Duration and distance without time

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ABSTRACT. We try to establish operational definitions of duration and distance at the atomic level, avoiding the concept of time flow which is a source of conceptual difficulties.

RESUME. Dans cet article, on s'interroge sur l'universalité de la définition de temps et d'espace. Il est proposé deux définitions opérationnelles liées aux propriétés de la matière, sans référence à des artefacts. La logique suivie conduit à des propriétés des durées et distances identiques à celles habituellement acceptées au niveau macroscopique, mais présentant quelques différences au niveau microscopique. En particulier, ces définitions conservent leur sens si elles sont appliquées au vide.

Introduction

Space and time are always used in physics, seldom discussed.

One is generally satisfied with the newtonian concepts, beautifully described in Newton's Principia [1] "absolute true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external..." and,

"absolute space in its own nature, without relation to anything external, remains always similar and immovable..."

However such definitions suffer from several flaws.

The first one, is that someone who does not know what time and space are will certainly not increase his knowledge by these definitions.

The second one is that it gives no clue to measurements but the indication that the measure of duration is obtained by means of motion [1].

Now, physics is universal. This means that the definitions it uses must be applicable anywhere in the universe and at any time.

As a historical example let us recall that when in 1790 the French "assemblee constituante" requested scientists to give a definition of the meter, they proposed the 40, 000, 000 th part of the terrestrial meridian precisely because every nation in the world was theoretically able to perform the necessary measurements which could lead to the actual realization of identical units of length [2].

Now the newtonian concepts of time and space do not provide us with universal operational definitions of duration and distance.

The concepts of units that we use are strongly dependent upon our physiology and location on a spinning planet rotating about the sun, as the etymology easily demonstrates.

The fact that nothing in newtonian space and time prevents us from measuring duration and distance in vacum has to be taken as a warning of the existence of difficulties since, in fact, such results are meaningless: a distance in vacuum is that of the measuring device as a duration is that separating two states of the clock one uses for that purpose.

Concerning duration an interesting problem arises. A duration is a measurement of the gap separating two events which are not simultaneous in general (our argument does not interfere with relativity).

We measure for instance the duration separating the passage of a particle between two detectors. We obtain a result in our present. Now, what is our past ? What, where is the state of the laboratory when the particle passed the first detector ? It is nowhere. There is no corner in the universe where that state now lies. That state is a mental reconstruction that we all perform using our observations of the present.

So a duration binds two states of a very different nature: one is the instant reality, the other a reconstruction from that reality. This is a very incoherent feature.

For all these reasons we have tried to find operational definitions of duration and distance which should be characterized by the following properties:

- They are defined in our present (or reality).

- They have a universal character.
- They have a meaning in vacuum and at any scale (micro and macroscopic).
- At macroscopic scale the definitions become equivalent to the newtonian ones.

This program can be achieved but at a certain price: some new concepts have to be introduced.

Moreover one must stress that the condition of universality forbids us from introducing artefacts. Our concepts have to be defined through the help of "freely organised matter" which we admit to have identical properties in our universe.

1. The objectivity of reality

- We consider two observers O, O' taking each all possible measurements in the universe at one instant. If there exists a transformation (Lorentz, for instance, in a flat universe) of one set of observations into the other we say that the observers are in the same real.
- We assume that in the sequence, ordered or not, of the instant observations made by O, one at least may be transformed to be identical to any instant observation of O'. This is what we assume to be the objectivity of the real.

1-1 Duration

The first question to set is "between what" ?

Since the real is our only source of measurements and knowledge, we must try to define our new concept from it.

We propose to operate in the following way.

We imagine that we part the universe into cells as any other observer can do.

In a first stage, we shall try to introduce a measurement of the otherness $(m - o)$ between two cells.

We are obviously not interested in distance (euclidian, for instance) between cells, neither in their relative orientation but rather in the difference of state of the matter and radiation inside the cells since properties of matter are independent from distance, orientation and state of uniform movement.

This difference of states of the cells depends upon the nature and precision of the measuring device that the observers use (and which we imagine to be identical). It is clear that different levels of accuracy in the instruments might change our judgment on the difference of states of two selected cells.

Happily we need not discuss the kind of measurement which we perform, but it is of interest to keep this particularity in mind.

In fact, what is important is that all cells have to be described at an equal level of accuracy.

If we consider now the set C of all the cells, we clearly see that we have defined a partition through the equivalence relation R: cell i and cell j contain matter in the same state (for the level of observation that we use).

Let $C_i, C_j, ...$ be the equivalence classes of the set C. We introduce now the notation Δ_{ij} for the measurement of otherness $(m - o)$ between C_i and C_j.

Thus, our idea is to define an $m - o$ between two particular cells i and j as a function defined on the classes of equivalence (according to the relation R) to which they each belong (in fact it is a function of $C \otimes C$ with real values).

Before obtaining an explicit expression for Δ_{ij}, one may already compare classical duration with Δ_{ij} and draw some conclusions. In a classical theory, one measures a duration between two different states of "the same" single cell. In general one state is in the past, C_p the other in the present C_R.

One of the states C_P is imaginary, C_R is real (in the present of the observer).

One needs a special instrument, a clock which in fact may be considered as belonging to the cell. One hase to compare a present reconstruction of the state C_P and of the clock with the actual state C_R of the cell and of the clock.

Something is supposed to have been streaming and we measure its quantity.

In our system, one needs to consider in the present, the cells C_P and C_R and find out their classes of equivalence. The measurement of otherness for these classes will give us Δ_{PR} the desired quantity.

C_P and C_R have to exist in our real, otherwise Δ_{PR} would not be defined. There is a deep connexion with Boltzman's ergodic assumption since we consider that a state which belongs to the history of a cell of our present, (classical point of view) also belongs to our real (present point of view).

- In this way, one works in the present, avoiding the mixing up of states with a different existential status.
- One avoids discussing the nature of time, its flowing etc.
- We avoid problems of scale (microscopic and macroscopic systems are treated in the same way).

The discussion presented here is model independant. The fundamental assumption is the existence of the possibility to enumerate states and particles belonging to identical cells of indifferent size and shape. Problems of separability do not interfere as long as one focusses his interest on the description of the cells.

- The definition is universal.
- It has meaning in vacuum (the cells concerned are simply empty).

What remains to be done is to find out an explicit formula for Δ_{ij} and discuss its relation to classical (newtonian) time.

Curious properties exist:

- It is completely meaningless to discuss the age of the universe since for that we would need two cells at least whereas the universe cell is represented by the totality.

The question of continuity of time is discussed in the following way. Let it be a particular cell. It belongs to the class C_i. Our form of thinking invites us to consider i as belonging to a sequence of states... $i-2, i-1, i, ...$ defined in the following way: one looks for the classes C_{i-1} and C_{i+1} which realise the minimum measurement of otherness with C_i with the condition: $\Delta_{i,i+1} \geq 0$ and $\Delta_{i,i-1} \geq 0$. For the next step, one uses C_{i+1} and so on.

The evolution or continuity comes from the consideration of the sequence of the cells labelled $...i-r, ..., i-1, i, i+1...$ as being real, the stage j being arbitrarily chosen inside the class.

1.2

Let us construct the $m-o$ function Δ_{ij}.

The first thing to be done is to describe the cells, so that one may then classify them according to the equivalence relation.

We think that the simplest would be the best and that means using elementary counting. We suppose that the contents of the cells i for instance is defined by a list of the number of different particles $q_{i_1}, q_{i_2}, ..., q_{i_u}, ...$ which it contains.

If this description proved to be insufficient, one might try to enrich it later on.

To simplify, the sequence of descriptive numbers of the cell $q_{i_1}, q_{i_1}, ...$ will be called Q_i.

Thus, with this very broad description, two cells i and j are equivalent if

$$Q_i = Q_j$$

Let $C_i, C_j...$ represent the classes of equivalent cells e.g. all the cells of C_i have the same Q_i.

Δ_{ij} will be the $m - o$ between any two cells belonging respectively one to C_i, the other to C_j.

We wish to have the following properties:

$$\Delta_{ij} = -\Delta_{ji} \qquad (1.2.1.\ 1))$$

$$\Delta_{ij} + \Delta_{jk} = \Delta_{ik} \qquad (1.2.2.\ 2))$$

which are fundamental properties of the classical time measurement.

Now one must understand the nature of the variables on which Δ_{ij} is dependent.

As mentioned, these variables have nothing to do with space variables, since geographical location is completely foreign to our problem. They must also be as simple as possible and enjoy a universal character.

A good candidate is the number of cells of each class i, j... This is a universal quantity, any observer will obtain the same results, and if one is able to part the universe into cells and sort the classes, one is also able to count the number of elements of these classes.

Due to possible problems of infinity, it might be better to consider variables limited in range as for instance the ratio of these numbers to the total number of cells, that is to say, the probability P_i that a cell belongs to the class C_i.

We will therefore make the hypothesis that Δ_{ij} depends on P_i and P_j but also on the quantities inside the cell for instance, e.g. the sequence

$$q_{i_1}, q_{i_2}, \cdots$$

which describes the number of different particles of C_i.

Thus, we set $\Delta_{ij} = \phi(Q_i; P_i \mid Q_j; P_j)$ (1.2.3) where P_i and P_j are the probabilities that a cell belongs to the class C_i or C_j.

3.1.3 Explicit determination of the $m - o$

To (1.2.1) and (1.2.2) one may add a further relation which arises from the definition of the $m - o$.

If we consider a given partition into cells, we obtain classes and an $m - o$ for these classes. Suppose now that one considers unions of a definite number of cells belonging to the same class. For instance one may consider new cells made up of the union of two cells belonging to the same class of the original partition. For such new cells, the "descriptive numbers" will be twice the original ones and the probabilities will be the squared of the previous ones.

However we consider that the $m - o$ for these new cells has to be the same as for the original ones. To express it in a classical way: two identical clocks should give the same duration as a single one.

If one writes

$$\Delta_{ij} = \phi(Q_i, P_i \mid Q_j, P_j) \tag{1.3.1}$$

as the explicit expression for Δ_{ij} as a function of the different "descriptive numbers" Q_i of the cell i for instance, we admit consequently that one has:

$$\Delta_{ij} = \phi(\lambda Q_i; P_i^\lambda \mid \lambda Q_j, P_J^\lambda) \tag{1.3.2}$$

where we have deliberately generalized the case for the new cells unions of two, to λ cells and in fact admit a "continuous" λ (as is well known, by subdivision of the cells this could be obtained for λ rational and by continuity of ϕ to λ real). Or

$$\Delta_{ij} = \phi(\lambda q_{1i}, \lambda q_{2i} \cdot; P_i^\lambda \mid \lambda q_{1j}, \lambda q_{2j}, \cdot; P_j^\lambda) \tag{1.3.3}$$

Using (1.2.1), (1.2.2) and (1.3.2), one may write:

$$\phi(Q; P \mid Q'; P') = -\phi(Q'; P' \mid Q; P) \tag{1.3.4}$$

$$\phi(Q; P \mid Q'; P') + \phi(Q'; P' \mid Q''; P'') = \phi(Q; P \mid Q''; P'') \tag{1.3.5}$$

$$\phi(\lambda Q; P^\lambda \mid \lambda Q'; P'^\lambda) = \phi(Q; P \mid Q'; P') \tag{1.3.6}$$

Introducing a fixed cell "o", one has

$$\begin{aligned}\phi(Q;P\mid Q';P') &= \phi(Q;P\mid Q_0;P_0)+\phi(Q_0;P_0\mid Q';P')\\ &= \phi(Q;P\mid Q_0;P_0)-\phi(Q';P'\mid Q_0;P_0)\end{aligned}\tag{1.3.7}$$

So that

$$\phi(Q;P\mid Q';P') = f(Q,P)-f(Q',P')\tag{1.3.8}$$

the property (1.3.2) allows us to write:

$$f(\lambda Q,P^{\lambda}) = f(Q,P)\tag{1.3.9}$$

Explicitly:

$$f(\lambda q_1,\lambda q_2,\cdot;P^{\lambda}) = f(q_1,q_2,\cdot;P)\tag{1.3.10}$$

deriving with respect to λ and setting $\lambda = 1$ one gets:

$$\sum_{i=1}^{n} q_i\frac{\partial f}{\partial q_i}+P\ln P\frac{\partial f}{\partial P}=0\tag{1.3.11}$$

To take into account the different role played by the q_i in the one hand and P on the other, we admit that in the solution the variables do separate. We set therefore:

$$f(q_1,\cdot q,\cdot;P)=\psi(q_1,\cdot q_u,\cdot)\phi(P)\tag{1.3.12}$$

(1.3.11) thus gives:

$$\sum_{i=1}^{n} q_i\frac{\partial\psi}{\partial q_i}(q_1\cdot q_u)\frac{1}{\psi(q_1,\dots q_n)}+P\ln P\frac{d\phi}{dP}\cdots=0\tag{1.3.13}$$

setting $P\ln P\frac{d\phi}{dP}\frac{1}{\phi}=\alpha$ we obtain

$$\sum_{i=1}^{n} q_i\frac{\partial\psi}{\partial q_i}(q_1,\dots,q_u)=-\alpha\psi(q_1,\dots q_n)\tag{1.3.14}$$

and

$$P\ln P\frac{d\phi}{dP}=\alpha\phi\tag{1.3.14}$$

that is to say:

$$\phi(P) = A(\ln P)^{\alpha} \tag{1.3.15}$$

There remains to be solved the equation

$$\sum_{i=1}^{n} q_i \frac{\partial \psi}{\partial q_i} = -\alpha\psi \tag{1.3.16}$$

where unfortunately we have no boundary conditions.

We discard solutions of the type:

$$\psi = \frac{B}{\pi_1^n q_i^{\alpha_i}} \quad \sum_1^n \alpha_i = \alpha \tag{1.3.17}$$

since 1) α will be shown to be equal to one 2) (1.3.17) presents difficulties when one q_i is zero which is a current situation (absence of particles of a certain type in a cell).

Up to now we have shown that

$$\Delta_{ij} = A[\psi(Q_i)(\ln P_i)^{\alpha} - \psi(Q_j)(\ln P_j)^{\alpha}] \tag{1.3.18}$$

1.4 Evaluation of α. Connexion with newtonian duration

Let us discuss the connexion between newtonian duration and $m-o$.

For that, we need a situation where it will be possible to read a clock on the one hand, and to measure an $m-o$ on the other.

The reading of a clock suffers no ambiguity: the duration in question is obtained by the difference in readings, on the same clock at different instants of time (classical newtonian theory).

We have to define an equivalence to this measurement in our theory in such a way that both methods are applicable.

We admit, for this purpose, the existence of a large amount N of identical clocks. Each clock will belong to a cell which will not contain anything else.

This will enable us to define the $m-o$ of two clocks -cells.

The observer will himself be provided with a clock arbitrarily taken from one of the cells and will be able to:

a) read two indications (e.g. j and j') on this clock at different instants of time (classical theory)

b) look for $m - o$ of two clocks reading j and j' at the same instant of time (whatever the instant defined by his own clock) (present theory).

The results should be the same.

a) is straightforward

b) demands an evaluation of the probabilities P_i.

We characterize now the distribution of the N clocks.

We remind that:

- the clocks are identical
- are not set at the same time since:
- there should be no priveleged instant of time in universe.

The observer at a certain instant picks out a clock. Let (j) be the reading of that particular clock which is, from now on, the observers' clock.

We define the distribution of the clocks by the number n_i^j of clocks (or cells) reading (i) when the clock of the observer reads (j).

The probabilities we are looking for will be obtained when the number N of clocks will go to infinity (or become very large).

$$P_I^J = \lim_{N \to \infty} \frac{n_i^j}{N} \tag{1.4.1}$$

Let us discuss n_i^j

- Since the clocks are identical, they beat at the same rate and one has:

$$n_i^j = n_{i+1}^{j+1} \tag{1.4.2}$$

- Since no particular instant of time is privileged, the observer should not be able to define his time by studying the distribution.

This is ensured by admitting:

$$\frac{n_i^j}{n_{i+1}^j} = \frac{n_i^{j+1}}{n_{i+1}^{j+1}} \tag{1.4.3}$$

Comparison between (1.4.2) and (1.4.3) shows that

$$\frac{n_i^j}{n_{i+1}^j} = \frac{n_{i-1}^j}{n_i^j} \quad \text{or} \quad \frac{n_{i+1}^j}{n_i^j} = \frac{n_i^j}{n_{i-1}^j} = \frac{n_{i-1}^j}{n_{i-2}^j} = \ldots = \frac{n_1^j}{n_0^j} = \omega_j \tag{1.4.5}$$

So that one may set:

$$n_i^j = a_j(\omega_j)^i \quad \text{with} \quad a_j = n_0^j \tag{1.4.6}$$

Using once again (1.4.3) and (1.4.2) we get:

$$\omega_j = \omega_{j+1} = \ldots = \omega \quad \text{and} \quad a_j = B\omega^{-j} \tag{1.4.7}$$

To obtain n_i^j and P_i^j the probability for the observer to find a clock reading i when his own reads j, we have a problem of normalisation since the total number of clocks is "very large". Since

$$n_i^j = a_j\omega^i = B\omega^{i-j} \tag{1.4.8}$$

We may compute B using:

$$N = \sum_{i=-k}^{i=l} n_i^j \tag{1.4.9}$$

and letting k, l tend to infinity afterwards. This gives:

$$N = \sum_{i=-k}^{i=l} B\omega^{i-j} \tag{1.4.10}$$

$$B = N\omega^j \frac{1-\omega}{\omega^{-k} - \omega^{l+1}} \tag{1.4.11}$$

Finally

$$P_i^j = \frac{n_i^j}{N} = \frac{B\omega^{i-j}}{N} = \omega^i \frac{1-\omega}{\omega^{-k} - \omega^{l+1}} \tag{1.4.12}$$

where k and l may be "very large".

Let us now get back to the observer who reads the time from his own clock: and obtains $j' - j$ from two different instants of time. This is a "classical newtonian" duration.

The observer compares it with the measurement for otherness of the cells containing at the same instant a clock reading j and a clock reading j'.

If the instant when the observer defines the $m - o$ be j_0 on his particular clock, the probabilities to find clocks reading j and j' will be respectively:

$$\omega^j \frac{1-\omega}{\omega^k - \omega^{l+1}} \quad \text{and} \quad \omega^{j'} \frac{1-\omega}{\omega^k - \omega^{l+1}} \tag{1.4.13}$$

Let us get back now to (1.3.18)

$$\Delta_{jj'} = A[\psi(Q_j)(\ln P_j)^\alpha - \psi(Q_{j'})(\ln P_{j'})^\alpha]$$

Since the clocks are physically identical, one may admit the equivalence of the sequences Q_j and $Q_{j'}$.

Consequently,

$$\Delta_{jj'} = A\psi(Q_j)[(\ln P_j)^\alpha - (\ln P_{j'})^\alpha]$$

If we introduce the values for P, P' that we have obtained (1.4.13), we see that

$$\Delta_{jj'} = A\psi(Q_j)[(\ln \omega^j \frac{1-\omega}{\omega^{-k} - \omega^{l+1}})^\alpha - (\ln \omega^{j'} \frac{1-\omega}{\omega^{-k} - \omega^{l+1}})^\alpha] \tag{1.4.14}$$

Now clearly, the classical result for such a time measurement is

$$\tau = K(j' - j) \tag{1.4.15}$$

where K is a constant and the only value of α which is compatible with it is $\alpha = 1$.

Thus we see that an agreement exists with the classical theory, provided we admit α to be equal to one, which we do from now on.

We may now write:

$$f(Q, P) = A\psi(Q) \ln P \tag{1.4.16}$$

where

$$\sum_i q_i \frac{\partial \psi}{\partial q_i} + \psi = 0 \tag{1.4.17}$$

We have no boundary conditions at our disposal as already mentioned and the only thing we may do is look for the simplest solution for (1.4.17).

For this, we set $\psi(q_1, q_2, \dots q_n) = \frac{1}{\xi(q_1, \dots q_n)}$ and (1.4.17) becomes

$$\sum_i^n q_i \frac{\partial \xi}{\partial q_i} = \xi \tag{1.4.18}$$

The simplest non trivial solution is obviously $\xi = \sum_1^n \beta_i q_i$.

If we take into account the remark concerning the vacuum (which now means that ξ may be zero only if all q_i are zero), we propose therefore, to set

$$\Delta_{ij} = \theta \left(\frac{\ln P_i}{\sum_l \beta_l q_{li}} - \frac{\ln P_j}{\sum_l \beta_l q_{lj}} \right) \tag{1.4.19}$$

where q_{li}, q_{lj} are the descriptive numbers for the cell i, j. We remain with the problem of the determination of the β_l. For that we discuss first the concept of distance.

2. Distance

We shall now take up the problem of distance which we shall of course discuss on the same line as that of duration.

One could argue that a discussion is useless since we have solved (from our point of view) the problem of Δ and since we know that the velocity of light is a universal constant. The time of travel of light from one point to another then becomes equivalent to the definition of a distance.

This definition is of course excellent when macroscopic distances are in question. Unfortunately it becomes extremely difficult when the distances are microscopic as shown by Salecker and Wigner [3].

From our point of view, it would be unreasonable to introduce a concept of distance which would be applicable to macroscopic distances (and thus to artefacts) and not to the only universal objects at hand, particles in the sense of §2.

We propose to proceed in a completely opposite way, introducing distance for microscopic objects in a way which enables us to obtain a correct concept when applied at macroscopic scale (i.e. for macroscopic objects).

We start by observing that this particular distance which we shall note δ cannot be defined between two points of space in vacuum since it is physically impossible to realize such type of measurement.

In fact if we try to measure a distance between two points of space, what we really obtain is the distance between two material points of the measuring device (i.e. not of the vacuum).

Therefore, what we should do is try to define a distance inside a physical system since this is what we measure in reality. This means, in fact, that the system itself has to be used as a measuring device.

These systems should be atoms or nuclei which are the smallest known objects and because it is possible to perform and repeat measurements.

Discussing these microscopic systems, one should ask where distance can be involved.

Let us consider a system S consisting of N identical atoms into which we try to define δ which should correspond to the classical distance. Let us discuss the system S itself.

The atoms of S cannot be confined, otherwise they would not be in a "universal configuration": the atoms cannot be interacting, since in that case other distances than those within an atom would manifest themselves (such as the mean free path). S is then a very subtle ensemble of non interacting identical particles. To introduce δ we need to measure something (and the most universal measurement is a counting).

If S contains atoms in a stable state, nothing is ever going to happen and we will make no measurement at all since no counting will ever take place.

Thus one of the atoms of S must be in an unstable state. However, it is clear that if there may happen transitions from one or several states to one or several others, we will have to deal with many dimensions. At last, what we must suppose at this stage is that the particles of S can be in two states only, s_1 and s_2 where transitions from s_1 to s_2 are allowed. A distance δ will therefore be bound to those two states s_1, s_2 and should be written $\delta(s_1, s_2)$. We cannot hope for more simplification with this approach.

Now, what can one measure ? One may count the number of atoms in the states s_1 and s_2. But this is a variable non reproducible number and so it cannot be $\delta(s_1, s_2)$.

We can also measure the duration between events concerning the evolution of S and this is of particular interest since at macroscopic scale we consider that the time of flight of a photon from one point M to another M' is a correct definition of the distance separating M from M'.

In view of this definition we might try to define the distance $\delta(s_1, s_2)$ as equal to the absolute value of the duration, which is needed for one single particle to pass from the state s_1 to the state s_2 by emission of a photon (in fact of a zero mass particle).

A first consequence of this definition is that a stable particle has no radius, since it is associated to one state only.

2.1 Expression for $\delta(s_1, s_2)$

Let us now define this duration. We wish to measure with our standards the duration separating the following states: one particle is excited at level 1 the same particle is on level 2.

In fact what we will do is get back to the previos definition of Δ to measure the $m - o$ between two states s_1 and s_2.

So, what we should do is consider the $m-o$ between cells containing a single atom in state s_1 and cells containing one atom only in the excited state s_2.

δ will be equal to the absolute value of the $m - o$ between these cells, multiplied by the velocity of light.

Therefore the definition could be:

$$\delta(s_1, s_2) = c\theta \mid \frac{1}{\beta_{i1}} \ln P_1 - \frac{1}{\beta_{i2}} \ln P_2 \mid \tag{2.1.1}$$

β_{i1}, β_{i2} are the β_i associated to s_1, s_2.

P_1, P_2 are the probabilities of observing one atom in the state s_1 resp. s_2 in a cell.

But, unfortunately, this definition is not universal since P_1 and P_2 depend on the size of the cells as one can easily check by going to the limit where the cells tend to the size of the universe and therefore P_1 and P_2 tend to zero (since there will remain only a single cell containing the universe !).

To get a result independent of the size of the cells, one has to introduce a limiting procedure. One might consider the limit, when the size of the cells are such that no cell contains more than one particle of each type.

To make things clear let us assume that the universe is finite. We could obtain, if N is the total number of cells and n_i the total number of particles of type i:

$$P_i = \frac{n_i}{N}$$

and

$$\delta = c\theta \mid \frac{1}{\beta_{i_1}} \ln \frac{n_1}{N} - \frac{1}{\beta_{i_2} \ln \frac{n_2}{N}} \mid \qquad (2.1.2)$$

However this definition implies problems since N is not a universal number and depends on the spatial distribution of particles i, whereas n_i are constants (total number of s_i states):

We might imagine that at different instants, because of the proximity of a few number of particles in the state s, one should have to double the number of cells to separate particles.

Formula (2.1.2) would give a different result which is unacceptable.

However (2.1.2) may be written:

$$\delta(s_1, s_2) = c\theta \mid \frac{1}{\beta_{i_1}} \ln n_1 - \frac{1}{\beta_{i_2}} \ln n_2 + c\theta(\frac{1}{\beta_{i_1}} - \frac{1}{\beta_{i_2}}) \ln N \mid$$

and it is obvious that $\delta(s_1, s_2)$ is independent of N if and only if

$$\beta_{i_1} = \beta_{i_2} \qquad (2.1.3)$$

For consistency reasons, we must therefore admit this result that β_i is constant for all electromagnetically excited states of a system. For instance, all excited states of an atom share the same β.

$\delta(s_1, s_2)$ simplifies to:

$$\delta(s_1, s_2) = \frac{c\theta}{\beta_{i,2}} \mid \ln \frac{n_1}{n_2} \mid = \frac{c\theta}{\beta_{i,2}} \mid \ln \frac{d_1}{d_2} \mid \qquad (2.1.4)$$

where we replace n_1/n_2 by the ratio of the densities of states d_1/d_2 and of course, $\beta_{i,2} = \beta_1 = \beta_2$. (2.1.4) is our final result for $\delta(s_1, s_2)$.

(1.4.19) the formula expressing Δ is unchanged, however one must keep in mind that β_i are constants for electromagnetically excited states, or to express it more briefly, over chemical species.

2.2. Properties of δ

Let us consider a system with its different states $s_1, s_2, s_3 \dots$ one has:

$$\delta(s_1,s_2)+\delta(s_2,s_3)=c\frac{\theta}{\beta}\mid \ln\frac{d_1}{d_2}\mid +c\frac{\theta}{\beta}\mid \ln\frac{d_2}{d_3}\mid\geq$$
$$\frac{c\theta}{\beta}\mid \ln\frac{d_1}{d_2}+\ln\frac{d_2}{d_3}\mid=\frac{c\theta}{\beta}\mid \ln\frac{d_1}{d_3}\mid=\delta(s_1,s_3) \qquad (2.2.1)$$

which corresponds to the triangle inequality.

- $\delta(s_i, s_j) = 0$ however has not for consequence that $s_i = s_j$ but simply that $d_i = d_j$ as one reads from the definition.
- δ infinite means d_i or d_j eqal to zero and is impossible since it corresponds to the complete non existence of a state. Therefore, infinite distances do not exist in that model.

3. Applications

Finally we would like to give a result which permits to decide on the physical (as opposed to conceptual) validity or interest of this present theory.

If our definition of distance is of any value, it has to offer results in accordance with what is observed in atomic theory, so that

$$\delta_{1,2}=\frac{c\theta}{\beta}\mid \ln\frac{d_1}{d_2}\mid \qquad (3.1)$$

has to correspond in some way to real atomic distances.

If we eliminate β by considering two excited states of the same atom, hydrogen for instance, one obtains labeling by $0, 1, 2$ the fundamental and two excited states:

$$\frac{\delta_{01}}{\delta_{02}}=\frac{\mid \ln d_0/d_1\mid}{\mid \ln d_0/d_2\mid} \qquad (3.2)$$

where d_i are the densities in the universe (outside stars since only non interacting particles are relevant here as noted above) of the corresponding states.

It is of course desirable to obtain other results. This unfortunately depends upon evaluations of P_i the probabilities of the states of a cell which is not easy in general.

Let us consider the case of the decay (in classical term) of a particle into two others:

$$A \to B + C$$

with the aim to calculate the duration separating two states with different numbers n and n' of A particles.

What we should do is evaluate the probability P_n to have n particles of type A in a cell.

We need also the probability $P_{n'}$ of finding n' particles of type A in a cell together with $n - n'$ couples of B and C particles with the correct momenta.

Moreover, the probability cannot be understood as resulting from the independent events:

- n' A particles are in the cell
- $(n - n')$ B and C particles are in the cell since there is of course a strong correlation between them.

There is a single case where one might try an evaluation.

Let us suppose that in the decay state B and C have the velocity of light. One may admit that the cells we are interested in contain respectively n A particles and n' A particles (B and C being unobserved).

There are few cases of this type the π_0 being an example.

The $m - o$ between cells with n π_0 and n' π_0 respectively is:

$$\Delta_{nn'} = \frac{\theta}{\beta}(\frac{1}{n}\ln P_n - \frac{1}{n'}\ln P_{n'})$$

if we admit that a Poisson type distribution represents the situation, one obtains

$$P_n = e^{-d}\frac{d^n}{n!}$$

where d is the density of π_0 in the cells. Then

$$\Delta_{nn'} = \frac{\theta}{\beta}[\frac{1}{n}\ln e^{-d}\frac{d^n}{n!} - \frac{1}{n'}\ln e^{-d}\frac{d^{n'}}{n'!}]$$

which simplifies with the help of Stirling's formula to

$$\Delta_{nn'} = \frac{\theta}{\beta}\ln\frac{n'}{n} + \epsilon_{nn'}$$

where

$$\epsilon_{nn'} = (\frac{1}{n} - \frac{1}{n'})(d + \ln\sqrt{2\pi}) + \frac{1}{2}(\frac{\ln n'}{n'} - \frac{\ln n}{n}) + 0(\frac{1}{n^2}, \frac{1}{n'^2})$$

can be neglected for large n and n'.

The result seems acceptably near the classical one.

It is then tempting to discuss the decay of an atomic excited state.

Unfortunately, the problem of $s^* \to s + \gamma$ is quite difficult since there is a strong correlation between the different states not to discuss the problem of the reversed reaction.

Indeed, one has to admit that within our scheme, it is not possible to distinguish, for instance

$$s^* \to s + \gamma$$

from

$$s + \gamma \to s^*$$

This is due to the fact that we consider instant states and cannot decide on the direction of a momentum or of a velocity (see for example: reference [4], where the same argument is developped on different but equivalent bases.

Consequently the probabilities to find respectively s states, s^* states and photons are strongly correlated.

As a matter of fact, it is the existence of this correlation, in this theory, which allows one to introduce an equivalent to the classical concept of the reaction

$$s^* \to s + \gamma$$

which has a meaning when time is flowing.

Since we do not know the distribution function, we cannot calculate the $m - o$ between the cells of interest.

The case of the π_0 was much easier to handle since we have implicitly admitted that 1)

$$\gamma + \gamma \to \pi_0$$

is very seldom observed 2) the γ's are not present in the cells "after decay" so that there is no problem of correlation.

Références

[1] Newton's Principia.
[2] See for instance *L'espace et le temps aujourd'hui*, chapter 8 discussion with J. Terrien - Le Seuil Editeur - Paris 1983.
[3] Salecker, H. and Wigner E.P. (1958), Physical Review, 109, 571.
[4] G. Karpman, *Sur une difficulté dans la définition microscopique d'unités d'espace et de temps*, Comptes Rendus de l'Académie des Sciences 1977, t. 284, série A, p. 97.

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