Maxwell's theory extended (Part 2)

Theoretical and pragmatic reasons for questioning the completeness of Maxwell's theory

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ABSTRACT. In this Part 2 of a two part examination of Maxwell's theory, the theoretical and pragmatic reasons for questioning the completeness of Maxwell's theory are examined. Part 1 (Barrett, 1990) addresses empirical reasons for questioning the theory's completeness, namely, effects demonstrating the physical significance of the A_μ potentials.

The Wu-Yang theory attempted the completion of Maxwell's theory of electromagnetism by the introduction of a nonintegrable (path dependent) phase factor (NIP) as a physically meaningful quantity. The introduction of this construct permitted the demonstration of A_μ potential gauge invariance and gave an explanation of the Aharonov-Bohm effect. The NIP is implied by the magnetic monopole and magnetic charge constructs.

The recently formulated Harmuth Ansatz also addresses the incompleteness of the Maxwell theory: an amended version of Maxwell's equations can be used to calculate e.m. signal velocities provided that (a) a magnetic current density, and (b) a magnetic monopole, are assumed.

The A_μ potentials are local operators mapping global spatiotemporal conditions onto the local e.m. fields. The effect of this operation is measurable as a phase change, if there is a second comparative mapping of differentially conditioned fields in a many-to-one (global-to-local summation). With coherent fields the possibility of measurement (detection) after the second mapping is maximized. The question of whether A_μ potentials can be propagated to long range can be answered affirmatively if dual field coherence is maintained.

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The Maxwell theory is incomplete due to the neglect of (1) a definition of the A_μ potentials as gauge-invariant, topology-geometry- and global-boundary-condition-dependent operators on the local fields of intensity ; and (2) a definition of the constitutive relations, not between medium-independent fields and matter, but between medium-independent fields and the topology of the vacuum. (1) and (2) are related. Addressing these issues extends the Maxwell theory to cover physical phenomena which cannot be presently explained by the theory.

RESUME. Dans cette deuxième partie de l'examen de la théorie de Maxwell, on étudie les raisons théoriques et pratiques de remettre en cause la complétude de la théorie de Maxwell. La partie 1 (Barrett, 1990) aborde les raisons empiriques, c'est-à-dire les effets démontrant le sens physique des potentiels A_μ.

La théorie de Wu-Yang tentait de rendre la théorie de Maxwell complète en introduisant un facteur de phase non intégrable (NIP) – dépendant du chemin – comme quantité physiquement significative. Une telle construction permettait la démonstration de l'invariance de jauge du potentiel A_μ et donnait une explication de l'effet Aharonov-Bohm. La NIP est impliquée par les constructions du monopôle magnétique et de la charge magnétique.

L'hypothèse récemment formulée par Harmuth s'adresses'attaque aussi au caractère incomplet de la théorie de Maxwell: on peut utiliser une version amendée des équations de Maxwell pour calculer les vitesses d'un signal e.m. à condition de supposer (a) une densité de courant magnétique, et (b) un monopôle magnétique.

Les potentiels A_μ sont des opérateurs locaux appliquant des conditions spatiotemporelles globales sur les champs é.m. locaux. L'effet de cette opération est mesurable comme changement de phase, s'il y a une deuxième application des champs avec des conditions différentes dans une sommation (global à local). Avec des champs cohérents on maximise la possibilité de mesure (détection) après la deuxième application. On peut répondre affirmativement à la question de savoir si les potentiels A_μ peuvent se propager à grande distance, si la cohérence du champ double est maintenue.

La théorie de Maxwell est incomplète parce qu'elle ne fournit pas (1) une définition des potentiels A_μ comme opérateurs sur les champs locaux d'intensité qui soient invariants de jauge et dépendent de la topologie et des conditions aux limites globales, ; (2) ni une définition des relations consitutives, non entre les champs indépendants du milieu et la matière, mais entre les champs indépendants du milieu et la topologie du vide. (1) et (2) sont liés. Aborder ces questions étend la théorie de Maxwell en lui permettant de couvrir des phénomènes physiques qui ne peuvent pas être expliquées pour l'instant par cette théorie.

CONTENTS

1. Introduction

In this Part 2, both the theoretical and pragmatic reasons for questioning the completeness of Maxwell's theory are examined. It is found that Maxwell's theory is only exact for the steady state signal and medium condition, and for local effects. The theory, unamended, is misapplied to transient phenomena and phenomena involving energy exchange between local centers through a global medium, i.e., the propagation of waves and wave packets, whether the initial and final states be at different or the same locations.

2. Theoretical reasons for questioning the completeness of Maxwell's theory

Wu and Yang (1975 ; also : Yang, 1970, 1974 ; Yang & Mills, 1954) interpreted the electromagnetic field in terms of a nonintegrable (i.e., path dependent) phase factor by an examination of Dirac's monopole field (Dirac, 1931, 1948). According to this interpretation, the Aharonov-Bohm effect is due to the existence of this phase factor whose origin is due to the topology of connections on a fiber bundle.

The *phase-factor*,

$$e^{\frac{ie}{hc}\int_C A_\mu dx^\mu} \tag{2.1}$$

according to this view, is physically meaningful, but not the *phase*,

$$\frac{ie}{hc}\int_C A_\mu dx^\mu, \tag{2.2}$$

which is ambiguous because different phases in a region may describe the same physical situation. The phase factor, on the other hand, can distinguish different physical situations having the same field strength but different action.

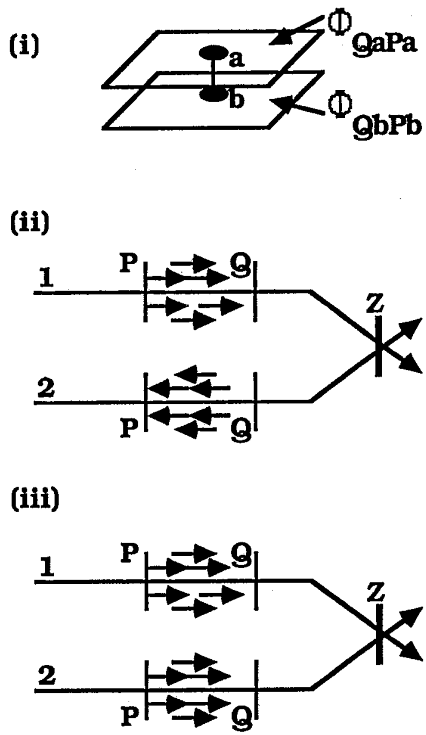

Figure 2.1. (i) The overlap area (Z in (ii) and (iii)) showing a mapping from location a to b. The phase factor $\Phi_{Q_aP_a}$ is associated with the e.m. field which arrived at Z through path 1 in (ii) and (iii) and $\Phi_{Q_bP_b}$ with the e.m. field which arrived at Z through path 2 in (ii) and (iii) ; (ii) In paths 1 and 2 the e.m. fields are conditioned by an A field between P and Q oriented in the direction indicated by the arrows. Note the reversal in direction of the A field in paths 1 and 2, hence $S(P) \neq S(Q)$ and $\Phi_{Q_aP_a} \neq \Phi_{Q_bP_b}$; (iii) Here the conditioning A fields are oriented in the same direction, hence there is no noticeable gauge transformation and no difference noticeable in the phase factors $S(P) = S(Q)$ and $\Phi_{Q_aP_a} = \Phi_{Q_bP_b}$. After Wu & Yang, 1975.

The phase factor for any path from, say, P to Q is :

$$\Phi_{QP} = e^{\frac{ie}{\hbar c}\int_P^Q A_\mu dx^\mu} . \tag{2.3}$$

For a *static magnetic monopole* at an origin defined by the spherical coordinate, $r = 0, \theta$ with azimuthal angle φ, and considering the region R of all space-time *other than this origin*, the gauge transformation in the overlap of two regions, a and b, is :

$$S_{ab} = e^{-i\alpha} = e^{\frac{2ige}{hc}\varphi} \quad , \quad 0 \leq \varphi \leq 2\pi \ , \tag{2.4}$$

where g is the monopole strength.

This is an allowed gauge transformation if, and only if :

$$2ge/hc = \quad \text{an integer} \quad = D \tag{2.5}$$

which is Dirac's quantization. Therefore

$$S_{ab} = e^{iD\phi}. \tag{2.6}$$

In the overlapping region there are two possible phase factors $\Phi_{Q_a P_a}$ and $\Phi_{Q_b P_b}$ and

$$\Phi_{Q_a P_a} S(P) = S(Q) \Phi_{Q_b P_b}, \tag{2.7}$$

which states that $(A_\mu)_a$ and $(A_\mu)_b$ are related by a gauge transformation factor.

The general implication is that for a gauge with any field defined on it, the total magnetic flux through a sphere around the origin $r = 0$ is independent of the gauge field and only depends on the gauge (phase) :

$$\int\int f_{\mu\nu} dx^\mu dx^\nu = (-ihc/e) \int \partial/\partial x^\mu (\ln S_{ab}) dx^\mu \ , \tag{2.8}$$

where the integral is taken around any loop around the origin $r = 0$ in the overlap between the R_a and R_b, as, for example, in an equation for a sphere $r = 1$. As S_{ab} is single valued, this integral must be equal to an integral multiple of a constant (in this case $2\pi i$).

Another implication is that if the A_μ potentials originating from, or passing through, two or more different local positions are gauge invariant when compared at another, again different, local position, then the referent providing the basis or metric for the comparison of the phase differences at this local position is a unit magnetic monopole. The unit monopole, defined at $r = 0$, is unique in not having any internal degrees of freedom (Weinberg, 1980). Furthermore, both the monopole and charges are topologically conserved, but whereas *electric* charge is topologically conserved in $U(1)$ symmetry, *magnetic* charge is only conserved in $SU(2)$ symmetry.

Usually, there is no need to invoke the monopole concept as the A_μ field is, as we have emphasized, treated as a mathematical, not physical, construct in contemporary classical physics. However, in quantum physics the wave function satisfies a partial differential equation coupled to boundary conditions. The boundary conditions in the doubly-connected region outside of the solenoid volume in an Aharonov-Bohm experiment results in the single valuedness of the wavefunction, which is the reason for quantization. Usually, e.g., in textbooks explaining the theory of electromagnetism as noted above, Stoke's theorem is written :

$$\int_C Adx = \int\int H \cdot ds = \int_S (\nabla \times A) \cdot n \; da, \tag{2.9}$$

and no account is taken of spacetime overlap of regions with fields derived from different sources having undergone different spatiotemporal conditioning, and no boundary conditions are taken into account. Therefore, no quantization is required.

There is not a lack of competing opinions on what the magnetic monopole implied by gauge-invariant A_μ potentials is (Bogomol'nyi, 1976 ; Montonen & Olive, 1977 ; Goddard & Olive, 1978 ; Atiyah & Hitchin, 1980 ; Craigie, 1986). The Dirac magnetic monopole is an anomalously-shaped (string) magnetic dipole at a singularity (Dirac, 1931, 1948). The Schwinger magnetic monopole is essentially a double singularity line (Schwinger, 1966, 1969). However, gauge-invariant A_μ potentials are the local manifestation of global constructs. This precludes the existence of *isolated* magnetic monopoles, but permits them to exist *globally* in any situation with the requisite energy conditioning. Wu and Yang (1975a,b, 1976), t'Hooft (1971, 1974), Polyakov (1974) and Prasad & Sommerfeld (1975) have described such situations.

Related to mechanisms of monopole generation is the Higgs field, Φ, approach to the vacuum state (Higgs, 1964a,b, 1966). The field, in some scenarios, breaks a higher-order symmetry field, e.g., $SU(2)$, G, into H of $U(1)$ form. The H field is then proportional to the electric charge.

There are at least five types of monopoles presently under consideration : (1) the Dirac monopole, a point singularity with a string source. The A_μ field is defined everywhere except on a line joining the origin to infinity, which is occupied by an infinitely long solenoid, so that $B = \nabla \cdot A$. Dirac's approach assumes that a particle has either electric or magnetic charge but not both. (2) Schwinger's approach, on the other hand, permits the consideration of particles with both electric and magnetic charge, i.e., dyons (Schwinger, 1966), 1969). (3) The 't Hooft-Polyakov monopole, which has a smooth internal structure but without the need for an external source. There is, however, the requirement for a Higgs field (Higgs, 1964a,b, 1966). The 't Hooft-Polyakov model can be put in the Dirac form by a gauge transformation (Goddard & Olive, 1978). (4) The Bogomol'nyi-Prasad-Sommerfeld monopole in which the Higgs field is massless, long range and with a force which is always attractive. (5) The Wu-Yang monopole requires no Higgs field, has no internal structure and is located at the origin. It requires multiply-connected fields. Brandt & Primack (1977) have shown that the singular string of the Dirac monopole can be moved arbitrarily by a gauge transformation. Therefore the Dirac and the Wu-Yang monopoles can be made compatible. The Higgs field formalism can also be related to that of Wu-Yang in which only the exact symmetry group appears.

Goddard & Olive (1978) demonstrated that there are two conserved currents for a monopole solution : the usual Noether current whose conservation depends on the equations of motion ; and a topological current whose conservation is independent of the equations of motion.

Yang (1983) showed that if spacetime is divided into two overlapping regions in both of which there is a vector potential A with gauge transformation between them in the overlap regions, then the proper definition of Stoke's theorem when the path integral goes from region, 1, to another, 2, is (Wu & Yang, 1976) :

$$\int_A^C A dx = \int_A^B A_1 dx + \int_B^C A_2 dx + \beta(B) \tag{2.10}$$

The β function is defined by the observation that in the region of overlap the difference of the vector potentials $A_1 - A_2$ is curl-less as both potentials give the same local electromagnetic field.

There are also general implications. Gates (1986) takes the position that all the fundamental forces in nature arise as an expression of gauge invariance. If a phase angle $\theta(x,t) = -(i/2)ln[\psi/\psi']$ is defined for quantum mechanical systems, then although the difference $\theta(x_1\ ,t) - \theta(x_2\ ,t)$ is a *gauge-dependent* quantity, the expression :

$$\theta(x_1\ ,t) - \theta(x_2\ ,t) + (e_0/hc)\int_{x_1}^{x_2} ds A(s,t) \tag{2.11}$$

is *gauge invariant*, (according to the Wu-Yang interpretation, the last expression should be $\exp[(e_0/hc)\int_{x_1}^{x_2} dsA(s,t)]$). Therefore, any measurable quantity which is a function of such a difference in phase angles must also depend on the vector potentials shown. Setting the expression (2.11) to zero gives a general description of both the Aharonov-Bohm and Josephson effects. Substituting $\exp[(e_0/hc)\int_{C\,x_1}^{x_2} A\cdot dl]$ for the final term gives a description of the Berry phase.

The phenomena described above are a sampling of a range which includes probably many yet to be discovered, or provided with the honorific title of an "effect". A unifying theme of all of them is that the physical effect of the A_μ potentials is only describable (a) when two or more fields undergo different spatiotemporal conditioning *and* there is also a possibility of cross-comparison (many-to-one mapping) or, equivalently, (b) in the situation of a field trajectory with a beginning (giving the field before the spatiotemporal conditioning) and an end (giving the field after the conditioning) *and* again a possibility of cross comparison. Setting boundary conditions to an e.m. field gives gauge invariance but without necessarily providing the conditions for detection of the gauge invariance. The gauge-invariant A_μ potential field operates on an e.m. field state to an extent determined by global symmetries defined by spatiotemporal conditions, but the effect of this operation or conditioning is only detectable under the global conditions (a) and (b). With no interfield mapping or comparison, as in the case of the solitary electromagnetic field, the A_μ fields remain ambiguous, but this situation occurs only if no boundary conditions are defined –an ambiguous situation even for the electric and magnetic fields. Therefore the A_μ potentials in all useful situations have a meaningful physical existence related to boundary condition choice– even when no situation exists for their comparative detection. What *is* different between the A_μ field and the electric and magnetic fields is that the ontology of the A_μ potentials is related to the spatiotemporal boundary conditions in a way in which the electric and magnetic fields are not. Due to this spatiotemporal (boundary condition) dependence, the operation of the A_μ potentials is a one-to-many, local to global mapping of individual e.m. fields, the nature of which is examined in section 3.2. The detection of such mappings is only within the context of a *second* comparative projection, but this time global-to-local.

This section addressed a theoretical reason for questioning the completeness of $U(1)$ symmetry, or Abelian, Maxwell Theory in the presence of *two* local fields separated globally. In the next section we examine a pragmatic reason : propagating velocities of e.m. fields in lossy media cannot be calculated in $U(1)$ Maxwell Theory. In this next section we show that the theoretical justification for physically defined A_μ potentials lies in the application of Yang-Mills theory, not to high energy fields, where the theory found its inspiration, but to low energy fields crafted by boundary conditions. This is a new application for Yang-Mills theory.

3. Pragmatic reasons for questioning the completeness of Maxwell's theory

3.1 Harmuth's Ansatz

According to Harmuth a satisfactory concept permitting the prediction of the propagation velocity of e.m. signals does not exist within the framework of Maxwell's theory (Harmuth, 1986a-c ; see also : Harmuth et al, 1987 ; Harmuth, 1987a,b ; Hussain, 1987 ; Wait, 1987 ; Kuester, 1987 ; Djordjvic & Sarkar, 1987 ; Hussain, 1987, Gray & Boules, 1987). The calculated group velocity fails for two reasons :(i) it is almost always larger than the speed of light for r.f. transmission through the atmosphere ; (ii) its derivation implies a transmission rate of information equal to zero. Maxwell's equations also do not permit the calculation of the propagation velocity of signals with bandwidth propagating in a lossy medium and all the published solutions for propagation velocities assume sinusoidal (linear) signals.

In order to remedy this state of affairs, Harmuth proposed an amendment of Maxwell's equations, which I shall call : the *Harmuth Ansatz* (Barrett, 1988 ; 1989a,b). The proposed amended equations are : Coulomb's Law (Equ.(1)) :

$$\nabla \cdot D = \rho_e \tag{3.1.1}$$

Maxwell's generalization of Ampère's Law (Equ.(2)) :

$$\nabla \times H = \frac{4\pi}{c} J_e + \frac{1}{c}\frac{\partial D}{\partial t} \tag{3.1.2}$$

Presence of free magnetic poles postulate :

$$\nabla \cdot B = \rho_m \tag{3.1.3}$$

Faraday's Law with magnetic monopole :

$$\nabla \times E + \frac{1}{c}\frac{\partial B}{\partial t} + \frac{4\pi}{c} J_m = 0 \tag{3.1.4}$$

Constitutive relations :

$$D = \epsilon E \qquad \text{(1.5(Part 1) and 3.1.5)}$$

$$B = \mu H \qquad \text{(1.7(Part 1) and 3.1.6)}$$

$$J_e = \sigma E \quad \text{(electric Ohm's law)} \qquad \text{(1.6(Part 1) and 3.1.7)}$$

$$J_m = sE \quad \text{(magnetic Ohm's law)} \tag{3.1.8}$$

where $(4\pi/c)J_e$ is electric current density ; $(4\pi/c)J_m$ is magnetic current density ; ρ_e is electric charge density ; ρ_m is magnetic charge density ; σ is electric conductivity and s is magnetic conductivity.

Setting $\rho_e = \rho_m = \nabla \cdot D = \nabla \cdot B = 0$, for free space propagation gives :

$$\nabla \times H = \sigma E + \epsilon \frac{\partial E}{\partial t} \tag{3.1.9}$$

$$\nabla \times E + \mu \frac{\partial H}{\partial t} + sH = 0 \tag{3.1.10}$$

$$\epsilon \nabla \cdot B = \mu \nabla \cdot B = 0 \tag{3.1.11}$$

and the following equations of motion :

$$\frac{\partial E}{\partial y} + \mu \frac{\partial H}{\partial t} + sH = 0, \tag{3.1.12}$$

$$\frac{\partial H}{\partial y} + \epsilon \frac{\partial E}{\partial t} + \sigma E = 0. \tag{3.1.13}$$

Differentiating equ.s (3.1.12) and (3.1.13) with respect to y and t permits the elimination of the magnetic field resulting in (Harmuth, 1986a, Equ. (21)) :

$$\frac{\partial^2 E}{\partial y^2} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} - (\mu\sigma + \epsilon s)\frac{\partial E}{\partial t} - s\sigma E = 0, \tag{3.1.14}$$

which is a two-dimensional nonlinear Klein-Gordon equation (without boundary conditions) in the sine-Gordon form :

$$\frac{\partial^2 E}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} - \alpha \sin(\beta E(y,t)) = 0 \; ; \tag{3.1.15}$$

$$\alpha \sin(\beta E(y,t))n - [\alpha\beta\frac{\partial E}{\partial t} - O(E)] \; ; \tag{3.1.16}$$

$$\alpha = \exp(+\mu\sigma) \; ; \tag{3.1.17}$$

$$\beta = \exp(+\epsilon\sigma), \tag{3.1.18}$$

where $(\partial^2 E/\partial y^2 - \mu\epsilon\partial^2 E/\partial t^2)$ is the nonlinear term and $(\alpha \sin(\beta E(y,t)))$ is the dispersion term. This match of nonlinearity and dispersion permits soliton solutions and the field described by Equ. (3.1.15) has a "mass", $m = v(b\alpha\beta)$.

Equ. (3.1.15) may be derived from the Lagrangian density :

$$\mathcal{L} = \frac{1}{2}\big[(\frac{\partial E}{\partial y})^2 - (\frac{\partial E}{\partial t})^2\big] - V(E), \tag{3.1.19}$$

where

$$V(E) = (\alpha/\beta)(1 - \cos\beta E). \tag{3.1.20}$$

The wave equation for E has a solution which can be written in the form :

$$E(y,t) = E_E(y,t) = E_0[w(y,t) + F(y)], \tag{3.1.21}$$

where $F(y)$ indicates that an electric step function is the excitation.

A wave equation for $F(y)$ is :

$$\frac{d^2 F}{dy^2} - s\sigma F = 0 \tag{3.1.22}$$

with solution :

$$F(y) = A_{00}e^{-yL} + A_{01}e^{y/L} \quad , \quad L = (s\sigma)^{-1/2} \tag{3.1.23}$$

Boundary conditions require $A_{01} = 0$ and $A_{00} = 1$, therefore :

$$F(y) = e^{-y/L} \tag{3.1.24}$$

Insertion of Equ. (3.1.21) into Equ. (3.1.14) gives (Harmuth, 1986a, Equ. (40)) :

$$\frac{\partial^2 w}{\partial y^2} - \mu\epsilon\frac{\partial^2 w}{\partial t^2} - (\mu\sigma + \epsilon s)\frac{\partial w}{\partial t} - s\sigma w = 0, \tag{3.1.25}$$

which we can again put into sine-Gordon form :

$$\frac{\partial^2 w}{\partial y^2} - \frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} - \alpha\sin(\beta w(y,t)) = 0\ ; \tag{3.1.26}$$

Harmuth (1986a) developed a solution of (3.1.21) by seeking a general solution of $w(y,t)$ using a separation of variables method (and setting s to zero after a solution is found). This solution works well, but we can now indicate another solution. The solutions to the sine-Gordon Equ. (3.1.15) is the hyperbolic tangent :

$$E(y) = 8\frac{\sqrt{\alpha\beta}}{\beta^2}\tan^{-1}\left[e^{\frac{[y-y_0-ct]}{\sqrt{1-c^2}}}\right] \tag{3.1.27A}$$

and

$$E(y) = -8\frac{\sqrt{\alpha\beta}}{\beta^2}\tan^{-1}\left[e^{\frac{[y-y_0-ct]}{\sqrt{1-c^2}}}\right] \tag{3.1.27B}$$

$$c = \sqrt{\frac{1}{\mu\epsilon}}.$$

It is also well-known that the sine-Gordon and the Thirring (Thirring, 1958, models are equivalent (Goddard & Olive, 1978) and that both admit two currents : one a Noether current, and the other a topological.

The following remarks may now be made : the introduction of $F(y) = e^{-y/L}$, according to the Harmuth Ansatz (1986a, p. 253) provides integrability. It is well known that soliton solutions require complete integrability. According to the present view, $F(y)$ also provides the problem with boundary conditions, the *necessary condition* for A_μ potential invariance. Equ. (3.1.24) is, in fact, a phase factor (Equ.s (2.1), (2.3), (2.4) and (2.6)). Furthermore, Equ. (3.1.21) is of the form of Equ. (2.11). Therefore the Harmuth Ansatz amounts to a definition of boundary conditions, i.e., obtains the condition of separate electromagnetic field comparison by overlapping fields, which permits complete integrability and soliton solutions of Maxwell's equations. Furthermore, it was already seen, above, that with boundary conditions defined, the A_μ potentials are gauge invariant implying a magnetic monopole and charge. It is also known that the magnetic monopole and charge constructs only exist under certain field symmetries. In the next section methods are presented for conditioning fields into those higher-order symmetries.

3.2 Conditioning the electromagnetic field to altered symmetry : Stokes' interferometers and Lie algebras

The theory of Lie algebras offers a convenient summary of the interaction of the A_μ potential operators with the E fields (Eisenhart, 1933 ; Hodge, 1959). The relevant parts of the theory are as follows. A manifold, $\mathcal{L}$, is a set of elements in one-to-one correspondence with the points of a vector manifold $\mathcal{M}$. $\mathcal{M}$ is a set of vectors called : points of $\mathcal{M}$. A Lie group, $\mathcal{L}$, is a group which is also a manifold on which the group operations are continuous. There exists an invertible function, $\mathcal{T}$, which maps each point x in $\mathcal{M}$ to a group element $X = \mathcal{T}(x)$ in $\mathcal{L}$. The group $\mathcal{M}$ is a global parametrization of the group $\mathcal{L}$.

If $\partial = \partial_x$ is the derivative with respect to a point on a manifold $\mathcal{M}$, then the Lie bracket is :

$$[a, b] = a \cdot \partial b - b \cdot \partial a = a \cdot \nabla b - b \cdot \nabla a, \tag{3.2.1}$$

where a and b are arbitrary vector-valued functions. Furthermore, with Λ signifying the outer product (Hestenes, 1987 ; Hestenes & Sobczyk, 1984), then :

$$[a, b] = \partial \cdot (a \Lambda b) - b \partial \cdot a + a \partial \cdot b, \tag{3.2.2}$$

showing that the Lie bracket preserves tangency.

The fundamental theorem of Lie group theory is : the Lie bracket $[a, b]$ of differential fields on any manifold is again a vector field. A set of vector fields, $a, b, c...$ on any manifold form a Lie algebra if it is closed under the Lie bracket and all fields satisfy the Jacobi identity :

$$[[a, b], c] + [[b, c], a] + [[c, a], b] = 0. \tag{3.2.3}$$

If $c = 0$, then

$$[a, b] = 0. \tag{3.2.4}$$

The A_μ potentials effect mappings, $\mathcal{T}_1$, from the global field to the E local fields, considered as group elements in $\mathcal{L}$; and there must be a second mapping, $\mathcal{T}_2$, of those separately conditioned E fields considered now global, onto a single local field for the $\mathcal{T}_1$ mappings to be detected (measurable). That is, in the Aharonov-Bohm situation (and substituting fields for electrons), if the E fields traversing the two paths are E_1 and E_2, and those fields before and after interaction with the A_μ field are E_{1i} and E_{1f} and E_{2i} and E_{2f} respectively, then $E_{1f} + E_{2f} = \mathcal{T}(E_{1i} + E_{2i})$, where $x_1 = E_{1i}$ and $x_2 = E_{2i}$ are points in $\mathcal{M}$, and $X = (E_{1f} + E_{2f})$ is considered a *group* point in $\mathcal{L}$ and $\mathcal{T} = \mathcal{T}_1 + \mathcal{T}_2, \mathcal{T}_1 = \mathcal{T}_1^{-1}$. In the same situation, although $E_{1f} - E_{2f} = \exp(ih/e) \int_C A_\mu dx^\mu = \Phi$, (i.e., the phase factor detected at Z in a separate second mapping, $\mathcal{T}_2$, in Figure 1 can be ascribed to a nonintegrable (path dependent) phase factor,) the influence of the first, $\mathcal{T}_1$, mapping or conditioning of $E_{1i} + E_{2i}$ by the A_μ operators along the separate path trajectories *preceded* that second mapping, $\mathcal{T}_2$, at Z. Therefore the A_μ potential field operators produce a mapping of the global spatiotemporal conditions onto local fields, which, in the case we are considering, are the separate $E_{1i} + E_{2i}$ fields. Thus, according to this conception, the A_μ potentials are local operator fields mapping the local-to-global gauge ($\mathcal{T}_1 : \mathcal{M} \to \mathcal{L}$), whose effects are detectable at a later spatiotemporal position only at an overlapping (X group) point, i.e., by a second mapping ($\mathcal{T}_2 : \mathcal{L} \to \mathcal{M}$), permitting comparisons of the differently conditioned fields in a many-to-one (global-to-local) summation.

If $a = E_{1i}$ and $b = E_{2i}$ where E_{1i} and E_{2i} are local field intensities and $c = A_\mu$, i.e., A_μ is a local field mapping ($\mathcal{T}_1 : \mathcal{M} \to \mathcal{L}$) according to gauge conditions specified by boundary conditions, then the field interactions of a, b and c, or E_{1i}, E_{2i} and A_μ are described by the Jacobi identity (Equ. 3.2.3). If $c = A_\mu = 0$, then $[a, b] = 0$. With the Lorentz gauge (or boundary conditions), the E_{1i}, E_{1f}, E_{2i} and E_{2f} field relations are described by $SU(2)$ symmetry. With other boundary conditions and no separate A_μ conditioning, the E_{1i} and E_{2i} fields (there are no E_{1f} and E_{2f} fields in this situation) are described by $U(1)$ symmetry relations.

The $\mathcal{T}_1$, $\mathcal{T}_2$ mappings can be described by classical control theory analysis and the A_μ potential conditioning can be given a physical waveguide interferometer representation (cf. Han & Kim, 1988). The waveguide system considered here is completely general in that the output can be phase, frequency and amplitude modulated. It is an adiabatic system (lossless) and only three of the lines are waveguides –the input, the periodically delayed line, and the output. Other lines shown are energy-expending, phase-modulating lines. The basic design is shown in Figure 3.2.1. In this Figure, the input is $E = \mathcal{E}\exp(i\omega t)$. The output is :

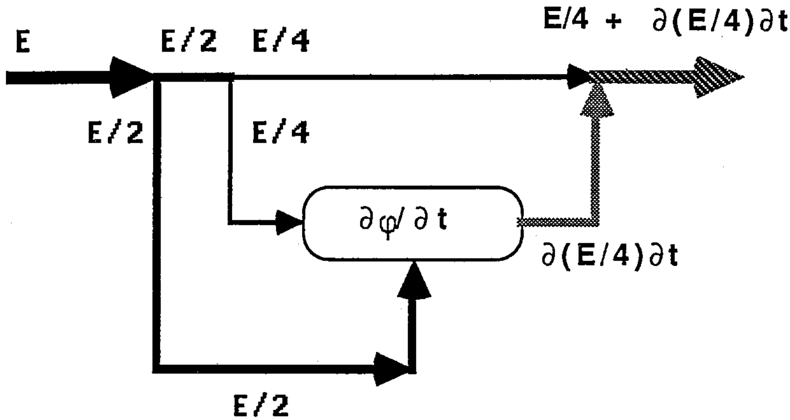

Figure 3.2.1. Waveguide system paradigm for polarization modulated ($\partial\Phi/\partial t$) wave emission. This is a completely adiabatic system in which oscillating energy enters from the left and exits from the right. On entering from the left, energy is divided into two parts equally. One part, of amplitude $\mathcal{E}/2$, is used in providing phase modulation, $\partial\Phi/\partial t$ –this energy is spent (absorbed) by the system in achieving the phase modulation ; the other part, of amplitude $\mathcal{E}/2$, is divided into two parts equally, so that two oscillating waveforms of amplitude $\mathcal{E}/4$ are formed for later superposition at the output. Due to the phase modulation of one of them with respect to the other, $0 < \varphi < 360°$, the output is of continuously varying polarization. The choice of wave division into two parts equally is arbitrary (From Barrett, 1989c).

$$E_{out(n=1)} = (\mathcal{E}/4)\exp(i\omega t) + (\mathcal{E}/4)\exp\{(i[\omega + \exp\{(i\varphi t)\} - 1]t\}, \quad (3.2.5)$$

where $\varphi = F(\mathcal{E}/2)$ and $\partial\varphi/\partial t = \dot{F}(\mathcal{E}/2)$.

The waveguide consists of two arms –the upper ($\mathcal{E}/4$) and the second ($\mathcal{E}/4$) with which the upper is combined. The lower, or third arm, merely expends energy in achieving the phase modulation of the second arm with respect to the first. This can be achieved by merely making the length of the second arm change in a sinusoidal fashion (i.e. producing a $\partial\varphi/\partial t$ with respect to the first arm), or it can be achieved electro-optically. Whichever way is used, one half the total energy of the system ($\mathcal{E}/2$)

is spent on achieving the phase modulation in the particular example shown in Figure 3.2.1. The entropy change from input to output of the waveguide is compensated by energy expenditure in achieving the phase modulation to which the entropy change is due.

One can nest phase modulations. The next order nesting is shown in Figure 3.2.2, and other, higher order nestings of order n, for the cases $\partial\varphi^n/\partial t^n$, $n = 2, 3...$ follow the same procedure. The input is again $E = \mathcal{E}\exp(i\omega t)$. The output is :

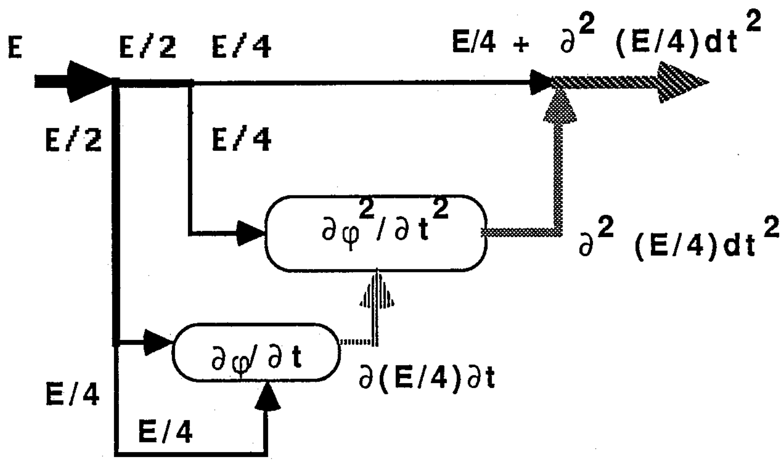

Figure 3.2.2. Waveguide system paradigm for polarization modulated ($\partial\varphi^2/\partial t^2$) wave emission. This is a completely adiabatic system in which oscillating energy enters from the left and exits from the right. On entering from the left, the energy is divided into two parts equally. One part, of amplitude $\mathcal{E}/2$, is used in providing phase modulation, $\partial\varphi^2/\partial t^2$ –this energy is spent (absorbed) by the system in obtaining the phase modulation ; the other part, of amplitude $\mathcal{E}/2$, is divided into two parts equally, so that two oscillating waveforms of amplitude $\mathcal{E}/4$ are formed for later superposition at the output. Unlike the system shown in Figure 3.2.1, the energy expended on phase modulating one of these waves is divided into two parts equally, of amplitude $\mathcal{E}/4$, one of which is phase modulated, $\partial\varphi/\partial t$, with respect to the other as in Figure 3.2.1. The energy of the superposition of these two waves is then expended to provide a second phase modulated $\partial\varphi^2/\partial t^2$ wave which is superposed with the nondelayed wave. Due to the phase modulation of one of them with respect to the other, $0 < \varphi < 360°$, the output is of continuously varying polarization. The choice of wave division into two parts equally is arbitrary (From Barrett, 1989c).

$$E_{out(n=2)} = \frac{\mathcal{E}}{4}e^{i\omega t} + \frac{\mathcal{E}}{4}e^{i[\omega+\exp\{i(\varphi_1+\exp(i\varphi_2 t)-1)t\}-1]t} \tag{3.2.6}$$

where $\varphi_1 = F_1(\mathcal{E}/4)$; $\varphi_2 = F_2(\mathcal{E}/4)$ and $\partial\varphi^2/\partial t^2 = \dot{F}_1 \cdot \dot{F}_2$.

Again, the waveguide consists of two arms –the upper $(\mathcal{E}/4)$ and the second $(\mathcal{E}/4)$ with which the upper is recombined. The lower two arms merely expend energy in achieving the phase modulation of the second arm with respect to the first. This again can be achieved by merely making the length of the second arm change in a sinusoidal fashion (i.e., producing a $\partial\varphi^2/\partial t^2$ with respect to the first arm), or it can be achieved electro-optically for visible frequencies. Whichever way is used, one half the total energy of the system $(\mathcal{E}/4 + \mathcal{E}/4 = \mathcal{E}/2)$ is spent on achieving the phase modulation of the particular sample shown in Figure 3.2.2.

Both the systems shown in Figures 3.2.1 and 3.2.2, and all higher order such systems, $\partial\varphi^n/\partial t^n$, $n = 1, 2, 3...$, are adiabatic with respect to the total field and Poynting's theorem applies to them all. However, the Poynting description, or rather limiting condition, is insufficient to describe these fields exactly and a more exact analysis is provided by the control theory picture shown here.

These waveguides we shall call Stokes' interferometers. The Stokes' equation is (Equ. 3.31 (Part 1)) :

$$\int_C A \cdot dl = \int_S (\nabla \times A) \cdot n \; da, \tag{3.2.7}$$

and the energy-expending lines of the two Stokes' interferometers shown are normal to the two wave guide lines. l is varied sinusoidally so we have :

$$\int_C A \sin\omega t dl = \int_S (\nabla \times A) \cdot n \; da = E_{out\ n=1} \quad \text{(Figure 3.2.1)} \tag{3.2.8}$$

$$\int_C A \sin\omega t dl = \int_S (\nabla \times A) \cdot n \; da = E_{out\ n=2} \quad \text{(Figure 3.2.2)} \tag{3.2.9}$$

The gauge symmetry consequences of this conditioning are shown in Figures 3.2.3 and 3.2.4. The potential, A_μ, in Taylor expansion along one coordinate is :

$$A = 1/4\ x^4 + 1/2ax^2 + bx + c, \tag{3.2.10}$$

with $b < 0$ in the case of E_{in} and $b > 0$ in the case of E_{out}. A Stokes' interferometer permits the E field to restore a symmetry which was broken before this conditioning. Thus the E_{in} field is in $U(1)$ symmetry and the E_{out} field is conditioned to be in $SU(2)$ symmetry form. The conditioning of the E field to $SU(2)$ symmetry form is the opposite of symmetry breaking. It is well-known that the Mawell theory is in $U(1)$ symmetry form and the theoretical constructs of the magnetic monopole and charge exist in $SU(2)$ symmetry form (Barrett, 1987b, 1988, 1989a,b).

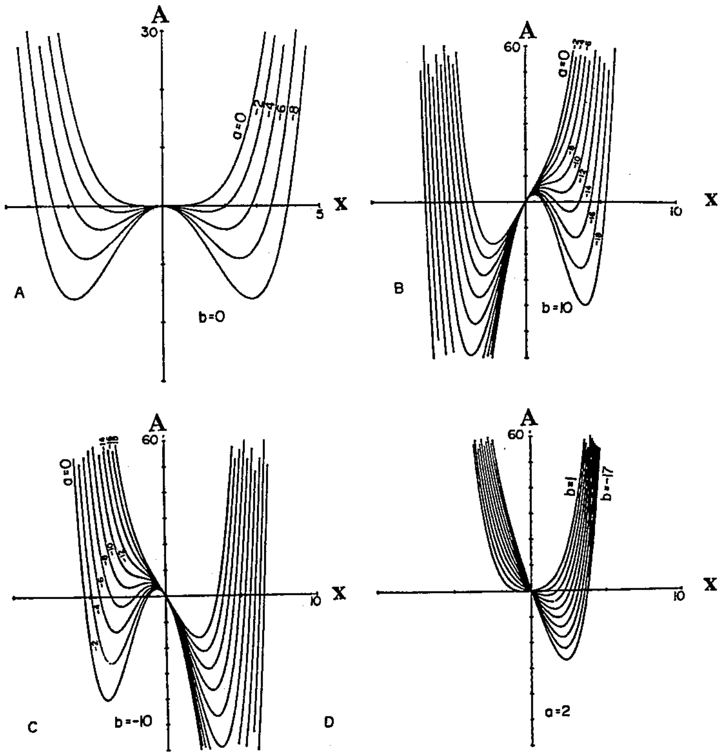

Figure 3.2.3. Plots of $A = 1/4x^4 + 1/2ax^2 + bx + c$. (A) $b = 0$ and various values of a ; (B) $b = 10$ and various values of a ; (C) $b = -10$ and various values of a ; (D) $a = 2$ and various values of b. After Barrett (1987a). For positive values of a, $SU(2)$ symmetry is restored. For negative values of a, symmetry is broken and $U(1)$ symmetry is obtained.

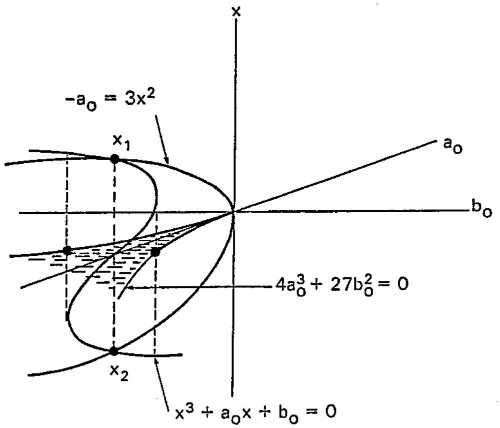

Figure 3.2.4. A representative system defined in the (x, a, b)-space. Other systems can be represented in the cusp area at other values of x, a and b. As in Figure 3.2.3, for positive values of a, $SU(2)$ symmetry is restored. For negative values of a, symmetry is broken and $U(1)$ symmetry is obtained. After Barrett (1987a).

Other interferometric methods beside Stokes' interferometer polarization modulators which restore symmetry are cavity waveguide interferometers. For example, the Mach-Zehnder and the Fabry-Perot are $SU(2)$ conditioning interferometers (Yurke et al (1986)) (Figure 3.2.5). The $SU(2)$ group characterizes passive lossless devices with two inputs and two outputs with the boson commutation relations :

$$[E_{1*}, E_{2*}] = [E_{1*}^{\dagger}, E_{2*}^{\dagger}] = 0, \tag{3.2.11}$$

$$[E_{1*}, E_{2*}^{\dagger}] = \delta_{12*}, \tag{3.2.12}$$

where $E^{\dagger}$ is the Hermitian conjugate of E and $*$ signifies both in (entering) and out (exiting) fields, i.e., before and after A_μ-conditioning. The Hermitian operators are :

$$\begin{aligned} \mathcal{J}_x &= \frac{1}{2}(E_1^{\dagger} E_{2^*} + E_{1^*} E2^{\dagger}) \\ &= \frac{1}{2}(A_1 \times B_{1IN} + B_{2IN} \times A_2) \\ &= (A \times B - B \times A), \end{aligned} \tag{3.2.13a}$$

$$\begin{aligned}\mathcal{J}_y &= -\frac{i}{2}(E_1^{\dagger}E_{2^*} - E_{1^*}E_2^{\dagger}) \\ &= -\frac{i}{2}(A_1 \times B_{1IN} - B_{2IN} \times A_2) \\ &= (A \cdot B - B \cdot A),\end{aligned} \tag{3.2.13b}$$

$$\begin{aligned}\mathcal{J}_z &= \frac{1}{2}(E_1^{\dagger}E_{1^*} - E_{2^*}E_2^{\dagger}) \\ &= \frac{1}{2}(A_1 \times E_{1OUT} - E_{2OUT} \times A_2) \\ &= (A \times E - E \times A),\end{aligned} \tag{3.2.13c}$$

$$\begin{aligned}i\mathcal{J}_z &= \frac{1}{2}(E_1^{\dagger}E_{1^*} + E_{2^*}E_2^{\dagger}) \\ &= -\frac{i}{2}(A_1 \times E_{1OUT} + E_{2OUT} \times A_2) \\ &= (A \cdot E - E \cdot A),\end{aligned} \tag{3.2.13d}$$

where the substitutions are :

$$\begin{aligned}E_{1^*} &= B_{2IN} \quad \text{and} \quad \times E_{1OUT}, \\ E_{2^*} &= \times B_{1IN} \quad \text{and} \quad E_{2OUT}, \\ E_1^{\dagger} &= A_1 \ , \\ E_2^{\dagger} &= \times A_2 \ ,\end{aligned} \tag{3.2.14}$$

satisfying the Lie algebra :

$$\begin{aligned}[\mathcal{J}_x \ , \mathcal{J}_y] &= i\mathcal{J}_z \ , \\ [\mathcal{J}_y \ , \mathcal{J}_z] &= i\mathcal{J}_x \ , \\ [\mathcal{J}_z \ , \mathcal{J}_x] &= i\mathcal{J}_y \ .\end{aligned} \tag{3.2.15}$$

The analysis presented in this section is based on the relation of induced angular momentum to the *eduction* of gauge invariance (see also Paranjape, 1987). One gauge invariant quantity or observable in one gauge or symmetry can be covariant with another in another gauge or symmetry. The Wu-Yang condition of field overlap, permitting measurement of $\Phi = \int_C \exp A_\mu dx^\mu$, requires coherent overlap. All other effects are observed either at low temperature where thermodynamic conditions provide coherence, or is a self-mapping, which also provides coherence.

Thus the question of whether classical A_μ wave effects can be observed at long range, reduces to the question of how far coherency of the two fields can be maintained.

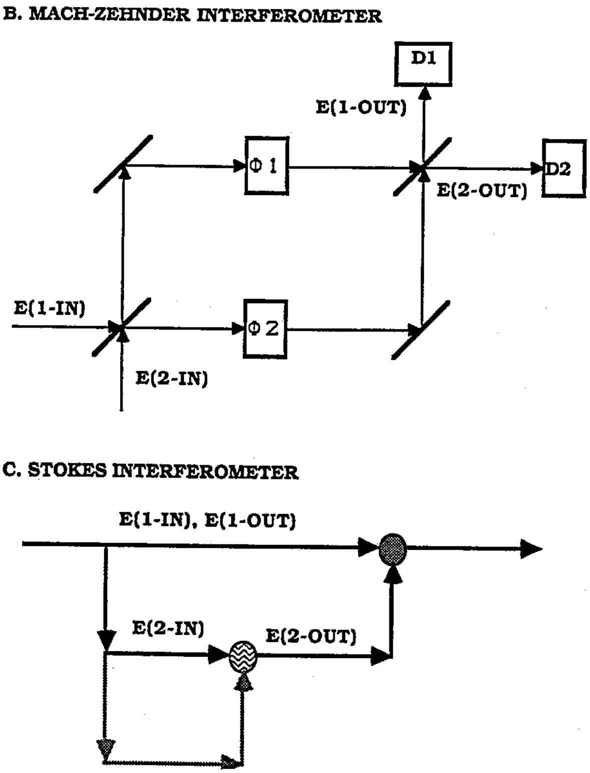

Figure 3.2.5. $SU(2)$ field conditioning interferometers : A. Fabry-Perot ; B. Mach-Zehnder ; C. Stokes. (From Barrett, 1989a).

Recently, Oh et al (1988) have derived the nonrelativistic propagator for the generalized Aharonov-Bohm effect, which is valid for any gauge group in a general multiply connected manifold, as a gauge artefact in the universal covering space. These authors conclude that (1), if a partial propagator along a multiply connected space ($\mathcal{M}$ in the present notation) is lifted to the universal covering space ($\mathcal{L}$ in the present notation) i.e., $\mathcal{T}_1 : \mathcal{M} \to \mathcal{L}$, then (2), for a gauge transformation $\mathcal{U}(x)$ of A_μ on the covering space $\mathcal{L}$, an Aharonov-Bohm effect will arise if (3), $\mathcal{U}(x)$ is not projectable to be a well-defined single-valued gauge transformation on $\mathcal{M}$, but (4), $A_\mu = \mathcal{U}(x)\partial_\mu \mathcal{U}(x)^{-1}$ (i.e., $\mathcal{T}_1\mathcal{T}_2^{-1}$) is nevertheless projectable, i.e., for a $\mathcal{T}_2 \; : \mathcal{T} \to \mathcal{M}$, in agreement with the analysis presented here. We have stressed, however, that the $A_\mu = \mathcal{T}_1 \; : \mathcal{M} \to \mathcal{L}$ have a physical existence, whether the $\mathcal{T}_2 \; : \mathcal{L} \to \mathcal{M}$ mapping is or can be performed or not. Naturally, if this second mapping is not performed, then no Aharonov-Bohm effect exists (i.e., no comparative mapping exists).

Although interferometric methods can condition fields into $SU(2)$ or other symmetric form, there is, of course, no control over the space-time metric in which those fields exist. When the conditioned field leaves the interferometer, at time $t = 0$, the field is in exact $SU(2)$ form. At time $t > 0$, the field will depart from $SU(2)$ form inasmuch as it is scattered or absorbed by the medium.

The gauge invariance of the phase factor requires a multiply connected field. In the case of quantum particles, this would mean wavefunction overlap of two individual quanta. Classically, however, *every* polarized wave is constituted of two polarized vectorial components. Therefore, classically, every polarized wave is a multiply connected field (cf. Merzbacher, 1962) in $U(1)$ symmetry. However, the extension of Maxwell's theory to $SU(2)$ form, i.e., nonAbelian Maxwell's theory, defines multiply connected local fields in a global covering space, i.e., in simply connected form. We next examine the Maxwell's equations redefined in $SU(2)$, nonAbelian, or simply-connected form.

3.3 Non-Abelian Maxwell equations

Using Yang-Mills theory (Yang-Mills, 1954), the non-Abelian Maxwell equations which describe $SU(2)$ symmetry conditioned radiation are :

Coulomb's Law :

$$\text{no existence in} \quad SU(2) \quad \text{symmetry} \tag{3.3.1}$$

Ampère's Law :

$$\frac{\partial E}{\partial t} - \nabla \times B + iq[A_0 \; , E] - iq(A \times B - B \times A) = -J \tag{3.3.2}$$

Absence of free magnetic monopoles :

$$\nabla \cdot B + iq(A \cdot B - B \cdot A) = 0 \tag{3.3.3}$$

Faraday's Law :

$$\nabla \times E + \frac{\partial B}{\partial t} + iq[A_0\ , B] + iq(A \times E - E \times A) = 0 \qquad (3.3.4)$$

Current relation :

$$\nabla \cdot D - J_0 + iq(A \cdot E - E \cdot A) = 0 \qquad (3.3.5)$$

Coulomb's law (3.3.1) amounts to an imposition of spherical symmetry requirements, as a single isolated source charge permits the choice of charge vector to be arbitrary at every point in space-time. Imposition of this symmetry reduces the non-Abelian Maxwell equations to the same form as electrodynamics, i.e., to Abelian form.

Harmuth's Ansatz is the addition of a magnetic current density to Maxwell's equations, an addition which may be set to zero after completion of calculations (Barrett, 1988). With a magnetic current density, Maxwell's equations describe a space-time field of higher order symmetry and consist of invariant physical quantities (e.g., the field $\partial_x F = J$), magnetic monopole and charge. Harmuth's amended equations are (Harmuth, 1986a, Equ.s (4)-(7)) :

$$\nabla \times H = \partial D/\partial t + g_e\ , \qquad (3.3.6a)$$

$$-\nabla \times E = \partial B/\partial t + g_m\ , \qquad (3.3.6b)$$

$$\nabla \cdot D = \rho_e\ , \qquad (3.3.6c)$$

$$\nabla \cdot B = \rho_m\ , \qquad (3.3.6d)$$

$$g_e = \sigma E, \qquad (3.3.6e)$$

$$g_m = sH, \qquad (3.3.6f)$$

where g_e, g_m, ρ_e, ρ_m and s stand for electric current density, magnetic current density, electric charge density, magnetic charge density and magnetic conductivity.

Comparing the $SU(2)$ formulation of the Maxwell equations and the Harmuth equations reveals the following identities (Barrett, 1988) :

$U(1)$ symmetry	,	$SU(2)$ symmetry	
$\rho_e = J_0$	,	$\rho_e = J_0 - iq(A \cdot E - E \cdot A) = J_0 - q\mathcal{J}_z$	(3.3.7a)
$\rho_m = 0$	,	$\rho_m = -iq(A \cdot B - B \cdot A) = -iq\mathcal{J}_y$	(3.3.7b)

$$g_e = J \quad , \quad g_e = iq[A_0\ , E] - iq(A \times B - B \times A) + J$$
$$= iq[A_0\ , E] - iq\mathcal{J}_x + J \tag{3.3.7c}$$
$$g_m = 0 \quad , \quad g_m = iq[A_0\ , B] - iq(A \times E - E \times A)$$
$$= iq[A_0\ , B] - iq\mathcal{J}_z \tag{3.3.7d}$$
$$\sigma = J/E \quad , \quad \sigma = \{iq[A_0\ , E] - iq(A \times B - B \times A) + J\}/E$$
$$= \{iq[A_0\ , E] - iq\mathcal{J}_x + J\}/E \tag{3.3.7e}$$
$$s = 0 \quad , \quad s = iq[A_0\ , B] - iq(A \times E - E \times A)/H$$
$$= \{iq[A_0\ , B] - iq\mathcal{J}_z\}/H \tag{3.3.7f}$$

It is well-known that only some topological charges are conserved (i.e., are gauge invariant) after symmetry breaking – electric charge is, magnetic charge is not (Weinberg, 1980). Therefore, the Harmuth Ansatz of setting magnetic conductivity (and other $SU(2)$ symmetry constructs) to zero on conclusion of signal velocity calculations has a theoretical justification. It is also well-known that some physical constructs which exist in both a lower and a higher symmetry form are more easily calculated for the higher symmetry, transforming to the lower symmetry after the calculation is complete. The observables of the electromagnetic field exist in a $U(1)$ symmetry field. Therefore the problem is to relate invariant physical quantities to the variables employed by a particular observer. This means a mapping of space-time vectors into space vectors, i.e., a space-time split.

This mapping is not necessary for solving and analyzing the basic equations. As a rule, it only complicates the equations needlessly. Therefore, the appropriate time for a split is usually after the equations have been solved. It is appropriate to mention here, the interpretation of the Aharonov-Bohm effect offered by Bernido & Inomata (1981). These authors point out that a path integral can be explicitly formulated as a sum of partial propagators corresponding to homotopically different paths. In the case of the A-B effect, the mathematical object to be computed in this approach is a *propagator* expressed as a path integral in the *covering space* of the background physical space. Therefore, the path-dependence of the AB phase factor is wholly of topological origin and the AB problem is reduced to showing that the full propagator can be expressed as a sum of partial propagators belonging to all topological inequivalent paths. The paths are partitioned into their homotopy equivalence classes, Feynman sums over paths in each class giving homotopy propagators, the whole effect of the gauge potential being to multiply these homotopy propagators by different gauge phase factors. The relevant point, however, with respect to the Harmuth Ansatz, is that the full propagator is expressed in terms of the covering space, rather than the physical space. The homotopy propagators are related to propagators in the universal covering manifold, leading to an expansion of the propagators in terms of eigenfunctions of a Hamiltonian on the covering manifold.

The approach to multiply connected spaces offered by Dowker (1972) and Sundrum & Tassic (1986) also uses the covering space concept. A multiply-connected space $\mathcal{M}$ and a universal covering space, $\mathcal{M}^*$, are defined :

$$\mathcal{M}^* \to \mathcal{M} = \mathcal{M}^*/\Gamma \ , \tag{3.3.8}$$

where Γ is a properly continuous, discrete group of isometries of $\mathcal{M}^*$, without fixed points and $\mathcal{M}^*$ is simply connected. Each group of $\mathcal{M}$ corresponds to n different points $q\gamma$ of $\mathcal{M}^*$, where γ ranges over the n elements of Γ. $\mathcal{M}^*$ is then divided into subsets of a finite number of points or "fibers", one fiber corresponding to one point of $\mathcal{M}$. $\mathcal{M}^*$ is a bundle or fibered space, and G is the group of the bundle. The major point, in the present instance, is that the propagator is given in terms of a matrix representation of the covering space $\mathcal{M}^*$. Harmuth calculates the propagation in the covering space where the Hamiltonian is self-adjoint. Self-adjointness means that non-Hermitian components are compensated (Schulman 1971). Thus, the propagation in the covering space is well-defined.

Consequently, Harmuth's Ansatz can be interpreted as : (i) a mapping of Maxwell's ($U(1)$ symmetrical) equations into a higher-order symmetry field (of $SU(2)$ symmetry) or covering space, where magnetic monopoles and charge exist ; (ii) solving the equations for propagation velocities ; and (iii) mapping the solved equations back into the $U(1)$ symmetrical field (thereby removing the magnetic monopole and charge).

4. Conclusions

The concept of the electromagnetic field was conceived by Faraday and set in a mathematical frame by Maxwell to describe electromagnetic effects in a space-time region. It is a concept addressing *local* effects. Action-at-a-distance (Newton) was replaced by contact-action (Descartes) when the field concept was adopted. That is, a theory

(Newton's) accounting for both local and global effects was replaced by a completely local theory (Descartes'). The local theory can address global effects with the aid of the Lorentz invariance condition, or Lorentz gauge. However, Lorentz invariance is due to a chosen gauge and chosen boundary conditions, and these are not a inevitable consequence of the Maxwell theory, which is a theory of only local effects.

According to this concept, the local field strength, $F_{\mu\nu}$, completely describes electromagnetism. However, due to the effects discussed, there is reason to believe that $F_{\mu\nu}$ does not describe electromagnetism completely. In particular, it does not describe *global* effects resulting in different histories of local spatiotemporal conditioning of the constituent parts of summed multiple fields.

Weyl (1918, 1919, 1928, 1939) first proposed that the electromagnetic field can be formulated in terms of an Abelian gauge transformation. But the Abelian gauge only describes local effects. Yang and Mills (1954) extended the idea to non-Abelian groups. The concepts of the Abelian electromagnetic field – electric charge, E and H fields, are explained within the context of the non-Abelian concepts of magnetic charge and monopole. The Yang-Mills theory **is** applicable to both local and global effects.

If the unbroken gauge group is non-Abelian, only some of the topological charges are gauge-invariant. The electric charge is, the magnetic charge is not (Weinberg, 1980).

The A_μ potentials have an ontology or physical meaning as *local* operators mapping onto *global* spatiotemporal conditions the *local* e.m. fields. This operation is measureable if there is a second comparative mapping of the conditioned local fields in a many-to-one fashion (multiple connection). In the case of a single local (electromagnetic) field, this second mapping is ruled out –but such an isolated local field is only imaginary, because the imposition of boundary conditions implies the existence of separate local conditions and thereby always a global condition. Therefore, practically speaking, the A_μ potentials always have a gauge-invariant physical existence. The A_μ potential gauge invariance implies the theoretical constructs of a magnetic monopole and magnetic charge, but with no singularities. These latter constructs are, however, confined to $SU(2)$ field conditioning, whereas the A_μ potentials have an existence in *both* $U(1)$ **and** $SU(2)$ symmetries.

The physical effects of the A_μ potentials are observable empirically at the quantum level (Effects 1-5, Part 1) and at the classical level (Effects 2,3 and 6, Part 1). The question of whether the A_μ potentials can propagate to a distance is a proper question and answerable inasmuch as questions of maintaining field coherence over large distances can be answered. Coherent fields can be obtained at low temperature in condensed matter systems and also in cavities (e.g., Mach-Zehnder). Coherency over large distances is maintained in the case of the laser. "Local" coherence could also be maintained by a wave packet propagating without dispersion or decrement.

The Maxwell theory of fields, restricted to a description of *local intensity* fields, requires no amendment at all. If, however, the intention is to describe both local and global electromagnetism, then an amended Maxwell theory is required in order to include the local operator field of the A_μ potentials, the integration of which describe the phase relations between local intensity fields of different spatiotemporal history after global-to-local mapping.

With only the constitutive relations of e.m. fields-to-matter defined (and not those of fields-to-vacuum), contemporary opinion is that the dynamic attribute of force resides in the medium-independent fields, i.e., they are fields of force. As the field-vacuum constitutive relations are lacking, this is somewhat surprising, giving rise to competing accounts of where force resides, e.g., not in the fields but in the matter (cf. Graneau, 1985).

Maxwell, of course, had two types of constitutive relations in mind : "...whenever energy is transmitted from one body to another in time, there must be a medium or substance in which the energy exists after it leaves one body and before it reaches the other ..." (Maxwell, 1891, Vol II, p. 493).

After removal of the ether from consideration, the fields have continued to exist as the classical limit of quantum mechanical exchange particles. However, those particles are in units of action, not force.

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