of the polarization modulated field emitted from the Figure 8 waveguide system paradigm, *twistor space* can be defined with elements:

$$Z^\alpha = [\omega^A] \tag{2.35}$$

which determines the output of Figure 8 *up to phase* (which is the necessary measure of exactness):

$$Z^\alpha \mapsto e^{i\theta}Z^\alpha. \tag{2.36}$$

If then we define:

$$p_a = \bar{\pi}_A\pi_{A'} \quad , \quad M_{ab} = i\omega^{(A}\bar{\pi}^{B)}\epsilon^{A'B'} - i\bar{\omega}^{(A'}\pi^{B')}\epsilon^{AB} \tag{2.37}$$

we can define, analogous to F_{ab}:

$$M_{ab} = \bar{\mu}^{AB}\epsilon^{A'B'} + \mu^{A'B'}\epsilon^{AB} \tag{2.38}$$

and an angular momentum twistor can be defined:

$$A_{\alpha\beta} = \begin{pmatrix} 0 & p_A^{B'} \\ p_B^{A'} & 2i\mu^{A'B'} \end{pmatrix} \tag{2.39}$$

$$\bar{A}_{\alpha\beta} = \begin{pmatrix} -2i\bar{\mu}^{AB} & p_{B'}^{A} \\ p_{A'}^{B} & 0 \end{pmatrix}$$

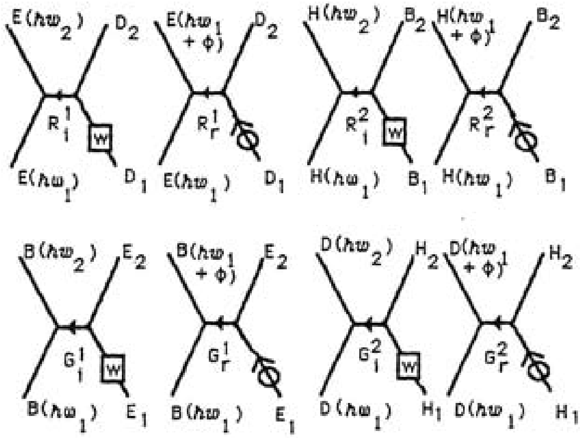
The anglar momentum twistor, $A_{\alpha\beta}$, is the description we have sought for the output of Figure 8, with the μ 's defined in terms of the spinors, ω^A, and $\pi_{A'}$, the p's in terms of the spinor, π_A, and the spinors themselves in terms of the e.m. field F_{ab} and the vector potential Φ.

We thus arrive at a definition of the output from the system of Figure 8 more rigorous than any based on Poynting's theorem alone. Unlike a tensor formalism, which provides an algebra, the spinor/twistor formalism provides a calculus. Furthermore, this definition permits the conclusion that the symmetry of such an electromagnetic field conditioned (polarization modulated) to be in twistor form is not $U(1)$, but $SU(2)$.

III. FIELD-METRIC INTERACTIONS

Given: (I) the magnetic monopole as an excited state of the neutrino and that the Majorana equation for the neutrino/monopole is a special case of the Dirac equation for the electron providing a (restricted) affine connection between the electromagnetic field and the space-time metric in that gauge invariance is demonstrated between the two (section I); and (II) the gravitational (neutrino) metric is composed of twistor forms and that the electromagnetic field, when polarization modulated, is in such a twistor form (Section II), we turn, in this section, to address the specific ways of obtaining both excited states (monopoles) and phase changes in the metric of the neutrino system, when the field is polarization modulated, i.e., the ways in which field and metric can interact. In this field-metric interactive condition, the field is assumed to be in twistor form, i.e., polarization modulated. Thus those cases, which do not address energy transfer but rather entropy changes, can be addressed.

Figure 17 shows the energy-momentum relations for the electromagnetic field which includes both electric and magnetic current densities, g_e and g_m [43-45], i.e., with a reformulation of the Maxwell equations to include magnetic charge. ϵ_r and ϵ_i and μ_r and μ_i are the real and imaginary parts of the dielectric constant and magnetic permeability respectively. Other real and imaginary components are as defined in the figure. The square with inserted "w" indicates an energy exchange. The circle with arrow indicates an excursion in metric parameter space, i.e., what we shall call an entropy exchange. Thus the figure describes the electromagnetic field affinities under polarization modulation in the case only of entropy exchanges.



R_i^1: $\partial \varepsilon_i/\partial t$ $\qquad$ R_i^2: $\partial \mu_i/\partial t$

R_r^1: $\partial \varepsilon_r/\partial t$ $\qquad$ R_r^2: $\partial \mu_r/\partial t$

G_i^1: $\mathrm{Im}[-\nabla \times E] = g_m$ $\qquad$ G_i^2: $\mathrm{Im}[\nabla \times H] = g_e$

G_r^1: $\mathrm{Re}[-\nabla \times E] = \partial B/\partial t$ $\qquad$ G_r^2: $\mathrm{Re}[\nabla \times H] = \partial D/\partial t$

$$\begin{aligned} \nabla \times H &= \partial D/\partial t + g_e \\ -\nabla \times E &= \partial B/\partial t + g_m \\ \nabla \cdot D &= \rho_e \\ \nabla \cdot B &= \rho_m \\ D &= \varepsilon E \\ B &= \mu H \end{aligned}$$

Figure 17.

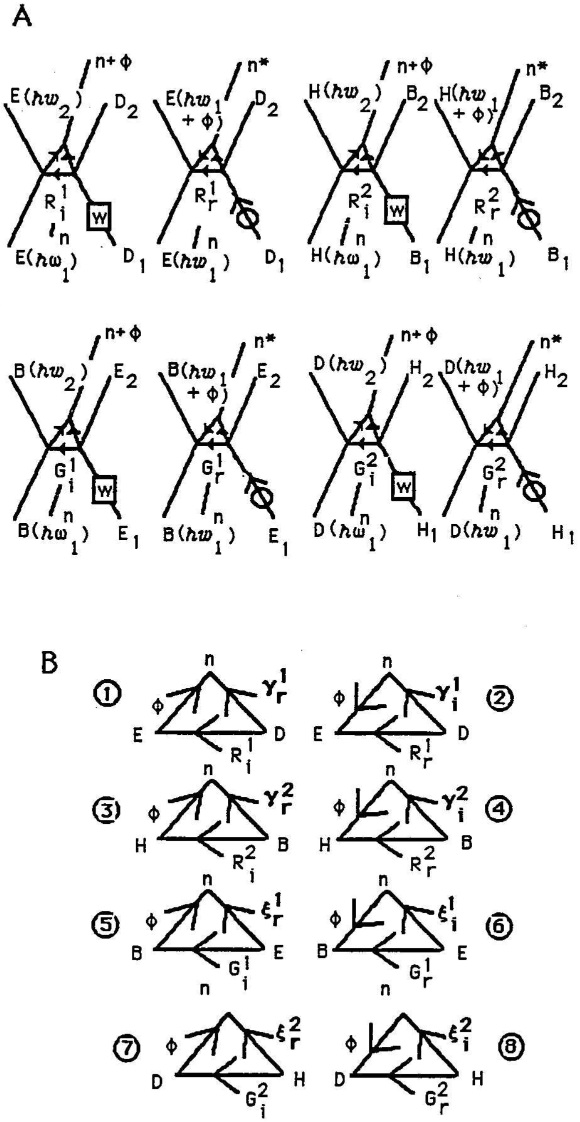


Figure 18.

Figure 18A describes the same affine relations of the electromagnetic field and the neutrino or metric field of space-time. The excited state of the neutrino (the magnetic monopole) for an entropy exchange between fields and involving polarization modulated fields is represented as: $n*$. The phase-changed state of the neutrino for an energy exchange between fields is represented as: $n + \phi$. In Figure 18B the connection coefficients and torsion forms are shown abstracted from Figure 18A. Forms 2 and 4 describe the conditions for the occurrence of Berry's phase [7-18], i.e., transport around a closed curve in the presence of a vector potential in parameter (momentum) space. Forms 6 and 8 describe the conditions for the occurrence of the Aharonov-Bohm effect [48-61] (and the related Altshuler, Aranov and Spivak (AAS) effect [46,47]), i.e., transport around a closed curve in the presence of a vector potential in ordinary space. Both the Berry and Aharonov-Bohm effects result in a change in phase of the wavefunction.

The theoretical justification for implicating neutrino states, i.e., states of the metric, in field energy transfer lies in the requirement for *entropy-energy balance conservation.* The need for conservation of information-energy during energy transfer has been shown by Balian [62-8] and Remler [69-70]. Balian has shown that the Hilbert space representation of quantum exchanges is discrete and does not retain all aspects of the Liouville-Hermite representation which is continuous and it is in the continuity wherein the information content lies. Furthermore, the symmetry of the laws of space-time (the metric) and of angular momentum was indicated in the principle of reciprocity of Born [71] and, more recently, by Ali [72].

The required conservation condition for energy-entropy balance is:

$$\Delta S + \Delta E = \Xi_{S+N}. \tag{3.1}$$

In Figures 17 and 18 an *energy transferral* is distinguished, represented by a "w", from: an *entropy transferral*, represented by a circle and arrow. Thus, in Figure 18B, the exchanges 1, 3, 5 and 7 are energy transferrals, and 2, 4, 6 and 8 are entropy transferrals. Now describe these transferrals in matrix notation, and designate A and B to be fields, and C to be discrete, noncontinuous, space-time, metric elements. If the case of *energy transfer* is considered, i.e., 1, 3, 5 and 7 in Figure 18B, and letting an asterisk "$*$" indicate an excited state, then the total representation is:

$$\begin{pmatrix} A* & A & A \\ A & A* & A \\ A & A & A* \end{pmatrix} \to \begin{pmatrix} A & & \\ & A & \\ & & A \end{pmatrix} \tag{3.2}$$

$$\begin{pmatrix} B & & \\ & B & \\ & & B \end{pmatrix} \to \begin{pmatrix} B* & & \\ & B* & \\ & & B* \end{pmatrix}$$

$$\begin{pmatrix} C & & \\ & C & \\ & & C \end{pmatrix} \to \begin{pmatrix} C & C & C \\ C & C & C \\ C & C & C \end{pmatrix}$$

If the A and B fields are considered independently of the metric element C, then although $E_A - E_B = 0$ and energy is conserved, yet $S_A - S_B \neq 0$, so consideration of the A and B fields alone does not indicate conservation of Ξ_{S+N}. However, if the fields A and B, and *also* the metric element C, is considered, then this energy transfer *is* conserved with respect to both E, S and Ξ_{S+N}.

Considering now the case of *entropy transfer*, or 2, 4, 6 and 8 of Figure 18B. The representation is:

$$\begin{pmatrix} A* & A & A \\ A & A* & A \\ A & A & A* \end{pmatrix} \to \begin{pmatrix} A & & \\ & A & \\ & & A \end{pmatrix} \tag{3.3}$$

$$\begin{pmatrix} B & & \\ & B & \\ & & B \end{pmatrix} \to \begin{pmatrix} B & B & B \\ B & B & B \\ B & B & B \end{pmatrix}$$

$$\begin{pmatrix} C & & \\ & C & \\ & & C \end{pmatrix} \to \begin{pmatrix} C* & & \\ & C* & \\ & & C* \end{pmatrix}$$

If the A and B fields are considered independently of the metric element C, then although $S_A - S_B = 0$ and entropy is conserved, yet $E_A - E_B \neq 0$, so consideration of the A and B fields alone again does not indicate conservation of Ξ_{S+N}. The lack of energy conservation in these situations is usually ascribed to the energy loss to the medium through which the field passed while being "conditioned" by the medium into angular momentum form. This energy loss is clearly accounted for in the adiabatic waveguide system of Figure 8 –the action of the energy-expending lines producing the polarization modulation is described by the $C \to C*$ condition of (3.3). The energy loss occurring in an electric field (i.e., an A field in (3.3)) when conditioned to generate a magnetic field (i.e., a B field in (3.3)), when the electric field is rotated by a medium, is balanced by an energy gain in a C element. That is to say, the excited state of n in 2, 4, 6 and 8 of Figure 18B is due to entropy-energy balance requirements. This excited state is also a metrical concept of statistical mechanics rather than a force or energy concept of quantum mechanics. In this way information theory notions define the space-time metric (cf. also Harmuth [73]).

The concept of entropy-energy balance denies both the possibility of the transfer of energy without a balancing transfer of entropy, and the transfer of entropy without a balancing transfer of energy. The entropy-energy balance does not, however, constrain two-field interactions, but rather: three-object interactions of two fields and their metric. The rationale for the unification of gravity (the metric) and forces lies, therefore, in quantum *and* statistical mechanical considerations, rather than quantum mechanical considerations alone. Unification of gravity and electromagnetic forces therefore addresses the energy-entropy connection. Thus the "excited state" of the neutrino (i.e., the magnetic monopole) is the result of the translation of energy into a form not usable as force but rather into metric "distortion" or reweighting. According to the viewpoint presented here, the magnetic monopole is not a "force" or energy concept. Therefore, it is not appropriate to search for the magnetic monopole as if it were.

The following remark is appropriate. After the eighteenth century physicist, Pierre de Maupertius, survived an expedition to Lapland to verify Newton's theory on the flattening of the earth near the poles, Voltaire joked to him: "Vous avez confirmé dans les pleins d'ennui ce que Newton connût sans sortir chez lui".

CONCLUSIONS

The analysis was based on the system of Figure 2 but can be generalized to Figure 3 and higher order systems, i.e., $\partial^n \phi / \partial t^n$, $n = 1, 2, 3, ...$ Extrapolation to higher order moments is thus possible. For example, Curtis [74] demonstrated that it is possible to obtain multipole spinors as elements of the tensor algebra over the ten dimensional vector space of spinors satisfying the twistor equation. These multipoles are identical to those of Geroch [75,76].

The fields generated by the polarization modulating system of Figure 2 are broken in symmetry with respect to a system with feedback. Similarly, the system of Figure 3 exhibits broken symmetry with respect to the system of Figure 2. Thus, the generalized system exhibits supersymmetry. It is significant, therefore, that supersymmetry was first realized nonlinearly on Majorana spinors. The nonlinear form of the O'Raifeartaigh model [77] is the simplest interacting model consisting of 3 chiral superfields and their anti-chiral partners which manifest breaking of supersymmetry [78]. The spontaneous breakdown of supersymmetry and internal symmetry has been studied for the interaction of N chiral scalar superfields [77]. It is now known that a spontaneous breakdown of supersymmetry does not occur for the interaction of a chiral scalar superfield with itself, but that it may occur for the interaction of a nonchiral, or gauge, superfield with two scalar superfields. The Lagrangian for N chiral scalar superfields in the absence of any gauge fields is a simple generalization of the Wess-Zumino Lagrangian [79] for one chiral scalar superfield [77]. Future work will address these results with respect to the generalized $\partial^n\phi/\partial t^n$, $n = 1, 2, 3, ...$ polarization modulated system.

The twistor formalism is only exactly applicable to the electromagnetic field conditioned by polarization modulation. Stated differently: a polarization modulated wave is only described exactly by an angular momentum twistor. The electromagnetic field without polarization modulation conditioning is well-known to be of $U(1)$ symmetry. After polarization modulation conditioning, it is of $SU(2)$ symmetry and thus of non-Abelian Yang-Mills form. Conditioning the $U(1)$ electromagnetic field into $SU(2)$ form in effect adds a degree of freedom to the field.

The major conclusion is that the relation of local fields and their metric is governed by an energy-entropy conservation condition modeled by an adiabatic polarization modulation waveguide. Experimental testing of this theory, in particular, the relations shown in Figure 18, can procede at radar, infrared and visible frequencies. While the necessary speed (Δt for $0 - 360°$ traversal, cf. Figure 7) of polarization modulation at optical frequencies (in the picosecond range) is somewhat difficult to obtain, it is easily obtained at radar and infrared frequencies.

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RESUME. Une nouvelle description du monopôle est présentée. Elle est fondée sur : 1) une séparation entre les champs, distincts du milieu dans lequel ils existent, et les relations entre champs et milieu, 2) le monopôle magnétique défini comme un état excité du neutrino (Lochak). A l'aide du formalisme des torseurs, il est démontré que l'interaction entre les champs électromagnétiques et la métrique d'espace-temps (métrique gravitationnelle ou éther) est un difféomorphisme du second ordre du potentiel vecteur $\vec{A}$ *sur la métrique, donné par le concept de paire neutrino-antineutrino (Lochak).*