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Tesla's nonlinear oscillator-shuttle-circuit (OSC) theory
compared with linear, nonlinear-feedback
and nonlinear-element electrical engineering circuit theory*

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ABSTRACT. Tesla's approach to electrical engineering addresses primarily the reactive part of electromagnetic field-matter interactions, rather than the resistive part. His approach is more comparable with the physics of nonlinear optics and many-body systems than with that of single-body systems. It is fundamentally a nonlinear approach and may be contrasted with the approach of mainstream electrical engineering, both linear and nonlinear. The nonlinear aspects of mainstream electrical engineering are based on feedback in the resistive field, whereas the nonlinearity in Tesla's approach is based on oscillators using to-and-fro shuttling of energy to capacitative stores through non-circuit elements attached to circuits. These oscillator-shuttle-circuit connections result in adiabatic nonlinearities in the complete oscillator-shuttle-circuit systems (OSCs). Tesla OSCs are reactive or active rather than resistive, the latter being the mainstream approach, therefore *device nonlinear susceptibilities* are possible using the Tesla approach.

As a development of this approach, 3-wave, 4-wave...n-wave mixing is proposed here using OSC *devices*, rather than laser-matter interactions. The interactions of oscillator-shuttles (OS) and circuits (C) to which they are attached as *monopoles* forming OSCs are not describable by Kirchhoff's and Ohm's laws. It is suggested that in the OSC formulation, floating grounds are functionally independent and do not function as common grounds. Tesla employed, rather, a concept of multiple grounds for energy storage and removal by oscillator-shuttles which cannot be fitted in the simple monolithic circuit format, permitting a many-body definition of the internal activity of device subsystems which act at different phase relations. This concept is the basis for his polyphase system of energy transfer.

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The Tesla OSCs are analogs of quaternionic systems. It is shown that more complex OSCs are analogs of more complex number elements (e.g., Cayley numbers and "beyond Cayley numbers"). The advantages of crafting energy in quaternionic, or $SU(2)$ group, and higher group, symmetry form, lie in : (1) parametric pumping with only a one drive system (*power control*) ; (2) control of the E field or Joule/cycle (*energy control*) ; (3) phase modulation at rates greater than the carrier (*phase control*) ; (4) reduction of noise in energy transmission (*noise control*) for communications ; and (5) reduction of power loss in power transmission. Engineering applications are suggested.

Finally, it is shown that Tesla's OSC approach is more appropriately (succinctly) described in A four potential form, than in E, H, B and D field form or by Ohm's law. That is, the boundary conditions are of crucial importance in defining the functioning of OSCs.

RESUME. La manière dont Tesla approche l'ingénierie électrique concerne principalement la partie réactive de l'interaction champmatière, plutôt que la partie résistive. Son approche est plus comparable à la physique de l'optique non linéaire et des systèmes à plusieurs corps qu'à celle de systèmes à un seul corps. L'approche de Tesla est basée sur des connections circuit-navette-oscillateur (OSC) permettant des susceptibilités nonlinéaires du dispositif. Les OSC de Tesla sont des analogues des systèmes de quaternions. Des OSC plus complexes sont des analogues de nombres plus complexes (par ex. nombres de Cayley). Il y a des avantages pratiques à mettre l'énergie sous forme de quaternions ou d'éléments de groupes de symétrie de type SU(2) ou plus élevé: par exemple, dans 1) le contrôle de puissance, 2) le contrôle d'énergie, 3) le contrôle de phase, 4) le contrôle du bruit. L'approche OSC de Tesla est décrite de manière plus appropriée sous la forme de quadripotentiel A plutôt que sous celle de champs de force.

I. Introduction

There is almost universal agreement that Nicolai Tesla approached electrical engineering from a different viewpoint than conventional circuit theory. There is, however, no agreement on the physical model behind his particular approach. In this paper I hope to show that Tesla's approach took advantage of the many possibilities of nonlinear interaction in joined *oscillator-shuttles*, and his nonlinear oscillator-shuttle-circuit approach, which we shall call the OSC approach, can be contrasted sharply with linear circuit theory. The particular nonlinearity of OSCs arises because

of the use of multiple independent floating "grounds" which provide separate energy storage capacitative repositories from which energy is oscillator-shuttled to-and-fro. The use of independent and noninteracting energy storage "cul-de-sacs" is a trademark of Tesla's work and sets it apart from linear circuit theory as well as nonlinear feedback theory (cf Tesla, 1956 ; Ford, 1985).

The OSC arrangement cannot be adequately described either by Kirchhoff's or Ohm's law. In section III, below, the field relations are derived for OSCs *within the constraints of the OSC arrangement considered as boundary conditions.*

The OSC arrangement is treated in section III as another method of *energy crafting* or conditioning similar to that of wave guides or other field-matter interactions. Viewed from this perspective, Tesla's OSC arrangements *offer methods to achieve macroscopic or device nonlinear interactions presently only achieved, with difficulty, in nonlinear optics* (cf Bloembergen, 1965, 1982 ; Bloembergen & Shen, 1964 ; Shen, 1984 ; Yee & Gustafson, 1978).

A clear distinction can be made between the adiabatic nonlinear oscillator-shuttle-circuits addressing the dynamics of the reactive field considered here and nonadiabatic circuits addressing the resistive field. For example, Chua and coworkers (Chua, 1969 ; Chua et al, 1986 ; Matsumoto et al, 1984, 1985, 1986, 1987a,b ; Kahlert & Chua, 1987 ; Rodriguez-Vasquez et al, 1985 ; Kennedy & Chua, 1986 ; Abidi & Chua, 1979 ; Pei et al, 1986) have described many nonlinearities in physical systems such as, e.g., four linear passive elements (2 capacitors, 1 inductor and 1 resistor) and one active nonlinear 2-terminal resistor characterized by a 3 segment piecewise linear $v - i$ characteristic. Such circuits exhibit bifurcation phenomena, Hopf bifurcation, period-doubling cascades, Rössler's spiral and screw type attractors, periodic windows, Shilnikov phenomenon, double scroll and boundary crisis. The tunneling current of Josephson-junction circuits can even be modeled by a nonlinear flux-controlled inductor (Abidi & Chua, 1979). However, in all these instances, (i) the nonlinear resistive elements require an energy source to the nonlinear resistor which is external to that of the circuit, (ii) the resistive field, not the reactive field, is the operative mode, and (iii) of course the physical system is a circuit, not an OSC.

Treatments of electrical circuits by the oriented graph approach (Ingram & Cramlet, 1944 ; Van Valkenburg, 1955 ; Seshu & Reed, 1961 ; Brayton & Moser, 1964a,b ; Rez & Seely, 1959 ; Branin, 1959, 1966 ;

Smale, 1972) have all commenced with a one-dimensional cell complex (i.e., a graph) with vertices and branches connecting them, as well as separable loops. Representing the connectivity relations of an oriented linear graph by a branch-vertex matrix $A = \sum a_{ij}$, the elements have values of $+1$, -1, and 0, depending on whether current is flowing into, out of, or stationary, at a particular vertex (i.e., $a_{ij} = (+1, -1, 0)$). This *linear* graph representation does not, however, take into account any representation (resulting from modulation) which does not conform to the values for a_{ij}, e.g., when a_{ij} takes on spinor values, that is, obeys the even subalgebra of a Clifford algebra.

There are, however, other approaches to circuit analysis which *are* compatible with Tesla OSCs. Kron (1938, 1939, 1944, 1945a,b, 1948) equated circuits with their tensor representations. Kron's methods were supported by Roth's demonstration (1955) that *network analysis is a practical application of algebraic topology.* Roth (1955a,b) showed that Kirchhoff's current law is the electrical equivalent of a homology sequence of a linear graph, and Kirchhoff's voltage law corresponds to a cohomology sequence, these sequences being related by an isomorphism corresponding to Ohm's law. The algebraic topology approach was enhanced considerably further by Bolinder (1957a,b, 1958, 1959a,b, 1986, 1987) who introduced three-dimensional hyperbolic geometrical transformations to circuit analysis and showed how partially polarized electromagnetic or optical waves can be transformed by Clifford algebra. Tesla OSCs also can be described in Clifford algebra terms. Below, the OSCs are described in quaternion algebra, which is the even subalgebra of a three-dimensional Clifford algebra with Euclidean metric.

In the immediately following section II the reader is introduced to Tesla OSCs shown in Tesla (1956, 1986) and Ford (1985), establishing the case of unique use of oscillator-shuttle (OS) arrangements joined in a monopole fashion, i.e., with one connection, to circuits (C), thereby forming OSCs. Simple OSC models are then related to the Tesla models highlighting the operating principles.

II. Some Tesla OSCs

There are unifying physical themes present in Tesla OSCs and antennas (Figures 1A-J). Figure 1A is the prototypical oscillator-shuttle (which we shall call OS) with the common ground situated between two inductances one of which is joined to a capacitive energy store indicated by the circle. The OS, imbedded in conventional circuits, (which we shall

refer to as Cs), (Figures 1B & 1C), we shall call an OSC. The frequency of the OS becomes the *signal* frequency (β) for the *pump* frequency (α) of the circuit, C, resulting in an *idler* frequency (γ) for the OSC, using the signal, pump, idler nomenclature of the theory of parametric excitation. However, as will be indicated below, this is *a unique form of adiabatic invariant parametric excitation* and distinct from the conventional form which requires energy expenditure in the signal as well as the pump. *The OSC only requires energy expenditure in the pump.*

Figures 1D, E & F are further examples of OSCs in which the primary coil (our pump or circuit inductance) is wound around the secondary (our signal or OS inductance). In Figures 1G & H in a variation, the signal, or secondary, OS inductance has two energy storage capacitors for shuttle operation and is coupled to the circuit by the primary field alone.

Figures 1I & J are pancake antennas which utilize *two principles* : (1) the OSC principle already discussed in which the pancake is an inductance OS for energy storage in the pancake coil, and, in the case of Figure 1J, even with a capacitance store at the vertex of the pancake ; and (2) E field overlap due to the in-plane winding of the pancake resulting in a many-to-one mapping of E fields and a strict boundary condition constraint. The pancake antenna is also a frequency-independent antenna.

Figures 2A-G are OSC diagrams illustrating in a more simplified fashion the principles exhibited in Figures 1A-J. Figure 2A represents the design of Figures 1B & C ; Figure 2B represents polarization modulation ; Figures 2C & D represents Figures 1B & C ; Figure 2E represents Figures 1G ; Figure 2F represents Figure 1H ; and Figure 2G represents Figures 1I & J.

In the following section III, the pump, signal and idler fields of the simplest, or Tesla, OSC are derived by a treatment in which the OSC arrangement is treated as another method of energy crafting or conditioning similar to waveguides or other field-matter interactions, i.e., with network theory subsumed under field theory.
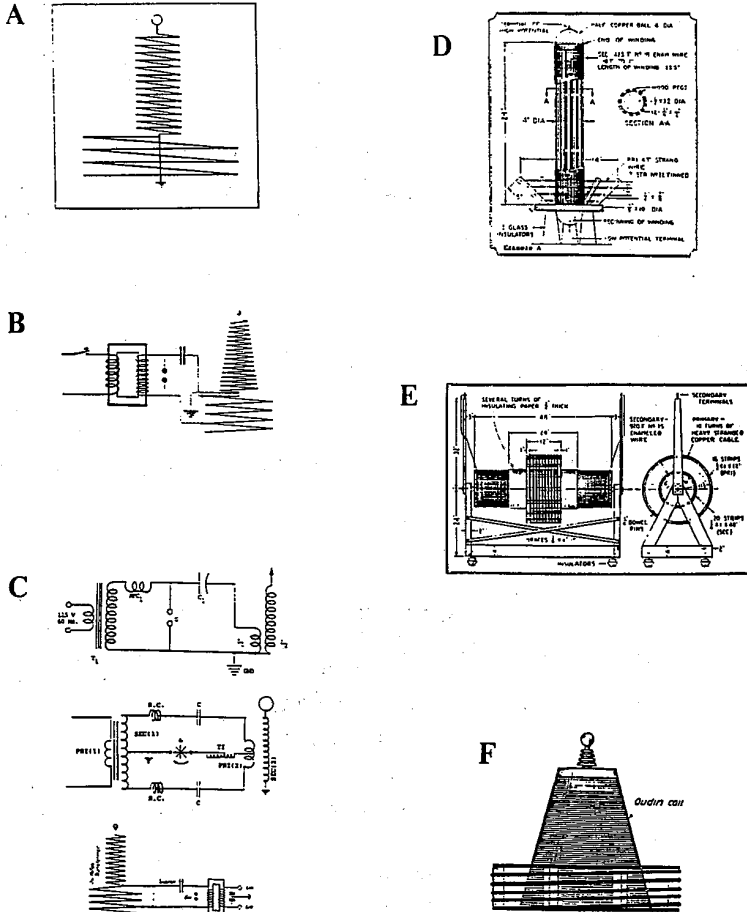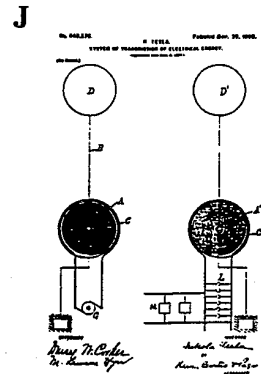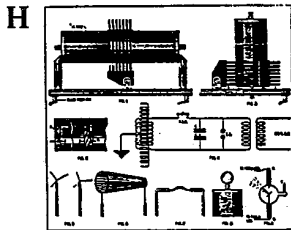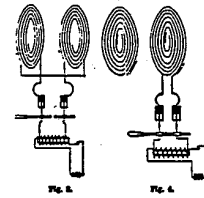
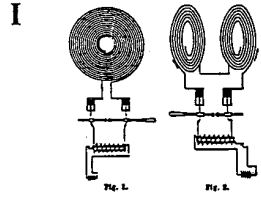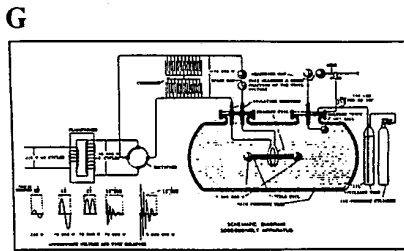**Figure 1.**

G

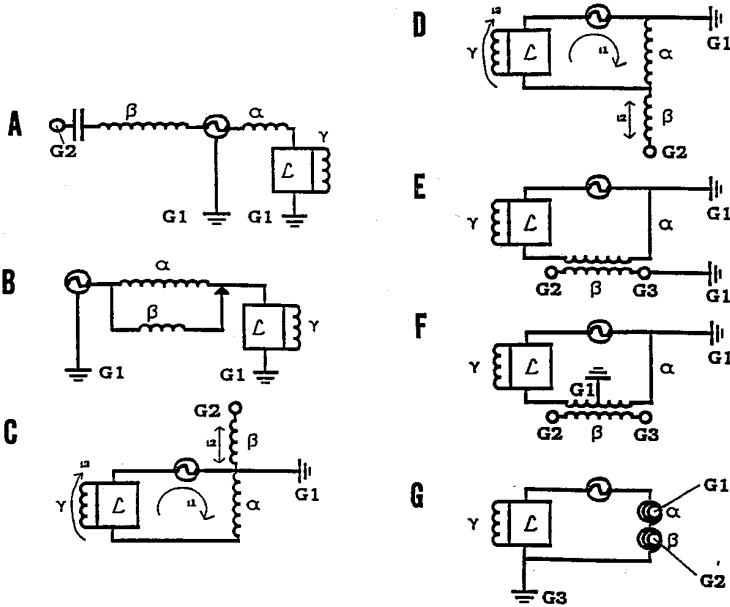**I**

H

J

Figure 1bis.

Figure 2.

III. Reactive versus resistive fields : analogy between oscillator-shuttle-circuits and coherent coupling between modes in a nonlinear optical waveguide

The operation of a many-body Tesla OSC system can be described by a model already used in nonlinear optics for describing radiation-matter interactions. Specifically, there exists an analogy between Tesla OSC theory and coherent coupling between modes in a nonlinear waveguide.

One can commence with the Maxwell's equations :

$$\nabla \times E = -i\omega\mu_0 H \tag{1}$$

$$\nabla \times H = i\omega D \tag{2}$$

$$\nabla \cdot D = 0 \tag{3}$$

$$\nabla \cdot \mu_0 H = 0 \tag{4}$$

and

$$D = \epsilon E, \tag{5}$$

if no free charge density is present and the medium is isotropic. By setting

$$\mu_0 H = \nabla \times A \tag{6}$$

and introducing this into the first Maxwell equation gives :

$$\nabla \times (E + i\omega A) = 0 \tag{7}$$

Therefore $E + i\omega A$ is the gradient of a scalar potential ϕ :

$$E = -i\omega A - \nabla\phi.$$

Introducing this into the second Maxwell equation gives :

$$\nabla \times (\nabla \times A) = \omega^2 \mu_0 \epsilon A - i\omega \mu_0 \epsilon \nabla\phi. \tag{9}$$

Using the identity for curl A, gives :

$$\nabla^2 A = -\omega^2 \mu_0 \epsilon A + \nabla[\nabla \cdot A + i\omega \mu_0 \epsilon \phi] - i\omega \mu_0 \phi \nabla \epsilon. \tag{10}$$

Using the Lorentz gauge :

$$\nabla \cdot A + i\omega \mu_0 \epsilon \phi = 0 \tag{11}$$

and with no source terms, gives :

$$\nabla^2 A + \omega^2 \mu_0 \epsilon A = 0, \tag{12}$$

which permit solutions of the form :

$$A = x\psi(x, y) \exp[-i\beta z], \tag{13}$$

$$A = y\psi(x, y) \exp[-i\beta z], \tag{14}$$

for media uniform along the z-direction. The scalar function then obeys the scalar wave equation :

$$\nabla_T^2 \psi + (\omega^2 \mu_0 \epsilon - \beta^2)\psi = 0, \tag{15}$$

where

$$\nabla_T = x\partial/\partial x + y\partial/\partial y. \tag{16}$$

From equ.s (8) and (11) we have :

$$E = -i\omega A - i\nabla(\nabla \cdot A)/\omega\mu_0\epsilon, \tag{17}$$

and the E and H fields for the x-polarized vector potentials are :

$$\begin{aligned}E = -i\omega[x(\psi + (1/\omega^2\mu_0\epsilon)\partial^2\psi/\partial x^2) + y(1/\omega^2\mu_0\epsilon)\partial^2\psi/\partial x\partial y \\ - i(\beta z/\omega^2\mu_0\epsilon)\partial\psi/\partial x]\end{aligned} \tag{18}$$

$$\mu_0 H = \nabla \times A = [-x \times \nabla_T + iy\beta]\psi\exp(-i\beta z). \tag{19}$$

The fields for the y-polarized vector potentials are similar.

We now introduce the impedance changes in a Tesla OSC, or the equivalent of a *device* nonlinear second-order susceptibility tensor $\chi^{(2)}$ due to the electrical control field and the particular waveguide conditionings of an OSC. (Higher-order device nonlinearities are considered below). If the frequency of the control or signal fields is designated : ω_s and that of the pump field is ω_p, then the impedance change caused by the signal or control field is :

$$\Delta Z_r = Z_0\chi^{(2)}E(\omega_p)E^*(-\omega_s), \tag{20A}$$

$$\Delta Z_i = Z_0\chi^{(2)}E(\omega_s)E^*(\omega_s), \tag{20B}$$

where Z, the complex impedance is :

$$Z(i\omega) = R + i(\omega L - 1/\omega C),$$

with magnitude :

$$Z = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]}.$$

With the signal or control field defined :

$$E(\omega_s, t) = E_s(t)\exp[i(\omega_s t - \beta_s t)], \tag{21}$$

the pump field defined :

$$E(\omega_p, t) = E_p(t)\exp[i(\omega_p t - \beta_p t)], \tag{22}$$

and the two fields coupled by the OSC device-generated nonlinear susceptibility, the following changes in the impedance occur :

$$\Delta Z_r = Z_0 \chi^{(2)} E_s^*(t) E_p(t) \exp[i(\omega_p - \omega_s)t - i(\beta_p - \beta_s)t], \qquad (23A)$$

$$\Delta Z_i = Z_0 \chi^{(2)} E_s^*(t) E_s(t). \qquad (23B)$$

If $\chi^{(2)}$ is purely real (inductive), then only phase changes are produced. If $\chi^{(2)}$ has an imaginary (resistive) component, then power transfer, and even gain, can be obtained for one of the inputs.

The idler field (Tesla coil load output) is then :

$$E_i(\omega_i, t) = \mid E_s(t) \mid^2 E_p(t) \exp[i(\omega_p - \omega_s)t - i(\beta_p - \beta_s)t]. \qquad (24)$$

where

$$(\beta_p - \beta_s)^2 = [\int \omega^2 \mu_0 A_0^* \cdot \Delta Z \cdot A dl]/[\int A_0^* \cdot A dl]. \qquad (25)$$

and E_s and E_p are defined by equ.s (17). Thus the three-body interactive system of E_i, E_s and E_p is defined in terms of the A vector potential :

$$E_i(\omega_i, t) = [\mid -i\omega A_s(t) - i\nabla(\nabla \cdot A_s(t))/\omega\mu_0\epsilon \mid^2]$$

$$X[-i\omega A_p(t) - i\nabla(\nabla \cdot A_p(t)/\omega\mu_0\epsilon]$$

$$X \exp[i(\omega_p - \omega_s)t - i(\beta_p - \beta_s)t]. \qquad (24)$$

The quaternionic impedance for the OSC is then :

$$Z_1 + i_2 Z_2 = R_1 + i_1(\omega_1 L_1 - 1/\omega_1 C_1) + i_2(R_2 + i_3(\omega_2 L_2 - 1/\omega_2 C_2)), \quad (26)$$

where the subscripts on i_1, i_2 and i_3 distinguish the separate field conditioning of the waveguide-like properties of the OSC and the subscripts on R, ωL and ωC distinguish circuit, C, elements (1) from oscillator-shuttle, OS, elements (2). As the waveguide properties of circuits, C, (1) are fundamentally different from oscillator-shuttles, OS, (2), distinguishing the $i_{x,y} (= \sqrt{-1}, x, y = 1, 2, 3 \ldots, i_x i_y = -i_y i_x, i_x$ anticommutes with i_y), is a necessary for distinguishing the OS and C dynamic interaction of the OSC total arrangement.

The distinguishing characteristics of higher order OSCs is in analogy to the dimensions of the number system. The dimensionality of the real numbers is 2^0 ; of complex numbers is 2^1 ; of quaternions is 2^2 ; of

Cayley numbers is 2^3 ; of "beyond Cayley numbers" is 2^4 ; etc. Each number system has an OSC device analog associated with a higher-order nonlinear susceptibility.

Quaternions are four-dimensional numbers. The Appendix reviews quaternion number interactions and gives Equ.(24) in quaternionic form. Figure 3 shows a mapping of a four-dimensional quaternionic signal in three-dimensions (one dimension representing two). Similar arguments apply to OSC for representations higher than quaternions, e.g., Cayley numbers and involve higher order device nonlinear susceptibilities. OSCs are shown in Figure 4A-C. Figure 4A is a quaternion OSC with dimension $2^2(SU(2)$ group symmetry), to which number associativity and unique division applies, but commutativity does not apply. Figure 4B is a Cayley number OSC with 2^3 dimension ($SU(3)$ group symmetry), to which unique division applies, but associativity and commutativity does not apply. Figure 4C is a number of 2^4 dimension ($SU(4)$ group symmetry), to which neither associativity, nor commutativity nor unique division applies.
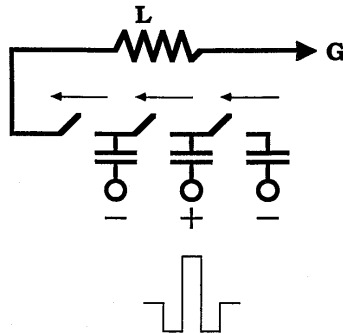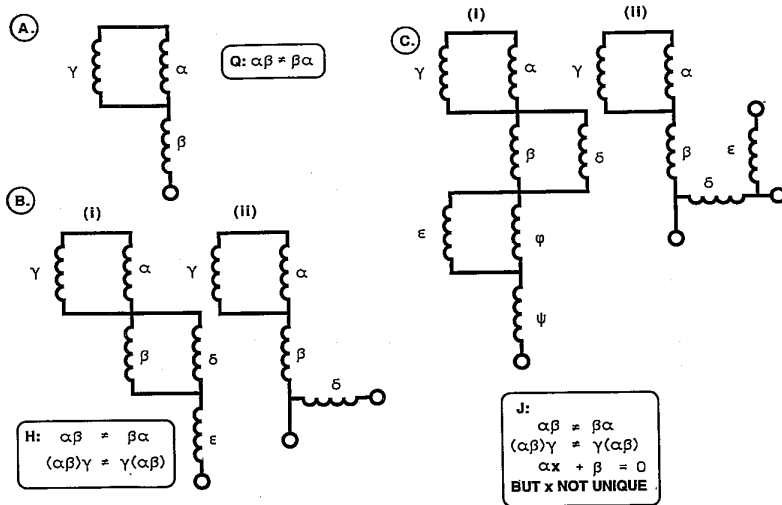

Figure 3.

Figure 4.

IV. The virtues of oscillator-shuttle-circuit (OSC) arrangements

Among the many virtues, or beneficial applications of the OSC arrangements, a nonexhaustive list would include the following :

1. Parametric pumping with a *one-drive* energy source system (*frequency of energy flow or power control, i.e., Manley-Rowe (1959) relations without an external drive other than a primary source*). In contrast, parametric pumping (three-wave mixing) in conventional circuit theory is restricted to two active systems (pump and signal). Even in the case of three-wave mixing in nonlinear optics, two beam sources are required (cf Kaup, 1976, 1980 ; Zakharov & Manakov, 1976). The OSC arrangement can also be contrasted with conventional harmonic generation, as the idler frequency generated is controlled by choice of signal and pump inductances which may not be harmonically related. In contrast to the conventional, parametric pumping using OSCs *requires only one active system* (the pump) permitting "energy bleeding" from that active system. The total OSC power flow for the second-order OSC system, W,

is given by the Manley-Rowe relation (Manley & Rowe, 1959) :

$$W = \{c^2/2\pi\}[\{k_{pT,z} \cos^2 \alpha_p \mid E_{pT} \mid^2 /\omega_p\} + \{k_{sT,z} \cos^2 \alpha_s \mid E_{sT} \mid^2 /\omega_s\}$$
$$+ \{k_{iT,z} \cos^2 \alpha_i \mid E_{iT} \mid^2 /\omega_i\}], \tag{27}$$

where $k_{pT,z}$, $k_{sT,z}$, $k_{iT,z}$ are the phases of the pump, signal and idler.
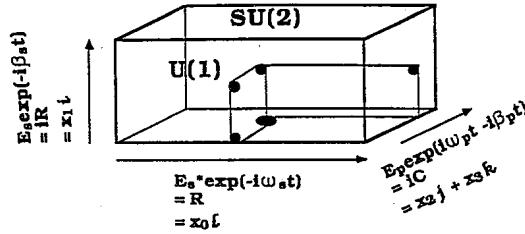
E_{pT}, E_{sT}, E_{iT} are the *total transmitted energies*, i.e., the powers, of the pump, signal and idler.

α_p, α_s, α_i are the angles between $E_T(\omega_i)$ and $E_{T\perp}(\omega_i)$.

That is, whereas the power, W, is an *adiabatic invariant* for OSC arrangements, it is not for conventional parametric circuits which require an external signal power source. In the case of conventional parametric circuits the Manley-Rowe relation applies to a conventional circuit and its power source, *together with* an external signal power source, i.e., the application is to a nonadiabatic device.

Higher-order OSC arrangements permit more complex dynamics. For example, due to parametric processes, third-order OSC device nonlinear susceptibilities permit "phase conjugate mirror" –like signal reception and communication with cancellation of noise-in-medium similar to that achieved with phase conjugate mirrors based on radiation-matter interaction.

2. E field control (*energy per cycle (Joules/cycle)control*). The OSC principles permit E field control as exhibited in frozen Hertzian wave generation (Figure 5) (For frozen Hertzian wave generation cf: Cronson, 1975 ; Zucker et al, 1976 ; Proud & Norman, 1978 ; Mathur et al, 1982 ; Chang et al, 1982, 1984).

$E_s exp(-i\beta_s t)$
$= iR$
$= x_1 i$

$SU(2)$

$U(1)$

$E_s^* exp(-i\omega_s t)$
$= R$
$= x_0 l$

$E_p exp(i\omega_p t -i\beta_p t)$
$= iC$
$= x_2 j + x_3 k$

$E_p exp(i\omega_p t -i\beta_p t) = $ **Pump**
$= iC$
$= x_2 j + x_3 k$

$E_s^* exp(-i\omega_s t) = $ **Signal (Real)**
$= R$
$= x_0 l$

$E_s exp(-i\beta_s t) = $ **Signal (Imaginary)**
$= iR$
$= x_1 i$

$R = E_s^* exp(-i\omega_s t)$
$\qquad\qquad = $ Signal (Real)
$C = R + iR = E_s^* exp(-i\omega_s t) + iE_s exp(-i\beta_s t)$
$\qquad\qquad = $ Signal (Real & Imaginary)
$g = C + iC = C + i[E_p exp(i\omega_p t) - iE_p exp(-i\beta_p t)]$
$\qquad\qquad = $ Pump

Idler $= x$
$x = x_0 l + x_1 i + x_2 j + x_3 k$

$x_0 = E_s^* exp(-i\omega_s t)$
$x_1 = E_s exp(-i\beta_s t)$
$x_2 = \gamma \overline{E_p} exp(i\omega_p t)$
$x_3 = \gamma \overline{E_p} exp(-i\beta_p t)$

Figure 5.

3. Phase modulation at rates greater than the carrier (*phase control)* is permitted by OSC arrangements because the OS or signal circuit is contrained always to be the signal oscillator –the load or idler always being a function of the circuit, C. Therefore even if the signal frequency of the OS is a higher frequency than that of pump signal of the primary, the power flow to the idler will be a modulation of the primary or pump frequency, rather than a modulation of secondary or signal frequency.

4. Noise reduction in communications transmission due to condi-

tioning of fields in higher order symmetry form (*noise control*). As an output from an OSC, i.e., a transmitted wave, is in higher-order group symmetry form (see section III above) and such higher-order symmetries have a low probability of occurrence naturally, environmental noise, which is of lower symmetry form (usually, $U(1)$) and has a high probability of natural occurrence, will be excluded from a receiver designed for OSC transmitted wave reception. Therefore in the case of communications, less noise will be processed statistically at a receiver designed for $SU(2)$ or higher group symmetry operation, resulting in enhanced signal-to-noise.

5.   A similar statistical argument holds for less loss in power transmission by higher-order symmetry energy crafting or conditioning. Higher-order group symmetry "receivers" will have enhanced reception over lower-order receivers, i.e., leakage to ground.

The type of antenna to be used with OSC devices is a small loop antenna which has an inductive reactance (cf. Smith, 1988). Such antennas are the *dual* of a short dipole which has a capacitative reactance. Thus a small loop antenna can be substituted for the β inductance of the OSC of Fig. 4A, for the ϵ or δ inductances of Fig. 4B, and for the ψ or ϵ inductances of Fig. 4C. Such antennas could be used in either the send or receive operation modes and the OSCs would function as "active media".

V. Appendix

Quaternions-overview

The algebra of quaternions is the even subalgebra of a three-dimensional Clifford algebra with Euclidean metric. A quaternion is :

$$x = x_0 l + x_1 i + x_2 j + x_3 k,$$

where the scalar multiplication is :

$$cx = cx_0 l + cx_1 i + cx_2 j + cx_3 k,$$

and the sum is :

$$x + y = (x_0 + y_0)l + (x_1 + y_1)i + (x_2 + y_2)j + (x_3 + y_3)k.$$

The product is :

$$xy = (x_0y_0 - x_1y_1 - x_2y_2 - x_3y_3)l + (x_0y_1 - x_1y_0 - x_2y_3 - x_3y_2)i$$
$$+ (x_0y_2 + x_2y_0 + x_3y_1 - x_1y_3)j + (x_0y_3 + x_3y_0 + x_1y_2 - x_2y_1)k,$$

and

$$i^2 = j^2 = k^2 = -l^2,$$
$$li = il = i \quad , \quad lj = jl = j,$$
$$lk = kl = k \quad , \quad ij = -ji = k,$$
$$jk = -kj = i \quad , \quad ki = -ik = j.$$

The set of quaternions is a division ring. It satisfies all the axioms for a field except the commutative law of multiplication.

Let : $x = E_x \exp[i\omega_x t - \beta_y t]$, $y = E_y \exp[i\omega_y t - \beta_y t]$, and

$$\overline{x} = x_0 l + x_1 i + x_2 j + x_3 k,$$

$$\overline{y} = y_0 l + y_1 i + y_2 j + y_3 k.$$

then :

$$[x, y] = xy - yx,$$

$$[x, y] = \begin{pmatrix} 0 & y_1x_2 - x_1y_2 & x_0y_2 - y_0x_2 & y_3x_2 - x_3y_2 \\ x_1y_2 - y_1x_2 & 0 & y_0x_1 - x_0y_1 & x_1y_3 - y_1x_3 \\ y_0x_2 - x_0y_2 & x_0y_1 - y_0x_1 & 0 & y_0x_3 - x_0y_3 \\ x_3y_2 - y_3x_2 & y_1x_3 - x_1y_3 & x_0y_3 - y_0x_3 & 0 \end{pmatrix}$$

If

$$x_0 = E_s^* \exp(i\omega_s t) \quad , \quad x_1 = E_s \exp(i\beta_s t),$$
$$x_2 = \sqrt{E_p} \exp(i\omega_p t) \quad , \quad x_3 = \sqrt{E_p} \exp(i\beta_p t),$$

then

$$\overline{x} = x_0 x_1 x_2 x_3 = \mid E_s \mid^2 E_p[\exp[i(\omega_p - \omega_s)t - i(\beta_p - \beta_s)t]]$$

or Equ.(24) of the text.

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