On the wave-particle duality

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ABSTRACT. When an entity in motion behaves unambiguously as a particle, it is unlikely for it to behave unambiguously as a wave simultaneously and vice versa. It may exist in an ambiguous state until a measurement is made.

RESUME. Quand une entité en mouvement se comporte sans équivoque comme une particule, il est improbable qu'elle se comporte simultanément, sans équivoque, comme une onde, et vice versa. Cela pourrait être le cas dans un état équivoque jusqu'à ce qu'une mesure soit faite.

According to de Broglie

$$p = \frac{h}{\lambda} \tag{1}$$

Differentiating p with respect to λ, one obtains from equation (1)

$$\frac{dp}{d\lambda} = -\frac{h}{\lambda^2} \tag{2}$$

Differentiating λ with respect to p, one obtains from equation (1)

$$\frac{d\lambda}{dp} = -\frac{h}{p^2} \tag{3}$$

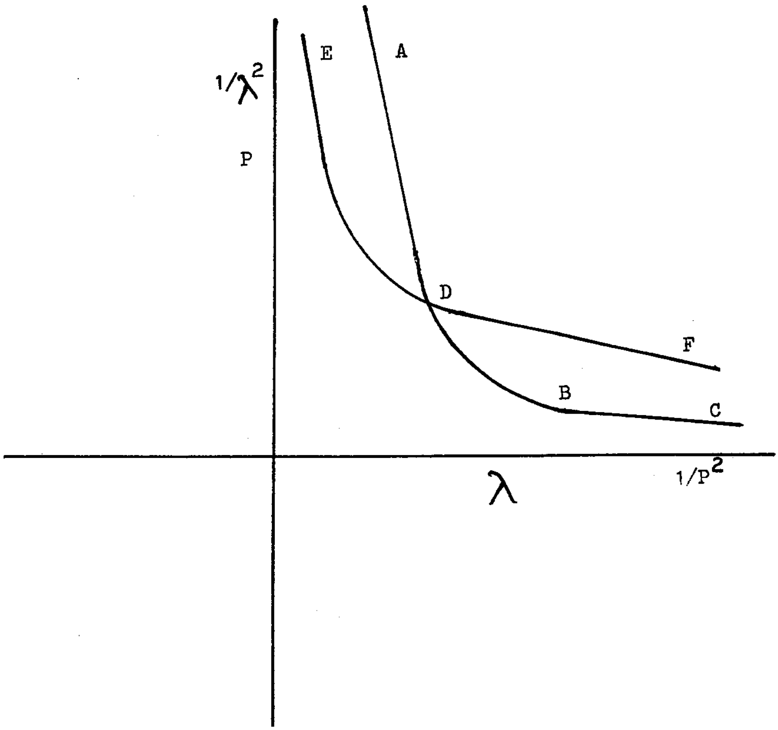

Figure 1.

Some arbitrary values of p are plotted against $1/p^2$ to obtain the curve $ADBC$ (Fig. 1), while the curve FDE (Fig. 1) is obtained by plotting some arbitrary values of λ against $1/\lambda^2$. The two curves intersect at the point D at which we have $dp/d\lambda = d\lambda/dp$ which means that $p^2 = \lambda^2$. As the units of p and λ are not the same, numerically p^2 cannot be equal to λ^2. One way of showing p^2 equal to λ^2 is to express these two quantities in terms of probabilities. For example, the motion of an entity may be described in terms of standing waves associated with it. The component of the amplitude of the associated wave in the x direction is given by

$$\psi = A\sin(2\pi x/\lambda) \tag{4}$$

in which λ is the wavelength of the associated wave.

Differentiating ψ twice with respect to x, one obtains from equation

(4)

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \tag{5}$$

Or,

$$\psi = \left(-\frac{1}{4\pi^2}\frac{d^2\psi}{dx^2}\right)\lambda^2 \tag{6}$$

According to equation (6), λ^2 is proportional to the probability of finding the entity in a region whose extent is of the order of λ along the x direction and may, therefore, be considered as the probability of the entity's wave nature.

Again, applying the appropriate operator to ψ in measuring the square of the component of momentum in the x direction, one obtains

$$-\frac{h^2}{4\pi^2}\frac{d^2\psi}{dx^2} = \frac{h^2}{\lambda^2}\psi \tag{7}$$

whence we have

$$p^2 = \frac{h^2}{\lambda^2} \tag{8}$$

As λ^2 is the probability of finding the entity in the region of length λ, p^2 may be considered as the probability of finding the same entity in the same region with momentum p. That is, p^2 may be considered as the probability of the entity's particle nature.

Let us return to equation (2). As one passes from D along the curve DE, the rate of decrease of momentum with increase of the probability of wave nature decreases till the momentum becomes certain (dp tends to a negligibly small quantity) when the probability of wave nature has attained a very large value. But, according to equation (8), when the probability of wave nature is very large, that of particle nature becomes negligibly small. This means that when the momentum is certain, the entity behaves unambiguously as a wave and that when the entity behaves as a wave, it is unlikely for it to behave as a particle simultaneously. Similarly, according to equation (3), as one passes from D along the curve DBC, the rate of decrease of wavelength with increase of the probability of particle nature decreases till the wavelength becomes certain ($d\lambda$ tends to a negligibly small quantity) when the probability of particle nature has attained a very large value. But, according to equation (8), when the probability of particle nature is very large, that of wave nature is negligibly small. Therefore, when the wavelength is certain, the entity behaves unambiguously as a particle and is unlikely to behave as a wave simultaneously. The point D may, therefore, be considered to represent an ambiguous state in which the entity is likely to exist till a measurement is made (for particle nature or wave nature).

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