On specific nonpoint-like properties of microparticles as implied by quantum dynamics

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ABSTRACT. Specific quantum-theoretic nonpoint-like properties of individual microparticles are discussed and related conceptual difficulties of the Conventional Interpretation (CI) and the 'Minimal' Statistical Interpretation (MSI) are pointed out. The MSI, nevertheless, has definite advantages before the CI in treating such properties as a consequence of a complex structure of the microparticle and creating a picture of essentially noninstantaneous measurement procedures based on the very dynamics of the theory.

RESUME. On discute de certaines propriétés non-ponctuelles au sens de la mécanique quantique et on souligne les difficultés conceptuelles associées, dans l'interprétation conventionnelle (CI), et dans l'interprétation statistique minimale (MSI). La MSI a cependant des avantages certains sur la CI quand il s'agit de traiter de telles propriétés comme une conséquence d'une structure complexe de la microparticule et de bâtir une image de méthodes de mesure essentiellement non instantanées, fondées sur la dynamique même de la théorie.

We shall examine here in a specific context along the general line of reasoning in our papers [1,2] certain nonpoint-like properties of individual particles as implied by quantum dynamics. These properties lead to certain conceptional difficulties in both the CI and the MSI that are worth mention, together with the differences in the way of treating wave-like effects in the said interpretations.

As wave-like properties are more or less of the same physical essence in both relativistic and nonrelativistic quantum mechanics (QM), we shall confine ourselves (for the sake of simplicity and in order to evade difficulties with Klein's paradox) to the one-dimensional case of nonrelativistic free motion along z of a particle of mass m. (The generalizations to the three-dimensional case being trivial).

As well know, $\psi(z,t), t>0$, can be obtained from formula

$$\psi(z,t) = \int_{-\infty}^{\infty} K(z,t;z',0)\psi(z',0)dz' \tag{1}$$

in which the propagator $K(z,t;z',0)$ is given for the case of free motion by

$$K(z,t;z',0) = (m/2\pi i\hbar t)^{1/2} \exp[im(z-z')^2/2\hbar t] \tag{2}$$

Assume now that the normalized initial state of motion $\chi(z,0)$ is concentrated entirely inside interval $I_1 = (z_1, z_2)$, $z_1 < z_2 \leq 0$, i.e. $\int_{z_1}^{z_2} \mid \chi(z,0) \mid^2 dz = 1$. An analogous assumption about interval $I_2 = (z_3, z_4)$ with $0 \leq z_3 < z_4$ will give an initial state $\phi(z,0)$ satisfying $\int_{z_3}^{z_4} \mid \phi(z,0) \mid^2 dz = 1$. Due to the linearity of the Schrödinger equation function

$$\psi(z,t) = \sqrt{b_1}\chi(z,t) + \sqrt{b_2}\phi(z,t) \quad , \quad t \leq 0 \quad , \quad b_1 > 0 \quad , \quad b_2 > 0 \tag{3}$$

is also a possible normalized state of motion when $b_1 + b_2 = 1$.

The postulate of nonrelativistic QM on momentum distribution yields the following expression for the momentum amplitude $a_1(p)$ assigned to function $\chi(z,0)$ (or, which is the same, for the freely evolving $\chi(z,t)$ at each $t \geq 0$)

$$a_1(p) = (2\pi\hbar)^{-1/2} \int_{I_1} \chi(z,0) \exp(-ipz/\hbar)dz \tag{4}$$

Analogously, for state $\phi(z,0)$ we have

$$a_2(p) = (2\pi\hbar)^{-1/2} \int_{I_2} \phi(z,0) \exp(-ipz/\hbar)dz, \tag{5}$$

so that the momentum amplitude assigned to $\psi(z,0)$ will be

$$a(p) = \sqrt{b_1}a_1(p) + \sqrt{b_2}a_2(p) \tag{6}$$

Replace z_3 by 1. Obviously,

$$a(p) = \sqrt{b_1}a_1(p) + \sqrt{b_2}a_2^{(0)}(p) \cdot \exp(-ipl/\hbar), \tag{7}$$

where $a_2^{(0)}(p)$ is the value of $a_2(p)$ for $z_3 = 0$ (at a fixed length of I_2 and a fixed course of ϕ inside I_2). Consequently, we have

$$| a(p) |^2 = b_1 | a_1(p) |^2 + b_2 | a_2^{(0)}(p) |^2 + \mathrm{Int}(p, 1) \tag{8}$$

where

$$\begin{aligned} \mathrm{Int}(p, l) = \sqrt{b_1 b_2} \{ &\mathrm{Re}[a_1(p) a_2^{(0)*}(p) \cos(pl/\hbar)] \\ &- \mathrm{Im}[a_1(p) a_2^{(0)*}(p) \sin(pl/\hbar)] \} \end{aligned} \tag{9}$$

is an l-dependent interference term. Eq. (9) gives a clearcut nonpoint-like effect in the one-particle state consisting of two spatially separated parts at $t = 0$. Namely, keeping z_1, z_2 and $\chi(z, 0)(z_1 \le z \le z_2)$ fixed as well we see that the momentum density assigned to $\psi(z, 0)$ depends on the distance $d_0 = z_3 - z_2 = 1 - z_2$ separating intervals I_1 and I_2 since $\mathrm{Int}(p, l) = \mathrm{Int}(p, d_0 + z_2)$ (z_2 fixed). The importance of $\mathrm{Int}(p, l)$ in (8) has no tendency of diminishing when l (equivalently d_0) $\to \infty$ because of the periodic dependence of $\mathrm{Int}(p, l)$ on l for any given p.

(The fast varying character of $\mathrm{Int}(p, l)$ as a function of p at large l may create the impression that wave-like properties would be unobservable when $l \to \infty$. However, this is not really so : according to quantum dynamics and its MSI any 'infinitesimal' variation Δp of p would be noticeable to time-of-flight technique in the limit of large t, when $t\Delta p/m \gg d_0$).

The above observations have the following explanation from the viewpoint of the wave interference picture in z-space given by the QM dynamical equation in the case of free motion. When d_0 is large enough, that is, $d_0 \gg l_1, l_2$, l_i standing for the length of interval $I_i (i = 1, 2)$, then one may regard d_0 as the characteristic spread of the initial one-particle(ensemble) state of motion. At sufficiently large times $t > 0$ (that indefinitely increase as $d_0 \to \infty$) the two parts $\chi(z, t)$ and $\phi(z, t)$ of the overall state $\psi(z, t)$ will begin to essentially overlap and a stable interference picture will set in at $t \to \infty$ (corresponding to distances $d_t \gg d_0$ –cf.[1,2] ; one may note that in the non-relativistic case there will exist no 'fine structure' of $| \psi(vt, t) |^2$ at $t \to \infty$). More exactly, we arrive at a picture in which $| \psi |^2$ is practically z-independent in interval $I_t = (vt, (v + \Delta)t)$ along the z-axis (v fixed) and $\int_{I_t} | \psi(z, t) |^2 \, dz \to$ const as $t \to \infty$. The value of the latter integral essentially depends on $\mathrm{Int}(p, 1)$, i.e. on the phase difference of the 'sub-waves' corresponding to each specific v. All this is a direct consequence of eqs. (1,2) as applied to $\psi(z,0)$ given by (3) at $t=0$; things become really vivid having in mind that $\exp(imz'^2/2\hbar t) \approx 1$ at $t \to \infty$.

Let us consider now the implications and the conceptional difficulties connected with this specific example of a wave that is initially split in two spatially isolated parts. As we have here a state of motion of an individual particle, the CI will insist that the particle is simultaneously present in both intervals I_1 and I_2 at $t=0$. However, in an instantaneous (according to the CI) act of checking at $t=0$ in which interval the particle actually is one may discover the particle 'as a whole' either in I_1 or in I_2. This means that an instantaneous process is assumed in which the particle (that was initially somehow simultaneously present in both intervals I_1 and I_2) has concentrated itself in one of these intervals only. Things are aggravated in this acausal picture by the possibility to choose an arbitrarily large distance between I_1 and I_2. One cannot remove the difficulty by asserting that the particle actually was in only one of these intervals at $t=0$ since this is inadmissible in the CI: the interference term $\mathrm{Int}(p,1)$ would be absent then.

In the terminology of de Broglie et al. [3] (who on their turn cite Schrödinger's opinion) the CI description of what happens in situations of this kind represents just 'wizardry'. The acausal character of the interpretation adduced is not due to the employing of nonrelativistic QM : the same phenomenon can be immediately formulated with the aid of Dirac states.

Consider now the same situation from the viewpoint of the MSI. Postulate P2 requires that the particle's "core" be objectively present either in I_1 or in I_2. But the wave interference property expressed by the presence of $\mathrm{Int}(p,1)$ in (8) compels us to assert that even when the "core" is present, say, in interval I_1 a certain wave property connected with the very existence of the particles somewhere in space is present in I_2. The dynamical equation of the theory says that the latter factor will sooner or later perturb the motion of the particle in the process of its spreading along z. In this point the ideology of the MSI is practically identical with that of de Broglie [4] in which the situation described is interpreted as giving evidence about the possible existence of *empty waves*, that is, waves which do not carry particles attached to them. The experimental search for empty waves (with negative results for the time being) has already begun [5]. Certain interpretational difficulties, however, still remain.

In order to demonstrate this consider, for simplicity, intervals I_1 and I_2 of equal lengths l_1 and l_2 along z. The particle is assumed to move freely inside I_1 and I_2, the region outside them being forbidden for it due to the presence of infinitely high potential barriers at the boundary points $z_1, \ldots, z_4$ (see above for notations). Let ψ be nonzero in both I_1 and I_2 and let its course in each interval correspond, e.g., to the ground energy eigenstate of motion in this interval (obviously, $E_{g_1} = E_{g_2} = E$). Assume that a check-up of particle position has not discovered the particle, say, in I_1. This means that the particle is now with probability one in I_2 and its state of motion there is certainly characterized by the same eigenenergy E. One must state as well that prior to measurement there was only an empty wave in I_1 which was then removed or destroyed by the process of measurement. (Really, if the barriers at the boundaries of I_2 are removed after the act of measurement, the wave function will spread along z without the presence of any nonzero interference term of the kind $\mathrm{Int}(p, 1)$). But the energy of the particle in I_2 has remained the same after the negative-result check-up that was carried out in I_1. Consequently, the presumable empty wave in I_1, while being capable of influencing the particle's motion, must nevertheless be assigned zero energy which is unacceptable indeed for an objective physical entity.

Clearly, this difficulty appears due to the assumption of unrestricted validity of the present-day QM dynamical equations. (A difficulty of this kind was considered in our work [6a]). In particular, the most essential assumption here was that of the *linearity* of these equations which leads to the inference that any linear combination of states (even spatially separated one-particle states) is a possible state. It should therefore be stressed that de Broglie's nonlinear interpretation of QM [4] incorporates the hypothesis of a finite range of the wave-like property of particles. Besides, our consideration in [6a,b] gives implications for a limited validity of the nonstationary QM equations of motion. (The further evolution of this consideration supports de Broglie's nonlinear interpretation, as it will be demonstrated in a future work). One may therefore hope that difficulties of the above kind would not be insurmountable in a more developed interpretation of QM.

It is worth mentioning that the infinite range of the microparticle's wave property in the linear Schrödinger's dynamics may be demonstrated in an even simpler way. Really, assume that there exists only one interval I confined by impenetrable walls and that our particle is, say, in a state of definite eigenenergy $E > 0$ in I. Then, irrespective of the magnitude of the length l of I, the probability to register (say, with the time-of-flight technique) at $t > 0$ an energy of the particle *exactly* equal to E (the barriers being switched off at $t = 0$) is equal to zero. Indeed, the possible energy values at $t > 0$ form a continuum in which nonzero probabilities are assigned to nonzero energy *intervals*, infinitesimal energy intervals having infinitesimal probabilities. Thus, according to the Schrödinger picture, even when $l \to \infty$ the particle inside the potential well will be 'aware' all the time of the presence of the barriers and its energy will undergo a nonzero variation with probability one after the falling of the barriers.

A modification of the above-described situation of spatially separated coherent parts of an overall state makes possible the evincing of other interesting facts and fundamental differences between the CI and the MSI. Examine, e.g., the following situation. Let the initial QM state be given by the normalized function $\Phi(z, 0) = \phi(z, 0)$ concentrated entirely inside interval I_2 (see above). Assume also that a totally reflecting mirror M (an absolutely impenetrable barrier $U = \infty$) of mass $\mu = \infty$ is located at $z \leq 0$, the motion along $z > 0$ being free. (All these idealizations are introduced for the sake of clarity only). The latter requirement leads to conditions

$$\Phi(0, t) = 0 \quad , \quad \int_0^\infty \mid \Phi(z, t) \mid^2 dz = 1 \tag{10}$$

which must now be satisfied by $\Phi(z, t)$ at all $t \geq 0$. The evolution of $\Phi(z, t), t > 0$, in the allowed region $0 < z < \infty$ can be obtained with the help of a well known mathematical trick. Namely, examine together with $\Phi(z, 0)$ a fictitious normalized state $\chi(z, 0) = -\Phi(-z, 0) = -\phi(-z, 0), z \leq 0$, and the time-evolution of these two states along z at $t > 0$ under the assumption of free motion along the *entire* z-axis $-\infty < z < \infty$. The physically meaningful region, certainly, remains $0 < z < \infty$ and, as obvious from the symmetry, we have a *normalized* solution

$$\Phi(z, t) = \chi(z, t) + \phi(z, t) \quad , \quad z > 0 \quad , \quad t \geq 0, \tag{11}$$

the role of $\chi(z, t)$ consisting in guaranteeing the zero boundary condition at $z = 0$ for all $t > 0$. ($\chi(z, t)$ and $\phi(z, t)$ stand here for the time-evolved of $\chi(z, 0)$ and $\phi(z, 0)$ under the assumption of free motion along the entire z-axis). In such a way the time-evolution of $\Phi(z, t), t > 0, z > 0,$

determined –in a more vivid language– by the interference of its nonreflected part with the part reflected by M, is equivalently determined by the superposition to the right of $z = 0$ of two fictitious normalized waves $\chi(z,t)$ and $\phi(z,t), t > 0$. Our problem is thus reduced to a particular case of the problem already examined here, the only difference being that, in order to obtain (II), $\psi(z,t)$ in (3) must now be replaced by $\Phi(z,t)$, b_1 and b_2 –by unity and one must examine positive z (equivalently– positive momenta p) only. Correspondingly, instead of (8) we have now

$$\mid a(p) \mid^2 = \mid a_1(p) \mid^2 + \mid a_2(p) \mid^2 + 2\,\mathrm{Re}[a_1(p)a_2^*(p)] \quad , \quad p > 0, \tag{12}$$

where $\mid a(p) \mid^2, p > 0$, is the normalized momentum density distribution assigned to $\Phi(z,t), t \to \infty$, that may be measured, as we know, by the time-of-flight method in the limit of large t, whereas $a_1(p)$ and $a_2(p)$, $p > 0$, are determined in the same manner as before (cf.(4) and (5), correspondingly). One can certainly determine in the case of free motion $a_2(p)$ for $p < 0$ as well with the aid of (5) and it is easily seen that, under our assumptions about $\Phi(z,0)$ and $\chi(z,0)$,

$$a_1(p) = -a_2(-p) \quad , \quad a(-p) = -a(p) \quad , \quad -\infty < p < \infty, \tag{13}$$

so that (12) can be rewritten as

$$\mid a(p) \mid^2 = \mid a_2(p) \mid^2 + \mid a_2(-p) \mid^2 - 2\,\mathrm{Re}[a_2(-p)a_2^*(p)] \quad , \quad p > 0 \tag{14}$$

This fact has an interesting consequence. The probability density distribution $A(T_p)$ for kinetic energy $T_p = p^2/2m$ in the case of an arbitrary freely evolving state with momentum density distribution $R_Q(p), -\infty < p < \infty$, can be obtained via the obvious formula $A(T_p)dT_p = [R_Q(p) + R_Q(-p)]dp, p > 0$, which gives

$$A(T_p) = (m/p)[R_Q(p) + R_Q(-p)] \quad , \quad p > 0 \tag{15}$$

Examine first the case of free evolution of $\phi(z,0)$ along the entire z-axis $-\infty < z < \infty$ at $t > 0$. The kinetic energy distribution $A'(T_p)$ for this case is obtained by replacing R_Q in (15) with $\mid a_2 \mid^2$:

$$A'(T_p) = (m/p)[\mid a_2(-p) \mid^2 + \mid a_2(p) \mid^2] \quad , \quad p > 0 \tag{16}$$

Examine now the evolution of the same $\phi(z,0)$ in the presence of our impenetrable barrier at $z \leq 0$, region $z > 0$ being free. Eq. (14) gives a kinetic energy distribution

$$A''(T_p) = \frac{m}{p} \mid a(p) \mid^2 = A'(T_p) - \frac{2m}{p}\,\mathrm{Re}[a_2(-p)a_2^*(p)] \quad , \quad p > 0 \tag{17}$$

We arrived at the following result : at $t \to \infty$ the kinetic energy distribution $A''(T_p)$ in the case of elastic scattering of (a part of) an initial wave packet $\phi(z,0)$ by an impenetrable barrier is essentially different from the one obtainable at a free evolution of $\phi(z,0)$ along $-\infty < z < \infty$. Only the average kinetic energy (as may be easily seen) remains the same in both cases. A phenomenon of this kind is impossible in classical mechanics, in which the presence or absence of M can in no way affect the kinetic energy distribution.

As we already know, this strange phenomenon can be explained in the ensemble evolution picture as follows. Momentum distribution (determining kinetic energy distribution too) can be obtained from position distribution in the cloud of particles after the appearing of a stable interference picture of position distribution at suitably moving (i.e. time-variable) positions $z_t = vt$, and in the case of presence or absence of the barrier at $z \leq 0$ the corresponding interference pictures are essentially different. (As it follows from the above discussion, in a more developed SI the latter conclusion may be substantially modified).

What is more important, however, is that time-of-flight technique makes conceptually possible the *simultaneous* measurement of momentum and kinetic energy distributions in the case of presence of an impenetrable barrier too. Indeed, a 'snapshot' of the position interference picture just described would simultaneously be, by the very essence of the time-of-flight technique, a 'snapshot' of coexisting momentum and kinetic energy distributions. But it is worth making things even clearer by employing the following nondestructive procedure of momentum distribution measurement via indirect position registration.

Assume that we want to check whether the particle is located or not in interval $I_t = (vt, (v+\Delta)t)$ of length $t\Delta$ along the z-axis (Δ and v fixed, t sufficiently large) by carrying out position measurement at t *outside* this interval. (Recall that position measurement is always quasi-instantaneous). If the particle is registered outside I_t the event is discarded. If not the event is not discarded and one knows practically with certainty that the particle's macromomentum and energy lie after measurement in intervals $(mv, m(v+\Delta))$ and $(mv^2/2, m(v+\Delta)^2/2)$, respectively. (Recall that, according to Heisenberg's uncertainty relation $\delta p_z \geq \hbar/2\delta z$ no matter whether one adheres to the CI or the MSI –the sense only in which this relation is interpreted is different in the said interpretations. In the limit $t \to \infty$ we have $\delta z = t\Delta \to \infty$, so that $\delta v = \delta p/m \ll \Delta$, which proves the above statement about

p and T_p). So choose a value of t for which $\delta z = t\Delta$ is as large as necessary. From (1) and (2) it immediately follows that, for an arbitrary normalized state $\Phi(z, 0)$ of the kind considered here, we shall have at this t a state $\Phi(z, t)$ of a practically constant modulus inside I_t (Δ being 'infinitesimal'). Besides, for any $z \in I_t$ one arrives, up to 'infinitesimal' terms, at equalities $-i\hbar\partial\Phi(z, t)/\partial z = mv\Phi(z, t)$ and $(-\hbar^2/2m)\partial^2\Phi(z, t)/\partial z^2 = (mv^2/2)\Phi(z, t)$. Consequently, $\Phi(z, t)$ behaves practically as a plane wave inside I_t, being proportional there to $\exp(imvz/\hbar) = \exp(ipz/\hbar)$ under the specified conditions. The above procedure thus cuts off at a chosen moment $t = \tau$, τ large, a segment of the overall wave which segment represents, for all practical purposes, a plane wave inside I_t and a wave of zero intensity outside I_t. At $t > \tau$ this wave train will move in the positive z-direction with a well defined velocity v, slowly extending its dimension due to the small scatter Δ of particle velocities about v (the 'uncertainty' scatter δv being arbitrarily small at large t compared to Δ as shown above). The MSI therefore makes it conceptually possible to select (at large t's) a group of particles (those inside our wave train) which have well defined momenta, hence energies, even in the presence of mirror M at the origin.

Now, according to the CI the coexistence of energy and momentum is impossible in the presence of M. Indeed, one may immediately see that $a(p)(m/p)^{1/2}$ (the squared modulus of $a(p)$ being given by (12) or (14) in the case examined) is the coefficient standing before $2i\sin(pz/\hbar)/(2\pi\hbar p/m)^{1/2}$, $z > 0$, $p > 0$, in the representation

$$\Phi(z, 0) = \int_0^\infty \frac{a(p)}{\sqrt{p/m}} \frac{2i\sin(pz/\hbar)}{\sqrt{2\pi\hbar p/m}} dT_p \tag{18}$$

The functions $2i\sin(pz/\hbar)/(2\pi\hbar p/m)^{1/2}$ are the correctly normalized (to δ-functions of energy) energy eigenfunctions in the presence of M, corresponding to eigenvalues $p^2/2m$, as shown in standard courses on QM [7]. According to the CI the (kinetic) energy probability density at *any* moment $t \geq 0$ will be given by (17), where now $A''(T_p) = (m/p) \mid a(p) \mid^2$, $p > 0$ (i.e. will be t-independent and instantaneously measurable at any chosen $t \geq 0$). However, the states to which the energy values $E = T_p$ belong, being proportional to $\sin(pz/\hbar) = [\exp(ipz/\hbar) - \exp(-ipz/\hbar)]/2i$, correspond to no definite (algebraic) value of momentum. (Only the momentum modulus is definite). The energy-momentum noncoexistence in this situation comes as no surprise in the CI since, formally, the momentum operator $-i\hbar\partial/\partial z$ and the Hamiltonian $(-(\hbar^2/2m)\partial^2/\partial z^2+$

an infinitely large potential term describing M) cannot certainly be regarded as commuting operators. (Interesting enough, $-i\hbar\partial/\partial z$ is not even selfadjoint in the presence of M [8]). But one sees than that the latter CI conclusion comes in direct contradiction with the inference on momentum-energy coexistence in the case examined, following from Schrödinger's dynamics itself and its MSI.

Up to here we examined states ψ that were initially concentrated in regions having sharp boundaries in position space. The above-discussed property of $\Phi(z,t)$ to practically represent at large t a plane wave of relevant amplitude in each its portion, corresponding to a given momentum p (or velocity v), remains obviously valid for an arbitrary free-motion state ψ too (i.e. in the absence of mirror M) and one can make use of this property in order to examine, for the sake of completeness, free ψ's with sharp boundaries in momentum space. Consider, say, the simplest possible such $\psi(z,t)$ whose momentum amplitude $a(p) \neq 0$ for $p \in I' = (p_1, p_2)$ and $p \in I'' = (p_3, p_4)$, $p_1 < p_2 < p_3 < p_4$, while everywhere outside I' and I'' $a(p)$ is exactly zero. Clearly, such a state will not possess sharp boundaries in z-space at the initial moment $t = 0$. One can see however that (as an immediate consequence of our discussion here ; cf. also [1]) this $\psi(z,t)$ will have a tendency to disintegrate, as t indefinitely increases, into two distinct wave packets. One of them will be practically concentrated at sufficiently large t in interval $I'_t = (v_1 t, v_2 t)$, $v_i = p_i/m$, $i = 1, 2$, and the other –in interval $I''_t = (v_3 t, v_4 t)$, $v_i = p_i/m$, $i = 3, 4$. The integral of $\mid \psi(z,t) \mid^2$ over the z-axis with excluded intervals I'_t and I''_t will tend to zero at $t \to \infty$ due to the fact that $a(p) = 0$ outside I' and I''. We thus arrive at a wave packet picture that de facto coincides with the one corresponding to de Broglie's outlook on measurement [3] in which the initial state ψ disintegrates in result of the measurement procedure into a set of wave packets, the position of each wave packet corresponding to a (set of) specific value(s) of the measured magnitude. (In our case position in I'_t or I''_t means that macroscopic particle momentum belongs to I' or I''_t, respectively ; certainly, there may exist an arbitrary number of intervals of the kind of I' and I'' and, besides, the lengths l' and l'' of I' and I'' can be arbitrarily small numbers). In the case of nonexistence of 'isolated' intervals of the kind of I' and I'' in p-space the overall state $\psi(z,t)$ will not disintegrate at $t \to \infty$ into 'isolated' wave packets in z-space but its behaviour at such t will nevertheless be classically understandable : $\mid \psi(vt,t) \mid^2$, $t \to \infty$, will be proportinal to $1/t$, whereas $\psi(z,t)$ will behave as a plane wave corresponding to momentum $p = mv$ in the vicinity of point $z = vt$. We thus see once again that the linearity of the Schrödinger equation is well suited for creating a noninstantaneous physical picture of measurement in which positions are connected with the values of the measured physical magnitudes as a consequence of the very dynamics of the theory. At the same time, as demonstrated above, linearity is not well suited for creating a physical model of an *individual* microparticle of finite spatial dimensions. But it should be stressed that, as it follows from our discussion in [2], even at this stage of development of the theory the MSI gives no evidence of any superluminal interactions, etc.

Références

[1] N.S. Todorov, Ann. Fond. L. de Broglie, (a) **13**, 183 (1988) ; (b) ibid., p. 357 ; (c) ibid., p. 381.

[2] N.S. Todorov, ibid., submitted together with the present paper.

[3] L. de Broglie, G. Lochak, A. Beswick, and J. Vassalo-Pereira, Found. Phys. **6**, 3 (1976).

[4] L. de Broglie, *Non-Linear Wave Mechanics. A Causal Interpretation*, Elsevier, Amsterdam, 1960.

[5] W. Mückenheim, P. Locai and B. Burghardt, Phys. Lett. A **127**, 387 (1988).

[6] N.S. Todorov, Int. J. Theoret. Phys., (a) **19**, 605 (1980) ; (b) ibid., p. 725.

[7] L.D. Landau and E.M. Lifshits, *Quantum Mechanics. The Nonrelativistic Theory*, Third ed., Moscow, 1974.

[8] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Springer, Berlin, 1932.

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