De Broglie's causal interpretations of quantum mechanics

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ABSTRACT. In this article we trace the history of de Broglie's two "causal" interpretations of quantum mechanics, namely the "double solution" and the "pilot wave" theories, at the two periods in which he developed them: 1924-27 and 1952 onwards. Examining the reasons for which he always preferred the first theory to the second, reasons that are mainly concerned with the question of the physical nature of the quantum wave function, we try to show the continuity and the coherence of his underlying vision.

RESUME. Dans cet article nous traçons l'histoire des deux interprétations "causales" de la mécanique quantique proposées par de Broglie, à savoir la théorie de la "double solution" et celle de "l'onde-pilote"; on les examinera aux deux époques de leur développement, 1924-27 et dès 1952. En examinant les raisons pour lesquelles de Broglie préférait toujours la première théorie à la seconde, raisons qui concernent essentiellement la question de la nature physique de la fonction d'onde quantique, nous essayons de montrer la continuité et la cohérence de sa vision fondamentale.

1. Introduction

In the sixty-five years which have elapsed since de Broglie's theoretical discovery of the wave-like aspect of particles, quantum mechanics has been yielding a most remarkable crop of experimental predictions, and came to be widely accepted as the basic dynamical theory of physics. However, in all these years, no common agreement has been reached as to its correct "interpretation", that is no physical world-view has been found with which it could be considered as solidary. In the course of the search for such an interpretation, a special attention has always been given to so-called “hidden-variables” schemes, of which the most interesting and most developed is undoubtedly what came to be known as the “pilot wave” theory. According to popular accounts of the kind which sometimes appear in the first chapters of physics textbooks, the “pilot wave” theory was first put forward by de Broglie, who quickly abandoned it because of the criticism levelled at his ideas at the fifth Solvay conference, and especially because of a devastating argument by Pauli. But later, after an initial rejection, he returned to that theory following its resurrection by Bohm in 1952. Some historical studies [1] give more precise details, but a similar impression of “inconstancy” in de Broglie’s position, and this impression might be partially responsible to the fact that while Bohm’s version of the “pilot wave” theory is widely acclaimed, not many people today care to read carefully de Broglie’s original publications on this subject. The purpose of this article is to show that this common account is inexact, and does not do justice to the clarity and coherence of de Broglie’s thought. We shall try to show that beneath the apparent “inconstancy” lies a single unified vision, which kept its basic lines stable over the years. This vision merits to be studied, as some of its elements might serve to clarify the difficult questions concerned with the interpretation of the quantum theory.

While evaluating de Broglie’s position, one should first pay attention to the fact that between 1926-1927 de Broglie developed not one, but two different interpretations of wave mechanics, to which he referred together as “causal theories”. The first of these interpretations, the “theory of the double solution”, indeed reflected de Broglie’s deeper convictions; but the second, “the theory of the pilot wave”, was never considered by him as more than a “provisory” and even a “truncated” approach. In the following sections, we shall try to trace the development of these theories, and to understand de Broglie’s different appreciations of them. As we shall see, this difference also explains de Broglie’s reaction to Bohm’s 1952 work: while still rejecting (and for similar reasons as before) Bohm’s resurrection of the “pilot wave” theory, de Broglie was now able to return to a slightly modified version of his original “double solution” idea, and to continue working on it in the last decades of his scientific career.

2. Motivation

In order to understand de Broglie’s vision of his “causal theories”, it is worthwhile to examine the reasons which, in his own account, motivated his interest in them. Usually, "supplementary" or "hidden" variables theories are thought of mainly as a way to restore determinism in the microphysical domain. Indeed, de Broglie's two "causal theories" are both deterministic, but it is interesting to note that determinism by itself is almost never cited by him as the major argument in their favour. Here as in the following, we shall distinguish between the two different periods in which de Broglie pursued his research on the "causal theories": the "early period" of 1924-1927, and the "later period" from 1952 onwards.

The early period of de Broglie's active interest in his "causal theories" immediately precedes Bohr's [2] presentation of the "complementarity" idea, with its subsequent acceptance by the major part of the physical community. At that time, de Broglie's suggestions were perceived as one among several attempts to interpret the recently-developed quantum formalism in terms of clear physical images [1]. Therefore, in order to make acceptable his own ideas, he had to show the weak points of the other approaches. Thus, in his talk at the fifth Solvay conference, de Broglie [Ref. 3, p. 112] criticises Schroedinger's interpretation, which regards the quantum wave function as a continuous distribution of matter and charge. Here, de Broglie's arguments are as follows: first, the many-particle wave function $\psi(x_1 \ldots x_N)$ is written as a function of $3N$-dimensional configuration space, and thus makes an explicit use of it. But the configuration space is defined by the coordinates $x_1 \ldots x_N$ of N "imaginary" point particles, which themselves do not appear in the theory; therefore, the explicit use of configuration space seems meaningless. Second, even by itself, the concept of configuration space does not have a clear physical interpretation – it is only an abstract mathematical construct. As we shall see, this second argument plays a major role in de Broglie's later reaction to the "pilot wave" theory.

In an earlier and more detailed article, de Broglie [4] criticises a "continuous" conception of the wave function similar to Schroedinger's on the grounds that it is incompatible with the (discrete) atomistic structure of matter. In the same article, the difference between de Broglie's deterministic theories and Born's purely probabilistic interpretation of the wave function is mentioned, but the concept of non-determinism itself is not criticised, so that any argument motivated by the quest for determinism rests at least implicit. Thus we may conclude that at the early period, de Broglie's concern is much more with the non-physical nature of configuration space and with the discrete structure of matter, than with the question of non-determinism.

This important feature of de Broglie's position remains also at the later period, although meanwhile his motivation obviously came to be based on a reaction against the prevailing Bohr-Heisenberg interpretation, of which non-determinism is one of the main features. For example, in his "exposé général" of the problem [Ref. 5, p. 1-22], de Broglie mentions Einstein's and Schroedinger's objections to the non-local character of the usual interpretation, and bases his search for a "causal theory" first on the desire to return to "clear, cartesian conceptions which respect the validity of the space and time framework", and then on the hope to eliminate the uneasy feeling caused to the "realist" physicist by a "subjectivist interpretation". To be sure, it does happen that de Broglie speaks in his scientific papers about the need to restore causality in the microphysical domain, like for example in a remark attached to a presentation of an argument by Einstein [6]; but even in such an occasion the case for causality appears as a side remark, which occupies a minor place in the overall argumentation. This fact is important, because as both of de Broglie's "causal theories" are deterministic, any motivation based only on the wish to restore causality could not explain the differences between his attitudes towards them. As we shall later see, from a point of view which emphasises the question of the non-physical nature of configuration space, there does exist an fundamental difference between the theory of the "double solution" and that of the "pilot wave".

3. The Early Period, 1924-1927

In this section, we shall trace the early development of de Broglie's two "causal theories". In order to make our account easier to follow and more in line with the later developments of the "pilot wave" theory, we shall present in most cases only the results which de Broglie obtains in the non-relativistic limit (which he usually calls "the Newtonian approximation"). Indeed, in de Broglie's accounts written at the "later period", he himself sometimes uses a non-relativistic presentation. However, if one wants to understand the complete evolution of de Broglie's thought (of which many features lie outside the scope of this article), it is important to remember that his vision is basicly relativistic. It is only the later discussion on the foundations of quantum mechanics that became centered on the non-relativistic formalism, finding it already difficult enough.

The earlier development of the "causal theories" continues the same lines of thought which have guided de Broglie's earlier (1923-24) work on the wave-like aspect of particles. In that work, de Broglie associated the energy and momentum of a moving particle with the frequency and wavelength of a wave. But as both of these quantities pertain only to the phase of the wave, the amplitude was left with no significant role, and indeed de Broglie first used the term "phase waves". In order to fill this conceptual gap, he published soon afterwards a short note [8], in which he suggested to associate with the moving particle an extended wave packet with an amplitude singularity. The location of the singularity somewhere within the wave packet represents the exact location of the particle, well-defined at each moment.

In a later note, de Broglie [9] applies a similar idea of an amplitude singularity to the case of moving light particles (that is, photons). But now he distinguishes between two different amplitudes of the same phase wave: first, there is the "classical" amplitude which obeys the usual wave equation, and second, there is the "real" amplitude which contains the moving singularity. As we shall presently see, this is the origin of the "double solution" approach. Several other features of de Broglie's later theories also appear in this note: the amplitude singularities are supposed to move along lines perpendicular to equal-phase surfaces of the wave, and their velocities (which may be different from c) are determined by the phase that acts as a "velocity potential". Both of these features, which are inspired by the classical Hamilton-Jacobi theory, as well as the extremely important principle of phase equality between the two amplitudes, also resemble similar features in de Broglie's earlier work. In addition, when there are many light particles described by the same "classical" amplitude, the density of their corresponding singularity points is equal to the classical density, that is to the squared "classical" amplitude. These results are explicitly derived from the supposition of a certain particular form of the singular amplitude, whose exact details will not interest us here.

At the same time, other theoretical physicists have made several important advances in the wave-mechanical description of non-relativistic matter particles, advances which seemed to go in a direction opposite to the one suggested by de Broglie: Schroedinger succeeded in accounting for atomic spectra by using continuous (that is, non-singular) solutions of his wave equation, and Born gave meaning to the wave amplitude (up to a normalisation constant) with his probabilistic interpretation. De Broglie's response came in another note [10], in which he further clarified the difference between the two wave amplitudes mentioned above, and applied it also to the case of material particles: in his view, the moving particle's location is represented by a singularity point of a "singular" solution of the wave equation, while the statistical distribution of many such particles is described by the squared amplitude of a "continuous" (that is, regular) solution of the same equation. This "continuous" solution is identified with Schroedinger's wave function, so that Born's probabilistic interpretation holds for it. The "singular" and the "regular" solution clearly correspond to what de Broglie earlier called the "real" and the "classical" amplitudes of light waves, and again they are supposed to have one and the same phase.

These ideas were put together in a longer article [4], which is the only full exposition of the "causal theories" published at that period. In this article, de Broglie bases his derivations on the relativistic Klein-Gordon wave equation, but we shall describe only the results he obtains in the non-relativistic limit. As in his previous articles, de Broglie proposes to associate with the movement of each quantum particle (or with a "cloud" of similar particles, that is with an ensemble) two distinct solutions of the same wave equation. First, the "singular" solution

$$u(x, y, z, t) = f \cdot e^{i\phi} \tag{1}$$

where we have adopted a complex form and $h = 2\pi$ units, in conformity with the habitual present-day notation. The amplitude function f is supposed to include a moving singularity, whose location indicates the exact place of the "material point". In addition, there is the "continuous" solution

$$\psi(x, y, z, t) = a \cdot e^{i\phi} \tag{2}$$

where the amplitude a is continuous and regular at all points. In the non-relativistic case, it is identical to the usual Schroedinger wave function. The "principle of the double solution" states that the two solutions ψ and u always have the same phase ϕ.

De Broglie now treats the case of a free moving particle. With the aid of some particular assumptions on the form of the singular amplitude and on its properties under a Lorentz transformation (this is a kind of reasoning typical of de Broglie, who used similar considerations in his earlier work on the wave-like aspect of particles), he arrives at the following expression for the velocity of the moving singularity point:

$$v = -\frac{1}{m}\,\overrightarrow{\mathrm{grad}}\,\phi \tag{3}$$

where m is the particle mass, and an additional $e\vec{A}/c$ term should be added in the case of an electromagnetic field. Except for the minus sign which is due to de Broglie's relativistic metrics (so that in the non-relativistic case, he arrives at the complex conjugate of the usual Schroedinger equation), this "law of motion" of the particle's position would also occupy a central place in later presentations of the "pilot wave" theory [11,12]. As de Broglie notes, and as Bohm would later remark, it bears a close analogy to the similar expression in the classical Hamilton-Jacobi theory. Now, he assumes that Eq. (3) holds also in the general case, and not only for a free particle.

Next, de Broglie considers the case of an ensemble ("cloud") of similar particles in what is usually called the same quantum state. He assumes that all of them are described by the same ψ-wave (that is, the "continuous" solution), but that their u-waves (the "singular" solutions) add together to form a function of ordinary space with many singular points, each one representing the position of a single particle. For the density ρ of the singularity points he writes down the expression

$$\rho(x, y, z) = Ka^2(x, y, z) = K|\psi|^2 \tag{4}$$

where K is a suitable constant. As the ψ-wave is identical to the usual quantum wave function, this equation enables de Broglie to retrieve Born's probabilistic interpretation, and thus to reproduce all the experimental predictions of the usual quantum formalism. For an ensemble of particles, ρ represents the particles density, and for a single particle, it describes its probability distribution for being actually located at the point (x, y, z).

What has been described until now is the "theory of the double solution". Summarising it at the end of his article, de Broglie remarks the central importance of the two equations (3) and (4), or their relativistic counterparts. However, as we have mentioned earlier, Eq. (3) has been derived from certain assumptions only for the case of a free particle, and its validity for the general case has been assumed without proof. De Broglie therefore suggests a second way of interpreting his two equation, which dispenses with the need to derive Eq. (3) simply by accepting it as a basic postulate of the theory. In this second scheme, the ψ-wave and the "material point" itself (and not the singular wave which would represent it) are considered as two distinct, but related, physical entities. Eq. (3) is then postulated as the "law of motion" which determines the velocity of the material particle; thus, the particle appears to be "guided" in its motion by the phase ϕ of the ψ-wave. For this reason, de Broglie calls this second scheme the "pilot wave" theory. As for Eq. (4), it is supposed to hold in the same manner as before. This second view obviously dispenses with the mathematical complexities involved with the calculation of the u-wave singular amplitude; but de Broglie still regards it only as "a provisory attitude". In the end, he believes, the particle should be reincorporated into the ondulatory phenomenon, and this would probably be achieved along lines similar to the "double solution" approach.

In order to understand de Broglie's position in this matter as well as his attitude at the later period, it is important to note that although the "double solution" and the "pilot wave" theories both rely on the two equations (3) and (4), there is a fundamental conceptual difference between them. As we have seen, the u-wave and the ψ-wave have developed from what de Broglie earlier called the "real" and the "classical" amplitudes, respectively. This means that from the beginning, de Broglie tended to regard only the u-wave as physically real. This vision could easily be maintained in the "double solution" scheme: one could go on interpreting the u-wave as the real physical entity, while considering the ψ-wave (that is, the usual quantum wave function) only as a fictitious mathematical construct, related to the statistical distribution of ensembles of particles. True, the phase ϕ appears in Eq. (3) as "guiding" the motion of the u-wave singularity point; but as the same phase is common to the two waves, one can maintain that this "physical" effect comes from the "real" u-wave, and not from the "fictitious" ψ-wave. However, such a view is no longer possible in the "pilot wave" framework. As the u-wave is replaced by a "material point" which carries no phase of its own, the appearance of ϕ in Eq. (3) must be ascribed to its role as the phase of the ψ-wave. But this means that the ψ-wave is considered as a physical field which determines the motion of the material point – that is, as a real physical entity.

This difference between the two "causal theories" becomes especially important for de Broglie in the many-particle case. As we have seen in the previous section, he criticised Schroedinger's "continuous" interpretation because in de Broglie's view, any function which is defined in the abstract $3N$-dimensional configuration space could not be regarded as a real physical entity. Obviously, the same argument also holds for his own ψ-wave, which is completely equivalent to Schroedinger's wave function.

Therefore, in order not to fall under the scope of his own criticism of Schroedinger's interpretation, de Broglie must regard the ψ-wave only as a mathematical abstraction. But as we have seen, this view can be maintained only in the "double solution" framework, and not in the "pilot wave". It is important to note that this is not only a matter of strategy: the whole sense of de Broglie's search for "clear, cartesian conceptions which respect the validity of the space and time framework" means that physical reality should be described by functions of the "real" ordinary space, and not of the "fictitious" configuration space. One may suppose that from the beginning, this was one of the main reasons for which de Broglie preferred to regard the "pilot wave" theory as only a provisory approach.

4. The Fifth Solvay Conference and its aftermath

In the Spring of 1927, Lorentz asked de Broglie to give an exposition on wave mechanics at the fifth Solvay Conference, which was to be held in Brussel in October of the same year. The young de Broglie, who was not yet much accustomed to make public appearances, was faced with a problem: at the stage which he has reached at that time, the "double solution" theory was not yet developed enough, and it seemed (as proved right by his work on it at the later period) that the calculations concerned with the singular u-wave would involve some immense mathematical difficulties. Being intimidated by the prospect of presenting to the world's greatest physicists such a difficult research plan, which he himself did not yet have the time to work in full, de Broglie decided to expose only the much simpler "pilot wave" theory. As he later realised [13], this was a mistake. By presenting what he himself considered only as "a provisory approach", and what he would later call a "truncated and unacceptable" form of his ideas, he has undermined his own position. Even in the limited framework of the "pilot wave" theory, de Broglie [3] presented in detail only the case of a single particle, so that the conceptual difficulties involved with the use of the configuration space did not have to be entered into. For the many-particle case, he expressed some general ideas which are clearly influenced by his "double solution" approach, but the theory of the "double solution" itself was not explicitly mentioned. According to this presentation, the ψ-wave in the configuration space is only a fictitious entity which represents a cloud (ensemble) of particles, and actually each particle should be assigned its own separate wave, which would be influenced by those of the other particles.

In addition, de Broglie repeated his contention that the "pilot wave" separation between particles and waves is only provisory, and that one should look for a possibility to incorporate the particle back into the ondulatory phenomenon by considering singular solutions of the wave propagation equations. As we mentioned in Sec. 2, he also criticised Schroedinger's "continuous" conception of the wave function, and the explicit use of configuration space.

The reaction of the other participants at the conference to de Broglie's ideas was unfavorable, except for a remark by Einstein who sympathised with the general direction of de Broglie's research, but did not enter into the details of the theory exposed. Even this was not of much help to de Broglie, because Einstein himself was isolated at this conference, of which most of the participants chose to align with Bohr's "complementarity" approach. Still, the only concrete objection to the "pilot wave" theory came from Pauli.

Pauli's argument, which was formulated only in a verbal form, is exposed in detail by several authors, including de Broglie himself [1,27]. Here we shall present it in a simpler formalism, which nevertheless keeps all of its relevant features. The argument is based on the example of a non-elastic collision between a particle and a rigid rotator analysed in detail in a 1926 article by Fermi, but as Pauli himself remarks, the same considerations apply for a much more general class of interactions between two quantum systems. In Fermi's treatment, both the particle and the rotator are initially assumed to be in well-defined energy states. We can therefore represent the state of the composite system which includes them both by the quantum state

$$\Psi_0 = \xi_0 \psi_0 \tag{5}$$

where ξ_0 is a monochromatic plane wave which represents the incoming particle with energy E_0, and ψ_0 is a well-defined angular momentum state of the rotator. After the collision, the state of the same composite system is a superposition

$$\Psi = \sum a_k \xi_k \psi_k \tag{6}$$

where ξ_k is a plane wave representing an outgoing particle with energy E_k, ψ_k is a well-defined angular momentum state of the rotator, and a_k is a numerical factor. Conservation of energy implies that the energy difference $E_k - E_0$ of the particle is equal to the energy difference between the states ψ_0 and ψ_k of the rotator. According to the usual prescription of ordinary quantum mechanics, a subsequent measurement would find the composite system in one of the superposed states of Eq. (6), so that both the rotator and the particle would each be found in a well-defined energy state. But as Pauli remarks, the superposition Ψ of final states would have as its phase ϕ a very complicated function of space, so that according to the "guidance" formula (3), both the particle's and the rotator's motions would undergo very strong fluctuations, and bear no resemblance to the smooth and regular motions predicted by the usual version of quantum mechanics. In particular, the rotor would not be found in a well-defined rotation state.

De Broglie answered Pauli's objection with two brief remarks. The example of Fermi's rotator, he said, concerns a composite system and thus involves the configuration space; in contrast, all of the examples he discussed in his lecture involved only single particles, whose wave functions are defined in ordinary space. In addition, he argued that in analogy with the corresponding situation in classical optics, one should consider not infinitely extended plane waves as in Pauli's argument, but only limited wave packets.

As Bohm (1952) would later show in detail, de Broglie's second remark indeed indicates a valid answer to Pauli's argument. If one considers the case of a wave packet with only a finite spatial extension for the incoming particle, then it is easy to show that at a certain time after the collision, the wave packets of the composite system particle plus rotator become separated in configuration space, and remain so afterwards. The exact "particle positions" of the composite system (that is, the locations in ordinary space of the singularity points in the "double solution" theory, or of the material points in the "pilot wave") correspond to a single "representative point" in configuration space, which enters one of these wave packets and cannot leave it once the different packets are separated. Thus, a subsequent measurement would find the outgoing particle and the rotator in (almost exactly, because of the finite extension of the wave packets) well-defined energy states. In addition, the generalisation of Eq. (3) to the configuration space implies that once the different wave packets become separated, they do not superpose and do not disturb the regular motion of the "representative point". On the other hand, at the short time before the wave packet separation, ordinary quantum mechanics also predicts strong irregular fluctuations, which correspond to the uncertainty principle.

As popular myth would have it, it was the devastating power of Pauli's argument which persuaded de Broglie to abandon his "pilot wave" theory. But we see that Pauli's argument does not hold, and that de Broglie was indeed on the right track answering it. Why then didn't he develop his response in detail? The reason for this is hinted at in his first remark. A detailed treatment of the collision problem would have forced him to extend the "pilot wave" theory to the many-particle case, and to make an explicit use of the 3N-dimensional configuration space – thus conferring upon this "fictitious" space a real physical status. Indeed, in the same article in which Bohm [11] gives the detailed answer to Pauli's argument, he also refers to the wave function in configuration space as a "six-dimensional but objectively real field", although some years later he would come to realise the problematics of this concept. But such an expression was completely inacceptable to de Broglie, who would not admit the reality of anything physical in configuration space. And obviously, he could not have adopted this line of argumentation without falling under the scope of his own criticism of Schroedinger's interpretation, thus undermining even further his already weak position.

There was one way out of this difficulty, and de Broglie indeed adopted it at the later period: return to the "double solution" theory, accept a detailed treatment in configuration space for the many-particle case as a practical prescription, but insist that eventually, all the results should be translated back into terms which involve the "real" physical entities – that is, u-waves in ordinary space. But in the circumstances which prevailed at the fifth Solvay conference, having already exposed only the "pilot wave" approach, a sudden switch to the "double solution" theory with its hugh mathematical complexities was out of the question. Thus, one may say that contrary to popular myth, de Broglie fell victim not to the force of Pauli's argument, but to his own hesitation between the two "causal theories".

The lack of sympathy with which de Broglie's ideas were greeted at the Solvay conference made a strong impression on him. Contemplating the situation after his return to Paris, he realised that for the reasons explained above, he could not maintain the "pilot wave" theory. But a return to the "double solution" scheme seemed to involve unsurmountable mathematical difficulties, of the kind that had played a crucial part in his original decision not to expose it at the Solvay conference. Also, having just been appointed as a professor at the Institut Henri Poincaré, de Broglie had to decide which theory to teach, and naturally, he felt reluctant to commit a whole generation of french scientists to his own personal ideas, which have just been rejected by the most distinguished members of the physdical community. Under these circumstances, Bohr's "complementarity" scheme with its reliance on Born's probabilistic interpretation of the wave function seemed a much more reasonable choice; de Broglie decided to subscribe to it, and to abandon his work on the "causal theories".

In a text published not very long afterwards, which reproduces the contents of the course at the Institut Henri Poincaré, de Broglie [Ref. 26, p.132] briefly presents the "pilot wave" theory only to dismiss it immediately as unacceptable. He gives two reasons for doing so. First, the double role of the wave function, whose phase ϕ "guides" the single particle (Eq. (3)), and whose amplitude a determines the probability density for its location (Eq. (4)), is problematic. The phase and the amplitude are mutually dependent, because they are determined together by the Schroedinger equation. But the amplitude, which is a probability function, depends on our knowledge of the initial state of the particle. Therefore, the "pilot wave" scheme seems to imply that our subjective knowledge affects the real particle's motion. A similar difficulty arises in connection with the "collapse" of the wave function following a quantum measurement: here, our knowledge that the particle is contained in one wave packet causes the other packets to vanish, and this is again hard to reconcile with a view of the wave function as a real physical entity. We see that again, it is the assumption of a physical nature of the wave function which seems to de Broglie as most problematic, and indeed he would retain the same argument at the later period, in order to explain why he prefers the "double solution" to the "pilot wave" theory [13]. Bohm, on the other hand, would answer the same difficulty by assuming that the wave function never "collapses"; and while it may be shown that the "non-collapsed" parts of the wave function do not influence the subsequent motion of the particle, their existence seems to be an extra conceptual burden for the theory [14].

De Broglie's second argument is based on the following observation: if energy and momentum are ascribed to the point particle in the usual manner (i.e. depending on the mass and the velocity of the particle as defined by Eq. (3)), then these quantities are generally not conserved even in the absence of any external field. The reason for this surprising feature is that in the general case, the phase ϕ of the wave function can have a very complicated form as a function of space, so that in accordance with Eq. (3), the velocity of the particle, and with it the energy and momentum, would undergo very strong and violent fluctuations. The resulting motion of the point particle seems to de Broglie as having a "small verisimilitude", and he also remarks that an energy or a momentum measurement would give the usual quantum-mechanical values, and not the momentary fluctuating values connected with the real point particle's motion. In the "pilot wave" framework, the source of this problem lies in the fact that Eq. (3) may be regarded as describing an action of the ψ-wave on the point particle, but there is no reaction of the particle on the wave, and this "action without reaction" leads to a non-conservation of energy and momentum. It is possible that a similar difficulty had impeded Einstein from ever publishing his "guiding field" idea, which seems to have been conceptually similar to the "pilot wave" theory [17], and this might explain some of his reservations about the exact details of de Broglie's ideas. Later, Bohm would try to solve the same problem by supposing that the particle exchanges energy with a hypothetical "subquantum level" [15], and as the same problem also arises in the "double solution" theory, de Broglie [6,28] would similarly be led to introduce at his later period some kind of a "hidden thermostat". Bell [16], on the other hand, would try to avoid the same difficulty by supposing that the "point particle" has only a location, but not energy and momentum in the usual sense.

We may further remark two interesting points in de Broglie's 1930 text. First, the many-particle case, with its concomitant problem of the configuration space, is not referred to. But we shall see it re-appear at the later period, when de Broglie again needed to distinguish between his two "causal theories". Second, the original "double solution" approach is only vaguely referred to in the introduction (p. XII). It is immediately dismissed not only because of the mathematical difficulties involved with the calculation of the singular solution, but also because "there exists serious objections against this point of view"; these objections are not explicitly specified.

Still, while rejecting the "pilot wave" theory as a description of physical reality, de Broglie points out that its mathematical formalism may be retained if one replaces the notion of a "cloud of particles" by a "cloud of probability elements". This leads to the usual "probability current" of ordinary wave mechanics.

5. The Later Period, 1951 Onwards

As we described in the previous section, the negative reaction to his ideas at the fifth Solvay conference caused de Broglie to abandon both of his "causal theories", and to adhere to the prevailing Bohr-Heisenberg view of the quantum theory. It is to this conception which he subscribed in his cours and publication, although looking backwards at that period, de Broglie [18] describes a growing "uneasy impression" with the probabilistic interpretation of the quantum formalism, and with Bohr's answers to Einstein's and Schroedinger's objections to it.

In the summer of 1951, de Broglie received a preprint of Bohm's [11] "hidden variables" paper, in which Bohm exposes a theory very much similar to de Broglie's "pilot wave", as presented at the Solvay conference. De Broglie's initial reaction to it was reserved. In a note published shortly after he received Bohm's paper [19], he repeats his objections to the "pilot wave" scheme. These are essentially similar to the arguments which appear in his 1930 text described in the previous section, plus the earlier argument concerning the fictitious nature of the configuration space. On the other hand, de Broglie admits that the "pilot wave" theory provides a valid answer to the EPR argument [20], and that its mere existence may serve as a counter-example against Von Neumann's [21] alledged "proof" of the impossibility of any successful "hidden variables" theory (see also Ref. 12).

It was only in the appendix to the second part of his article, added to the published version, that Bohm gave his answer to Pauli's argument of the fifth Solvay conference, described in the previous section. This fact was acknowledged by de Broglie [22] in a later note, which also repeats his arguments against the physical nature of the wave function. But in the meanwhile, de Broglie's attention was drawn by his young collaborator J.-P. Vigier to the similarity between the original "double solution" scheme and a 1927 work of Einstein on the motion of singularity points in a gravitational field, a similarity which went unnoticed by de Broglie at that time. Encouraged by this discovery as well as by Bohm's work, de Broglie suggests in this note a return to the "double solution" idea in which, as we saw in Sec. 3 , the quantum wave function does not have to be regarded as a real physical entity.

But the original "double solution" scheme had now to be revised. In the meantime, it became clear that the quantum wave equation cannot accomodate for both a regular and a singular solution having the same frequency. De Broglie therefore modified his idea: instead of considering two distinct solutions of the same wave equation, he now suggested that the "singular" u-wave would be the solution not of the usual quantum wave function, but of a slightly different equation, which would contain some additional non-linear terms. These additional terms would be negligible in all cases except for the case of very strong amplitudes, and on the other hand, the strict mathematical singularity of the u-wave would be replaced by a very small region of strong but finite amplitudes, outside of which the usual linear quantum wave equation and the Schroedinger wave function would be a valid approximation. Thus, the exact position of the particle would no more be represented by a mathematical singularity, but the particle itself would be costituted by a small non-dispersing region of very strong wave amplitudes, held together by the additional non-linear terms. This idea, which (as de Broglie [27] was quick to note) is similar to a "soliton" solution of non-linear wave equations, strongly resembles Einstein's conceptions on the relations between waves and particles [18]. It has the interesting consequence that in cases involving very small distances, in which the non-linear terms are important, one could expect some deviations from the experimental predictions of ordinary quantum mechanics.

These ideas were put forward in a subsequent book, whose title expresses de Broglie's [27] new research plan: "A tentative of a Causal and Non-Linear Interpretation of Quantum Mechanics". Here, de Broglie repeats his arguments against the consideration of the quantum wave function as a real physical entity: the norm of the wave function is arbitrary, in the general case it is defined in the fictitious configuration space, and it reflects our subjective knowledge and "collapses". For these reasons, de Broglie still rejects the "pilot wave" theory as presented at the fifth Solvay conference, and as resurrected by Bohm. In its stead, de Broglie proposes the new non-linear version of the "double solution" approach.

De Broglie's new research plan did not have much influence on the large part of the physical community, and in his later years he, like Einstein, found himself marginalised. But with a small group of young collaborators at the Institut Henri Poincaré – G. Lochak and J. Andrade E Silva among others – he continued to work on his "double solution" theory. Developing an idea similar to Bohm and Vigier's [23], he was now led to consider additional random (what Bohm calls "subquantum") microscopic fluctuations, which would account for the probability distribution of Eq. (4), achieved as an equilibrium state. These later developments [6,24,25,28] lie outside the scope of the present article.

6. Conclusions

In this article we traced the developement of de Broglie's two "causal theories": the "double solution" and the "pilot wave". As we saw, de Broglie's thought shows a continuity throughout all of the changing phases of its development, and one can trace a clear coherent line from his early work on the wave-like aspect of particles to the new developments of the "double solution" theory in his later years. Also, we saw that his underlying vision kept his basic features unchanged: his main concern was always with finding a coherent description of physical reality in terms of functions defined in ordinary (but obviously relativistic) space, and not in terms of the "fictitious" configuration space. For this reason, he never accepted seriously the "pilot wave" theory, if not as a "provisory approach" towards the "double solution" framework, in which he firmly believed. These features of de Broglie's thought are valid both for the early and for the later period. Together they constitute a unified and coherent vision, which resembles Einstein's view in many of its features. This vision merits to be studied even by those who do not accept the exact details of de Broglie's later research plan, because as an acceptable alternative to the usual viewpoint, it may raise new questions and give new insights for the clarification of the complex problems involved with the interpretation of quantum mechanics.

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