# Does Planck's constant vary versus time ?

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ABSTRACT. Experiment shows that a certain number of ratios built up upon the fundamental constants ($e$, $m$, the charge and the mass of electron resp. ; $\hbar$, the quantum constant ; $k$ the Boltzmann constant ; $M$, the proton mass) have practically remained constant since $10^{10}$ years. The limits of past variability obtained for these ratios do not exclude variations for the constants themselves. It is sufficient that $\hbar \propto e^2 \propto m \propto k$. Assuming that the stability of an atom arises from the balance between the radiated energy and that absorbed arising from the other systems which constitute the universe, the connection with the cosmological models shows that $\hbar$ would decrease as the curvature radius $R$ of the universe increases. The present rate of variation of $\mid \hbar \mid$ would be approximately equal to $10^{-10}$year$^{-1}$.

*RESUME. L'expérience montre qu'un certain nombre de rapports construits sur les constantes fondamentales (la charge e de l'électron, sa masse m, la constante quantique $\hbar$, la constante de Boltzmann k, la masse M du proton) sont restés pratiquement constants depuis $10^{10}$ années. La limite de la variabilité de ces rapports n'exclut pas des variations pour les constantes elles-mêmes. Il suffit que $\hbar \propto e^2 \propto m \propto k$. Postulant que la stabilité d'un atome résulte de la balance entre l'énergie rayonnée et celle absorbée provenant des autres systèmes qui constituent l'univers, l'utilisation de modèles cosmologiques montre que $\hbar$ est une fonction décroissante du rayon de courbure de l'univers. Le taux de variation de $|\hbar|$ serait actuellemnt de $10^{-10}$ année$^{-1}$.*

## 1. Introduction

The question, asked in such an abrupt manner, may appear as rather preposterous, since $h$ is a constant, by definition ! In fact, the question hides a much deeper question, the one of the physical signification of the Planck constant, and, consequently, of quantum mechanics itself. This theory postulates that $\hbar = h/2\pi$ is a constant, as well as the light velocity $c$. The remarkable agreement between the conclusions deduced from quantum mechanics and those from experiment for atomic and molecular systems, justifies this hypothesis, so that it is usual to put $\hbar = c = 1$ in equations. In fact, if we examine the equations more closely, we see that $h$ never appears alone. It is always associated with the electron mass, $m$, or with the electron charge, $e$, or also with Boltzmann's constant, $k$, so that the problem of the variability of $\hbar$ has not to be separated from the one of the other fundamental constants. Various authors (Eddington [1], Gamov [2], in particular) have envisaged that the values of the fundamental "constants" can vary versus time. In spite of a certain number of negative results [3, 4, 5] and categorical claims [6, 7], the problem cannot be considered as being definitively resolved.

In a theoretical point of view, the question of the variability of the fundamental constants has to be considered within the framework of the interaction theories. It is well known that four fundamental types of interaction exist, namely the electromagnetic, weak and strong interactions, and gravitation. The three first interactions are described in a satisfactory manner by the standard $U(1) \times SU(2) \times SU(3)$ model [8] which introduces three dimensionless coupling constants, one for each type of interaction. Recent works [8] seem to show that these three types of interaction can be included within a more general framework $SU$ [5] (the Grand Unified Theory). According to this theory, these interactions would arise from a unique interaction which would govern all phenomena when the interaction energies are higher than $10^{15}$ GeV. Below this value, the three interactions can be considered as practically decoupled. Moreover, researches are in progress with a view to built up a more general theory which would include gravitation [9]. Owing to the fact that the coupling constants can be connected with the fundamental constants we consider (e.g. the electromagnetic coupling constant is precisely equal to the fine structure constant $\alpha = e^2/\hbar c$), the problem of the variability of these fundamental "constants" presents a recrudescence of interest.

In a rather unexpected manner, these theories have permitted to throw a bridge between cosmology and physics of elementary particles [10]. Indeed, in the beginning of the universe (just after the big bang) the density of matter was very great, so that a unique interaction existed. But very rapidly, owing to the expansion, the decoupling between the various types of interaction occured, so that, at the present time, we can consider that the four fundamental interactions are independent of one another. The weak interactions and the strong interactions present a very local character (they are practically confined inside the nuclei or in their vicinity), so that, at the present time, given the very weak average density of matter in the universe, both these interactions have not any effect on the general behavior of the universe, which is essentially governed by long-range interactions, more precisely by gravitation. The electromagnetic forces, indeed, are negligible given the electroneutrality of the universe. On the contrary, the electromagnetic interactions are governed by the behavior of the universe and, consequently, can be tackled only within the framework of a theory taking the whole universe into account. The aim of this paper is, first, to show the connections which exists between the values of the various fundamental constants, and, second, to propose a model which allows to foresee the variation rate of these constants.

## 2. Experimental data

No direct experiment is able to detect the variation of the values of $e$, $m$, $h$ and $k$. The conclusions which have been claimed concerning the constants themselves cannot be retained. The case of $\hbar$ is typical in this regard. Experiment [6] shows that the energy of photons of a given color, does not depend on their age. From which, the authors conclude that $\Delta h/h \sim -3 \times 10^{-12}$ per year, value which is not significantly different from zero. In fact, this experiment only shows that the law $E = h\nu$ is remained valid since the early universe. The photons under consideration exhibiting the same frequency $\nu$, given $h$ corresponds to the present value, it is obvious that the same value of $E$ must be obtained !

Another example of an excessive conclusion is given by a work concerning the Boltzmann constant. The fossil radiation which fills the universe corresponds very exactly to that of a black body at $2.9K$ [7]. This only proves that the ratio $\tau k/\hbar$ ($\tau$ = time unit) is remained constant. No conclusion can be drawn concerning the values of $\hbar$ and $k$.

In fact, the problem of the variability presents a double difficulty. First, it is necessary to utilize time-intervals sufficiently long for a significative variation to be observed. Second, the measurements have to be performed in a time-invariant unit system. Given the units we use can vary as the fundamental constants themselves, only the values of dimensionless ratios can bring significative informations.

At the present time, we can resume the situation as follows :

a – The study of the relative doublet splittings of optical emission lines in radiogalaxies shows that for the fine structure constant $\alpha = e^2/\hbar c$.

$$\mid \frac{1}{\alpha}\frac{d\alpha}{dt} \mid < 5 \times 10^{-18} \text{year}^{-1} [3, 11]$$

b – The comparison between the redshifts of optical hydrogen lines and that of the $21cm$ line arising from the hyperfine structure, shows that for $P = \alpha^2 \gamma m/M$ (where $M$ and $\gamma$ are the mass and the gyromagnetic ratio of proton respectively) :

$$\mid \frac{1}{P}\frac{dP}{dt} \mid < 2 \times 10^{-14} \text{year}^{-1} [12]$$

c – The comparison between the optical redshifts for hydrogen atom and those corresponding to heavier atoms indicates that $m/M$ had between 0.7 and 1.5 times its present value ca. $10^{10}$ years ago [13].

d – Astronomical observations corresponding to a 200 year period seem to prove that $G$ is very slowly decreasing :

$$\frac{1}{G}\frac{dG}{dt} = -(7 \pm 2) \times 10^{-11} \text{year}^{-1} [14].$$

Recent works give $-5 \times 10^{-11}\text{year}^{-1}$ as the upper limit for the relative rate of variation [15].

e – The study of desintegration products in the natural fission reactor of Oklo [16, 3] shows that the ratio $\beta = g_f M^2 c/\hbar^3$ ($g_f$ = Fermi's constant) connected with the weak interactions, is remained practically constant

$$\mid \frac{1}{\beta}\frac{d\beta}{dt} \mid < 10^{-12} \text{year}^{-1}.$$

These weak limits on the variations can, of course, incite us to think that the ratios under consideration are time-invariant. In fact, only for $\alpha$, the conclusion seems to be indisputable. Besides, this latter is in complete agreement with the interaction theories which identify $\alpha$ with the electromagnetic coupling constant. Concerning the other results, we will make four remarks :

i. The weak variability of $P$ ($b$) is often interpreted [3] as proving that of the ratio $m/M$. Such a conclusion requires the invariance of $\gamma$, which is questionable. In fact, $P$ corresponds to the ratio $\mu_p/\mu_e$ ($\mu_p$ and $\mu_e$ being the magnetic momenta of proton and electron respectively). So that the results obtained for $P$ would be rather an argument for the time-invariance of the ratio of the "sizes" of the particles.

ii. The results concerning $G$ have been obtained by assuming that the planet (or moon) masses –i.e. practically the proton mass– are time-invariant, so that the conclusions, in fact, do concern the product $GM$.

iii. According to a Cartan theorem [17], the product of $G$ by the total mass of the universe is constant, so that, if the mass of this latter is remained constant during its evolution, $G$ is a time constant, and the variability observed for $GM$ has to be ascribed to the proton mass. A priori, the mass of proton must depend on its charge. Nevertheless, given that this mass essentially arises from the strong interactions between the quarks which constitute this nucleus, in so far as these interactions are independent of the size of the universe owing to their very short range, we will put

$$M \sim M_s + bm \quad , \quad (b \sim 1) \tag{1}$$

($M_s$ = contribution arising from the strong interactions). Consequently $\dot{M} \sim \dot{m}$, so that $\mid \dot{m}/m \mid < 10^{-7}\text{year}^{-1}$. This does not bring any positive information concerning the past variability of $m$.

iv. The values of $m/M$ quoted in Ref [13] seem to show that between 10 and $13 \times 10^9$ years, the weakest values, correspond to the oldest ages, so that $m/M$ would have been *increasing* versus time at this epoch, the variation rate being *ca.*$2 \times 10^{-10}\text{year}^{-1}$. New experimental data would be necessary to make the actual variation law precise.

## 3. First consequences

As we have said, the notion of variability requires the existence of time-independent reference units. In this paper, we will use the Gaussian system, so that only three units have to be defined, namely those of mass, length and time.

On the atomic scale, the two following quantities appear

$$\begin{aligned} &\text{i a length : } a = \tfrac{\hbar^2}{me^2} \\ &\text{ii a duration : } \tau = \tfrac{e^2}{mc^3} \end{aligned} \tag{2}$$

Given that $e^2/\hbar$ = constant, both $a$ and $\tau$ are proportional to $e^2/m$. Even if this ratio varies, in a system which utilizes $a$ and $\tau$ as reference units, all speeds –that of light in particular– keep the same values. Moreover, if we consider a pendulum of a given length ($\propto a$), within a gravitation field of a given acceleration ($\propto a/\tau^2$), its period is proportional to $e^2/m$. In other words, the macroscopic scale of time defined by this device is identical with the atomic scale $\tau(2)$.

On the other hand, we can define a temperature scale from phase transitions of a given crystal. Given that the cohesive energy of a crystal is proportional to $e^2/a$, i.e. to $m$, $kT$ appears as being proportional to $m$. Consequently, if we adopt a time-invariant temperature scale, $k$ is proportional to $m$. This explains the result obtained from the fossil radiation. Indeed, according to (2), the ratio $\hbar\omega/kT \sim \hbar/k\tau$ remains constant.

In conclusion, experiment shows that

$$e^2 \propto \hbar \quad , \quad a \propto \tau \propto \frac{e^2}{m} \quad , \quad k \propto m \tag{3}$$

On the other hand, astrophysics brings a very important information. According to the general relativity theory [18], the expanding universe involves a red-shift for all the radiations. More precisely, a radiation whose wave length is equal to $\lambda$ at its emission time, parvenes to us with a wave length

$$\lambda' = \lambda \frac{R_0}{R_e} \tag{4}$$

$R_0$ being the present curvature radius of the universe, and $R_e$, that at the emission time. Now $\lambda \sim a$, so that the spectral ratio is equal to

$$\tilde{\zeta} = \frac{\lambda'}{\lambda} = \frac{\left(\frac{a}{R}\right)_e}{\left(\frac{a}{R}\right)_0} \tag{5}$$

Given the red-shift, we must conclude that $a/R$ decreases versus time.

If we admit that the redshifts arise only from the expanding universe, i.e. that $\tilde{\zeta}$ is equal to the ratio $R/R_e$, $a$ must remain constant (equal to the present value of the Bohr radius, $a_0$) and, consequently, according to (1), we can write

$$e^2 \propto m \tag{6}$$

Such a relation would insure the time-invariance of $a$ and $\tau$ (2), which can be used as reference units, and that of many fundamental electron properties, namely : the Compton wave-length ($\hbar/mc$), the Thomson cross-section ($\propto e^4/m^2$), the frequency $\omega_0 = 2mc^2/\hbar$ which corresponds to the ($e^+$, $e^-$) pair creation. Moreover, it would be in agreement with a purely electromagnetic origin of electron. But, in fact, no experimental feature allows us to adopt the hypothesis (6). Only the coherence of the results concerning the history of the universe, which are implicitely deduced from the time-invariance of $a_0$ (1), can be considered as an argument for this hypothesis.

Whatever that may be, we see that we can admit a time-dependence of the fundamental "constants" provided that certain relationships are respected, namely those given in (3) and, probably, in (6). But how to know whether these "constants" effectively vary, and, if yes, why ?

## 4. A working hypothesis

More and more the fact that no system can be considered as being strictly isolated in the universe, seems to be taken into account. In fact, the idea is not new. As far back as 1924, Slater [19] wrote "Any atom may, in fact, be supported to communicate with other atoms, by means of a virtual field radiation". We can also quote the Stochastic Electrodynamics [20] which assumes that all charged particles radiate energy during their motion, and receive energy from a random electromagnetic field which would fill the whole universe (vacuum field). Let us recall that quantum electromagnetics theory is also based on the existence of a vacuum field.

As a working hypothesis, we will admit that the stability of a system (e.g. an atom) arises from the balance between the radiated energy and that absorbed arising from the other systems which constitute the universe [21].

Given the properties of the field corresponding to this exchange obviously depend on the size of the universe, the expanding universe appears as a possible cause for the variation of the fundamental constants, of $h$ in particular.

## 5. Connection with cosmology

It is well known that, after a brief but tumultuous period in which particles and the first nucleons appeared, about twenty billion years ago, the universe entered a quieter period characterized by a regular expansion which, according to general relativity, is governed in the Roberton-Walker metric by the following equations [22]

$$\begin{cases} 2\dfrac{\ddot{R}}{R} + \dfrac{\dot{R}^2}{R^2} + \dfrac{8\pi G}{c^2}p + \dfrac{kc^2}{R^2} = \Lambda \\ \dfrac{\dot{R}^2}{R^2} - \dfrac{8\pi G}{3}\rho + \dfrac{kc^2}{R^2} = \dfrac{\Lambda}{3} \end{cases} \tag{7}$$

where $\rho$ is the total density (assumed to be the same in any point of the space at a given time –the universe is homogeneous and isotropic) ; $p$, the total pressure ; $K$ the universe curvature $(+1, -1 \quad \text{or} \quad 0)$ ; $R$ the curvature radius (or the scale factor if $K = 0$). $G$ and $\Lambda$ are the gravitational constant and the cosmological constant respectively.

Two limit cases are generally envisaged. For large values of $R$, the universe is assimilated to a pressureless fluid ($p = 0$). This is the Friedmann-Lemaître model [23]. On the contrary, as the universe was very condensated, the matter density was completely negligible with respect to the radiation energy (radiation universe)

$$\rho_r = \frac{3}{c^2}p_r \tag{8}$$

where $p_r$ is the radiation pressure.

For the intermediate values of $R$, it is necessary, of course, to introduce a density equal to the sum of the matter density, $\rho_m$, and the radiation density, $\rho_r$. In all these cases, from (7), we obtain

$$\dot{R} = R_0 H_0 \sqrt{D(x)} \tag{9}$$

where $D(x)$ is a certain expression depending, on the one hand, on the values of $R$, $\dot{R}$ and $\ddot{R}$ (at the present time $R_0$, $\dot{R}_0$ and $\ddot{R}_0$ resp.), and on the other, both on the present density $\rho_0$ in the universe and on the temperature $T_0$ of the background radiation. $H_0 = \dot{R}_0/R_0$ is the Hubble constant, $x = R/R_0$. Within the model we consider, we must introduce the density $\rho_e$ corresponding to the exchanges between the systems. Nevertheless, like $\rho_r$, $\rho_e$ becomes negligible when $R$ is sufficiently large. Consequently, our model asymptotically tends, as $t$ increases, to the Friedmann-Lemaître model in which $D(x)$ (9) reduces as follows

$$D(x) = \frac{2(q_0 + \lambda_0)}{x} + (1 - 2q_0 - 3\lambda_0) + \lambda_0 x^2 \tag{10}$$

where $Kc^2/\dot{R}_0^2 = 2q_0 - 1 + 3\lambda_0$, $q_0 = -R_0\ddot{R}_0/\dot{R}_0^2$ and $\lambda_0 = \Lambda/3H_0^2$ [18].

In order to find the relationship between $R$ and $\hbar$, we will write that the energy radiated by an atom is equal to that received from the other atoms of the universe.

Apart from an unessential numerical coefficient, according to classical electromagnetism, the power radiated by an atom assimilated to an electron in circular orbit of radius $a$ around the corresponding nucleus, is the following [20]

$$I_e = \frac{e^2\omega^4 a^2}{c^3} = \frac{c^2\alpha^5\hbar}{a^2} \tag{11}$$

$\omega$ being the rotation frequency $\alpha c/a$.

Moreover, owing to the finiteness of the light velocity, and the expansion of the universe, the number of atoms which can be observed from a given point of space at a given time is finite (cosmological horizon). Let $N$ be the number of observable atoms corresponding to the ratio $\zeta$.

The number $dN$ of atoms located in the distance range $(\zeta, \zeta + d\zeta)$, according to Ref [24], can be written as follows

$$dN = n_0 H_0^{-3} f(\zeta) d\zeta \tag{12}$$

$n_0$ being the number of atoms per volume unit, $H_0$ the Hubble constant at the present time and $f(\zeta)$ a function depending on the adopted cosmological model ($K = 0, +1, -1$) but whose explicit expression is unessential for our problem (Vide infra Eq 14).

Moreover the flux $\phi$ arriving at the point under consideration is proportional to $H_0^2$

$$\phi = H_0^2 g(\zeta) \tag{13}$$

$g(\zeta)$ –as $f(\zeta)$– being a function depending on the model, but which it is unnecessary to make clear.

Consequently, the total flux at the point we consider arising from all the atoms of the universe ($1 < \zeta < \infty$) is

$$\Phi = \frac{n_0}{H_0} \int_1^\infty g(\zeta) f(\zeta) d\zeta \propto \frac{n_0}{H_0} \tag{14}$$

(The integral is convergent, if not, the brillance of the sky would be infinite).

In order to write the balance energy, we will partition the universe into cells, each of them containing *one* atom. The surface of a cell is typically proportional to $R_0^2$, so that the power which penetrates into one cell is proportional to

$$\frac{n_0 R_0^2}{H_0} = \frac{n_0 R_0^3}{\dot{R}_0} \tag{15}$$

Now, in the absence of continuous matter creation [25], $n_0 R_0^3$ remains constant versus time, so that, whatever the epoch we consider may be, this power (15) is proportional to $(\dot{R}_0)^{-1}$.

Let us write that this power is equal to that radiated by the atom located inside the corresponding cell, i.e. to $I_e$ (11), we obtain

$$\hbar_0 \propto (\dot{R}_0)^{-1} \tag{16}$$

This relationship established for the present time, is valid for all the times, so that we obtain the following law

$$\hbar \dot{R} = \hbar_0 \dot{R}_0 \tag{17}$$

The behavior of $\hbar$ for $R \to \infty$ depends on the cosmological model we adopt. According to (9) and (10) : i. if $\lambda_0 = 0$ and $q_0 \geq 1/2$ or $\lambda_0 < 0$, $\hbar \propto e^2 \to \infty$ after a finite or infinite time, which is physically unacceptable, ii. if $\lambda_0 = 0$ and $q_0 < 1/2$, $\hbar$ tends to a finite value, iii. if $\lambda_0 > 0$, $\hbar$ tends to zero.

Recent works concerning the space-distribution of quasars lead to optimized parameters $q_0$, $\lambda_0$ (10) which would seem to show that the universe is finite ($K = +1$) and in eternal expansion ($\Lambda > 0$) [26] :

$$q_0 = -1.1 \pm 0.1 \quad \text{and} \quad \lambda_0 = 1.2 \pm 0.1 \tag{19}$$

Moreover, $1.4 < 10^{18} H_0 (s^{-1}) < 3.4$ [27]. From which it results that, at the present time,

$$(\dot{\hbar}/\hbar)_0 \sim -10^{10} \text{year}^{-1}$$

With $q_0 = -1.05$ and $\lambda_0 = 1.25$ (the universe is $20 \times 10^9$ years old), for $x = R/R_0 = 1/3$, we obtain $\hbar \sim 1.2\hbar_0$, $\hbar$ exhibiting a maximum equal to $1.5\hbar_0$ for $x \sim 0.5$. These results agree, on the one hand, with the values obtained for $m/M$ ($\propto \hbar$ if we assume that $M$ remains constant) which are located between 0.7 and 1.5, and, on the other, with the fact that $m/M$ seems to be increasing versus time for $x \sim 0.3$.

Models with $\lambda_0 = 0$ can also lead to acceptable results: e.g. for $x = 0.3$, $q_0 = 0.2$ gives $\hbar = 0.7\hbar_0$ and $q_0 = 0.1$, $0.85\hbar_0$, $\hbar$ being increasing without exhibiting a maximum. But the values obtained for the age of the universe from these values of $q_0$ are obviously too large ($> 30 \times 10^9$ years). Therefore, a priori, this case has to be excluded.

For very small values of $R$, eq.(17) becomes unapplicable, the atoms are ionized. We have to deal with a plasma.

## 6. Weak interactions

In so far as we can consider that $\beta = g_f M^2 c/\hbar^3$ is a time constant [3], and that the proton mass is remained practically unchanged or, at least, tends to a finite limite which corresponds to the strong interactions, when $R \to \infty$ (the electromagnetic contribution tends to zero), $g_f$ behaves as $\hbar^3$, i.e. tends to zero when $R \to \infty$.

In other words, the "weak" interactions would be decreasing much more quickly than the electromagnetic interactions, which would be consistent with the *GUT* which foresees that in the early universe, the intensities of both these interactions were of the same order of magnitude. In any cases, the weak and electromagnetic interactions clearly appear as being strongly connected.

## 7. Conclusion

The chief point which emerges from this study is the strong interdependence between the values of the fundamental constants. But, conversely, this interdependence is a serious obstacle to any attempt to detect any variation of these constants, so that, after all, it may be asked whether the problem of the variation of the constants possesses any physical meaning.

The analysis we present, based on a general energy balance in the universe, is a possible way to approach the problem. Our hypothesis which can seem as being *risqué* with respect to the conventional quantum formalism, nevertheless, leads to interesting results. In particular, it allows to understand the origin of the Planck constant, this latter appearing as the coupling coefficient between a system and the rest of the universe [28]. Owing to its origin, this coefficient must necessarily be proportional to $e^2$. On the other hand, the fact that all the electric charges of the particles are multiples of the $d$-quark charge ($e/3$), can be explained by a resonance at the scale of the universe whose size is variable. Therefore, it is logical to find that $e^2$ and the other constants which are linked to it, vary versus time, even if this variation is not detectable.

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