Maxwell's theory extended (Part 1)

Empirical reasons for questioning the completeness of Maxwell's theory
Effects demonstrating the physical significance of the A potentials

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ABSTRACT. This paper examines critically the treatment of the vector and scalar potentials in classical Maxwell theory as mere mathematical conveniences with no physical significance. In Part 1 the experimental evidence is reviewed for a number of well-established effects, demonstrating that the A fields *do have physical significance* as local-to-global operators or gauge fields, in precisely constrained topologies. The A fields are physically meaningful in topologies of SU(2), and higher, symmetry form. The conventional classical Maxwell theory is of U(1) symmetry with Abelian commutation relations. Extended to SU(2), or higher, symmetry form, Maxwell's theory possesses non-Abelian commutation relations. In Part 2, this paper applies an adapted Yang-Mills interpretation of *low* energy fields – an approach previously applied to *high* energy fields. This adaptation is permitted by precise definition of the boundary conditions of those low energy electromagnetic fields.

RESUME. Cet article présente une étude critique de la manière dont sont traités, en théorie classique de Maxwell, les potentiels scalaire et vecteur, comme de simples commodités mathématiques, sans signification physique. Dans la partie 1, on rappelle les preuves expérimentales d'un certain nombre d'effets bien établis, démontrant que les champs A ont vraiment une signification physique *en tant qu'opérateurs permettant de passer du point de vue local au point de vue global, ou champs de jauge, dans des topologies contraintes de façon précise. Les champs A ont un sens physique dans des topologies de type de symétrie SU(2) ou supérieure. La théorie classique conventionnelle de Maxwell est de symétrie U(1), avec des relations*

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de commutation abéliennes. Etendue à une symétrie de type SU(2), ou supérieure, la théorie de Maxwell possède des relations de commutatin non abéliennes. Dans la partie 2, cet article applique une interprétation de type Yang-Mills à des champs de basse *énergie – une approche utilisée jusqu'ici pour les* hautes *énergies. Cette adaptation est permise grâce à une définition précise des conditions aux limites de ces champs électromagnétiques de basse énergie.*

PROLEGOMENA

In this Part 1 of a two part examination of Maxwell's theory, effects demonstrating the physical significance of the A_μ potentials are examined. Part 2 (Barrett, 1990) adresses the theoretical and pragmatic reasons for questioning the completeness of Maxwell's theory.

A number of physical effects strongly suggest that the Maxwell field theory of electromagnetism is incomplete. These effects address fields(s)-free electron (F-FE), field(s)-conducting electron (F-CE), field(s)-particle (F-P), wave-guide-field (WG-F), conducting electron-field(s) (CE-F) and rotating frame-field(s) (RF-F) interactions. A list of these experimentally observed effects, all of which involve the A_μ potentials in a physically effective role, includes: 1. The *Aharonov-Bohm* and *Altshuler-Aronov-Spivak* effects (F-FE & F-CE): Ehrenberg and Siday, Aharonov and Bohm, and Altshuler, Aronov and Spivak predicted experimental results by attributing physical effects to the A_μ potentials. Most commentaries in classical field theory still show these potentials as mathematical conveniences without gauge invariance and with no physical significance. However, many experiments have demonstrated the phase changes predicted.

2. The *Berry-Aharonov-Anandan-Pancharatnam* phase rotation effect (WG-F) and (F-P): In the WG-F version, the polarization of light is changed by changing the spatial trajectory adiabatically. The Berry-Aharonov-Anandan phase has also been demonstrated at the quantum, as well as the classical level. This phase effect in parameter (momentum) space is the correlate of the Aharonov-Bohm effect in metric (ordinary) space, both involving adiabatic transport.

3. The *Josephson effect* (CE-F): both at the quantum and macrophysical level, the free energy of the barrier is defined with respect to an A_μ potential variable (phase).

4. The *quantum Hall effect* (F-CE): gauge invariance of the A_μ vector potential, being an exact symmetry, forces the addition of a flux

quantum to result in an excitation without dependence on the electron density.

5. The *De Haas-Van Alphen* effect (F-CE): the periodicity of oscillations in this effect is determined by A_μ potential dependency and gauge invariance.

6. The *Sagnac effect* (RF-F): exhibited in the well-known and well-used ring laser gyro, this effect demonstrates that the Maxwell theory, as presently formulated, does not make explicit the constitutive relations of free space, and does not have a built-in Lorentz invariance as its field equations are independent of metric.

The A_μ potential has been demonstrated to be a physically meaningful construct at the quantum level (Effects 1-5), at the classical level (Effects 2,3 and 6), and at relatively long range in Effect 2. In the F-CE and CE-F cases (Effects 1,3-5), the effect is limited by the temperature-dependent electron coherence length with respect to the device/antenna length.

Formerly, treatment of the A_μ potentials as anything more than mathematical conveniences was prevented by their obvious lack of gauge invariance. However, gauge invariance for the A_μ potentials results from situations in which fields, firstly, have a history of separate spatiotemporal conditioning and then, secondly, are mapped in a many-to-one, or global-to-local, fashion (in holonomy). Such conditions are satisfied by A_μ potentials with boundary conditions, i.e., the usual empirically encountered situation. Thus, with the correct geometry and topology (i.e., with stated boundary conditions) the A_μ potentials always have physical meaning. This indicates that Maxwell's theory can be extended by the appropriate use of topological and gauge symmetrical concepts.

The A_μ potentials are local operators mapping global spatiotemporal conditions onto the local e.m. fields. The effect of this operation is measurable as a phase change, if there is a second comparative mapping of differentially conditioned fields in a many-to-one (global-to-local summation). With coherent fields the possibility of measurement (detection) after the second mapping is maximized. The question of whether A_μ potentials can be propagated to long range can be answered affirmatively if dual field coherence is maintained.

SYNOPSIS FOR PARTS 1 and 2

1. Maxwell's theory is a linear theory in which the scalar and vector potentials are arbitrary, defined by choice of gauge, and have only mathematical, not physical, significance.

2. However, in quantum theory, potentials do have physical significance, and a number of physical phenomena –both classical and quantum mechanical– indicate that the $A_\mu, \mu = 0, 1, 2, 3$, do possess physical significance and have physical effects.

3. Maxwell's linear theory is of $U(1)$ symmetry form with Abelian commutation relations. It can be extended to include *physically meaningful* A_μ effects by its reformulation in $SU(2)$ or nonAbelian form. The theory is then nonlinear.

4. When reformulated in nonAbelian form, Maxwell's theory addresses not only *local* phenomena, but also *global* phenomena, by means of the A_μ potentials used as *local-to-global operators*.

5. The approach presented in this paper adopts a Yang-Mills orientation –previously applied to high energy fields– to low-energy fields. The adaptation is permitted by precise definition of boundary conditions on the low-energy fields.

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1. INTRODUCTION

There are a number of different reasons for questioning the completeness of Maxwell's theory of electromagnetism (Maxwell, 1871). It

is well known that there is an arbitrariness in the definition of the A vector and scalar potentials, which, nevertheless, have been found very useful when used in calculations with boundary conditions known. The reasons for questioning completeness are due to experimental evidence (section 3, Part 1), theoretical (section 2, Part 2), and pragmatic (section 3, Part 2).

An examination of the Maxwell theory may begin with the well-known Maxwell equations: Coulomb's Law:

$$\nabla \cdot D = 4\pi\rho; \tag{1.1}$$

Maxwell's generalization of Ampère's Law:

$$\nabla \times H = (4\pi/c)J + (1/c)\partial D/\partial t; \tag{1.2}$$

the absence of free *local* magnetic poles postulate:

$$\nabla \cdot B = 0; \tag{1.3}$$

and Faraday's Law:

$$\nabla \times E + (1/c)\partial B/\partial t = 0. \tag{1.4}$$

The constitutive relations of the medium-independent fields to matter are well-known to be:

$$D = \epsilon E, \tag{1.5}$$

$$J = \sigma E, \tag{1.6}$$

$$B = \mu H. \tag{1.7}$$

Because of the postulate of an absence of free local magnetic monopoles (Equ.(1.3)), the following is permitted:

$$B = \nabla \times A, \tag{1.8}$$

but the vector potential is thus always arbitrarily defined, because the gradient of some scalar function, Λ, can be added leaving B unchanged, i.e., B is unchanged by the gauge transformations:

$$A \to A' = A + \nabla\Lambda \quad , \quad \phi \to \phi' = \phi - (1/c)\partial\Lambda/\partial t. \tag{1.9}$$

This arbitrary definition of the potentials means that *any* gauge chosen is arbitrary, or, an appeal must be made to boundary conditions for any choice.

Now Equ. (1.8) permits a redefinition of Equ. (1.4):

$$\nabla \times (E + (1/c)\partial A/\partial t) = 0 \quad \text{(Faraday's law rewritten)}, \tag{1.10}$$

which means the quantity in brackets is the gradient of a scalar function, ϕ, so:

$$E + (1/c)\partial A/\partial t = -\nabla\phi \quad , \qquad \text{or} \quad E = -\nabla\phi - (1/c)\partial A/\partial t; \tag{1.11}$$

and the Maxwell equations (1.3) and (1.4) can be redefined by (1.8) and (1.11).

Maxwell equations (1.1) and (1.2) can also be written as:

$$\nabla^2\phi + (1/c)\partial(\nabla \cdot A)/\partial t = -4\pi\rho, \tag{1.12}$$

$$\nabla^2 A - (1/c^2)(\partial^2 A/\partial t^2) - \nabla(\nabla \cdot A + (1/c)\partial\phi/\partial t) = -(4\pi/c)J. \tag{1.13}$$

Since the gauge conditions (Equ.s (1.9)) are arbitrary, a set of potentials (A, ϕ) can be chosen so that:

$$\nabla \cdot A + ((1/c)\partial\phi/\partial t) = 0. \tag{1.14}$$

This choice is called the Lorentz condition or the Lorentz gauge. Equ.s (1.12) and (1.13) can then be decoupled to obtain:

$$\nabla^2\phi + (1/c^2)\partial^2\phi/\partial t^2 = -4\pi\rho, \tag{1.15}$$

$$\nabla^2 A - (1/c^2)\partial^2 A/\partial t^2 = -(4\pi/c)J, \tag{1.16}$$

which is useful, because the Maxwell equations are then independent of the coordinate system chosen. Nonetheless, as A and ϕ are not gauge invariant, the original choice of the Lorentz gauge is arbitrary –a choice which is not an inevitable consequence of the Maxwell theory– and the resultant from that choice, namely Equ.s (1.15) and (1.16) is equally arbitrary.

Then again, the arbitrariness of Equ.s (1.9) is useful because it permits the choice:

$$\nabla \cdot A = 0. \tag{1.17}$$

Equ. (1.12), which is Maxwell equation (1.1), then permits:

$$\nabla^2\phi = -4\pi\rho, \tag{1.18}$$

which is the instantaneous Coulomb potential, and hence condition (17) is called the Coulomb or transverse gauge because the wave equation for A can be expressed in terms of the transverse current:

$$\nabla^2 A - (1/c^2)\partial^2 A/\partial t^2 = -(4\pi/c)J_t \tag{1.19}$$

where $J_t = J - J_l$. This is a useful thing to do when no sources are present, but, again, as A and ϕ are not gauge invariant, i.e., considered to have no physical meaning, the original choice of the Coulomb gauge is arbitrary, and so is the resultant from that choice, namely Equ. (1.19).

For all that, the absence of gauge invariance (physical meaning) of the A vector potential and the ϕ scalar potential may seem a fortunate circumstance to those using the Maxwell theory to calculate predictions. These potentials have long been considered a fortunate mathematical convenience, but *just* a mathematical convenience, with no physical meaning. These constructs lack gauge invariance, a defining characteristic of physical, rather than merely mathematical, constructs. What then is meant by a gauge and gauge invariance?

2. WHAT IS A GAUGE?

In 1918 Weyl (1918, see also Yang, 1986) treated Einstein's general theory of relativity as if the Lorentz symmetry were an example of *global* symmetry but with only *local* coordinates defineable, i.e., the general theory was considered as a *local* theory. A consequence of Weyl's theory is that the absolute magnitude or norm of a physical vector is not treated as an absolute quantity but depends on its location in space-time. This notion was called scale (Mass-stab) or gauge invariance.

This concept can be understood as follows. Consider a vector at position x with norm given by $f(x)$. If the coordinates are transformed, so that the vector is now at $x + dx$, the norm is $f(x + dx)$. Using the abbreviation $\partial_\mu = \partial/\partial^\mu$, $\mu = 0, 1, 2, 3$ and expanding to first order:

$$f(x + dx) = f(x) + \partial_\mu f dx^\mu. \tag{2.20}$$

If a gauge change is introduced by a multiplicative scaling factor, $S(x)$, which equals unity at x, then

$$S(x + dx) = 1 + \partial_\mu S dx^\mu. \tag{2.21}$$

If a vector is to be constant under change of location then:

$$Sf = f + [\partial_\mu S] f dx^\mu + [\partial_\mu f] dx^\mu, \tag{2.22}$$

and, on moving, the norm changes by an amount

$$[\partial_\mu + \partial_\mu S] f dx^\mu. \tag{2.23}$$

Weyl identified $\partial_\mu S$ with the electromagnetic potential A_μ.

However, this suggestion was rejected (by Einstein) because the natural scale for matter is the Compton wavelength, λ, and as the wave description of matter is $\lambda = h/mc$, then if, as is always assumed, the wavelength is determined by the particle's mass, m, and with h and c constant (according to the special theory of relativity), λ cannot depend on position without violating the special theory. When made aware of this reasoning, Weyl abandoned his proposal. So the term: gauge change, originally meant: change in length, and was withdrawn from consideration for this particular metric connotation shortly after its introduction.

But the term did not die. "Gauge invariance" managed to survive in classical mechanics because, with the potentials arbitrary, Maxwell's equations for the E, B, H and D fields have a built-in symmetry and such arbitrary potentials became a useful mathematical device for simplifying many calculations in electrodynamics, as we have seen. Nevertheless, the gauge invariance in electromagnetism for the E, B, H and D fields was regarded as only an "accidental" symmetry, and the lack of gauge invariance of the electromagnetic vector and scalar potentials was interpreted as an example of the well known arbitrariness of the concept of the potential in classical mechanics.

But this arbitrariness in the concept of the potential did, and does not, exist in quantum mechanics. The electromagnetic vector and scalar potentials were viewed in quantum mechanics in yet another way. Upon the development of quantum mechanics, Weyl and others realized that the original gauge theory could be given a new meaning. They realized that the phase of a wavefunction could be a new *local* variable. Instead of a change of scale or metric, for which it was originally introduced, a gauge transformation was reinterpreted as a change in the phase of the wavefunction:

$$\psi \to \psi e^{-ie\lambda}, \tag{2.24}$$

and the gauge transformation for the potential A_μ became:

$$A_\mu \rightarrow A_\mu - \partial\lambda/\partial x_\mu. \tag{2.25}$$

Equ.s (2.24) and (2.25) together ensure that the Schrödinger formulation for a single charged particle in an electromagnetic field remains invariant to phase changes because they self-cancel. Thus any change in location, for that *single* charged particle, which produces a change in the phase (Equ. (2.24)), is compensated by a corresponding change in the potential (Equ. (2.25)). Therefore Weyl's original idea, reinterpreted, was accepted, and the potential in quantum mechanics was viewed as a *connection* which relates phases at different locations. Nevertheless, this use and interpretation did not carry over into classical mechanics and a schizoid attitude exists to this day regarding the physical meaning of the potentials in classical and quantum mechanics. In classical mechanics the potentials were, up until recently, viewed as having only an arbitrary mathematical, not physical, meaning, as they seemed to lack gauge invariance. In quantum mechanics, however, they *are* viewed as gauge invariant and as possessing a physical meaning. It is an aim of the present review to show that in classical mechanics the potentials can also be taken to have, under special circumstances, a physical meaning, i.e., possess the required gauge invariance.

A major impetus to rethink the physical meaning of the potentials in classical mechanics came about from the experiments examined in the next section.

3. EMPIRICAL REASONS FOR QUESTIONING THE COMPLETENESS OF MAXWELL'S THEORY

3.1 Aharonov-Bohm and Altshuler-Aronov-Spivak effects

Beginning in 1959 Aharonov and Bohm (1959, 1961-3) challenged the view that the classical vector potential produces no observable physical effects by proposing two experiments. The one which is most discussed is shown in Figure 3.1.1. A beam of monoenergetic electrons exits from the source at X and is diffracted into two beams by the two slits in the wall at $Y1$ and $Y2$. The two beams produce an interference pattern at Z which is measured. Behind the wall is a solenoid, S, the B field of which points out of the paper. The absence of a free local magnetic monopole postulate (Maxwell equation (3) above) predicts the magnetic

field outside the solenoid to be zero. Before the current is turned on in the solenoid, there should be the usually expected interference patterns seen at Z. Aharonov-Bohm predicted that if the current is turned on and due to the differently directed A fields in paths 1 and 2 indicated by the arrows in Figure 1, additional phase shifts should be discernible at Z. This prediction was confirmed experimentally (Chambers, 1960; Boersch et al, 1960; Möllenstedt et al, 1962; Matteucci & Pozzi, 1985; Tonomura et al, 1982, 1983, 1986; Tonomura & Callen, 1987) and the evidence has been reviewed by Berry (1980), Peshkin (1981), Olariu & Popescu (1985) and Horvathy (1986). Aharonov and Casher (1984) have extended the theoretical treatment of the AB effect to neutral particles with a magnetic moment; and Botelho and de Mello (1985) have analyzed a non-Abelian Aharonov-Bohm effect in the framework of pseudoclassical mechanics.

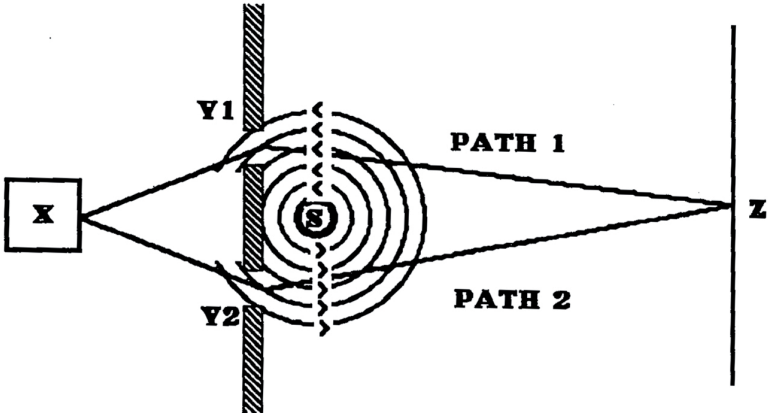

Figure 3.1.1. Two-slit diffraction experiment of the Aharonov-Bohm effect. Electrons are produced by a source at X, diffracted by the slits at $Y1$ and $Y2$ and their diffraction pattern is detected at Z. The solenoid is between the slits and directed out of the page. The different orientations of the A field at the points of interaction with the two paths are indicated by the arrows $>$ and $<$ following the right-hand rule.

One explanation of the effect is as follows. Let ψ_0 be the wavefunction when there is no current in the solenoid. After the current is turned on the Hamiltonian is:

$$H = (1/2m)(-ih\nabla - eA)^2, \tag{3.1.1}$$

and the new wavefunction is:

$$\psi = \psi_0 \exp[-ieS/h], \tag{3.1.2}$$

where, S, the flux, is:

$$S = \int_C A \cdot dx, \tag{3.1.3}$$

which is the quantum analog of the classical action evaluated along the paths 1 and 2. At point Z the wavefunctions of the two electron beams are:

$$\begin{aligned} \psi_1 &= \psi_0 \exp[-eS_1/h], \\ \psi_3 &= \psi_0 \exp[-eS_2/h], \end{aligned} \tag{3.1.4}$$

and the phase difference is:

$$(e/h)(S_1 - S_2) = (e/h)\big[\int_1 A \cdot dx - \int_2 A \cdot dx\big] = 2\pi(e/h)\phi. \tag{3.1.5}$$

By Stoke's theorem, this is directly proportional to the magnetic flux, $\phi = \int_C A \cdot dx$, in the solenoid.

However, the phase difference given by Equ. (3.1.5) is not single-valued. Therefore, the value of the phase change will only be determined to within an arbitrary multiple, n of $2\pi e\phi/h$, where n is the number of times the measured charge circulated the solenoid.

The Aharonov-Bohm (AB) effect was confirmed experimentally in the originally proposed field-free electron (F-FE) situation (cf. Chambers, 1960). More recent experiments address the appearance of the effect in the field-conduction electron (F-CE) situation. This situation is also not strictly the same as in the originally proposed Aharonov-Bohm experiment in another respect –the magnetic flux is produced by a large solenoid surrounding the influenced condensed matter, usually a loop or a cylinder– so that the B field is not set to zero within the material. However, the preponderance of the B field is always in the hole encompassed by that cylinder or ring, and the magnetic field causes only secondary effects in the material. Under these conditions the Aharonov-Bohm effect is seen in normal metal (Webb et al, 1985, 1987; Benoit et al, 1986; Washburn et al, 1985, 1987; Chandrasekhar et al, 1985; Datta et al, 1985; Tonomura et al, 1982, 1983, 1986; Cavalloni & Joss, 1987), bulk Mg (Sandesara & Stark, 1984); semiconductors (Datta et al, 1986, Datta & Bandyopadhyay, 1987); and on doubly connected geometries

on GaAs/AlGaAs heterostructures (Timp et al, 1987). The effect has been seen in structures such as: cylindrical Mg films (Sharvin & Sharvin, 1981; Gijs et al, 1984) and Li films (Altshuler et al, 1982), wire arrays (Pannetier et al, 1984; Bishop et al, 1985), arrays of Ag loops (Umbach et al, 1986), small metal loops (Webb et al, 1985; Chandrasekhar et al, 1985) and MBE-grown double quantum wells (Datta et al, 1985).

Bandyopadhyay et al (1986) and Datta & Bandyopadhyay (1987) have also discussed a novel concept for a transistor based on the electrostatic Aharonov-Bohm effect in MBE-grown quantum wells, where the current is modulated by quantum interference of electrons in two contiguous channels of a gate voltage. They predict that transistors based on this effect will have power-delay products orders of magnitude better than those of existing devices such as MODFETs and Josephson junctions. The transconductance will also be much higher than that of MODFETS. Unlike previous experimental treatments which assumed diffusive transport with negligible inelastic scattering. Datta & Bandypadhyay (1987) assume ballistic transport and perfect symmetry in the arms of the interferometer and in the voltage along the interferometer or two channel structure.

Now, the AB (F-CE) effect is temperature-dependent as coherent transport is required. The effect has only been seen at very low temperature. Measurements were made on parallel GaAs quantum wells at $4.2K$ and below (Datta et al (1985)); on $860nm - i.d.$ Au loops at $0.003K$ (Webb et al, 1987) and $0.05K < T < 0.7K$ (Webb et al, 1985; Washburn et al, 1985), on $75nm - o.d.$ Sb loops at $0.01 < T < 1K$ (Milliken et al, 1987) and at $0.04K$ (Washburn et al, 1987) and on Ag loop arrays at $4.2K$ (Umbach et al, 1986). Measurements on $1.5 - 2.0$ micron diameter Mg cylinders of length 1 cm were made at $1.12K$ (Sharvin & Sharvin, 1981). The Thouless scaling parameter, V, or the sensitivity of energy levels to a change in the phase of the wavefunctions at the boundaries (Edwards & Thouless, 1972; Lee et al, 1987) implies that the necessary energy correlation range for small rings is accessible in the temperature range $0.0001 - 10K$ (Stone & Imry, 1986).

What is remarkable is that these experiments on the (F-CE) Aharonov-Bohm effect demonstrate that the effect *can* occur in *disordered* electrical conductors if the temperature is low enough. The effect in metals is a small magnetoresistance oscillation superimposed on the ohmic resistance in multiply-connected conductors at low temperatures (Altshuler et al, 1981; Sharvin & Sharvin 1981; Stone & Imry,

1986). *This means that the conducting electrons must possess a high degree of phase coherence (internal correlation) over distances larger than the atomic spacing or the free path length.* It was initially thought that the effects of finite temperature and the scattering from, and collision with, impurities, would cause incoherence and prevent the observation of the Aharonov-Bohm effect in bulk samples (Imry, 1986). The metal loops used measure, e.g., less than a micron in diameter and less than 0.1 microns in line thickness. Therefore, the electron is thought to be represented by a pair of waves –one traveling around the ring in the clockwise direction, and the other in the opposite direction, but following the time-reversed path of the first wave. Thus, although each wave has been scattered many times, *each wave collides with the same impurities*, i.e., acquires the same phase shifts, resulting in constructive interference at the origin. *The total path length of both waves is twice the circumference of the ring, meeting the requirement that the phase coherence of the electrons be larger than the circumference of the ring*, or, the transport through the metals arms considered as disordered systems is determined by the eigenvalues of a large random matrix (Imry, 1986).

Thus, the conductance, G, of a one-dimensional ring in the presence of elastic scattering is (Landauer, 1970):

$$G = \frac{2e^2}{h}\frac{t}{1-t}, \tag{3.1.6}$$

and an Aharonov-Bohm flux applied to the ring results in periodic oscillations of G, *provided that the phase coherence length of the ring is longer than the size of the system.*

A related effect is the Altshuler, Aronov and Spivak (AAS) effect (Altshuler, Aronov & Spivak, 1981). These authors considered an ultrathin normal metal cylindrical shell of moderate length but very small transverse dimensions at low temperature and how the magnetoresistance would depend on the intensity of magnetic flux axially threading the cylinder. They concluded that it would be an oscillating function of the total flux with a period of $h/2e$, i.e., the same as the flux of the superconductive state. The analogous "flux quantum" of the Aharonov-Bohm effect is h/e (Webb et al, 1985, 1987) and differs from the AAS situation which involves coherent "backscattering". The AAS effect has been observed in a $1000\mathring{A}$ thick magnesium layer on a quartz fiber several millimeters long (Sharvin & Sharvin, 1981). More recent treatments of the AAS effect (Büttiker et al, 1983, 1985; Stone & Imry, 1986) are based

on the quantum mechanical transmission (t) coefficients of electrons and, unlike the original AAS treatment, find an h/e periodic component as well as the $h/2e$ harmonic. Raising the temperature above a crossover, T_c, changes the flux periodicity of magnetic resistance oscillations from h/e to $h/2e$, where T_c is determined by the energy correlation range hD/L^2, where D is the elastic diffusion constant, L is the length of sample and the quantity hD/L^2 is the Thouless scaling parameter V for a metal.

The AAS effect arises because of a special set of trajectories –time-reversed pairs which form a closed loop– which have a fixed relative phase for any material impurity configuration. These trajectories do not average to zero and contribute to the reflection coefficients which oscillate with period $h/2e$. The h/e oscillations of the AB effect, on the other hand, arise from oscillations in the transmission coefficients and can at higher temperature average to zero. Below T_c both contributions are of order e^2/h (Stone & Imry, 1986).

Xie and DasSarma (1987) studied both the AB and AAS effects in the transport regime of a strongly disordered system in which electron transport is via a hopping process, specifically, via variable-range-hopping transport. Their numerical results indicate that only the $h/2e$ (AAS) flux-periodic oscillations survive at finite temperatures in the presence of any finite disorder.

The results of the metal loop experiments demonstrated that elastic scattering does not destroy the phase memory of the electron wave functions (Webb et al, 1987; Washburn & Webb, 1986). Although the flux periodicity in a condensed matter system due to the Aharonov-Bohm effect would not be surprising in a superconductor, the same periodicities in *finite* conductors is remarkable (Xie & DasSarma, 1987). Numerical simulation of variable-range-hopping conduction (Xie & DasSarma, 1987) only finds AB oscillations ($\Phi_0 = h/e$) in hopping conductance when a metal ring is small and at low temperature. At the large ring limit and higher temperature AAS ($\Phi_0 = h/2e$) oscillations survive – a finding consistent with the experimental findings of Polyarkov et al (1986). A suggested reason for the retention of long range phase coherence is that the phase memory is only destroyed exponentially as e^{-L/L_i}, where L_i is a "typical inelastic scattering length" and the destruction depends on the energy changes in the hopping process dependent on long wavelength, low energy acoustic phonons. Search for an explanation for both AB and AAS effects has resulted in consideration of systems

neither precisely quantum mechanical nor classical, but inbetween, i.e., "mesoscopic".

"Mesoscopic" systems have been studied by Stone (1985) in which the energy and spacing is only a few orders of magnitude smaller than kT at low temperatures. The prediction was made, (Stone, 1985), that large Aharonov-Bohm oscillations should be seen in the transport coefficients of such systems. Such systems have a sample length which is much longer than the elastic mean free path, but shorter than the localization length. The magnetic field through a loop connected to leads changes the relative phase of the contribution from each arm of the loop by $2\pi\phi/\phi_0$, where $\phi_0 = hc/e$ is the one electron flux quantum and ϕ is the flux through the hole in the loop –but only if the phase dependent terms do not average to zero. In the mesoscopic range, if inelastic scattering is absent, these phase-dependent contributions do not self-average to zero.

Washburn et al (1985) and Stone & Imry (1986) demonstrated experimentally that the amplitude of aperiodic and periodic conductance fluctuations decrease for the F-CE Aharonov-Bohm effect with increasing temperature. There is a characteristic correlation energy:

$$E_C = \pi h D/2L^2, \tag{3.1.7}$$

where D is the diffusion constant of the electrons, L is the minimum length of the sample length. If thermal energy $k_B T > E_C$, the conductance fluctuations decrease as $(E_C/k_B T)^{1/2}$. The conductance fluctuations also decrease when L_Φ, the phase coherence length, is shorter than the length, L or the distance between voltage probes, the decrease being described by a factor $\exp(-L/L_\Phi)$ (Milliken et al, 1987). This gives a conductance fluctuation:

$$\Delta G n (L_\Phi/L)^{3/2}. \tag{3.1.8}$$

In condensed matter, therefore, the Aharonov-Bohm effect appears as the modulation of the electron wave functions by the A_μ potential. The phase of the wave function can also be changed by the application of an electric field (Washburn et al, 1987), in which case the electric field contributes to the fourth term in the four-vector product $A_\mu(dx)^\mu$ which contains the scalar potential ϕ associated with transverse electric fields and time. The phase shift in the wavefunction under field influence is:

$$\Delta\varphi = \int e\phi dt/h. \tag{3.1.9}$$

Experiments on Sb metal loop devices on silicon substrate have demonstrated that the voltage on capacitative probes can be used to tune the position (phase) of h/e oscillations in the loop. Thus, there appears two ways to modulate the phase of electrons in condensed matter : application of the A_μ potential by threading magnetic flux between two paths of electrons; and also by application of a scalar potential by means of a transverse electric field. Aharonov-Bohm fluctuations in metal loops are also not symmetric about $H = 0$. Four probe measurements yield resistances which depend on the lead configurations (Benoit et al, 1986).

In summary, the AB and AAS effects, whether F-FE or F-CE, demonstrate that the phase of a *composite* particle's wavefunction is a physical degree of freedom which is dependent on differences in A_μ potential influences on the space-time position or path of a *first* particle's wave function with respect to that of another, *second*, particle's wavefunction. But the *connection*, or *mapping*, between spatiotemporally different fields or particles which originated at, or passed through, spatiotemporally separated points or paths with differential A_μ potential influences, is only measurable by many-to-one mapping of those different fields or particles. By interpreting the phase of a wavefunction as a *local* variable instead of the norm of a vector, electromagnetism can be interpreted as a *local* gauge (phase) theory, if not exactly, then very close to the way Weyl originally envisioned it to be.

Below, the interaction of the A_μ field, whether vector or potential, with field-free electron, field-conducting electron, field-wave guide, field-neutral particle and field-rotating frame will be referred to as (F-FE), (F-CE), (F-WG), (F-P) and (F-RF) interactions. Whereas the AB and AAS effects are either F-FE and F-CE effects and might be considered "special" in that they involve quantum mechanical particles, i.e., electrons, in the next section we examine a phase rotation which can only be considered classical. Nonetheless, the same conclusion, that the A_μ potentials possess physical effects, applies.

3.2 Berry-Aharonov-Anandan-Pancharatnam phase rotation effect

Wu & Yang (1985), when addressing the Aharonov-Bohm effect, argued that the wave function of a system will be multipled by a non-integrable (path-dependent) phase factor after its transport around a closed curve in the presence of an A_μ potential *in ordinary space*. The

Berry-Aharonov-Anandan-Pancharatnam (BAAP) phase, another non-integrable phase factor, arises from the adiabatic transport of a system around a closed path in *parameter (momentum) space*, i.e., the BAAP phase is the Aharonov-Bohm effect in parameter space (Pancharatnam, 1956; Berry, 1984a,b; 1985; 1987a,b; Wilkinson, 1984a,b; Chiao & Wu, 1986; Haldane, 1987; Chiao & Tomita, 1987). The WG-F version of this effect has been experimentally verified (Tomita & Chiao, 1986) and the phase effect in general interpreted as due to parallel transport in the presence of a gauge field (Simon, 1983). The effect exists at both the classical and quantum levels (cf. Thomas, 1988).

There has been, however, an evolution in understanding concerning what causes the BAAP effect. Berry (1984) originally proposed a geometrical (beside the usual dynamical) phase acquisition for a non-degenerate quantum state which varies adiabatically through a circuit in parameter space. Later, the constraint of adiabaticity was removed (Berry, 1987b) and also the constraint of degenerate states (Wilczck & Zee, 1984). Then Aharonov and Anandan (1987) showed that the effect can be defined for any cyclic evolution of a quantum system. Bhandari and Samuel (1988) have also pointed out that Berry's phase is closely connected with a phase discovered by Pancharatnam (1956). These authors also demonstrated that unitary time evolution of a system is not essential for the appearance of the phase by the measurement of the phase change in one beam of a laser interferometer as the polarization state of light is taken along a closed circuit on the Poincaré sphere. Thus current thought is that the history of "windings" of a particle is "remembered", or registered and indicated, by changes in phase in a quantum mechanical particle's state, or in a classical wave's polarization.

The BAAP effect, in its F-P version, has been observed in NMR interferometry experiments (Suter et al, 1987, 1988) and using ultracold neutrons (Richardson et al, 1988); in coherent states (Giavarini et al, 1989a,b); optical resonance (Ellinas et al, 1989); and the degenerate parametric amplifier (Gerry, 1989). The BAAP effect is also seen in a classical waveguide-field (WG-F) version.

In its classical WG-F version, the helicity or polarization state, σ, is (Chiao & Wu, 1986):

$$\sigma = s \cdot k, \tag{3.2.1}$$

where s is a spin or helicity operator and k is the direction of propagation (k_x, k_y, k_z). If τ is the optical path length, then $\mid k(\tau), \sigma >$ is the

spin or polarization state. Interpreted classically, the constrainment of k to remain parallel to the axis of a waveguide is due to the linear momentum being in that direction. This means that a waveguide can act as a polarization rotator. Furthermore, as helicity (polarization), σ, is adiabatically conserved, s is also constrained to remain parallel to the local axis of the waveguide. Therefore, the topology of a waveguide, e.g., a helix-shape, will constrain k and also s to perform a trajectory C on the surface of a sphere in the parameter space (k_x, k_y, k_z) which prescribes the linear momentum. Thus the topology of the constrained trajectory of radiation progressing between two *local* positions has a *global* effect indicated by a polarization (spin) change. If $\gamma(C)$ is the BAAP phase, and $\beta = \exp[i\gamma(C)]$ is a phase factor, the final polarization state after progression along a constrained trajectory, i.e., "momentum conditioning" instigated by changes in topological trajectories, is:

$$\sigma_2 = \beta \cdot s \cdot k, \tag{3.2.2}$$

where the subscript indicates a second location on the trajectory. Whether the "momentum conditioning" is merely a concomitant effect to the topological progression differences along the waveguide, has yet to be determined.

As a monopole is theoretically required at $k = 0$, due to the radial symmetry of the parameter space and resulting singularity, a solid angle, $\Omega(C)$, can be defined on a parameter space sphere with respect to the origin $k = 0$. Thus, $\Omega(C)$ can be said to define the "excited states" of the monopole at $k = 0$. Therefore:

$$\sigma_2 - \sigma_1 = \beta \cdot s \cdot k \cdot \sigma_1 = \sigma_1 \Omega(C) - \sigma_1 = \gamma(C). \tag{3.2.3}$$

The question can then be asked: what conservation law underlies the BAAP phase? A clue is provided by Kitano et al (1987), who point out that the BAAP phase can also be seen in discrete optical systems which contain no waveguides at all, e.g., in a configuration of (ideal or infinitely conducting) mirrors. Now mirrors do not conserve helicity; they reverse it and the local tangent vector, t, must be replaced by $-t$ on alternate segments of the light path. Mirror configurations of this type have been used in a laser gyro (Chow et al, 1985). This suggests that changes of acceleration –whether along a waveguide, or in mirror reflection– under equivalence principle conditions is the compensatory change which matches changes in the BAAP phase, giving the conservation equation:

$$\gamma(C) + \int_C A \cdot dl = 0 \tag{3.2.4}$$

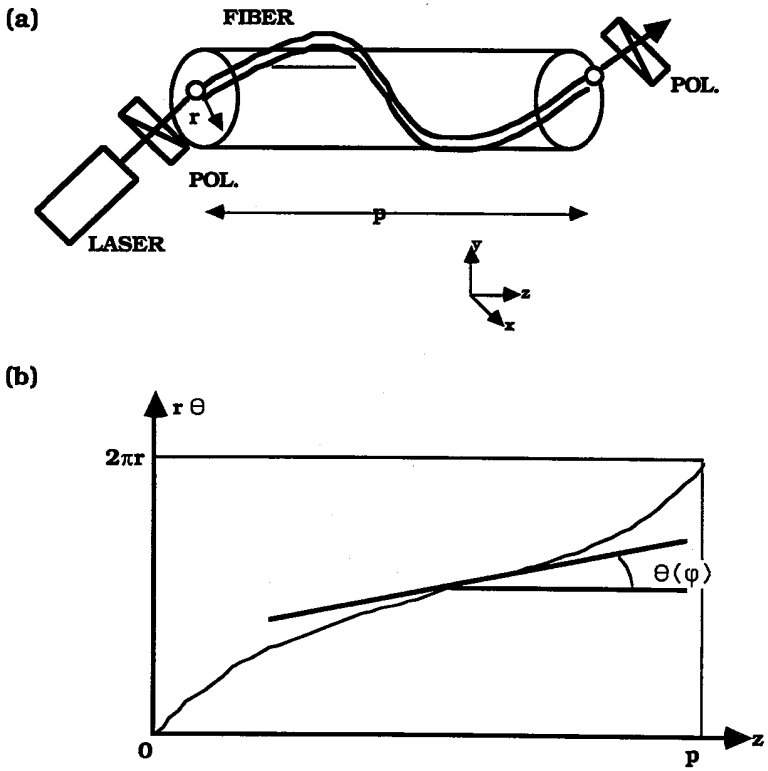

Figure 3.2.1. (a) Experimental setup; (b) geometry used to calculate the solid angle in momentum space of a nonuniformaly wound fiber on a cylinder. After Tomita & Chiao, 1986.

That the effect can occur in classical mechanical form is witnessed by changes in polarization rotation resulting from changes in the topological path of a light beam. Tomita & Chiao (1986) demonstrated effective optical activity of a helically wound single-mode optical fiber in confirmation of Berry's prediction. The angle of rotation of linearly polarized light in the fiber gives a direct measure of the BAA phase *at the classical level.* (Hannay (1985) has also discussed the classical limit of the BAAP phase in the case of a symmetric top). This classical effect arises from the overall geometry of the path taken by the light and is thus a *global topological effect* independent of the material properties of the fiber. The optical rotation is *independent of geometry* and therefore may be said to

quantify the "*topological charge*" of the system, i.e., the helicity of the photon, which is a relativistic quantum number.

Referring to Figure 3.2.1, the fiber length is:

$$s = [p^2 + (2\pi r)^2]^{1/2}, \tag{3.2.5}$$

and the solid angle in momentum space $\Omega(C)$ spanned by the fiber's closed path C, a circle in the case considered, is:

$$\Omega(C) = 2\pi(1 - \cos\theta). \tag{3.2.6}$$

The BAAP phase is:

$$\gamma(C) = -2\pi\sigma(1 - p/s), \tag{3.2.7}$$

where $\sigma = \pm 1$ is the helicity quantum number of the photon.

By wrapping a piece of paper with a computer generated curve on a cylinder to which the fiber is fitted, and then unwrapping the paper, the local pitch angle, or tangent to the curve followed by the fiber can be estimated to be (Figure 3.2.1(b)):

$$\theta(\varphi) = \tan^{-1}(r d\varphi/dz), \tag{3.2.8}$$

which is the angle between the local waveguide and the helix axes. In momentum space, $\theta(\varphi + \pi/2)$ traces out a closed curve C, the fiber path on the surface of a sphere. The solid angle subtended by C to the center of the sphere is:

$$\Omega(C) = \int_0^{2\pi} [1 - \cos\theta(\varphi)] d\varphi. \tag{3.2.9}$$

The BAAP phase is then, more properly:

$$\gamma(C) = -\sigma\Omega(C). \tag{3.2.10}$$

Tomita & Chiao thus demonstrated the linear relation between the angle of rotation of linearly polarized light, and the solid angle $\Omega(C)$ subtended by C at the origin of the momentum space of the photon.

More recently, Chiao et al (1988) have demonstrated a topological phase shift in a Mach-Zehnder interferometer in which light travels along nonplanar paths in two arms. They interprete their results in terms of the Aharonov-Anandan phase and changes in projective Hilbert space,

i.e., the sphere of spin directions of the photon, rather than parameter (momentum) space. The hypothesis tested was that the evolution of the state of a system is cyclic, i.e., that it returns to its starting point, adiabatically or not. Thus the C in Equ. (3.2.10) is to be interpreted as a closed circuit on the sphere of spin directions.

Chiao & Wu (1986) consider the BAAP phase rotation effects to be "topological features of the Maxwell theory which originate at the quantum level, but which survive the correspondence principle limit ($h \to 0$) into the classical level". The effect is viewed as a classical effect by other authors (e.g., Berry (1987) and Segert (1987a,b)). Segert (1987a,b) views the evolution of the polarization vector as determined by a connection on the tangent bundle of the two-dimensional sphere. *The effect is then viewed as non-Abelian.* Segert (1987b) describes the situation with a family of Hamiltonian operators, $H_0 + k \cdot V$, where H_0 is rotationally invariant, V is a vector operator and k varies over the unit vectors in R^3.

The BAAP effect even appears in quantum systems constrained by molecular geometry. For example, Delacrétaz et al (1986) verified the BAAP phase in the molecular system Na_3. They supposed a system in an eigenstate $C(r,t)$ responding to slowly varying changes in its parameters $R(t)$, such that the system remains in the *same eigenstate apart from an acquired phase.* If the parameters, $R(t)$, completed a circuit in parameter space, then that acquired phase is not simply the familiar dynamical phase, $[(ih)^{-1}E(R(t)]dt$, but is, rather, an additional geometrical phase factor $\gamma_n(c)$. The origins of this additional phase factor depend only on the geometry of the parameter space and the topology of the circuit traversed. Therefore, adiabatic excursions of molecular wave functions in the neighborhood of an electronic degeneracy results in a change of phase. That is, if the internuclear coordinates of a wave function traverse a circuit in which the state is degenerate with another, then the electronic wave function acquires an additional phase, i.e., it changes its sign. This change was predicted by Herzberg & Longuet-Higgins (1963; Longuet-Higgens, 1975; Mead & Truhlar, 1979) and is a special case of the BAAP phase applying to a large class of molecular systems exhibiting conical intersections. Delacrétaz et al (1986) reported the evidence for half-odd quantization of free molecular pseudorotation and offered the first experimental confirmation of the sign-change theorem and a direct measurement of the phase. The BAAP phase has also been observed in fast-rotating superfluid nuclei, i.e., oscillations of pair-transfer matrix elements as a function of the angular velocity (Nikam & Ring, 1987) and in neutron spin rotation (Bitter & Dubbers, 1987).

The BAAP effect thus gives credence to the view that the A_μ potentials register physical effects both at the classical and quantum mechanical levels. That such a role for these potentials exists at the quantum mechanical level is not new. *It is new to consider the A_μ potentials for such a role at the classical level.* One may ask how the schism in viewing the A_μ potentials came about; that is, why are they viewed as physical constructs in quantum mechanics, but as merely arbitrary mathematical conveniences in classical mechanics? *The answer is that whereas quantum theory is defined with respect to boundary conditions, in the formal presentation of Maxwell theory boundary conditions are undefined.* Stoke's theorem demonstrates this.

3.3 Stoke's theorem re-examined

Let us examine Stoke's theorem of classical electromagnetism which relates diverging potentials on line elements to rotating potentials on surface elements. Thus, Stoke's theorem describes a local-to-global field relationship.

If $A(x)$ is a vector field, S is an open surface, C is the closed curve bounding S, dl is a line element of C, n is the normal to S and C is traversed in a right-hand screw sense relative to n it is well-known that:

$$\int_C A \cdot dl = \int_S (\nabla \times A) \cdot n da. \tag{3.3.1}$$

From which it can be seen that Stoke's theorem, as described, takes no account of: (i) spacetime overlap in a region with fields derived from different sources; and (ii) boundary conditions.

This neglect of boundary conditions in the Stoke's theorem of classical mechanics can be contrasted with the situation in quantum mechanics. In quantum mechanics the wave function satisfies a partial differential equation coupled to boundary conditions because the Schrödinger equation describes a minimum path solution to a trajectory between two points. The boundary condition in the doubly connected (overlap) region outside of the shielded volume in an Aharonov-Bohm experiment is the reason for the single valuedness of the wavefunction, and also the reason for quantization. The situation is also different with spatial symmetries other than the usual, Abelian, spatial symmetry.

A non-Abelian Stokes theorem is (Goddard & Olive, 1978):

$$h^{-1}(dh/ds) = ie \int_0^1 g^{-1} G_{ij} g(\partial r^i/\partial t)(\partial r^j/\partial s) dt \tag{3.3.2}$$

where $h(s)$ is a path-dependent phase factor associated with a closed loop and defines a closed loop $r(s,t)$, $0 \leq t \leq 1$, s fixed, in the $U(1)$-symmetry space, H (equivalent to A_μ); G is a gauge field tensor for the $SU(2)$ non-Abelian group; and g is magnetic charge. Here, the boundary conditions, i.e., the path dependencies, are made explicit, and we have a local field (with $U(1)$ symmetry) to global field (with $SU(2)$ symmetry) connection.

In classical electromagnetism therefore, Stoke's theorem appears merely as a useful mathematical relation between a vector field and its curl. *In gauge theory, on the other hand, an amended Stoke's theorem would provide the value for the net comparative phase change in the internal direction of a particle traversing a closed path, i.e., a local-to-global connection.*

Lest it be thought that the Aharonov-Bohm effect exists only as a quantum effect with no relevance to classical behavior, the relation of the A_μ potential to the properties of bulk condensed matter is examined in the following section. A more complete definition of Stoke's theorem is given in section 4 below (Equ. 4.10).

Therefore use of Stoke's theorem has a price: that of the (covert) adoption of a gauge for local-to-global connections. Stoke's theorem applies directly to propagation issues, which are defined by local-to-global connections. Such connections are also required in propagation through matter. Thus there is a requirement for Stoke's theorem in any realistic definition of macroscopic properties of matter, and in the next section we see that the physical effects of the A_μ potentials exist not merely in fields traversing through various connecting topologies, but in radiation-matter interactions.

3.4 Properties of bulk condensed matter. Ehrenberg & Siday's observation

In the Aharonov-Bohm F-FE situation, when the size of the solenoid is much larger than the de Broglie wavelength of the incident electrons, the scattering amplitude is essentially dominated by simple classical trajectories. But the classical manifestation of quantum influences is not peculiar to the Aharonov-Bohm effect. For example, macroscopic quantum tunneling is observable in Josephson tunnel junctions in which the phase difference of the junction can be regarded as a macroscopic degree of freedom, i.e., a classical variable (Martinis et al, 1987; Clarke et al, 1988).

Even without known quantum influences or quantum mechanical explanation, there is a classical justification for the A_μ potential as a physical effect. For example, on the basis of optical arguments, and seeking a definition of the refractive index, a macroscopic variable, the A_μ potential must be chosen so as to satisfy Stoke's theorem, *thereby removing any arbitrariness with respect to gauge.* Furthermore, an argument, originating with Ehrenberg and Siday (1949), shows that a gauge-invariant A_μ potential is *presupposed* in any definition of the refractive index.

This argument is a derivation of the refractive index based on *Fermat's Principle*: in any optical medium, a scalar quantity, e.g., the refractive index, finite everywhere in space, can be defined so that the line integral in the three-dimensional space taken between any fixed points must be an extremum which passes through these points. The optical path along a given line connecting a point 1 and 2 is:

$$\int_1^2 mds = \int_1^2 [mv + (A_\mu n)]ds, \tag{3.4.1}$$

where n is the unit vector in the direction of the line; v is the velocity of the electron; and m is its mass. Defined in this way, an unambiguous definition of the refractive index indicates the *necessity* of a unique (gauge invariant) definition of the A_μ potential. Stated differently: an unambiguous definition of the refractive index implies defining the boundary conditions through which test radiation moves. These boundary conditions define a definite gauge and thereby definite A_μ potentials.

This is an example of physical A_μ-dependent effects (the refractive index) in radiation propagating through matter from, say, a point A to a point B. In the next section we see that A_μ effects can occur when two fields, say A and B, are in sufficient close proximity. This is the Josephson effect, and we see again that the A_μ potential functions as a local-to-global operator.

3.5 Josephson effect

Josephson (1962, 1964, 1965, 1974) predicted that a d.c. voltage, V, across the partitioning barrier of a superconductor gives rise to an alternating current of frequency:

$$\omega = 2eV/h. \tag{3.5.1}$$

The equivalent induced voltage is (Bloch, 1968):

$$V = (1/c)d\phi/dt, \tag{3.5.2}$$

where ϕ is the magnetic flux through a superconducting ring containing a barrier. The circulating current, I, exhibits a periodic dependence upon ϕ:

$$I(\alpha) = \Sigma_n a_n \sin 2\pi\alpha, \tag{3.5.3}$$

where

$$\alpha = \phi/(hc/e). \tag{3.5.4}$$

The validity of Equ. (3.5.4) depends upon the substitution of

$$p - eA/c \tag{3.5.5}$$

for the momentum, p, of any particle with charge and with a required gauge invariance for the A potential.

The phase factor existing in the junction gap of a Josephson Junction is an exponential of the integral of the A potential. The fluxon, or the decrementlessly conducting wave in the long Josephson Junction and in a SQUID is the equivalent of an A-wave in one dimensional phase space. The phenomenological equations are:

$$\partial\varphi/\partial x = (2ed/hc)H_y; \tag{3.5.6}$$

$$\partial\varphi/\partial t = (2e/h)V; \tag{3.5.7}$$

$$J_z = j \sin\varphi + \sigma V, \tag{3.5.8}$$

where φ is the phase difference between *two* superconductors; H is the magnetic field in the barrier; V is the voltage across the barrier; $d = 2\lambda + l$; λ is the penetration depth; and l is the barrier thickness.

If the barrier is regarded as having a capacitance, C, per unit area, then Equ. (3.5.6) and Maxwell's equations give:

$$\Big[\frac{\partial^2}{\partial x^2} - \frac{1}{\mathbf{c}^2}\frac{\partial^2}{\partial t^2} - \frac{\beta}{\mathbf{c}^2}\frac{\partial}{\partial t}\Big]\varphi = \frac{1}{\lambda_0^2}\sin\varphi, \tag{3.5.9}$$

where $\mathbf{c}^2 = c^2/4\pi dC$ is the phase velocity in the barrier, $\lambda_0^2 = h\mathbf{c}^2/\delta\pi edj$ is the penetration depth and $\beta = 4\pi d\mathbf{c}^2\sigma = \sigma/C$ is the damping constant. Anderson (1964) demonstrated that solutions of this equation, representing votex lines in the barrier, are obtained as solutions of:

$$\partial\varphi^2/\partial x^2 = (1/\lambda_0^2)\sin\varphi, \tag{3.5.10}$$

which, except for sign, is the equation of a pendulum.

The Josephson effect is remarkable in the present context for three reasons: (i) with well-defined boundary conditions (the barrier), the phase, φ, is a well-defined gauge-invariant variable; and (ii) an equation of motion can be defined in terms of the well-studied pendulum (cf Barrett, 1987a), relating a phase variable to potential energy; (iii) the "free" energy in the barrier is (Lebwohl and Stephen, 1967):

$$F = (hj/2e) \int dx[(1 - \cos\varphi) + 1/2\lambda_0^2(\partial\varphi/\partial x)^2 + 1/2(\lambda_0/c)^2(\partial\varphi/\partial t)^2], \tag{3.5.11}$$

an equation which provides a (free) energy measure in terms of the differential of a phase variable. The Josephson effect, like the A-B effect, demonstrates the registration of physical influences by means of phase changes. The Josephson phase, also like the AB phase, registers field influences.

Jaklevic et al (1965) studied multiply-connected superconductors utilizing Josephson junction tunneling and modulated the supercurrent with an applied magnetic field. The interference "fringes" obtained were found to occur even when the magnetic flux is confined to a region not accessible to the superconductor, i.e., there occurs vector potential modulation of superconducting electron drift velocity. As always, the superconductive state had global phase coherence, indicating that the modulation effect studied was a local (A_μ) influence on global phase effects (i.e., the phase order parameter in the barrier).

In the next effect examined, the quantized Hall effect, we see that the effect is crucially dependent upon the gauge invariance of the A_μ potential. The result of this A_μ gauge invariance is powerfully significant: *an independence of the quantization condition on the density of mobile electrons in a test sample.*

We saw this independence, above, in examining the remarkable independence in preservation of phase coherence in electrons over distances larger than the atomic spacing or the free path length in the F-CE Aharonov-Bohm effect. In both cases, the primacy and importance of macroscopic, and "mesoscopic", effects are indicated.

3.6 Quantized Hall effect

The quantized Hall effect (von Klitzing et al, 1980; Stormer & Tsui, 1983) has the following attributes: (1) there is the presence of a Hall

conductance σ_{xx} in a two-dimensional gas within a narrow potential well at a semiconductor-heterostructure interface e.g., in MOS, quantum well and MOSFET; (2) the temperature is low enough that the electrons are all in the ground state of the potential well and with the Fermi level being between the Landau levels; (3) the conductance is quantized with plateau having $\sigma_{xy} = nh/e^2$ (n an integer) for finite ranges of the gate voltage in which the regular conductance is severely reduced; (4) together with the well-known Hall effect (1879) condition (a magnetic field perpendicular to the plane and an electric field in the plane and the electrons drifting in the direction EXB) the energy associated with the cyclotron motion of each electron takes on quantized values $(n + 1/2)h\omega_c$, where ω_c is the cyclotron frequency at the imposed magnetic field and n is the quantum number corresponding to the Landau level.

The Aharonov-Bohm flux, or A-wave, can be generated in such 2-dimensional systems and be increased by one flux quantum by changing the phase of the ground state wave function around the system. The quantized Hall effect is thus a macroscopic quantum Hall phenomenon related to the fundamental role of the phase and the A_μ potential in quantum mechanics.

An important and crucial feature of the quantized Hall effect is the lack of dependence of quantization (integral multiples of e^2/h on the density of the mobile electrons in the sample tested (but rather on the symmetry of the charge density wave, cf. Tsui et al 1982). Underlying this lack of dependence is a required gauge invariance of the A_μ potential. For example, Laughlin (1981) has shown that the current around a metallic loop is equal to the derivative of the total electronic energy, U, of the system with respect to the magnetic flux through the loop, i.e., with respect to the A_μ potential pointing around the loop:

$$I = (c/L)\partial U/\partial A. \tag{3.6.1}$$

As this derivative is nonzero only with phase coherence around the loop, i.e., with an extended state, Equ. (3.6.1) is valid only if:

$$A = nhc/(eL), \tag{3.6.2}$$

i.e., only with a gauge invariance for A.

With a gauge invariance defined for A, and with the Fermi level in a mobility gap, a vector potential increment changes the total energy, U,

by forcing the filled states toward one edge of the total density of states spectrum and the wave functions are affected by a vector potential increment only through the location of their centers. Therefore, gauge invariance of the A potential, being an exact symmetry, forces the addition of a flux quantum to result in only an excitation or deexcitation of the total system (Laughlin, 1981). Furthermore, the energy gap exists *globally* between the electrons and holes affected by such a perturbation in the way described, rather than in specific local density of states. Thus, the Fermi level lies globally in a gap in an extended state spectrum and there is no dependence of Hall conductivity on the density of mobile electrons.

Post (1982a,b, 1983) has also implicated the vector potential in the conversion of the voltage/current ratio of the quantized Hall effect into a ratio of period integrals. If V is the Hall voltage observed transversely from the Hall current I, the relation is:

$$V/I = \int_C A / \int_C G = Z_H = \quad \text{quantized Hall impedance,} \tag{3.6.3}$$

where G defines the displacement field D and the magnetic field H. The implication is that:

$$V/I = \int_0^T V dt / \int_0^T I dt, \tag{3.6.4}$$

where

$$\int_0^T V dt = \int_C A \quad \text{– the quantization of magnetic flux,} \tag{3.6.5}$$

$$\int_0^T I dt = \int_C G \quad \text{– the quantization of electric flux,} \tag{3.6.6}$$

and T is the cyclotron period.

Aoki & Ando (1986) also attribute the universal nature of the quantum Hall effect, i.e., the quantization in units of e^2/h at $T = 0$ for every energy level in a finite system, to a topological invariant in a mapping from the gauge field to the complex wave function. These authors assume that in the presence of external Aharonov-Bohm magnetic fluxes, the vector potential A_0, is replaced by $A_0 + A$, where $A = (A_x, A_y)$.

In cylindrical geometry, a magnetic flux penetrates the opening of the cylinder and the vector potential is thought of as two magnetic fluxes, $(\phi_x, \phi_y) = (A_x L, A_y L)$ penetrating inside and through the opening of a torus when periodic boundary conditions are imposed in both x and y directions for a system of size L. According to the Byers-Yang theorem (Byers & Yang, 1961), the physical system assumes its original state when A_x or A_y increases by Φ_0/L, where $\Phi_0 = hc/e$, the magnetic flux quantum.

The next effect examined, the De Haas-Van Alphen effect, also pivots on A_μ potential gauge invariance.

3.7 De Haas-Van Alphen Effect

In 1930 W.J. de Haas and P.M. van Alphen observed what turned out to be susceptibility oscillations with a changing magnetic field which were periodic with the reciprocal field. Landau showed in the same year that for a system of free electrons in a magnetic field, the motion of the electrons *parallel* to the field is *classical*, while the motion of the electrons *perpendicular* to the field is *quantized*; and Peierls showed in 1933 that this holds for free electrons in a metal (with spherical Fermi surface). Therefore, the free energy of the system and thus the magnetic moment $M = \partial F/\partial H$ oscillates with the magnetic field H. This oscillation is the major cause of the de Haas-van Alphen effect.

In 1952 Onsager showed that the frequencies of oscillations are directly proportional to the extremal cross-sections of the Fermi surface perpendicular to the magnetic field. If p is the electronic momentum and

$$[p - (e/c)A] \tag{3.7.1}$$

is the canonical momentum (cf. Equ.s (3.5.5), section 3.5 Josephson effect), then:

$$\int_C (p - (eA/c) \cdot dl) = (n + \gamma)h, \tag{3.7.2}$$

where n is an integer and γ is a phase factor. The relation of the A vector potential and the real space orbit is:

$$\int_C A \cdot dl = \int \nabla \times A \cdot d\mathcal{A} = H\mathcal{A}, \tag{3.7.3}$$

where $\mathcal{A}$ is the area of the orbit in real space. Furthermore, electron paths in momentum space have the same shape as those in real space

but changed in scale and turned through 90°, due to the Lorentz force relation: $\partial p/\partial t = (e/c)(v \times H)$.

Therefore, as (i) the area of the orbit in momentum space is $\mathcal{A} = (n + \gamma)(ehH/c)$, and (ii) the susceptibility is $-(1/H)(\partial F/\partial H)$ which is periodic in $(1/H)$ with period $\Delta(1/H) = 2\pi e/ch\mathcal{A}$, there is a direct influence of the A vector potential on the de Haas-van Alphen effect due to the phase factor dependence (Equ. (3.7.2)). *Thus the validation of Equ.s (3.7.1) and (3.7.2) requires A_μ potential gauge invariance.*

We have now examined two effects pivoting on A_μ potential gauge invariance. *This gauge invariance implies flux conservation, i.e., a global conservation law.* The next effect examined, the Sagnac effect, makes explicit the consequences of this global conservation.

3.8 Sagnac effect

G. Sagnac in 1913 (Sagnac 1913, 1914) demonstrated a fringe shift by rotating an interferometer (with a polygonal interference loop traversed in opposite senses) at high speed (Figure 3.8.1).

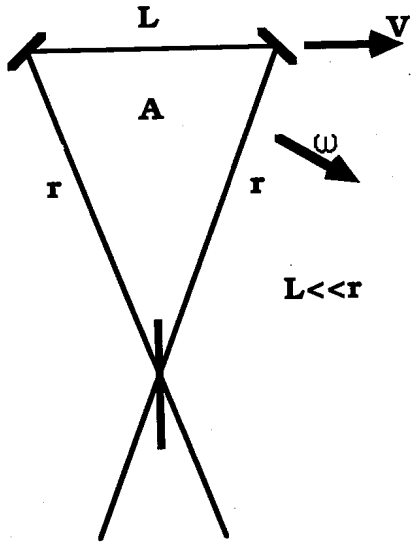

Figure 3.8.1. The Sagnac interferometer in which the center of rotation coincides with the beam splitter location. The Sagnac phase shift is independent of the location of the center of rotation and the shape of

the area. The phase shift along L is independent of r. After Silvertooth, 1986.

Einstein's general theory of relativity predicts a phase shift proportional to the angular velocity and to the area enclosed by the light path –not because the velocity of the two beams is different, but because they each have their own time. However, the AB, AAS and the Berry's phase rotation effects deny Lorentz invariance to the electromagnetic field as any field's natural and inevitable implication, i.e., Lorentz invariance is not "built-in" to the Maxwell theory– it is a gauge implied by special A_μ potential conditions, i.e., special boundary conditions imposed on the electromagnetic field. Therefore, the Einstein interpretation pivots on unproven boundary conditions and the effect is open to other, competing, explanations (cf. Forder, 1984).

A different explanation is offered by the Michelson & Gale (1925; Michelson, 1924, 1925) experiments. These investigators predicted a phase shift more simply on the basis of a difference in the velocity of the counter propagating beams and the earth rotating in a stationary ether without entrainment. (It should be noted that the beam path in the well-known Michelson-Morley 1886 interferometer does not enclose a finite surface area. Therefore this experiment cannot be compared with the experiments and effects examined in the present review, and, in fact, according to these recent experiments, no fringe shift can be expected as an outcome of a Michelson-Morley experiment, i.e., the experiment was not a test for the presence of an ether).

Post (1967) argues that the Sagnac effect demonstrates that the space-time formulations of the Maxwell equations do not make explicit the constitutive properties of free space. The identification: $E = D$, $H = B$, in the absence of material polarization mechanisms in free space is the socalled *Gaussian field identification* (Post, 1978). This identification is equivalent to an unjustified adoption of Lorentz invariance. However, the Sagnac effect and the well-used ring laser gyro on which it is based indicate that in a rotating frame the Gaussian identity does not apply. This requirement of metric independence was proposed by Van Dantzig (1934). In order to define the constitutive relations between the fields E and B constituting a covariant six-vector $F_{\lambda\nu}$, and the fields D and H, constituting a contravariant six-vector, $G^{\lambda\nu}$, the algebraic relation (Post, 1978):

$$G^{\lambda\nu} = 1/2\chi^{\lambda\nu\sigma\kappa}F_{\lambda\nu} \tag{3.8.1}$$

was proposed, where $\chi^{\lambda\nu\sigma\kappa}$ is the constitutive tensor and Equ. (3.8.1) is the constitutive map. The generally invariant vector d'Alembertian (wave equation) is:

$$\partial_\nu \chi^{\lambda\nu\sigma\kappa} \partial_\sigma A_\kappa = 0, \tag{3.8.2}$$

indicating the vector potential dependence.

The pivotal role of the vector potential is due to the flux conservation, which is a *global* conservation law (Post, 1974, 1982a). The *local* conservation law of flux:

$$dF = 0 \tag{3.8.3}$$

excludes a role for the A potential (F is inexact). Only if:

$$\int_c F = 0 \tag{3.8.4}$$

is it possible to state $dA = F$ (F is exact). In other words, $dF = 0$ implies $\int_c F = 0$ only if the manifold over which F is defined is compact and simply connected, e.g., 1-connectedness (contractable circles), 2-connectedness (contractable spheres), and 3-connectedness (contractable 3-spheres).

Post (1972) argues that the constitutive relations of the medium-free fields E and H to the medium left out treatment of free space as a "medium". If C is the differential 3-form of charge and current density, then the *local* conservation of charge is expressed by:

$$dC = 0 \tag{3.8.5}$$

and the *global* definition is:

$$C = dG. \tag{3.8.6}$$

The Post relation is in accord with the symmetry of space-time and momentum-energy required by the reciprocity theory of Born (1949) and, more recently, of Ali (1985) and Ali & Prugovecki (1986).

In summary, we have examined now

(i) The Aharonov-Bohm and Altshuler-Aronov-Spivak effect in which changes in the A_μ potential at a third location indicates differences in the A_μ field along *two* trajectories at two other locations.

(ii) The BerryAharonovAnandanPancharatnam effect in which changes in polarization defined by the A_μ potential at a point A is different

from a point B due to topological winding of the trajectory between *two* points A and B.

(iii) Stoke's theorem which requires precise boundary conditions for *two* fields: the local and global fields, for exact definition in term of the A_μ potential.

(iv) Ehrenberg and Siday's derivation of the refractive index which describes propagation between *two* points in matter and which requires gauge-invariance of the A_μ potential.

(v) The Josephson effect which requires the A_μ potential as a local-to-global operator connecting *two* fields.

(vi) The quantized Hall effect which requires gauge invariance of the A_μ potential in the presence of *two* fields.

(vii) The De Haas-van Alphen effect which requires gauge invariance of the A_μ potential in the presence of *two* fields.

(viii) The Sagnac effect which requires flux conservation, i.e., gauge invariance of the A_μ potential in the comparison of *two* fields: before and after movement.

Therefore all these effects pivot on a physical definition of A_μ potentials. In the next section we examine theoretical reasons for questioning the completeness of Maxwell's theory and the reason for the physical effectiveness of the A_μ potential in the presence of *two* fields.

4. Conclusions

The A_μ potentials have an ontology or physical meaning as *local* operators mapping onto *global* spatiotemporal conditions the *local* e.m. fields. This operation is measurable if there is a second comparative mapping of the conditioned local fields in a many-to-one fashion (multiple connection). In the case of a single local (electromagnetic) field, this second mapping is ruled out –but such an isolated local field is only imaginary, because the imposition of boundary conditions implies the existence of separate local conditions and thereby always a global condition. Therefore, practically speaking, the A_μ potentials always have a gauge-invariant physical existence. The A_μ potentials' gauge invariance implies the theoretical constructs of a magnetic monopole and magnetic charge, but with no singularities. These latter constructs are, however, confined to $SU(2)$ field conditioning, whereas the A_μ potentials have an existence in *both* $U(1)$ and $SU(2)$ symmetries.

The physical effects of the A_μ potentials are observable empirically at the quantum level (Effects 1-5) and at the classical level (Effects 2,3 and 6). The question of whether the A_μ potentials can propagate to a distance is a proper question and answerable inasmuch as questions of maintaining field coherence over large distances can be answered. Coherent fields can be obtained at low temperature in condensed matter systems and also in cavities (e.g., Mach-Zehnder). Coherency over large distances is maintained in the case of the laser. "Local" coherence could also be maintained by a wave packet propagating without dispersion or decrement.

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