Considerations of an oscillating spiral universe cosmology

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ABSTRACT. It is proposed that if the spiral configuration of galaxies is explicable in terms of the equations of motion of its constituent stars, *as an expression of global laws of nature*, then the universe as a whole may be similarly described in terms of the motions of its constituent galaxies with a similar spiral dynamics. With the functional form of the spiral paths in terms of Fresnel integrals, taken from solutions of equations in general relativity (from previous analyses of galactic configurations) the density of the universe at the 'big bang' stage is determined. It is found to depend, numerically, on the neutron lifetime and the period of oscillation of the universe as a whole. There is some concluding discussion of the implications of this analysis of the matter of the universe at the 'big bang' stage vis à vis the black hole state of matter.

RESUME. Un modèle cosmologique de l'univers dont les galaxies sont dans une configuration spirale oscillante, peut être déduit, d'une manière dynamique, de lois globales en relativité générale. Les solutions des équations du mouvement, dans une factorisation en quaternions des équations du champ tensoriel d'Einstein, sont des intégrales de Fresnel. La densité de l'univers est déterminée à partir de ces solutions au stade du big bang, suivant la durée de vie du neutron. On discute en conclusion les implications de cette analyse de l'état de la matière de l'univers au stade du big bang face à l'état de type trou noir de la matière.

1. INTRODUCTION

A feature of the universe that is usually taken for granted in present day studies is based on the 'cosmological principle'. This is the idea that all of the matter of the universe is distributed in space both homogeneously and isotropically. This idea is clearly false in the 'local domain', as we can see in astronomical observations of the nonhomogeneous and anisotropic structures of the spiral galaxies, the elliptical galaxies, the large variation of mass distributions of stars and comets and other interstellar fragments within our galaxy and the neighboring galaxies, the clustering of galaxies throughout the universe, and so on. Nevertheless, the present day astronomical data reveal that *on the average* the galaxies and other matter of the universe are indeed distributed globally homogeneously and isotropically, in accordance with the 'cosmological principle'. The concensus of opinion among presentday astronomers and astrophysicists in then that the local deviations from such uniformity of the matter of the universe are only nearby fluctuations in an otherwise smoothly distributed system of galaxies, other interstellar matter and radiation.

One of the strong corroborations of the cosmological principle was the discovery by Penzias and Wilson [1] that indicated the universe to be permeated by electromagnetic radiation, distributed isotropically with a blackbody radiation spectrum near $3°K$. Because of the uniformity of this radiation, compared with the nonisotropic distribution of stars in our galaxy, this has been interpreted as radiation that is a remnant of the primordial fireball, at the time of the 'big-bang' –that is, radiation associated with the universe as a whole, rather than a locally distributed radiation field of the Milky Way alone.

In contrast with the assertion of the 'cosmological principle', I have been investigating the idea that the laws of nature that we deduce for the local domain (whether in elementary particle physics or the physics of the matter of the galaxy) are an indication of the laws of nature for the global domain as well. Thus it is proposed that the anisotropic, nonhomogeneous distribution of stars in the local domain of our own galaxy, the Milky Way, and our neighboring galaxies, must be the way it is because of some universal dynamical laws that indeed apply to all domains of the universe, including the domain of the totality of the universe – cosmology.

A striking example of an anisotropic, nonhomogeneous distribution of stellar matter is that of the spiral galaxies –most of the stars being near a plane and rotating about a perpendicular to this plane. The idea about the universality of the dynamical laws then implies that the cause-effect relations that are to explain the spiral shapes of the individual galaxies may also be responsible for a spiral distribution of galaxies in the universe, as a whole.

This idea must then assume that the present day astronomical data do not yet entail sufficient resolution that could detect the difference between such global anisotropy and inhomogeneity and the isotropic, homogeneous distribution of galaxies of the universe, as postulated by the 'cosmological principle'. [Such an assumption as the latter is similar to that of Galileo, in the 17th century, who also did not have sufficient resolution in his measurements to detect the actual deviation from the spherical uniformity of the Milky Way, which was discovered to be the case about a century later, by Herschel [2]].

It should be noted at this stage of the discussion that the spiral distribution of galaxies of the universe, if it exists, does not in the context of Einstein's general relativity imply the existence of an absolute axis of symmetry about which the (spiral) universe must rotate. What it does imply is that from any particular galactic frame of reference one must observe a rotation of the galaxies about *some* axis. However, where this axis is in the universe and the period of rotation of the galaxies of the universe as a whole, must be subjective elements of knowledge about the universe. That is, these are functions of the reference frame from which they may be observed. The application of the space-time transformations of general relativity theory to any other galactic reference frame would then predict the observed location of the universe's rotation axis and its period from that frame.

For the very same reason, the principle of general covariance predicts that the 'time' of the 'big-bang' is not an absolute measure. This is in contrast with some present day views that take the reference frame of the universe as a whole to be absolute. Here, it is rather that the 'time' of the 'big bang' is a subjective measure, in accordance with the principle of general covariance. That is to say, if the astronomer in our own reference frame, according to his/her observations of the expansion of the universe, claims that the 'big bang' happened about 2×10^{10} years ago, then observers from other galactic frames of reference could claim that the 'big bang' happened 3×10^{15} years ago, while still others in a different frame could claim that it was only 6×10^{5} years ago.

The claim that I am making here is that when Einstein's theory of general relativity is taken to its logical extreme, there can be no absolute space nor absolute time in *any* domain of physics, including the cosmological domain of the universe as a whole. Thus, the theory of general relativity cannot accept the idea of an 'absolute beginning', singularly, as some cosmologies assert –at least within the context of cosmology *as a physical theory of matter*.

In regard to space, per se, the theory of relativity rules out the possibility of an absolute center or rotation axis or absolute motion. The measurements of dynamical features of matter in terms of these entities are then strictly subjective, relative to the reference frame from which they are determined. This conclusion of Einstein's relativity theory (whether in its special or its general form) is, of course, in contrast with Newton's classical theory, wherein there is an absolute center of space and time and an absolute motion of the things of the universe. Newton then described rotation with respect to this absolute space and time. Einstein's relativity theory, in contrast, as observed from any galactic frame of reference, is a covariant concept –with nothing absolute except the laws of nature. Rather than a theory of matter in terms of the 'things' of the Newtonian approach, this is a continuous field theory of the universe wherein *rotation*, per se, is represented in terms of a *covariant rotation field* [3].

As I have indicated above, there is not yet sufficient resolution in the astronomical data on a cosmological scale to indicate if there may be a bona fide frame-dependent spiral distribution of the galaxies of the universe as a whole or their relative rotation with respect to the universe as a nonhomogeneous, anisotropic distribution of matter, as contrasted with the alleged isotropic, homogeneous distribution of galaxies according to the 'cosmological principle'. Nevertheless, a full exploitation of the principle of general covariance of Einstein's theory implies that there is no requirement for reflection symmetries in space-time in the structures of the laws of nature ; thus there is no reason in this theory for excluding reflection non-symmetric (frame-dependent) planar distributions of galaxies, spread out spirally and rotating collectively about an axis perpendicular to this plane. All that the generally relativistic cosmology does require is that the dynamical laws of the matter of the universe as a whole must be in one-to-one correspondence in all possible frames of reference, as determined by any particular observer (i.e. that the laws of the universe must be generally covariant).

It then follows that future empirical results that could further corroborate the basis of Einstein's theory of general relativity would be astronomical observations that verify that indeed the galaxies do rotate relative to the universe as a whole. Such a confirmation must necessarily entail improved resolution of the astronomical data on a cosmologically global scale.

2. A SPIRAL OSCILLATING UNIVERSE

In earlier research, I have demonstrated that by fully exploiting the symmetry requirements of the theory of general relativity, particularly paying attention to the fact that there is no requirement for reflection symmetry in space and time, the symmetric tensor formalism of Einstein's original formulation *factorizes* into the product of a quaternion formalism and its conjugated formalism [4]. The generalized metrical field –the quaternion variable $q^{\mu}(x)$– has more degrees of freedom than the original symmetric metric tensor field $g^{\mu\nu}(x)$, where x denotes the independent space-time variables that these fields are mapped into. The new field, $q^{\mu}(x)$, is *geometrically*, a four-vector ; however, each of its four vector components is, *algebraically*, a quaternion. This form of the metrical field then has 16 independent components, rather than the 10 components of the symmetric tensor metric field $g^{\mu\nu}(x)$. The added generality follows from the fact that the quaternion field variables are not covariant with respect to reflections, as are the symmetric tensor solutions $g^{\mu\nu}(x)$ of the original tensor formulation of Einstein's general relativity.

Starting with the factorization of the Riemannian invariant metric into its quaternion and conjugated quaternion forms, we have :

$$ds^2 \equiv ds d\tilde{s} \leftrightarrow g^{\mu\nu} dx_{\mu} dx_{\nu} \to \begin{cases} ds = q^{\mu} dx_{\mu} \\ d\tilde{s} = \tilde{q}^{\mu} dx_{\mu} \end{cases} \tag{1}$$

Algebraically, the quaternion metrical field behaves like a second-rank spinor of the type $\psi \times \psi^*$, where the variable $\psi(x)$ transforms in general relativity as a two-component spinor field mapped in a curved space-time. The conjugated quaternion field $\tilde{q}^{\mu}(x)$ is simply a reflection of $q^{\mu}(x)$ (in space or time, depending on the convention chosen for the conjugation). Thus, there is in reality a single metrical field, $q^{\mu}(x)$, with 16 independent field components. It followed from the removal of the space and time reflection symmetry elements from the underlying group –leaving a purely continuous group of general relativity (the 'Einstein group')– which is all that was required of the symmetry group of general relativity theory at the outset.

The factorization of the Riemannian metric in eq. (1) led, in turn, to a factorization of Einstein's tensor field equations, transforming in the factorized version as the quaternion field variable $q^{\mu}(x)$, as well as yielding a more general expression for the geodesic equation in the curved space-time [5]. One of the interesting astrophysical implications of this generalization was the prediction of a new sort of model of a pulsar [6].

In regard to the focus of this paper, an important prediction of the generalized geodesic equation, following from the quaternion factorization of the Riemannian metric, is that this equation, as the equation of motion of a star, as a 'test body' in the background of the stars of a host galaxy, predicts a spiral path (within some reasonable approximations). The equation of motion of the star within the galaxy that we are going to apply to the galaxy of the universe as a test body has the following form [7] :

$$(\ddot{\xi}^x)^2 + (\ddot{\xi}^y)^2 = A^2 t^2 \tag{2}$$

This differential equation is then to represent the motion of a galaxy in the background of all of the other galaxies of the universe, moving in a two-dimensional plane, labelled as $(x - y)$. The coordinate solutions of eq. (2), previously applied to the star's motion within a single galaxy, was found to be as follows :

$$x(t) = \xi^x(t) + \frac{1}{2}a^x t^2 \quad , \quad y(t) = \xi^y(t) + \frac{1}{2}a^y t^2$$

where t is a time measure relative to any observer's reference frame and (a^x, a^y) are the (constant) measured acceleration components of the galactic reference frame, relative to the observer, that contains the test body (the galaxy in the cosmological problem).

The galaxy's spiral trajectory then follows from the *Fresnel integral* solutions of eq. (2) :

$$\begin{aligned} \xi^x(t) &= c\Big\{\int_0^t \cos[(A/2c)\tau^2]d\tau - t\Big\} \\ \xi^y(t) &= c\int_0^t \sin[(A/2c)\tau^2]d\tau \end{aligned} \tag{3}$$

These solutions incorporate the boundary conditions for an oscillating model for the universe, whose constituent galaxies obey :

$$x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0$$

This result then predicts that at maximum density of the matter of the universe, a given galaxy would move outwards along a spiral path with ever decreasing density of the universe, until a minimum density is reached, at the inflection point of the motion. The galaxy would then turn around and spiral inward again toward maximum density of the universe, when it would turn around again at the second inflection point, proceeding to spiral outward, and so on in oscillatory fashion.

Thus we see that this is a model of the universe whereby at the 'beginning' of any particular cycle, i.e. at the time of the 'big bang' of that cycle, all of the matter of the universe is maximally dense and in an unstable state, *having reached this stage from a preceding implosion*, in spiral fashion. When the implosion of the cosmos had increased the density to such a point that the repulsive forces (in accordance with the non-positive definite affine connection terms in general relativity) exceeded its collective attractive forces, the inflection point of the collective motion sets in, changing the overall implosion to an overall explosion. [Both the repulsive and the attractive forces are always present, according to the geometrical fields of general relativity that play the role of 'external force' on a test body. The latter is in terms of the non-positive definite affine connection coefficients. At matter density sufficiently great, the repulsive force components would dominate while at lesser densities (and relative speeds) the attractive force components dominate [6]].

Thus, after an explosion of the universe sets in, i.e. the 'big bang', after the maximum density of the matter of the universe had been reached, the matter density would proceed to decrease. This would then continue until the matter of the universe would be sufficiently rarefied for the collective attractive force component to dominate once again, pulling all of the matter (galaxies) together in an implosion, in spiral fashion –until the maximum density is reached at the inflection point, changing the implosion to an explosion (i.e. changing the contraction of the universe into an expansion) and so on, in oscillatory fashion.

3. DENSITY OF THE UNIVERSE AT THE BIG BANG

Within the context of the proposed cosmological model, an interesting question is the following : What is the matter density of the universe just when its starts its expansion ? The answer may be approximated from the spiral solutions (3) of the equations of motion (2), expressed in the reference frame of the universe. That is we assume that the 'observer' is at the hub of the universe when it starts its expansion, in each cycle. In this frame of reference, $a^x = a^y = 0$.

The extent of the universe at this initial stage is then given by $R(t_0)$, according to equation (3) :

$$\begin{aligned} R^2(t_0) &= [x^2+y^2]_{t_0} = [(\xi^x)^2+(\xi^y)^2]_{t_0} \\ &= c^2\Big\{\Big(\int_0^{t_0}\cos[(A/2c)\tau^2]d\tau - t_0\Big)^2 + \Big(\int_0^{t_0}\sin[(A/2c)\tau^2]d\tau\Big)^2\Big\} \end{aligned} \tag{4}$$

With the time parameter t_0 in this expression, representing the time when the expansion begins, $R(t_0)$ would then be extent of the universe at the time of the 'big bang' (of any particular cycle). The idea I wish to propose at this stage of the analysis is that it should be physically reasonable to take this initial time to be the lifetime of the neutron, $t_n \simeq 930$ sec. The reason for this choice, $t_0 = t_n$, is that before the matter of the universe explodes outward, all of it is collectively a single charge-neutral matter field, stable against individual particle decay –just as the constituent neutron of nuclear matter is stable against decay, so long at it is bound in that matter. But when the neutron is released from the primordial matter of the universe to be sufficiently free of its influence, it decays according to the weak interaction scheme,

$$n \to p + e + \overline{\nu} \quad , \quad (t_n \simeq 930 \quad \text{sec} \quad) \tag{5}$$

This decay of the freed neutron into electrically charged matter then starts off the formation of the elements and thence to the stars and galaxies of the universe, as we presently observe them.

With $t_0 = t_n$ and with $A/2c = \omega^2 = 4\pi^2/T^2$, the (squared) extent of the universe at this initial time, $R^2(t_n)$, given in eq. (4), depends on the period of oscillation of the universe, T. Since $(t_n/T)^2 << 1$, the right-hand side of eq. (4) may be expressed to first order in the ratio (t_n/T), thus approximating the sine function with its argument and the cosine function with unity (thus cancelling the first term of the right-hand side of eq. (4)). With this approximation, and then integration, we obtain the following result :

$$R(t_n) = 4\pi^2 c t_n^3/3T^2 \quad \text{cm} \tag{6}$$

The inflection point in time, when the explosion changes to an implosion, or vice versa, corresponds to $R(0) = R(T')$, ocurring at $T' = T/\sqrt{2\pi}$. The magnitude of T' is, observationally, the order of magnitude of twice the time since the last 'big bang' happened. The contemporary astronomical data indicate that the latter time is the order of 2×10^{10} years [9]. Thus, in eq. (6), we may take :

$$T = \sqrt{2\pi}T' \simeq \sqrt{2\pi}(4 \times 10^{10} \quad \text{years} \quad) \simeq \sqrt{2\pi}(1.4 \times 10^{18} \quad \text{sec} \quad)$$

Using this value, along with $t_n = 930$ sec, in eq. (6), we find that

$$R(t_n) = 2.6 \times 10^{-17} \quad \text{cm} \tag{7}$$

(which is about one thousand times smaller than the Compton wavelength of a proton !)

According to the conceptual basis of the theory of general relativity, one should not consider $R(t_n)$ as the radius of a ball of material of the entire universe, sitting in empty space ! For, according to the conceptual basis of Einstein's theory, there is no empty space, as a 'thing-in-itself' ! The significance of the extremely small value for $R(t_n)$ is rather in relation to the extremely large value of the *matter density* of the universe, when it starts its expansion phase in each cycle of its oscillatory dynamics.

To estimate the magnitude of this initial density of the universe at the 'big bang' phase, we may take note of its relation to the present day perceived density and extension of the observable universe, ρ and R. Following from the assumption that the total mass of the universe is invariant with respect to the expansion and contraction of the universe as a whole, we have :

$$\rho(t_n) = [R/R(t_n)]^3 \rho \tag{8}$$

Using the following values for these parameters from modern astronomy [10]

$$\rho \simeq 2 \times 10^{-31} \quad \text{gm/cm}^3 \qquad , \quad R \simeq 1.8 \times 10^{10} l - y = 6 \times 10^{28} \quad \text{cm}$$

eqs. (7) and (8) give :

$$\rho(t_n) \simeq 1.3 \times 10^{105} \text{gm/cm}^3$$

This is the result that was sought –the estimate of the matter density of the universe at the time when the presently observed expansion started, in this particular cycle of an oscillating, spirally evolving universe.

Though this value of the initial matter density of the universe is very large compared with the presently observed matter density, or even compared with the density of nuclear matter, it is indeed finite ! The point here is that this result is in accord with Einstein's requirement of his theory of general relativity as a general theory of matter– that *all* of the features of matter must be non-singular, from the physics of elementary particles to that of cosmology.

It is interesting to note that the value obtained from this analysis for the initial density, $\rho(t_n)$, is many orders of magnitude greater than the minimum matter density of a black hole. [If we assume, along with the present day thinking about black holes, that its density is the order of the mass of a star, say the sun, M_S, divided by the volume of a sphere that has a radius equal to the star's Schwarzschild radius, $2GM_S/c^2$, then we find that the density of black hole matter is the order of only $10^{16} gm/cm^3$]. What it is that is especially interesting here is that if we define a black hole at the outset as a quantity of matter with a density so great that no matter or radiation can escape from it –even a single 'freed' neutron– because of its associated closed geodesics in space and time, then with the magnitude of $\rho(t_n)$ being much greater than the black hole density (by 89 orders of magnitude), no big bang could occur in the first place !

The question of whether or not a black hole state of matter exists in general relativity has never been rigorously determined, either from the experimental or the theoretical side. Theoretically, the answer will depend on whether or not there are stable solutions of the full inhomogeneous form of Einstein's field equations, i.e. including the source terms, (or its quaternion generalization, discussed in this paper), that correspond to a family of closed geodesics. Some preliminary work on this problem has been investigated recently by Pazameta [11].

4. SUMMARY

The primary theoretical view that led to the cosmological predictions of this paper was that by fully exploiting the group structure of Einstein's theory of general relativity, i.e. its *algebraic logic*, one arrives in natural fashion at a factorization of his second-rank, symmetric tensor form to a quaternion metrical field equation, whose solutions entail 16 components to determine the metrical features of space and time

[4]. The factorization is entirely analogous to that of the Klein-Gordon equation, when the reflection transformations are removed from the underlying Poincaré group of special relativity. The latter, of course, led to the pair of coupled, two-component spinor equations (the Majorana form), that in turn led to the bispinor form of the Dirac equation, when reflection symmetry was recovered. In any case, in both examples of factorization due to removing reflection symmetry from the underlying covariance group, one comes to a more general type of field, that is, with extra degrees of freedom and thus extra predictions for the theory.

The quaternion generalization of general relativity, in turn, leads to equations of motion of the matter elements of the universe (its constituent galaxies as 'test bodies') that predict a spiralling, oscillatory motion, described by *Fresnel integral* solutions. It is important that there are no singularities in these solutions, as required by Einstein's original interpretation of his theory.

The distribution of galaxies of the universe in a spiral configuration, and rotating about an axis perpendicular to the plane of rotation is contrary to the assertion of the 'cosmological principle'. But there is no reason, within the context of general relativity theory, that compels the matter of the universe to be homogeneously and isotropically distributed. All that Einstein's general relativity does require is that the laws of nature, governing the fundamental nature of matter under all possible circumstances, must be uniform with respect to their expression in any reference frame relative to any other. That is, the laws of nature are asserted to be totally *objective*. This is the fundamental axiom that underlies the theory of general relativity –the principle of general covariance.

As a final remark, it is noted that Galileo believed in the homogeneity and isotropy of the matter of the universe –which to him was our galaxy, the Milky Way. This was in part because of his belief in some of the philosophical views of Plato, but probably to a greater extent because of what he saw in the night sky. Similarly, at our present stage of scientific knowledge, there are several 'local hints' about the natural appearance of non-reflection symmetric and non-isotropic forces in nature, that go beyond Galileo's belief and beyond the presently held 'cosmological principle'. Of course, in elementary particle physics, the case of non-reflection symmetric Hamiltonians to represent the weak interaction is well known and has been seminal toward further understanding of the nature of elementary matter and its implicit forces. Thus, taking the hint from the domain of micromatter to be applicable in the cosmological domain, it should not be surprising to discover that indeed the universe may in fact be evolving in oscillatory fashion, along spiral trajectories, with a theoretical representation in general relativity that lacks reflection symmetry in space or time.

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