Realistic explanation of wave-corpuscle duality

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ABSTRACT. Realistic explanation of wave-corpuscle duality of particles is given qualitatively and quantitatively, based on proper methodology. The experiments on particle diffraction and Wheeler's delayed-choice experiments unambiguously demonstrate that an individual particle has no wave properties, both particle diffraction and particle interference being of essentially statistical nature. It is shown that those wave phenomena are determined by the same formal wave equation that holds for any genuine physical waves, but is a probability-related one in essence.

RESUME. Une explication réaliste de la dualité onde-corpuscule des particules est donnée sur les plans qualitatif et quantitatif, à partir d'une méthodologie correctement adaptée au problème. Les expériences de diffraction de particules et les "delayed choice experiments" de Wheeler montrent sans ambiguïté qu'une particule individuelle n'a pas de propriétés ondulatoires, la diffraction et l'interférence de particules étant toutes deux fondamentalement de nature statistique. Nous montrons que ces phénomènes ondulatoires obéissent à la même équation formelle qui régit toute onde réellement physique, tout en étant reliée aux probabilités de façon essentielle.

1. Introduction

The issue of wave-corpuscle duality of particle behavior has inevitably arisen, when the wave properties of particles, suggested by L. de Broglie, had been confirmed experimentally. Particles seem to have, according to the experiments, both corpuscular and wave properties. The former are revealed when particles move in sufficiently definite paths, the latter manifest themselves through such typical wave phenomena as diffraction and interference. One usually concludes from the experiments that both are inherent in every individual particle, and it is precisely this inconceivable to common sense conclusion which causes the bewilderment : How can one and the same thing both move as a particle and propagate as a wave ?

The issue could be settled neither in the framework of classical physics nor in that of conventional quantum mechanics (CQM), and has provoked long-lasting debates that have not ceased till now [1]. The pile of works dealing with it and related matters is getting bigger and bigger, involving new ideas, thought experiments, real experiments, etc., and no end seems to be foreseen to this even ongoing process. Many works reflect the perplexities related to the understanding of quantum mechanics (QM) and the confusion caused by Bell's inequalities, but none of them answers the vital question of why QM yields correct predictions, whereas it seems incompatible with the idea of real existence of the outer world. And that is not at all by accident.

The fact of the matter is that taken for granted by most physicists CQM actually consists of two quite different parts. The first one responsible for predictions contains specific mathematical methods used in practical calculations and is basically correct. The second part of CQM, which is its interpretation as the theory of measurements, is wrong, the defects of CQM as a whole being of essentially methodological nature [2].

The importance of proper methodology for clearing up seemingly unsolvable issues may perhaps be well-known to many. Of special interest now is the case of QM. Recently the methodological clue was found that made it possible to get a straightforward construction of *probabilistics* (the science of probability) and *probabilistic physics* (the application of probabilistics to physics), starting with the explicit definition of probability [2]. Two particular interconnected domains of probabilistic physics are *classical statistical mechanics* and *probabilistic quantum* mechanics (PQM). While the mathematics used in PQM is basically the same as in CQM, PQM has made it possible to clear up many issues whose solution might seem rather hopeless [2]. The methodological principle meant is as simple as one only can be, and perhaps just because of that its importance (were it known) has slipped great thinker's attention. It reads : It is necessary to distinguish *strictly* between *concrete objects* and *abstract objects.*

Unlike concrete objects, abstract objects do not exist in reality and merely are images of the former ones. But the names of both generally coincide, which often leads to a muddle and unceasing debates. This fact explains why that methodological principle is of such an extreme importance.

Any experiment is performed on concrete objects. Hence, experimental statistics deals just with them. Theoretical considerations obviously concern abstract objects. Hence, so does probability theory. Experimental statistics is linked to probability theory by the fact that statistical (relative) frequencies of events are, in a large enough number of random tests, the approximate values of the pertinent probabilities.

PQM's methodology has easily explained (qualitatively) the wave-corpuscle duality of particle behavior in a fully realistic way [2]. A careful examination of particle diffraction (PD) removes the above mentioned bewilderment. It turns out that there is no such *one* (real) thing that both moves as a particle and propagates as a wave. It is precisely every *concrete* particle which moves as a particle, as it should, whereas no real thing involved in PD does propagate as a wave. The so-called "wave properties" are connected with the probability distributions related to pertinent *abstract* particles, which are revealed (approximately) through the experimental statistical distributions of the corresponding concrete particles, in a long enough series of experiments (random tests). If a few *concrete* particles only are used in a diffraction experiment, no distinguishable diffraction pattern appears. So, only (real) corpuscular properties are inherent in every individual (*concrete*) particle, whereas the (unreal) "wave properties" are related to *abstract* particles. And this fact removes the above stated challenge to common sense.

In the present paper, PD is discussed in more detail, and its quantitative explanation is outlined. The particle interference (PI), which can be given a similar (realistic) qualitative explanation, is examined in connection with Wheeler's delayed-choice experiments [3], and its quantitative explanation also is outlined.

2. The nature of PD

Diffraction is known to occur when a series of any kind of running waves of one and the same frequency encounters impermeable obstacles, the diffraction pattern being determined mainly by the diffracting-system-incident-beam geometry and the wave length of the wave involved, no matter what its nature is. For a genuine physical wave process, the diffraction pattern, i.e., the spatial intensity distribution, is governed mainly by the pertinent wave equation and boundary conditions. The wave equation for any wave process is of one and the same form

$$\nabla^2 f = (1/v_p^2)\partial^2 f/\partial t^2, \tag{1}$$

where ∇ is del operator, ∇^2 is Laplacian, v_p is the phase velocity of waves, and f is a pertinent quantity whose magnitude squared, $\mid f \mid^2$, determines the diffraction pattern : the intensities in the latter are proportional to the corresponding values of $\mid f \mid^2$. Hence, the diffraction patterns should also be alike for any wave processes, provided the respective boundary conditions coincide, even though the nature and type of the waves may be altogether different, as in the cases of acoustic and light waves, for instance. This should obviously be true for PD as well, which brings up the question : What is the nature of PD ? If the conventional belief that wave properties are inherent in an individual (concrete) particle is correct, then another question arises : Does it mean that a concrete particle has some property f obeying (1) ?

To answer this question, let us recall that a diffraction pattern is given by the solution to (1). Therefore, f should have first- and second-order derivatives with respect to spatial coordinates and, hence, must be continuous in space. If now the answer to the question is "yes", the diffraction pattern must remain the same, regardless of the intensity of the beam, gradually weakening as the beam does. But that is not the case, as the experiments show.

The well-known fact that a diffraction pattern does not depend on the particle beam intensity should be understood correctly. That is only true when the total energy diffracted is the same. This means that the weaker the beam, the longer the experiment duration should be, in order to get the same picture. But if only a few concrete particles are used, no continuous diffraction picture appears at all. Instead, some point marks on the display-screen can be noticed, which are left by the concrete particles involved. Thus, the answer to the above question is "no". No specific wave property f is possessed by a concrete particle, which might obey (1). And what is more, actually there is no diffraction pattern for particles at all in the true sense of the notion. When the number of concrete particles is small, the picture obtained is not obviously a diffraction patter –it is not continuous, and the point marks left by the concrete particles are somehow dispersed over the screen. When the number of the concrete particles involved is large enough, the picture looks like a diffraction pattern, but actually it is not, for it consists of point marks as well. The experimental made-of-points diffraction pattern may, however, be regarded as an approximation to some continuous one which represents the solution to a certain wave equation (1) involving a certain quantity f. This is true for any particles, including photons.

New questions now arise : What is equation (1) in case of particles ? Does it describe some physical process or something else ? What is the meaning of the quantity f there ?

Before answering the questions, let us dwell on PD-experiments a little more. An experiment performed with a very weak intensity beam should last continuously for some long time T in order to get a diffraction pattern of desirable brightness. Suppose the experiment is interrupted a large number n of times, so that in every time interval τ_j ($j = 1, 2, ..., n + 1$) only a few concrete particles pass through the diffractometer. If the conditions of the experiment are the same for all τ_j, and $\Sigma\tau_j = T$, then the final result will be practically the same as for the continuous experiment. This result does not depend on the durations of the intermissions between the intervals either, and if it is possible to record the results for each interval on a separate display-screen and then combine them for all the intervals, the pattern obtained will practically coincide with one for continuous experiment of the same duration, even though on every screen separate point marks only may be found. And what is more, suppose the experiment is performed on a separate diffractometer in every interval τ_j. If the conditions of the experiment are the same for all $n+1$ diffractometers, the combined result will also reproduce practically the same diffraction pattern.

These considerations show that a diffraction experiment on particles looks like a typical statistical experiment, in which concrete particles, being subjected to independent random tests, hit different points on the screen by chance ; the statistical (relative) frequency of hitting an area in the vicinity of a point being an approximate value of the probability that a particle will do so. Thus, the quantitative explanation of PD is a probabilistic problem in physics and should, hence, be treated by methods of probabilistic physics, particularly by those of PQM [2].

3. Quantitative treatment of PD

In a PD-experiment we deal with a beam of *concrete* particles encountering impermeable obstacles and the *experimental statistical distribution* of the coordinates of diffracted *concrete* particles. In the theoretical treatment of PD we are interested in finding the pertinent *probability* *distribution* of the coordinates for the corresponding *abstract* diffracted particle. It follows directly from PQM's reasoning [2] that the probability distributions of physical quantities for an abstract physical system, which conform to real motion of the corresponding concrete physical systems, are determined by the solutions to the pertinent Schrödinger equation

$$\hat{E}\psi = \hat{H}\psi \tag{2}$$

In the PD-experiment, a concrete particle is free in any part of space available to it. Hence, (2) should refer to abstract free particle. In the general case, the Hamiltonian function H of a concrete (classical) free particle is

$$H = c(p^2 + m_0^2 c^2)^{1/2} \tag{3}$$

where m_0 and p are its rest mass and momentum magnitude, respectively, and c is the ultimate velocity of special relativity. Since in the coordinate-time representation the operators for particle moment p_α ($\alpha = x, y, z$) are $\hat{p}_\alpha = -i\hbar\ \partial/\partial\alpha$ and $\hat{E} = i\hbar\ \partial/\partial t$, (2) takes the form

$$c(-\hbar^2\nabla^2 + m_0^2 c^2)^{1/2}\psi = i\hbar\ \partial\psi/\partial t \tag{4}$$

We get from this, for $\partial/\partial t$ and $\partial/\partial\alpha$ commute ($\alpha = x, y, z$),

$$\hbar^2 c^2 \nabla^2 \psi = m_0^2 c^4 \psi + \hbar^2 \partial^2\psi/\partial t^2 \tag{5}$$

Recall now that diffraction refers to a stationary state of the abstract free particle, determined by a definite value E of energy and, hence, the corresponding definite value p of the momentum magnitude, connected with E by $p^2 = (E^2 - m_0^2 c^4)/c^2$. Therefore, $\hbar^2\partial^2\psi/\partial t^2 = -E^2\psi$, and $\psi = -E^{-2}\hbar^2\partial^2\psi/\partial t^2$. Substituting this for ψ in the first term of the right-hand side of (5), we obtain

$$\nabla^2\psi = c^{-2}(1 - m_0^2 c^4/E^2)\partial^2\psi/\partial t^2 \tag{6}$$

By introducing $(E^2 - m_0^2 c^4)/c^2 E^2 \equiv 1/v_p^2$, we finally get

$$\nabla^2\psi = (1/v_p^2)\partial^2\psi/\partial t^2, \tag{7}$$

i.e., the wave equation (1) with

$$v_p = E/p \tag{8}$$

defining the phase velocity and $f = \psi$.

Thus, we now have answers to all the queations raised in Sect. 2. Equation (7) is one we have been seeking. It does not describe a physical wave process, but is related to the probability distribution of particle coordinates in a stationary state of the corresponding abstract particle with definite values of energy and momentum magnitude. Function ψ in it is an auxiliary mathematical quantity whose magnitude squared, $\mid \psi \mid^2$, determines the probability density ρ of particle coordinates, according to the equation : $\mid \psi \mid^2 / (\psi, \psi) = \rho(x, y, z)$.

Equation (7) is valid for any particles, for it does not contain any parameters characterizing their nature. It follows from the corresponding Hamiltonian equations, (3), and $E = H$ that the constant velocity magnitude $v = (v_x^2 + v_y^2 + v_z^2)^{1/2}$ of a particle related to the state under consideration is

$$v = [(\partial H/\partial p_x)^2 + (\partial H/\partial p_y)^2 + (\partial H/\partial p_z)^2]^{1/2} = c^2 p/E \tag{9}$$

In view of this and (8), we get

$$vv_p = c^2 \tag{10}$$

Equation (7) is both a wave equation and a modified Schrödinger equation (4). Any solution to (4) is a solution to (7) as well. Consider, for instance, a solution to (4) of the form

$$\psi(x, y, z) = A \exp[2\pi i(pr - Et)/h], \tag{11}$$

where p and r are the momentum vector and radius vector of a particle, respectively. Then the pertinent solution to (7) is

$$\psi(x, y, z) = A \exp[2\pi i(nr/\lambda - \nu t) \tag{12}$$

where λ and ν are the "wave length" and "wave frequency", respectively, and n is the unit vector in the "propagation direction". By comparing these two solutions, which actually are merely two different representations of one and the same solution in respective terms, we get $\lambda = h/p$ – the famous de Broglie relation defining the "wave length"– and $\nu = E/h$, which is the definition of "frequency". In view of (8), the usual condition

$$v_p = \lambda\nu \tag{13}$$

is obviously satisfied.

Important practical methods of the diffraction pattern calculation are based on the Huygens-Fresnel principle applicable to any physical waves. At the same time, since a diffraction pattern is determined by the pertinent wave equation (1) with boundary conditions, i.e., by the mathematical description of the physical system involved, to find the diffraction pattern it would be enough to solve the corresponding mathematical problem. However, the problem is generally very complicated, and only some simple cases can be solved analytically. It is essential, therefore, that the practically verified principle of Huygens-Fresnel has been proved to result directly from the corresponding wave equation [4]. Hence, the practical methods based on that principle can be regarded as the techniques for the approximate solution of the corresponding mathematical problem, regardless of its origin. This means that the calculation of PD-patterns for any particles can also be performed with the aid of these methods that have originally been developed for the cases of sound and light [5], [6]. This concerns also the case of the famous "two-slit Gedankenexperiment" which is a typical diffraction experiment as well [7].

4. Wheeler's delayed-choice experiment and the nature of PI

The particular delayed-choice experiment proposed by Wheeler [3] deals with photons, but its possible results are to be valid for any particles. Here are its outlines.

An appropriate device allows one to perform the experiment in two different versions which will be called for short : "open" and "closed". In the "open" version, a photon encounters a beam splitter (BS) and can either pass through it or be reflected at it in a perpendicular direction. A detector at the end of each route counts the number of photons that have passed it. This version, hence, displays the corpuscular properties of photons.

In the "closed" version, appropriate mirrors put on both above routes make them change their directions and intersect at a second BS, which each photon can again either pass through or be reflected at in a perpendicular direction. It turns out that the ratio of the numbers of photons passing both final routes, registered by corresponding detectors, depends on the difference between their optical paths (or phases), which implicates a kind of interference. This version thus reveals the wave properties of photons. The fact that interference occurs when even one only photon is in the apparatus at any time, is regarded usually as the evidence that wave properties are possessed by an individual photon (particle) which interferes with itself and, hence, should travel both routes simultaneously.

The results obtained from both versions seem incompatible : the "open" version demonstrates that a photon can travel only one of two available routes ; the "closed" one implies that it should do both routes simultaneously.

Wheeler's delayed-choice experiment was supposed to answer the question : What is in reality ? Does a photon travel only one of two available routes or both ? His idea was to choose which of two versions was to be performed by moving the second BS in or taking it out, after the photon had already encountered the first BS. In Wheeler's words, this will have "an unavoidable effect on what we have a right to say about the already past history of that photon".

Subtle delayed-choice experiments have been realized on photons [8], whose results may seem strange to one who does not distinguish between concrete and abstract objects ; but one who does can easily predict them. It has turned out that the outcome does not depend on whether the choice is made before *or* after the photon has encountered the first BS. This means, if we recall that the experiments are carried out on *concrete* photons, that "interference" occurs while each *concrete* photon travels just one path. This *fact* decisively disproves the above mentioned common prejudice that in case of interference each photon travels both routes simultaneously and interferes with itself. And that is the *experimental* answer to Wheeler's question.

So, the delayed-choice experiments realized also prove unambiguously that *concrete* photons have corpuscular properties only. But what then is the nature of the interference-like effects observed in those experiments ? To answer this question we should recall that these effects are revealed through the dependence of the ratio of the numbers of concrete photons passing the two final routes on the performance of the "closed" version. This indicates that "interference" is of essentially statistical nature, just like in the case of PD, the experimental statistical data obtained on concrete photons conforming to pertinent probability distributions related to the corresponding abstract photons. Different performances of the "closed" version result in different experimental statistical distributions corresponding to different probability distributions, and that is the qualitative explanation of the "interference" in the case under consideration.

Now, since the statistical data in delayed-choice experiments are registered by detectors disposed on the final routes *behind* the pertinent devices, the particular processes proceeding within the devices themselves are of no immediate interest to us –the latter are merely "black boxes". Of interest to us are only the parts of the "open" and "closed" systems closely adjacent to the corresponding detectors, and photons can be regarded as free particles there. This fact drastically facilitates the treatment of the problem and allows one to deal with the general case valid for any particles.

The delayed-choice experiments can schematically be represented now as follows. Let two interswitchable devices I and II correspond to the above "open" and "closed" versions, respectively. Device I emits concrete free particles of one kind in two different directions 1 and 2 *at random.* The numbers of particles emitted for a certain time interval in each direction is recorded by the corresponding counters C_1 and C_2. If the time interval is large enough, the ratios of these numbers to their sum give the measured (approximate) values of the corresponding probabilities. The concrete free particles recorded belong to the corresponding subsets A_1 and A_2 of the whole set $A = A_1 + A_2$ of particles emitted by device I, whereas the probabilities are related to the pertinent abstract free particles. If the emission is so weak that there is a long enough time interval between any two consecutive counts, the records made by both counters never coincide in time, and it is precisely this fact which has been found when performing the "open" version on photons.

Device II, whose use results in some interference-like effects, emits concrete free particles of the same kind in two different directions 3 and 4 *at random*, these particles belonging to the corresponding subsets A_3 and A_4, respectively, of the whole set $A' = A_3 + A_4$ of particles emitted by the device. The relative numbers of concrete particles recorded by the corresponding counters C_3 and C_4 for a long enough time interval are the measured (approximate) values of the corresponding probabilities related to the abstract free particles for device II.

In performing the experiment, one can arbitrarily switch either device on at any time, the results obtained obviously being determined by which one of them was switched on at the very last moment when the recorded concrete particles were emitted. This explains the primary result of the delayed-choice experiments realized, namely, that delaying one's choice of the experimentation mode has no effect on the outcome.

5. Quantitative treatment of PI

If subset A_3 in Wheeler's case consists of concrete photons that either passed through or were reflected at both BS's, then subset A_4 does of concrete photons that either passed through the first BS and were reflected at the second one or *vice versa*. The interference-like effects on either final route can then be treated as if the corresponding concrete photons have been emitted by two different sources.

Considering the general case of any particles, we assume there are two sources, a and b, emitting *at random* concrete free particles of one kind (with equal energies E and, hence, equal momentum magnitudes p), which are elements of two respective sets S_a and S_b. A question now arises : What is the probability distribution of the free particle coordinates in this case ?

To answer it one should find the ψ-function describing the state of the abstract free particle corresponding to the set $S = S_a + S_b$, which conforms to the same values of E and p.

If there is only one source, a or b, and, hence, only one set, S_a or S_b, the state of the corresponding abstract particle is described by ψ_a or ψ_b, respectively, which are of the form given in (11)-(12). The ψ-function sought should be a solution to the Schrödinger equation (4) with the same values of E and p, and hence, a solution to the wave equation (7) with the pertinent values of λ and ν. At the same time, in the absence of either source, it should give a description of the state of the remaining abstract particle. The ψ-function $\psi = \psi_a + \psi_b$ satisfies both requirements and, hence, can be taken as the one sought.

Let us write $\psi_a = A\exp(if_a)$; $\psi_b = A\exp(if_b)$. Then

$$| \psi |^2 = 2 \mid A \mid^2 [1 + \cos(f_a - f_b)] \tag{14}$$

Now, in the general case, $f = 2\pi(nr/\lambda - \nu t) + \varphi$, where φ is a phase. Hence, $f_a - f_b = 2\pi(n_a - n_b)r/\lambda + \Delta\varphi$, where $\Delta\varphi = \varphi_a - \varphi_b$, and we have for the probability distribution of particle coordinates

$$\rho \propto \{1 + \cos[2\pi(n_a - n_b)r/\lambda + \Delta\varphi]\}, \tag{15}$$

instead of a uniform probability distribution in case of one source. And this quantitative explanation of PI is in line with all the experimental results known. In Wheeler's experiments, $n_a = n_b$. Hence, for any r,

$$\rho \propto 1 + \cos \Delta\varphi \tag{16}$$

6. Concluding remarks

The realistic explanation of PD and PI outlined above provides quantitative probabilistic predictions concerning both interconnected phenomena. The phenomena themselves turn out to be of merely statistical nature –they can be revealed only when a large enough number of concrete (individual) particles are subjected to the corresponding properly arranged random tests. The noticed in Sect.1 connection between probability theory and experimental statistics predetermines that the experimental statistical distributions of any random variables, in particular, of any physical quantities, should be in line with the calculated probabilistic predictions, provided the latter are correct. If, for instance, the probability for a particle to hit some spot on the screen is exactly zero, no concrete particle can do so. And whenever the PD and PI experiments are carried out, their results indeed are generally in agreement with the predictions, which corroborates the correctnes of the probabilistic treatment presented here.

It should be emphasized again and again that no mystic "wave-corpuscle duality" inherent in a concrete particle exists in reality. A concrete particle possesses corpuscular properties only, whereas the so-called "wave properties" are connected with the probability distribution of particle coordinates in the pertinent state of the corresponding abstract particle, and hence, refer to the latter. Thus, there is no contradiction between the corpuscular properties of concrete particles which do exist in the real world and the "wave properties" related to an abstract particle that only exists in our imagination (in our mind), the wave equation (7) being related to the latter, but not to some genuine physical process.

One should also notice that in a stationary state described by a solution to (4), a definite value of the momentum magnitude p corresponds to the abstract particle involved, which is determined by the energy value E for the state, whereas the momenta p_x, p_y, and p_z have only definite probability distributions. This means that all the concrete particles gradually forming the PD-pattern have one and the same momentum magnitude (and, hence, velocity magnitude), while having generally different momenta p_α satisfying the condition $\Sigma p_\alpha^2 = p^2$.

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