Massive chiral fermions : a natural account of chiral phenomenology in the framework of Dirac's fermion theory

G. Ziino

Istituto di Fisica dell'Università,
via Archirafi 36, I-90123 Palermo (Italy)

ABSTRACT. We assume a strictly invariant definition of the Dirac parity operator under fermion $\rightleftharpoons$ antifermion exchange. We see that the opposite-intrinsic-parity condition then requires two opposite-mass Dirac equations for the fermion and the antifermion. This leads us to introduce an asymptotically left-handed (fermion) and right-handed (antifermion) "chiral" field, as just an alternative basis in the internal space spanned by the new pair of charge-conjugate "Dirac" fields. Hence a "dual" intrinsic model of a spin $-1/2$ massive fermion is drawn : it predicts the coexistence of two anticommuting general varieties of conserved charges, namely a scalar variety, responsible for parity-invariant phenomenology, plus a pseudoscalar one, responsible for chiral phenomenology. In this light, 'CP-symmetry' is seen to be nothing but P-symmetry ; and a spontaneous 'CP-violation' mechanism is also derived, that should work in any single process occuring via both scalar- and pseudoscalar-charge interactions. We show, at last, that our scheme automatically yields Weyl's one for a merely left-handed neutrino and a merely right-handed antineutrino, further assigning them the special meaning of pure pseudoscalar-charge objects. Some general consequences as regards magnetic monopoles are briefly discussed too.

RESUME. On propose une formulation de la théorie des fermions de Dirac à l'aide d'équations de Dirac à masses opposées et charges conjuguées, qui permet l'introduction naturelle de champs à masse et à propriétés chirales et qui inclut automatiquement le schéma de Weyl du neutrino à deux composantes. On déduit un modèle intrinsèque "dual" d'un fermion massif de spin 1/2 : il prédit la coexistence de deux espèces générales anti-commutantes de charges qui se conservent, à savoir une espèce scalaire, responsable d'une phénoménologie invariante de parité, plus une espèce pseudo-scalaire, responsable d'une phénoménologie chirale. Grâce à cet éclairage, on voit que *la "symétrie CP" n'est autre que la P-symétrie; on en tire aussi un mécanisme de "violation CP" spontané.*

In the ordinary general formulation of Dirac's fermion theory, the charge-conjugate (fermion and antifermion) free wave functions $\varphi_f(\vec{r},t)$, $\varphi_{\overline{f}}(\vec{r},t)$ are assumed to be (coincident) solutions of the same (positive mass) Dirac equation and are further demanded (in agreement with experience) to have an *opposite* intrinsic parity. That can actually be got provided parity P is represented in the fermion and antifermion four-spinor spaces by two *different* (and opposite) operators : taking, e.g., the stationary solution $\varphi(\vec{r},t) \sim u(\vec{p})\exp[i(\vec{p}\cdot\vec{r}-Et)]$ $(\hbar=1, E>0)$, one has to put (because of the identity $u_{\overline{f}}(\vec{p}) = u_f(\vec{p})$)

$$P : u_f(0) \to U_P u_f(0) = \eta u_f(0) \quad , \quad u_{\overline{f}}(0) \to \overline{U}_P u_{\overline{f}}(0) = -\eta u_{\overline{f}}(0)$$

where $U_P = \eta\gamma^0$ $(\eta = \pm 1)$ and $\overline{U}_P = -U_P$.

As is well-known, the simple (P-invariant) Dirac fermion model cannot account for the peculiar 'handedness' shared by all fermions in weak-interaction processes : parity 'violation' must then be expressly invoked, with no adequate theoretical support to such a phenomenology. In the zero-mass limit, moreover, Dirac's scheme itself is to be drastically turned into Weyl's two-component one, by ruling out those further (right-handed neutrino and left-handed antineutrino) solutions having no actual physical counterpart.

In the present paper a slightly modified formulation of Dirac's theory is proposed, which is, on the contrary, both predicting an alternative "chiral" behaviour of fermions and naturally reducing (for zero mass) to Weyl's scheme. It leads, likewise, to a more complex internal model of a spin $-1/2$ point fermion, that seems also to clarify the physical origin of chiral phenomenology.

Our starting-hypothesis is the following one : parity P should anyhow be represented by a *unique* operator, say $U_P(=\eta\gamma^0)$, no matter whether the fermion or antifermion four-spinor space is involved. This *strictly* answers the invariance requirement due to (Dirac) symmetry under fermion $\rightleftharpoons$ antifermion exchange. If the opposite-intrinsic-parity condition is applied, it is immediate to see that φ_f and $\varphi_{\overline{f}}$ can no longer be coincident : in particular, the two (positive energy) four-spinors $u_f(\vec{p})$, $u_{\overline{f}}(\vec{p})$ should be such that

$$P : u_f(0) \to U_P u_f(0) = \eta u_f(0) \quad , \quad u_{\overline{f}}(0) \to U_P u_{\overline{f}}(0) = -\eta u_{\overline{f}}(0). \quad (1)$$

Assuming the fermion to have a (proper) mass $m > 0$, one can easily check that form invariance of the Dirac Hamiltonian H will then require u_f and $u_{\overline{f}}$ to be eigenspinors of the respective Hamiltonians

$$H_f \equiv H(\vec{p}, m) \quad , \quad H_{\overline{f}} \equiv H(\vec{p}, -m) \tag{2}$$

where (in units of c)

$$H(\vec{p}, \pm m) = \vec{\alpha} \cdot \vec{p} + \beta(\pm m) \tag{3}$$

($\beta = \gamma^0$). So, φ_f and $\varphi_{\overline{f}}$ should now obey the *opposite*-mass Dirac equations

$$(i\gamma^\mu \partial_\mu - m)\varphi_f = 0 \quad , \quad (i\gamma^\mu \partial_\mu - \overline{m})\varphi_{\overline{f}} = 0 \quad , \quad (\overline{m} = -m) \tag{4}$$

($\mu = 0, 1, 2, 3 \quad , \quad \gamma^k = \gamma^0 \alpha^k \quad , \quad k = 1, 2, 3$) [1] [1,2]. Of course, proper-mass sign has nothing to do with energy sign (since $E^2 = \vec{p}^2 + m^2$) : thus a real antifermion at rest may have, as usual, a positive energy eigenvalue $E = m$ [3].

In support to (2) a general classical argument can be further advanced, that exploits the Stueckelberg-Feynman interpretation of negative-energy motion [4,5]. Let a particle, with four-momentum $(-p^\mu)$, be carrying out a backward space-time displacement $(-dx^\mu)$ and covering a world-line segment ds. It will clearly be equivalent to an antiparticle, with four-momentum p^μ, traveling in the *opposite* way, namely carrying out the forward displacement dx^μ and covering the world-line segment $(-ds)$. The particle and antiparticle four-velocities will be coincident, $[(-dx^\mu)/ds] = [dx^\mu/(-ds)]$. So, *if* $(-p^\mu) \equiv m[(-dx^\mu)/ds]$ (for the particle), *then* $p^\mu \equiv (-m)[dx^\mu/(-ds)]$ (for the antiparticle).

By exploiting eqs.(4) we may introduce an "*intrinsic parity" operator* P_{in} such that

$$P_{in} : \varphi_f \to \frac{i}{m}\eta\gamma^\mu \partial_\mu \varphi_f = \eta\varphi_f \quad , \quad \varphi_{\overline{f}} \to \frac{i}{m}\eta\gamma^\mu \partial_\mu \varphi_{\overline{f}} = -\eta\varphi_{\overline{f}}. \tag{5}$$

---

[1] The metric here adopted is $(+ - - -)$. Note that, owing to the anticommutation relations $\gamma^\mu\gamma^\lambda + \gamma^\lambda\gamma^\mu = 2g^{\mu\lambda}$ ($g^{\mu\lambda}$ being the metric tensor), the product $(i\gamma^\mu\partial_\mu - m)(i\gamma^\mu\partial_\mu - \overline{m})$ gives just the Klein-Gordon operator :

$$(i\gamma^\mu\partial_\mu - m)(i\gamma^\mu\partial_\mu - \overline{m}) = -\partial^\mu\partial_\mu - m^2.$$

On wave functions with $\vec{p} = 0$, (and $E = m$) P_{in} will act the same as P : in that case, $i/m\gamma^\mu\partial_\mu\varphi = \gamma^0\varphi$. Recalling, moreover, the anticommutation relations $\{\gamma^5, \gamma^\mu\} = 0$ (where $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$) we see that charge conjugation C should now be given (apart from an arbitrary phase factor) the *covariant* expression

$$C : \varphi_f \rightleftharpoons \varphi_{\overline{f}} = \gamma^5\varphi_f, \tag{6}$$

in close analogy with the 'proper-mass reversal' operation introduced by Costa de Beauregard [6].

On passing to field formalism, wave functions $\varphi_f(x^\mu), \varphi_{\overline{f}}(x^\mu)$ may be replaced by two anticommuting charge-conjugate free-field operators such as

$$\psi_f(x^\mu) \equiv \psi(x^\mu, m) \quad , \quad \psi_{\overline{f}}(x^\mu) \equiv \psi(x^\mu, \overline{m}) \tag{7}$$

that will themselves satisfy equations (4). Setting $\eta = 1$, we shall have then

$$\left.\begin{array}{rll} P_{in}: & \psi_f \to \psi_f \quad , & \psi_{\overline{f}} \to -\psi_{\overline{f}} \\ C: & \psi_f \rightleftharpoons \psi_{\overline{f}} & \end{array}\right\} \tag{8}$$

where

$$\psi_{\overline{f}} = \gamma^5\psi_f \quad , \quad \overline{\psi}_{\overline{f}} = -\overline{\psi}_f\gamma^5 \tag{9}$$

$\overline{\psi}$ being the Dirac adjoint of ψ. By virtue of (9) we may further write

$$\psi_f = 2^{-1/2}(\psi_f^{ch} + \psi_{\overline{f}}^{ch}) \quad , \quad \psi_{\overline{f}} = 2^{-1/2}(-\psi_f^{ch} + \psi_{\overline{f}}^{ch}) \tag{10}$$

and define *the new fields*

$$\psi_f^{ch} \equiv 2^{-1/2}(1 - \gamma^5)\psi_f \quad , \quad \psi_{\overline{f}}^{ch} \equiv 2^{-1/2}(1 + \gamma^5)\psi_{\overline{f}} \tag{11}$$

which are both satisfying the mere (Klein-Gordon) equation $(\partial^\mu\partial_\mu + m^2)\psi^{ch} = 0$. In this way, an asymptotically left-handed (fermion) *"chiral" field*, $\psi_f^{ch}(x^\mu)$, and an asymptotically right-handed (antifermion) one, $\psi_{\overline{f}}^{ch}(x^\mu)$, are introduced, as together making up an *alternative basis* in the (two-dimensional) internal space spanned by "Dirac" fields $\psi_f(x^\mu), \psi_{\overline{f}}(x^\mu)$ themselves. [2]

[2] As is well-known, only *massless* chiral fields can, on the contrary, be defined in the ordinary scheme.

In such a framework, parity 'violation' [7] seems to find a *natural* theoretic interpretation. First, we can put (for an either neutral or charged current)

$$\overline{\psi}_f \gamma^\mu (1-\gamma^5)\psi_f \equiv \overline{\psi}_f^{ch} \gamma^\mu \psi_f^{ch} \quad , \quad \overline{\psi}_{\overline{f}} \gamma^\mu (1+\gamma^5)\psi_{\overline{f}} \equiv \overline{\psi}_{\overline{f}}^{ch} \gamma^\mu \psi_{\overline{f}}^{ch} \tag{12}$$

($\overline{\psi}^{ch} = \psi^{ch\dagger}\gamma^0$), so that the '$V-A$' (fermion) and '$V+A$' (antifermion) currents are now to be reread as mere chiral-field 'V' currents. Likewise, by exploiting eqs.(9), one can trivially explain the phenomenological absence of any '$V+A$' fermion, or '$V-A$' antifermion, current :

$$\begin{aligned} \overline{\psi}_f \gamma^\mu (1+\gamma^5)\psi_f &= \overline{\psi}_{\overline{f}} \gamma^\mu (1+\gamma^5)\psi_{\overline{f}} \quad , \\ \overline{\psi}_{\overline{f}} \gamma^\mu (1-\gamma^5)\psi_{\overline{f}} &= \overline{\psi}_f \gamma^\mu (1-\gamma^5)\psi_f . \end{aligned} \tag{13}$$

But what is more, parity 'violation' itself can be formally re-interpreted as an effect *quite obeying*, on the contrary, parity symmetry : the weak fermion current is now *C-invariant* and then will be acted upon by P_{in} (or P) just *the same* as by CP_{in} (or CP),

$$\left.\begin{array}{lll} P_{in}: & \psi_f^{ch} \rightleftharpoons \psi_{\overline{f}}^{ch} & \\ C: & \psi_f^{ch} \to -\psi_f^{ch} \quad , & \psi_{\overline{f}}^{ch} \to \psi_{\overline{f}}^{ch} \end{array}\right\} \tag{14}$$

Thus, to sum up, 'CP-symmetry' is here *nothing but P-symmetry*, as can already be checked by looking at the peculiar identities (13).

Of course, the ordinary (scalar charge) fermion model cannot physically account for all that ; hence a *unusual* intrinsic model of a spin $-1/2$ point fermion is also to be expected. To this aim, we may conveniently employ state vectors and denote the "fermion" and "antifermion" Dirac states by the respective unit kets $\mid f>$ and $\mid \overline{f}>$ ($\equiv C \mid f>$) such that

$$M \mid f> = m \mid f> \quad , \quad M \mid \overline{f}> = -m \mid \overline{f}> \tag{15}$$

$M(\equiv mP_{in})$ being the (proper) mass "operator". If the vacuum state is assigned an even P- and C-eigenvalue, the internal transformation (10) can then read

$$\begin{aligned} \mid f> &= 2^{-1/2}(\mid f^{ch}> + \mid \overline{f}^{ch}>) \quad , \\ \mid \overline{f}> &= 2^{-1/2}(-\mid f^{ch}> + \mid \overline{f}^{ch}>) \end{aligned} \tag{16}$$

where the new basis ($| f^{ch} >, | \overline{f}^{ch} >$) is made up by two "chiral" states with opposite chirality eigenvalues. Note, in particular, that $| f^{ch} >$ and $| \overline{f}^{ch} >$ seem just to define those "fermion" and "antifermion" states which should be the natural eigenstates of the 'electroweak' isospin third component entering the Weinberg-Salam theory [8]. Starting from

$$\left.\begin{array}{rl} P_{in}: & | f > \rightarrow | f > \quad , \quad | \overline{f} > \rightarrow - | \overline{f} > \\ C: & | f > \rightleftharpoons | \overline{f} > \end{array}\right\} \tag{17}$$

we infer that *conversely*,

$$\left.\begin{array}{rl} P_{in}: & | f^{ch} > \rightleftharpoons | \overline{f}^{ch} > \\ C: & | f^{ch} > \rightarrow - | f^{ch} > \quad , \quad | \overline{f}^{ch} > \rightarrow | \overline{f}^{ch} > \end{array}\right\} \tag{18}$$

Hence, C and P_{in} are in turn acting as fermion $\rightleftharpoons$ antifermion exchange operators, with the meaning of *scalar*- and *pseudoscalar*-charge conjugation, respectively (where by "charge" any additive internal quantity is understood). Let Q and Q^{ch} denote these general (scalar and pseudoscalar) charge varieties, such that $\{C, Q\} = \{P_{in}, Q^{ch}\} = 0$. States $| f >, | \overline{f} >$ are looking like *pure* Q-eigenstates, whereas $| f^{ch} >, | \overline{f}^{ch} >$ like *pure* Q^{ch}-eigenstates :

$$\begin{array}{c} < f \mid Q^{ch} \mid f > = < \overline{f} \mid Q^{ch} \mid \overline{f} > = 0 \quad , \\ < f^{ch} \mid Q \mid f^{ch} > = < \overline{f}^{ch} \mid Q \mid \overline{f}^{ch} > = 0. \end{array} \tag{19}$$

It is immediate to see, correspondingly, that Q and Q^{ch} *anticommute* :

$$\{Q, Q^{ch}\} = 0 \quad , \quad [Q^{ch}, Q^2] = [Q, (Q^{ch})^2] = 0. \tag{20}$$

We may therefore establish what follows : *Scalar and pseudoscalar charges borne by a spin-*1/2 *massive fermion cannot be simultaneously specified except in magnitude, since they would be alternately subject to a maximal uncertainty in sign.*

This statement seems to find a quite general application. As a matter of fact, one peculiar charge pair $Q = F$, $Q^{ch} = F^{ch}$ can be introduced,

$$F \equiv f P_{in} \quad , \quad F^{ch} \equiv -f^{ch} C, \tag{21}$$

with *non*-zero (real) eigenvalues f, f^{ch} for *every* "fermion" state of the type $\mid f>$ or $\mid f^{ch}>$, respectively : F and F^{ch} may in particular be identified with two anticommuting (scalar and pseudoscalar) "fermion number" varieties. So, *neither* of the charge sets $\{Q\}, \{Q^{ch}\}$ should ever be empty for a spin $-1/2$ fermion with mass.

Let us try to go even farther. With reference to the (Dirac) field basis $(\psi_f, \psi_{\overline{f}})$ the fermion and antifermion free Lagrangian densities will read

$$\Lambda_f \equiv \Lambda(\sigma_f, \partial_\mu \sigma_f; m) \quad , \quad \Lambda_{\overline{f}} \equiv \Lambda(\sigma_{\overline{f}}, \partial_\mu \sigma_{\overline{f}}; \overline{m}) \quad , \quad (\sigma = \psi, \overline{\psi}) \tag{22}$$

where Λ is P_{in}-invariant and $\Lambda(\sigma_f, \partial_\mu \sigma_f; m) = \frac{1}{2}[i\overline{\psi}_f \gamma^\mu \partial_\mu \psi_f + H.c.]$ $-m\overline{\psi}_f \psi_f$. If setting $\psi_{\overline{f}} = \gamma^5 \psi_f$ (and $\overline{\psi}_{\overline{f}} = -\overline{\psi}_f \gamma^5$) it is easily seen that $\Lambda_{\overline{f}} = \Lambda_f$. Thus Λ is also invariant under C (as given by (6)) and may well be assumed to be scalar under rotations in the internal space spanned by $(\psi_f, \psi_{\overline{f}})$, the new Lagrangian density Λ^{ch} will be obtained by merely substituting (10) into Λ. Hence, regardless of its internal state, the fermion (or antifermion) can be assigned a *unique*, both P_{in}- and C-invariant, free Lagrangian. This actually ensures the *simultaneous* conservation of F and F^{ch} (as defined by (21)): [3] re-defining Λ in the equivalent symmetrized form

$$\Lambda = \frac{1}{2}(\Lambda_f + \Lambda_{\overline{f}}) \tag{23}$$

we can write the correspondent Hamiltonian density as

$$\mathcal{H} \equiv \dot{\sigma}_f \frac{\partial \Lambda}{\partial \dot{\sigma}_f} + \dot{\sigma}_{\overline{f}} \frac{\partial \Lambda}{\partial \dot{\sigma}_{\overline{f}}} - \Lambda = \frac{1}{2}(\mathcal{H}_f + \mathcal{H}_{\overline{f}}) \tag{24}$$

$(\dot{\sigma} \frac{\partial \Lambda}{\partial \dot{\sigma}} \equiv \dot{\overline{\psi}} \frac{\partial \Lambda}{\partial \dot{\overline{\psi}}} + \frac{\partial \Lambda}{\partial \dot{\psi}} \dot{\psi})$ where [4]

$$\mathcal{H}_f = \dot{\sigma}_f (\partial \Lambda_f / \partial \dot{\sigma}_f) - \Lambda_f \quad , \quad \mathcal{H}_{\overline{f}} = \dot{\sigma}_{\overline{f}} (\partial \Lambda_{\overline{f}} / \partial \dot{\sigma}_{\overline{f}}) - \Lambda_{\overline{f}} \tag{25}$$

---

[3] Yet, there cannot clearly be any superselection rule holding *together* for $(\mid f>, \mid \overline{f}>)$ and $(\mid f^{ch}>, \mid \overline{f}^{ch}>)$.

[4] Actually, as it will result $\mathcal{H}_{\overline{f}} = \psi^\dagger_{\overline{f}}[\vec{\alpha} \cdot \vec{p} + \beta(-m)]\psi_{\overline{f}}$, it is immediate to see that $\mathcal{H}_{\overline{f}} = \mathcal{H}_f = \mathcal{H}$.

thus obtaining not only $[F, \mathcal{H}] = 0$, but also $[F^{ch}, \mathcal{H}] = 0$.

In the light of all that, the following general conclusion may be drawn: *By its own nature, a single spin $-1/2$ fermion with mass should together bear two anticommuting varieties of conserved charges, namely both a scalar variety and a pseudoscalar one under space reflection.*

Such a fermion is indeed an *intrinsically "dual"* object : it may at most be looking *either* like a pure scalar-charge eigenstate, $\mid f >$, associated with a (Dirac) free field ψ_f, *or* like a pure pseudoscalar-charge eigenstate, $\mid f^{ch} >$, associated with a (chiral) free field ψ_f^{ch}. On the other hand, because of F as well as F^{ch} conservation, states $\mid f >$ and $\mid f^{ch} >$ can only provide two *partial* internal pictures of the same fermion, the former being a P_{in}-invariant one (*as if* no net pseudoscalar charge were present) and the latter a C-invariant one (*as if* no net scalar charge were present). The *true* operation which takes the fermion into the antifermion will anyhow be, therefore, the *total* charge conjugation

$$C_{tot} \equiv CP_{in} \tag{26}$$

where

$$C_{tot} \mid f >= C \mid f > \quad , \quad C_{tot} \mid f^{ch} >= P_{in} \mid f^{ch} > . \tag{27}$$

From the dynamical viewpoint, this new fermion model seems just to underlie the actual coexistence of both a parity- and a chirality-invariant phenomenology : *Any given massive fermion will appear in its "partial" state $\mid f >$ or $\mid f^{ch} >$ according to whether "seen" through a pure scalar- or pseudoscalar- charge interaction, respectively.* More precisely, it is, first, evident that transformation (10) may be set down not only for free fields, provided that matter $\rightleftharpoons$ antimatter exchange is symmetrically extended to the *whole* interacting system : for instance, if ψ_f refers to a spin $-1/2$ point fermion with electric charge $(-e)$ (and mass m) in the presence of an external electromagnetic four-potential A_μ,

$$i\gamma^\mu[\partial_\mu + i(-e)A_\mu]\psi_f = m\psi_f,$$

then $\psi_{\overline{f}} \equiv \gamma^5\psi_f$ will refer to the corresponding antifermion (with electric charge e and mass $\overline{m} = -m$) in the presence of the C-conjugate external four-potential $(-A_\mu)$,

$$i\gamma^\mu[\partial_\mu + ie(-A_\mu)]\psi_{\overline{f}} = \overline{m}\psi_{\overline{f}}. \tag{28}$$

Let us, in general, denote by $\Lambda_{int}^{Q,Q^{ch}}$ the total (Q and Q^{ch}) interaction Lagrangian density (for the fermion) *plus* its C_{tot}-conjugate ; we may thus put at last (omitting for brevity's sake, the external fields to which the fermion and antifermion ones are symmetrically coupled)

$$\Lambda_{int}^{(Q,Q^{ch})} = \Lambda_{int}^{(Q)}(\psi_f, \psi_{\overline{f}}, \overline{\psi}_f, \overline{\psi}_{\overline{f}}) + \Lambda_{int}^{(Q^{ch})}(\psi_f^{ch}, \psi_{\overline{f}}^{ch}, \overline{\psi}_f^{ch}, \overline{\psi}_{\overline{f}}^{ch}) \qquad (29)$$

where $\psi_f, \psi_{\overline{f}}$ on the one hand, and $\psi_f^{ch}, \psi_{\overline{f}}^{ch}$ (as given by (11)) on the other, will *still* be related by (10).

On the ground of such a model, the 'weak' charge seems clearly to be an example of a pseudoscalar charge ; and 'CP-symmetry' is formally reducible to P-symmetry for the reason that *"chiral" fermions are equivalent to pure pseudoscalar-charge objects.* Yet, P-breakdown is not always forbidden in principle : a removal from the (bare) 'weak' basis $(\psi_f^{ch}, \psi_{\overline{f}}^{ch})$ by a small angle $\epsilon(\mid \epsilon \mid \ll \pi/4)$ would already be able to cause it, keeping valid the *mere* symmetry under C_{tot}. In that case, ψ_f^{ch} and $\psi_{\overline{f}}^{ch}$ should be replaced, to first order in ϵ, by the fields

$$\psi'_f \simeq \psi_f^{ch} + \epsilon\psi_{\overline{f}}^{ch} \quad , \quad \psi'_{\overline{f}} \simeq -\epsilon\psi_f^{ch} + \psi_{\overline{f}}^{ch} \qquad (30)$$

(such that $C_{tot} : \psi'_f \to \psi'_{\overline{f}}$, $C_{tot}^{-1} : \psi'_{\overline{f}} \to \psi'_f$). This could happen only under the influence of an "effective" charge

$$Q_{eff} \simeq Q^{ch} + 2\epsilon Q \qquad (31)$$

where the single (pseudoscalar and scalar) charges Q^{ch} and Q are formally expressible as $Q^{ch} = -q_{eff}C$ and $Q = q_{eff}P_{in}$, q_{eff} being the Q_{eff} eigenvalue associated with ψ'_f. If for $\epsilon \to 0$ a purely 'weak' process is restored, we may then (to first order in ϵ) identify Q^{ch} with the weak charge and correspondingly set $q_{eff} = q_{weak}$ (q_{weak} being the weak-charge eigenvalue). The apparent presence of the "mixed" charge (31) –to which there should correspond an additional Lagrangian term $\Lambda_{int}^{(Q_{eff})}(\psi'_f, \psi'_{\overline{f}}, \overline{\psi}'_f, \overline{\psi}'_{\overline{f}})$– would therefore mean that the actual process is also occuring via an interaction due to a *scalar* charge, whose magnitude is almost $(2|\epsilon|)^{-1}$ times smaller than q_{weak} : the *only* corrective effect (of order ϵ) so produced would just be the P-breaking one expressed by (30). Within the present scheme, such a P (but *not* C_{tot}) failure is clearly the same as a 'CP-violating' effect in the ordinary language. The special circumstances considered would then give rise to a natural *'CP-violation' mechanism*, which however, consistently, would *not* break at all the "true" symmetry between fermions and antifermions (i.e., the one under C_{tot}). Such a mechanism should in particular be working (at the quark level) in the 'weak' transition $K^0 \rightleftharpoons \overline{K}^0$, provided also a 'superweak' force due to a *scalar* charge be responsible for that process.[5]

So far, $m \neq 0$ has been assumed ; it is left now to analyse the limiting case of *massless* spin $-1/2$ fields. For $m = 0$ equations (4) become coincident, and transformation (10) can be rewritten (in Weyl's representation) as

$$\psi_f = \psi_\nu = 2^{-1/2} \begin{pmatrix} \psi_\nu^L \\ \psi_{\overline{\nu}}^R \end{pmatrix} \quad , \quad \psi_{\overline{f}} = \psi_{\overline{\nu}} = 2^{-1/2} \begin{pmatrix} -\psi_\nu^L \\ \psi_{\overline{\nu}}^R \end{pmatrix} \tag{32}$$

where ψ_ν^L and $\psi_{\overline{\nu}}^R$ are two-component fields *such that*

$$\psi_\nu^L = 2^{-1/2}(1-\gamma^5)\psi_\nu \quad , \quad \psi_{\overline{\nu}}^R = 2^{-1/2}(1+\gamma^5)\psi_{\overline{\nu}} \tag{33}$$

the former being related to a *left-handed* neutrino and the latter to a *right-handed* antineutrino. This leads automatically to Weyl's scheme, owing to the natural *absence* of one further independent pair of chiral-field solutions (related to a right-handed neutrino and a left-handed antineutrino) : since $\psi_{\overline{\nu}} = \gamma^5\psi_\nu$, we shall merely have

$$\psi_\nu^R \equiv 2^{-1/2}(1+\gamma^5)\psi_\nu = \psi_{\overline{\nu}}^R \quad , \quad \psi_{\overline{\nu}}^L \equiv 2^{-1/2}(1-\gamma^5)\psi_{\overline{\nu}} = -\psi_\nu^L . \tag{34}$$

Such a result, quite in line with (13), is also a manifest expression of the fact that *'CP-symmetry' obeyed by neutrinos would be nothing but P-symmetry.* Actually,

$$\gamma^0 \begin{pmatrix} \psi_\nu^L \\ \psi_{\overline{\nu}}^R \end{pmatrix} = \begin{pmatrix} \psi_{\overline{\nu}}^R \\ \psi_\nu^L \end{pmatrix} , \tag{35}$$

---

[5] In this regard, the parameter ϵ in the text is not to be confused with that one usually mixing the two CP-eigenstates K_1^0, K_2^0. How the ordinary 'CP-violating' phenomenology of the neutral kaon can actually be re-interpreted as a pure P-violating one is shown in a work in progress. The point is that (in view of the quark constitution of mesons) the usual relation $\mid \overline{K}^0 >= CP \mid K^0 >$ should now have a "dual" reading : $\mid K^0 >$ and $\mid \overline{K}^0 >$ may alternately stand for two P-invariant (and C-conjugate) or C-invariant (and P-conjugate) "partial" states. The 'CP-violating' quark mixing (30) (in the text) would therefore leave the above relation *unaltered* : it would occur in a "plane" *orthogonal* to the one spanned by $(\mid K^0 >, CP \mid K^0 >)$, thus involving *no* actual mixing of K_1^0 and K_2^0 states.

whence it just follows that

$$P: \qquad \psi_\nu^L \rightleftharpoons \psi_{\bar{\nu}}^R. \tag{36}$$

So, neutrino and antineutrino would merely be the *ordinary* mirror image of each other [1]. An identical result is reached by Costa de Beauregard in ref.[6], on the ground of a manifestly covariant definition of 'CPT' operation. To get an insight into (36), it should first be noticed that the previously seen "dual" fermion model cannot go well any longer : P_{in}, as defined by (5), becomes meaningless for $m = 0$, and the same applies to the scalar fermion-number operator $F(\equiv fP_{in})$. Thus, "neutrino" and "antineutrino" states are bound to be *permanent eigenstates of scalar-charge conjugation* C (with opposite eigenvalues). Hence we draw at last the conclusion that *neutrinos* (if really massless) *should bear charges of the pseudoscalar type only* (to be in particular identified with their leptonic fermion numbers). Such peculiar intrinsic model gives an account of the neutrino 'screw' nature and further clarifies the physical reason why neutrino $\rightleftharpoons$ antineutrino exchange can now be accomplished by applying P alone.

The theory here proposed seems also to have some general consequences as regards magnetic monopoles, the pseudoscalar character of the magnetic charge being well-known [9-11]. First, in close connection with what was recently pointed out by Lochak [11], a magnetic monopole carried by a fermion should really be observable only when the fermion is looking like a *chiral* particle. Another remarkable consequence is that the anticommutivity property (20), if in particular referred to the electric and magnetic charge operators, would provide a quantum motivation for Maxwell's equation div $\vec{H} = 0$ (with $\vec{H}$ generated by electric monopoles) : according to (20), the expectation value of the magnetic charge should always appear *vanishing* for electric-charge eigenstates with non-zero eigenvalues. Of course, also the converse should be true ; and then we may expect an analogous equation, div $\vec{E} = 0$, for the electric field $\vec{E}$ generated by magnetic monopoles. All that seems to suggest a "dual" theory of magnetic and electric monopoles, whose formulation clearly deserves a whole treatment apart.

The author wishes to thank Prof. O. Costa de Beauregard for his kind appreciation of the subject here developed, and Profs. A.O. Barut and G. Lochak for fruitful discussions. He is grateful also to Dr. S. Mercurio for his constant encouragement and help.

Références

[1] G. Ziino, Lett. Nuovo Cimento 13 (1975) 95 ; and in : Tachyons, Monopoles and Related Topics (Proceedings of the First Session of the Interdisciplinary Seminars, Erice 1976), ed. E. Recami (North-Holland, Amsterdam, 1978) p. 261. Such references already contain an early rough approach to the scheme here developed.

[2] A formally equivalent Dirac-equation pair existing in the literature is to be quoted –see A.O. Barut, Phys. Rev. Lett. 20 (1968) 893 ; J.D. Edmonds Jr. Found. Phys. 5 (1975) 239– in which the (electron and positron) masses, even though still of the same sign, appear multiplied by the corresponding (lepton) fermion numbers (of opposite sign).

[3] For a general discussion of the gravitational aspects of a negative mass, see H. Bondi Rev. Mod. Phys. 29 (1957) 423. As further concerns the gravitational coupling of a spin $-1/2$ particle, see O. Costa de Beauregard, C.R. Acad. Sci. Paris 307 (série II) (1988) 457. Owing to the still positive rest energy eigenvalue $E = m$ (which is, by the way, what is truly required by 'CPT' symmetry) the antigravity hypothesis does not seem to be necessarily implied in our case).

[4] E.C.G. Stueckelberg, Phys. Rev. 74 (1948) 218.

[5] R.P. Feynman, Phys. Rev. 76 (1949) 769.

[6] O. Costa de Beauregard, Found. Phys. 12 (1982) 861 ; and in *The Wave-Particle Dualism*, eds. S. Diner et al. (D. Reidel Publishing Company, 1984) p. 485. Proper-mass reversal is indeed equivalent to 'strong' (space-time) reflection: see J.J. Sakurai, *Invariance Principles and Elementary Particles* (Princeton University Press, Princeton, New Jersey, 1964).

[7] T.D. Lee and C.N. Yang, Phys. Rev. 104 (1956) 254 ; R.P. Feynman and M. Gell-mann, Phys. Rev. 109 (1958) 193.

[8] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264 ; A. Salam, in : Elementary Particle Theory, Nobel Symposium 8, ed. N. Svartholm (Stockholm, 1968) p. 367.

[9] P. Curie, J. Phys. 3 (1894) 415.

[10] A.O. Barut, Phys. Lett. 38B (1972) 97 ; 46B (1973) 81.

[11] G. Lochak, Int. Jour. Theor. Phys. 24 (1985) 1019 ; Ann. Fond. L. de Br. 12 (1987) 135.

(Manuscrit reçu le 21 décembre 1988, révisé le 1er mars 1989)