On position-velocity coexistence in a 'minimal' statistical interpretation of the Dirac electron states

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ABSTRACT. A 'minimal' statistical interpretation (MSI) of quantum mechanics (QM) resting on two postulates (the postulate of the fundamental character of the concept of statistical ensemble in a probabilistic theory and de Broglie's postulate asserting objective existence of positions of microparticles) makes possible the obtaining of nontrivial physical results in the frame of Dirac's dynamics. Namely, our consideration gives a 'vivid' confirmation of Schrödinger's statement on the existence of two kinds of velocities in the relativistic spin-1/2 case : micro and macrovelocities. Heisenberg's position-momentum uncertainty relation is shown to apply to an ensemble of macrovelocities, so that the idea of the possible coexistence of positions, velocities and wave-like properties of individual microparticles finds a definite support in our approach. It is demonstrated too that measurement is, generally, a noninstantaneous procedure in QM as a consequence of the very dynamics of the theory. Interpretational difficulties connected with the definition of an acceptable position operator and with a theorem due to Hegerfeldt are easily resolved in the context of the MSI when applied to the case of relativistic spin-1/2 particles.

RESUME. Une interprétation statistique "minimale" (MSI) de la mécanique quantique, reposant sur sur 2 postulats (le postulat du caractère fondamental du concept d'ensemble statistique dans une théorie probabiliste et le postulat de De Broglie affirmant l'existence objective des positions des corpuscules) permet d'obtenir des résultats physiques non triviaux dans le cadre de la dynamique de Dirac. En effet, nos raisonnements donnent une confirmation frappante de l'énoncé par Schrödinger de l'existence de deux sortes de vitesses dans le cas relativiste à spin 1/2*: vitesses micro- et macroscopiques. On montre que la relation d'incertitude d'Heisenberg sur la position et l'impulsion s'applique à un ensemble de vitesses macroscopiques,* *de sorte que l'idée de la coexistence possible de positions, de vitesses et de propriétés ondulatoires de corpuscules individuels trouve confirmation dans notre approche. On démontre aussi que la mesure est, en général, un processus non instantané en mécanique quantique, en conséquence de la dynamique même de la théorie. Les difficultés d'interprétation reliées à la définition d'un opérateur de position acceptable et à un théorème dû à Hegerfeldt sont aisément résolues dans le contexte de la MSI, quand on les applique au cas des particules relativistes de spin* 1/2.

I. Introduction and preliminaries

The MSI of QM, set forth in refs. [1a,b,c], represents a variant of the so called Statistical Interpretation (SI) of QM. The latter term encompasses, in fact, a number of different interpretations of QM some of which were considered in [1]. The common feature of most of them is that wave-like and particle-like properties of the QM object (microparticle) are not mutually exclusive but coexist. This idea is most clearly expounded, in our opinion, in the works of its inventor L. de Broglie (cf., say, refs. [2-4], and also [5]). According to him the QM object represents a 'core' (of very small dimensions and carrying practically the entire mass of the particle) imbedded in a wave process with which it interacts. These two components of the QM object serve to explain, respectively, its particle-like (local point-like) and wave-like (non point-like) properties that exist simultaneously in the above picture. The MSI too adopts the idea that microparticles objectively possess a characteristic that may be called "position". In fact, this interpretation rests on two basic postulates :

P1. (the fundamental postulate of any SI) Any physical theory of a statistical (stochastic, probabilistic) nature must essentially employ the concept of a relevant statistical ensemble of noninteracting copies of a given physical situation to which the statistical predictions of the theory apply.

P2. (de Broglie's postulate) A (reasonably) well defined position at every moment of time objectively exists for any microparticle.

(The present author does not insist on P2 in the case of massless 'particles'. The present paper deals with massive spin-1/2 particles).

Besides, we shall adhere to the interpretation of $|\psi(\vec{r},t)|^2$ as the position density distribution at point $\vec{r}$ and moment t. (In Dirac's case $|\psi(\vec{r},t)|^2 = \psi^+(\vec{r},t)\psi(\vec{r},t) = \sum_{i=1}^4 \psi_i^*(\vec{r},t)\psi_i(\vec{r},t)$, where $\psi_i(\vec{r},t)$, $i = 1,\ldots,4$, are the four components of 4-spinor ψ). But, as a consequence of P2, $|\psi|^2$ is no longer just the probability density *to find* the particle at point $\vec{r}$ in an act of position measurement at moment t as the Conventional Interpretation (CI) insists (which implies a sudden reduction to pointlike dimensions at moment t of a strange, indefinite object –the microparticle of the CI). Rather, $|\psi|^2$ is the *actual*, *objective* position density distribution $\rho_Q(\vec{r},t)$ in the 'cloud' representing the quantum ensemble (q-ensemble) corresponding to the given situation according to P1. In other words, in the MSI $|\psi|^2$ is the probability density for the particle *to be* at point $\vec{r}$ at moment t, position measurement being only a check-up of an objectively existing magnitude.

To summarize, in the MSI positions objectively exist at any moment t and probabilities refer to q-ensembles of systems and not necessarily to individual systems.

A natural question arises at this point : some of the existing models [6-13] (the recent refs. [10-13] dealing precisely with the spin-1/2 relativistic case) not only incorporate the above assumptions but contain additional physical hypotheses too concerning possible properties of the subquantum level, the role of the so called quantum potential, etc. One might then get the impression that the MSI is just a truncated variant of the more detailed approaches and ask why is it necessary ? The answer may be found in [1] : such models essentially rest on the quantum potential concept that appears in a Hamilton-Jacobi type primary picture of particle ensemble motions in which position(s) and time determine uniquely particle velocities. The quantum potential so obtained possesses certain unusual properties that are difficult to reconcile with physical intuition [1]. Even more, certain recent developments of such SIs [14] interpret "with causal nonlocal superluminal quantum potential interactions the EPR paradox, quantum statistics and N-body behaviour in E_4...") Having in mind, however, that nonlocality appears sometimes to be just seeming [1c] and that a more general Liouvillian picture (in which the $\vec{v}$-distribution can be an arbitrary nonsingular function of $\vec{v}$ at a given moment t and position $\vec{r}$) makes possible the discerning of classical-type ensemble motions without the necessity of a recourse to the quantum potential concept (as inferrable from the discussion in [1b], p.p. 368-9, and [1c]), we find no warrant for the sound character of 'nonminimal' statements as that just cited and should not like to connect the discussion of important problems as the conceptual admissibility of $\vec{r}-\vec{v}$

coexistence with the quantum potential conception. The MSI offered here is meant exactly as such an approach.

In order to find an answer to the problem of interest within the frame of the MSI one should keep in mind that position $\vec{r}$ and velocity $\vec{v}$ are concepts of a classical type. One should therefore examine first the consequences of $\vec{r}-\vec{v}$ coexistence in a classical ensemble (c-ensemble) of noninteracting "copies of the same particle" and then compare the results with those for a corresponding q-ensemble of particles (cf. requirement P1 and ref. [1a]). So examine a c-ensemble whose characteristic spread in $\vec{r}$-space about an 'average' point $\vec{r}=\vec{a}$ at the initial moment $t=t_0$ is of the order of d_0 (denoted as $\sim d_0$; for convenience we choose $t_0=0$ and $\vec{a}=0$). Irrespective of the character of the initial $\vec{v}$-distribution (at $t=0$), one can measure it by employing, say, the following prescription : Remove all external physical fields so that they be zero in the entire interval $0<t<\infty$. The c-ensemble will then evolve freely (with an unvarying $\vec{v}$-distribution) at $t>0$ and at sufficiently large t velocity will be obtainable to an arbitrarily high precision from formula $\vec{v}_t=\vec{r}_t/t$, $\vec{r}_t$ being the objective position of the measured particle at moment t. (The error $\sim d_0/t$ in this definition of $\vec{v}$ will tend to zero as $t\to\infty$). An indication that t is 'sufficiently large' would be a t^{-3}-law for position density $\rho_C(\vec{r}_t,t)$ at the movable point $\vec{r}_t=\vec{v}_t$ for practically all values of $\vec{v}$. Indeed, in $\vec{v}$-space almost all the particles possessing velocities inside a small volume $\Delta^3 v=\Delta v_x\Delta v_y\Delta v_z$ enclosing a given 'point' $\vec{v}$ will be located in volume $\Delta^3 r_t=t^3\Delta^3 v$ in $\vec{r}$-space *at sufficiently large* t (corresponding to sufficiently large distances $|\vec{r}_t|=d_t\gg d_0$ from the origin). The above method of $\vec{v}$-measurement is known as the *time-of-flight technique*. It rests on the c-ensemble picture since a 'snapshot' of positions at large t's would simultaneously be a 'snapshot' of coexisting velocities which are automatically derivable from position values using the one-to-one law $\vec{r}_t=\vec{v}t$ of $\vec{r}-\vec{v}$ correspondence.

Clearly, due to the objective character of position density in the MSI (cf. P2), $\vec{r}-\vec{v}$ coexistence in QM, if admissible, must entail an anlogous behaviour of $|\psi(\vec{r},t)|^2$ at $t\to\infty$. Our problem can therefore be reformulated here as a check-up of the possibility to employ time-of flight velocity measurement technique in the case of relativistic spin-1/2 particles. There are two points that should be kept in mind in the way :

(1) In c-ensembles with a one-to-one $\vec{r}-\vec{v}$ correspondence position density $\rho_C(\vec{r}_t,t)$ ($\vec{r}_t=\vec{v}t$) and velocity density $R_C(\vec{v})$ (t-independent for free motion) are linked via formula

$$\rho_C(\vec{r}_t,t)\Delta^3 r_t=R_C(\vec{v})\Delta^3 v \quad , \quad \Delta^3 r_t=t^3\Delta^3 v \tag{1.1}$$

In such a way only one of the distributions ρ_C and R_C may be treated as independent, the other being a direct consequence of the 'independent' one. The combination of P2 with the MSI of $|\psi(\vec{r},t)|^2$ imparts special significance to position distribution in QM, so in the case of an admissible $\vec{r}-\vec{v}$ coexistence velocity (equivalently momentum) QM distributions will be a consequence of position distribution in the limit of large t. Denoting by ΔP_t the position probability assigned to volume $\Delta^3 r_t$ at moment t and having in mind the above consideration we see that $\vec{r}-\vec{v}$ coexistence in the MSI would entail formula

$$R_Q(\vec{v}) = \frac{1}{\Delta^3 v} \lim_{t\to\infty} t^3 \int_{\Delta^3 v} |\psi(\vec{v}t,t)|^2 d^3 v = \lim_{t\to\infty} \Delta P_t/\Delta^3 v \qquad (1.2)$$

for the QM velocity distribution $R_Q(\vec{v})$ under our assumptions $t_0 = 0$, $\vec{a} = 0$ (see above) ; each one of the Δv's in $\Delta^3 v = \Delta v_x \Delta v_y \Delta v_z$ is arbitrarily small, i.e. 'infinitesimal'. In such a way t-independence of the r.h.s. of (1.2) in the limit $t \to \infty$ would indicate admissibility of the $\vec{r} - \vec{v}$ coexistence conception, whereas its numerical value makes possible a check-up of the validity of the QM postulate on momentum distribution in an arbitrary state (normalized to unity) by comparing this value with the one predicted by QM. We shall see below that the two values coincide. (Cf. also ref. [1] for a nonrelativistic treatment of this problem).

(ii) Eventual $\vec{r} - \vec{v}$ coexistence would mean in no way that we treat QM particles as simple material pints of a classical kind. (As we know, QM particles possess nonlocal properties, de Broglie's model offering one of the possible variants of explaining these). The QM particle may perform a complicated, e.g. 'trembling', motion due to its complex structure even in the absence of external force fields, so its velocity measured by the time-of-flight technique in the limit $t \to \infty$ would generally be a *macroscopic (time-averaged) velocity*, having little in common with its *microvelocity* (i.e. the objective velocity of the particle's 'core' in models as de Broglie's, which velocity exists as a direct consequence of P2). Schrödinger [15] was the first to discern between these two possible velocity characteristics of the relativistic spin-1/2 QM particle (and to introduce the very concepts of macrovelocity and microvelocity) as he noticed the possibility of a 'Zitterbewegung' of the particle in the dynamics of the Dirac equation. (The main features of this consideration are reproduced in Dirac's book [16]). Schrödinger's discussion was based on examining operator average values. Our consideration will produce a more direct picture of what happens by examining position density itself at suitable moving imaginary points in $\vec{r}$-space. We shall see that the $\vec{r}$-distribution of free spin-1/2 relativistic particles has an almost regular 'pulsating structure' along an arbitrary chosen axis in $\vec{r}$-space in the limit of large t in marked contrast with the classical case (and also with the case of relativistic spin-0 QM particles, as it will be shown in a future paper), where $\vec{r}$-distributions are practically uniform in this limit.

II. Position-velocity coexistence for Dirac particles

We shall carry out here the program set forth in Sec. I.

In the system of natural units $\hbar = c = 1$ Dirac's equation for 4-spinor $\psi(\vec{r}, t)$ reads [17]

$$(-i\gamma^{\mu}\partial_{\mu} + m)\psi(X) = 0 \tag{2.1}$$

in the absence of potential fields ; here $\mu = 0, 1, 2, 3$, $X = (x^{\mu}) = (x^0, x^1, x^2, x^3) = (t, x, y, z)$, $\gamma^0 = \beta$, $\gamma^k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}$, $k = 1, 2, 3$, σ_k being the well known Pauli matrices and the signature of the metric tensor being, as usually, $(+ - --)$. The solution of eq. (2.1) under the initial condition $\psi_{in} = \psi(X') = \psi(\vec{r}', 0)$ may be represented at $t > 0$ as [17]

$$\psi(X) = -i \int S(X - X')\gamma^0 \psi(\vec{r}\,', 0) d^3r', \tag{2.2}$$

where

$$S(X) = -(i\gamma^{\mu}\partial_{\mu} + m)\Delta(X) \tag{2.3}$$

and, at $t > 0$,

$$\Delta(X) = -\frac{1}{2\pi}[\delta(X^2) - \frac{m^2}{2}\theta(X^2)\frac{J_1(m\sqrt{X^2}}{m\sqrt{X^2}}], \tag{2.4}$$

where $X^2 = x^{\mu}x_{\mu}$, θ is the well known function $\theta(\xi) = 0$, $\xi < 0$, $\theta(\xi) = 1$, $\xi > 0$, and J_1 –the Bessel function of index 1.

For simplicity, we shall examine first the case of an arbitrary Dirac state ψ_{in} with sharp boundaries, that is $\psi_{in} = 0$ outside a region of a finite 'diameter' d_0 in $\vec{r}$-space. (The generalization of the results to arbitrary ψ_{in} is given in the Appendix). Having in mind that, by the very sense of the problem, one is interested in $|\psi(\vec{r}, t)|^2$ at positions $\vec{r} = \vec{v}t$,

$|\vec{v}| < 1$, that vary linearly with t (cf. Sec. 1), one easily sees that in this case the δ-function may be replaced by zero and θ-by unity in eqs. (2.3) and (2.4) at sufficiently large t. Consequently, at these t,

$$\psi(\vec{v}t,t) = \frac{im}{4\pi}\left[i(\gamma^0\frac{\partial}{\partial t} - \gamma^1\frac{\partial}{\partial(v_x t)} - \gamma^2\frac{\partial}{\partial(v_z t)} - \gamma^3\frac{\partial}{\partial(v_z t)} + m\right]\cdot \int \frac{J_1[m(t^2-(\vec{v}t-\vec{r}\,')^2)^{1/2}]}{(t^2-(\vec{v}t-\vec{r}\,')^2)^{1/2}}\gamma^t\psi(\vec{r}\,',0)d^3r' \quad (2.5)$$

(Operator $\partial/\partial t$, certainly, does not act on the terms under the square root containing $\vec{v}t$). In the limit $t \to \infty$ the argument of J_1 will be large and one may employ the well known asymptotic expression

$$J_1(\kappa) = (2/\pi\kappa)^{1/2}\cos(\kappa - 3\pi/4) + O(1/\kappa^{3/2}) \quad (2.6)$$

for J_1 at large $\kappa = m[t^2 - (vt - r')^2]^{1/2}$; as usually, $O(\xi)$ denotes a magnitude for which $\xi^{-1}O(\xi)$ is bounded as $\xi \to \infty$.

The volumes $\Delta^3 r_t$ and $\Delta^3 v$ of interest were defined in Sec. 1. One may easily see that our consideration leads to two kinds of terms in volume $\Delta^3 r_t$: (ess)-terms that are essential in the formulae adduced below and (inf)-terms that can be neglected compared to (ess)-terms having in mind that we are interested in (arbitrarily) large t and infinitesimal $\Delta v_x, \Delta v_y, \Delta v_z$. [The (inf)-terms contain higher-order powers of $1/t$ and Δv_i compared to the (ess)-terms]. Choose, for convenience, the positive z-axis along the direction of the given velocity $\vec{v}$ of interest, i.e. $\vec{v} = (0,0,v)$. One can see then that the partial derivatives with respect to $(v_x t)$ and $(v_y t)$ in eq. (2.5) as well as $\partial\psi(\vec{v}t,t)/\partial(v_x t)$ and $\partial\psi(\vec{v}t,t)/\partial(v_y t)$ at point $\vec{r} = (0,0,vt)$ are (inf)-terms compared to the corresponding partial derivatives with respect to t and $(v_z t) = (vt)$. This means, in particular, that the behaviour of $\psi(\vec{v}t,t)$ resembles more and more closely that of a $\vec{v}$-directed plane wave in the vicinity $\Delta^3 r_t$ of $\vec{r}_t = \vec{v}t$ (for an arbitrary $\vec{v}$) as $t \to \infty$. On the other hand, at our specific choice of the z-direction, one obtains in this limit expressions for $\sin(\kappa - 3\pi/4)$ and $\cos(\kappa - 3\pi/4)$ of the kind $\sin(\cdot) \approx \sin\alpha t\cos(mvz'/w) + \cos\alpha t\sin(mvz'/w)$ (an analogous formula holding for $\cos(\cdot)$), where $w = (1-v^2)^{1/2}$, $\alpha = mw - 3\pi/4t$, (inf)-terms being disregarded. Having all this in mind and, besides, the fact that $mv/w = p_z = p$ (p thus being the relativistic momentum) and introducing two four-component magnitudes $F(p)$ and $G(p)$ (with components $F_\nu(p)$, $G_\nu(p)$, $\nu = 1,\ldots,4$) via

$$F(p) = (2\pi)^{-3/2}\int \psi(\vec{r}\,',0)\cos pz' d^3r' \quad (2.7)$$

$$G(p) = (2\pi)^{-3/2} \int \psi(\vec{r}\,', 0) \sin pz' d^3r' \tag{2.8}$$

one arrives, up to (inf)-terms, at

$$\begin{aligned}
&|\psi(0,0,vt,t)|^2 = \\
&\quad (m/t)^3 w^{-5} \Big\{ F^+F + G^+G - 2w\,\mathrm{Im}(F_1G_1^* + F_2G_2^* - F_3G_3^* - F_4G_4^*) \\
&\quad + 4v\,\mathrm{Re}\big[(F_1^*F_3 - F_2^*F_4)\sin^2\alpha t + (G_1^*G_3 - G_2^*G_4)\cos^2\alpha t\big] \\
&\quad + v\cos 2\alpha t\big[v(G^+G - F^+F) + 2w\,\mathrm{Im}(F_3G_1^* - F_1G_3^* + F_2G_4^* - F_4G_2^*)\big] \\
&\quad + 2v\sin 2\alpha t\big[v\,\mathrm{Re}\,F^+G + \mathrm{Re}(F_1^*G_3 + F_3G_1^* - F_4G_2^* - F_2^*G_4) \\
&\quad + w\,\mathrm{Im}(F_1^*F_3 - F_2^*F_4 + G_1G_3^* - G_2G_4^*)\big]\Big\}
\end{aligned} \tag{2.9}$$

We are interested in fact in the time-behaviour of the total probability

$$\Delta P_t = t^3 \int_{\Delta^3 v} |\psi(\vec{v}t, t)|^2 d^3v \tag{2.10}$$

for objective particle position (P2) inside volume $\Delta^3 r_t = t^3\Delta^3 v$ (cf. Sec. 1). Having in mind that, in this volume and in the limit of large t, $|\psi(\vec{v}t,t)|^2$ is practically constant in the plane perpendicular to $\vec{v}$ (in our case the (x,y)-plane or, more precisely, the points in this plane whose distance from the z-axis is $\ll vt$), one obtains that, up to (inf)-terms,

$$\Delta P_t = t^3 \Delta v_x \Delta v_y \int_v^{v+\Delta v} |\psi(0,0,vt,t)|^2 dv \tag{2.11}$$

In order to carry out the integration in eq. (2.11), examine expression (2.9) for $|\psi|^2$ in the limit $t \to \infty$. At such t the integration of the terms containing $\sin 2\alpha t$ and $\cos 2\alpha t$ will obviously give a result that tends to zero due to their fast oscillations as functions of $v_z = v$ ($\Delta v_z = \Delta v$ being fixed and F, G being regarded as practically constant in interval Δv, which can always be achieved for sufficiently small $\Delta v's$), whereas $\sin^2 \alpha t$ and $\cos^2 \alpha t$ should be replaced, up to (inf)-terms, by their average values $= 1/2$. One thus obtains

$$\begin{aligned}
\Delta P &= \lim_{t\to\infty} \Delta P_t \\
&= mw^{-5}\Delta^3 v[F^+F + G^+G + 2v\,\mathrm{Re}(F_1^*F_3 + G_1^*G_3 - F_2^*F_4 - G_2^*G_4) \\
&\quad - 2w\,\mathrm{Im}(F_1G_1^* + F_2G_2^* - F_3G_2^* - F_4G_4^*)]
\end{aligned} \tag{2.12}$$

(up to (inf)-terms and for an argument p of F and G).

In such a way the averaging of the fastly varying in the $\vec{v}$-direction microscopic density $|\psi(\vec{v}t,t)|^2$ $(t \to \infty)$ over volume $\Delta^3 r_t$ containing point $\vec{r}_t = \vec{v}t$ yields in $\Delta^3 r_t$ a suitable average position density distribution

$$< |\psi(\vec{v}t,t)|^2 >_{\Delta^3 r_t} = \Delta P_t / \Delta^3 r_t \tag{2.13}$$

for which ΔP_t is time-independent at large t. As we know from Sec. 1 this is nothing else than objective position-macrovelocity coexistence for Dirac particles. But inspection of the precise microscopic expression (2.9) for the position density of free Dirac particles reveals a fundamental difference between their motion and that of simple relativistic classical points of the same mass : $|\psi(0,0,vt,t)|^2$ generally possesses an almost regular 'fine structure' along z, consisting of a set of pulsations of a very slowly varying period with distance at any given sufficiently large moment t, a structure of this sort being certainly absent in the classical case as pointed out in Sec. 1. Besides, one may notice that, up to a factor of t^{-3}, $|\psi|^2$ possesses a practically regular time-structure too about point $\vec{r}_t = \vec{v}t$ in the limit of large t as it undergoes pulsations of angular frequency $2m(1-v^2)^{1/2}$. Both these kinds of pulsations are superimposed on a 'stable' background $= (m/t)^3 w^{-5}[F^+F + G^+G - 2w\,\mathrm{Im}(F_1G_1^* + F_2G_2^* - F_3G_3^* - F_4G_4^*)]$. We shall return to this shortly. What should be evident from the above brief remarks is that free Dirac particles do not move in a simple uniform fashion.

It remains now to check the validity of the QM postulate on velocity distribution in Dirac's case, i.e. to check whether

$$R'_Q(\vec{v}) = R_Q(\vec{v}) = \Delta P / \Delta^3 v \tag{2.14}$$

where $R'_Q(\vec{v})$ is the velocity distribution obtainable from the QM postulate on $\vec{p}$-distribution, and $R_Q(\vec{v})$ and ΔP are defined by eqs. (1.2) and (2.14), respectively. In order to do this recall that the QM momentum distribution postulated for spin-1/2 particles is defined as

$$R'_Q(\vec{p}) = \sum_{i=1}^{4} |a^{(i)}(\vec{p})|^2 \tag{2.15}$$

where

$$a^{(i)}(\vec{p}) = (2\pi)^{-3/2}(\frac{E_p + m}{2E_p})^{1/2} \int u^{(i)+}(\vec{p})\psi(\vec{r}\,',0)e^{-i\vec{p}\vec{r}\,'} d^3r' \tag{2.16}$$

E_p denoting the positive magnitude $(m^2+p^2)^{1/2}$, the conjugate spinors $u^{(i)+}(\vec{p})(i=1,\dots,4)$ being given (for the z-directed momenta $\vec{p}=(0,0,p)$ of interest) by

$$u^{(1)+}(\vec{p})=(1,0,p/(E_p+m),0) \quad , \quad u^{(2)+}(\vec{p})=(0,1,0,-p/(E_p+m))$$

$$u^{(3)+}(\vec{p})=(-p/(E_p+m),0,1,0) \quad , \quad u^{(4)+}(\vec{p})=(0,p/(E_p+m),0,1) \tag{2.17}$$

(As well known, $u^{(1)}$ and $u^{(2)}$ are employed in the case of positive energy $\mathcal{E}_p=E_p$ and helicity ±1, respectively, while $u^{(3)}$ and $u^{(4)}$ correspond to negative energy $\mathcal{E}_p=-E_p$ and helicity ±1, respectively ; as always nowadays, the negative $\mathcal{E}_p$ are treated just as certain quantum numbers and not as actual energies).

Having in mind that in the case of negative $\mathcal{E}_p$ the direction of $\vec{p}$ is opposite to that of the corresponding velocity [17] (Pauli was probably the first to notice this as it may be infered from a remark of O. Klein [18]), we see that the density of interest is not $R'_Q(\vec{p})$ (2.15) but

$$R''_Q(\vec{p})=a^{(1)}(\vec{p})^2+a^{(2)}(\vec{p})^2+a^{(3)}(-\vec{p})^2+a^{(4)}(-\vec{p})^2 \tag{2.18}$$

since, for a fixed $|\vec{p}|$, the four terms in the r.h.s. of (2.18) correspond to the same velocity $\vec{v}$. Applying (2.16) (with momentum $\vec{p}$ for $i=1,2$ and $-\vec{p}$ for $i=3,4$) and equality $R'_Q(\vec{v})=(m^3/w^5)R"_Q(\vec{p})$, one can calculate $R''_Q(\vec{v})$ in a straightforward manner. The result coincides precisely with the content of eq. (2.14). Q.E.D.

The above results, obtained for spinors ψ with sharp boundaries in position space, are generalized for arbitrary initial states $\psi(\vec{r},0)$ in the Appendix.

Let us return once again to the microscopic formula (2.9) for $|\psi(\vec{v}t,t)|^2$, $\vec{v}t=(0,0,vt)$, which expression is valid now for an arbitrary initial state, normalized to unity. A straightforward computation employing definition (2.16) shows that if the initial state is composed of free-motion eigenfunctions corresponding to energies of a fixed sign only, any 'fine structure' along z for the chosen (arbitrary) direction of the z-axis will be absent in the limit $t\to\infty$. In the case when $\psi(\vec{r},0)$ incorporates $\mathcal{E}_p$'s or both signs the said structure will necessarily be present in the asymptotic form of $|\psi(\vec{r},t)|^2$, $t\to\infty$. Its period $\delta z(v)$ along z is constant for all practical purposes for such t and is determined by the requirement on the phase of $\sin 2\alpha t$ and $\cos 2\alpha t$ to undergo a variation

$= 2\pi$ along $\delta z(v)$ at the given moment t. (Recall that we are interested in values $z = z_t = vt$, so that $\delta z(v) = t\delta v$). For the corresponding $\delta v \ll \Delta v \ll v$ this requirement leads to

$$\delta v = (1 - v^2)^{1/2}/mtv \tag{2.19}$$

Consequently, the $\delta z(v)$ in question is time-independent and equal to

$$\delta z = (1 - v^2)^{1/2}/mv = \lambda(p)/2, \tag{2.20}$$

where $\lambda(p)$ is easily seen to be de Broglie's wave length corresponding to relativistic momentum $p = mv/(1 - v^2)^{1/2}$. (Note in passing that, in our units, $1/m$ is equal to the Compton wave length, so for the larger part of interval $0 \leq v < 1$ the characteristic length δz in (2.20) will be $\sim 1/m$. One should not interpret, however, $1/m$ as an actual 'diameter' of the spin-1/2 particle since the actual characteristic length in fact is $\lambda(p)/2$ and it indefinitely increases as $v \to 0$. Note also that even if one would interpret $\delta z(v)$ as a certain unavoidable indefiniteness of position along z, the product $\delta z \delta v$ of position-macrovelocity 'uncertainties' can be made arbitrarily small at large t since $\delta z(v)$ is t-independent, whereas $\delta v \sim 1/t$ –cf. (2.19)).

In such a way what actually happens in the volume $\Delta^3 r_t$ of interest is, generally, the following. We have a set of pulsations of $|\psi|^2$ there, whose period along z is a very slowly varying function of z about any point $z_t = vt$, $t \to \infty$, the structure of each individual period of $|\psi|^2$ becoming more and more regular in this limit due to the dying-off of 'deforming' (inf)-terms with the course of time. At the same time, the number of the pulsations of $|\psi|^2$ in $\Delta^3 r_t$ indefinitely increases as $t \to \infty$ which leads to the appearing of a well defined averaged density (2.13) in $\Delta^3 r_t$. Obviously, the same value (2.13) can be obtained by averaging over a single period in $\Delta^3 r_t$ at $t \to \infty$.

As for the time-pulsations of $|\psi(\vec{v}t, t)|^2$ about the movable (imaginary) point $\vec{r}_t = \vec{v}t$ in $\vec{r}$-space, their angular frequency $\omega(v)$ is equal, in the usual Gaussian units, to

$$\omega(v) = 2mc^2(1 - v^2/c^2)^{1/2}/\hbar \tag{2.21}$$

and they will also exist only when $\mathcal{E}_p$'s of both signs participate in $\psi(\vec{r}, 0)$. $\omega(v)$ can be interpreted in a straightforward fashion : $\omega(0) = 2mc^2/\hbar$ is a characteristic frequency in the frame of reference K_0 in which the particle is, on the average, at rest, so in our frame of reference in which K_0 moves with velocity v along z the frequency will undergo relativistic time-contraction given by the well known factor $(1 - v^2/c^2)^{1/2}$.

Clearly, both $\delta z(v)$ and $\omega(v)$ represent certain characteristics in space and time of the Zitterbewegung in our ensemble picture. As mentioned, they are more immediate and concrete than those obtained by examining the time-evolution of QM operators and their average values (loc. cit.).

III. Discussion and conclusion

The admissibility of objective $\vec{r} - \vec{v}$ coexistence is rejected in the CI on the basis of arguments as the well known one due to Heisenberg (discussed, say, by Ballentine [19] and, in detail, by Jammer [20]). Heisenberg maintains that apparent $\vec{r} - \vec{v}$ coexistence refers only to the past (with regard to the moment of measurement) but not to the results of realizable measurements themselves which must always respect inequality

$$\delta z \delta p_z \geq \hbar/2 \tag{3.1}$$

$$[\delta z = (< z^2 > - < z >^2)^{1/2} \quad \text{and} \quad \delta p_z = (< p_z^2 > - < p_z >^2)^{1/2}],$$

so "Occam's razor" cuts off the $\vec{r} - \vec{v}$ coexistence idea as superfluous. Clearly, arguments of this sort are purely philosophical, hence a matter of taste. From such a general standpoint our argument represents a counterargument suitable for different tastes. We shall enumerate and discuss here some of its advantages.

(i) It preserves essential features of the usual image of massive particles and at the same time enters in no contradiction with the QM postulate on momentum distribution, explaining at that its macroscopic essence. The MSI thus implies in a manner avoiding the unacceptable features of its 'nonminimal' alternatives that the typical QM properties of microparticles should not necessarily be attributed to qualities that transcend human imagination.

Indeed, we saw that the momentum (equivalently velocity) distribution predicted by the QM postulate for spin-1/2 particles is 'visible' in a 'snapshot' of objective (P2) position distribution at $t \to \infty$, thus referring to an *ensemble* of macroscopic velocities. At that, the product $\delta z \delta v$ can be made arbitrarily small for *individual* macrovelocities as demonstrated in the discussion of eqs. (2.19) and (2.20). Heisenberg's uncertainty relation thus applies (as clear from the very ensemble essence of the *average* magnitudes figuring in it) to the position-macrovelocity distribution in the entire q-ensemble of interest but not to individual simultaneous $(\vec{r}, \vec{v})$-measurements which can be arbitrarily precise. Besides, as postulate P2 turned out to be sufficient to make it possible to discern the difference between macro- and microvelocities, one is justified in asserting that the q-ensemble macroscopic property expressed by ineq. (3.1) would not necessarily come in contradiction also with the fact that an individual microparticle which is pointlike all the time would automatically possess an objective trajectory. Indeed, the latter concept is microscopic, hence outside the range of competence of ineq. (3.1). It is thus more or less a matter of belief rather than of strict logic to rule out the possibility of objective existence of trajectories on the basis of ineq. (3.1).

(ii) Our consideration shows that time-of-flight technique is a perfectly lawful macrovelocity measurement procedure in the spin-1/2 case too in the frame of relativistic dynamics. But it is not instantaneous as the postulated procedures in the CI [21]. On the contrary, in order to measure with the aid of this technique the momentum (or velocity) distribution assigned to $\psi(\vec{r}, t_0)$ (i.e. to state ψ at the initial moment t_0) one should only take care that no physical fields (if previously present) exist at moment $t \geq t_0$ and, after a sufficiently large lapse of time that characterizes the actual duration of the macrovelocity measurement procedure one should just register the objective position (P2) of the particle. What we have here is, therefore, a physical picture of a noninstantaneous, objective position evolution in a pertinent q-ensemble in which the very QM dynamics of the physical system evinces with the course of time (thus without any instantaneous state reductions at t_0) a definite macrovelocity distribution as a direct consequence of position distribution at $t \to \infty$. We thus arrive at a more general conception about measurement of an arbitrary QM magnitude. Namely, measurement is generally not carried out instantaneously at a given moment t_0 of interest but just begins at t_0 by creating at $t \geq t_0$ suitable conditions for a definite macroscopic evolution of the QM state, which requirement is necessary for the very definition of the pertinent physical magnitude. (The concept of macromomentum, say, by its very definition has, generally, no sense at all at t_0 or at moments t for which $t - t_0$ is small enough to preclude the fulfilment of the necessary requirement $d_t \gg d_0$). This conception is pretty close to de Broglie's conception on measurement [4,5] via suitable position registering. An interesting difference from the outlook of the latter in the specific case examined is that one needs no special analyzer in the time-of-flight setup [1] : empty space itself plays the role of an analyzer here.

It should be mentioned, though, that in a very specific sense position and velocity measurements may be thought of as incompatible given a definite moment t_0 of interest. Really, if one is interested in objective position at t_0, one must measure it (quasi)instantaneously at that moment whereas, as we know, velocity distribution assigned to $\psi(\vec{r}, t_0)$ is measured at t's for which $t - t_0 \to \infty$. But the above detailed consideration shows that this 'incompatibility' has nothing to do with the incompatibility conception of the CI, being just a consequence of definitions that would be essentially the same in *c-ensemble* velocity measurements by the time-of-flight technique.

(iii) The MSI offers a certain resolution of long-standing difficulties. Note first that de Broglie's postulate P2, conjoined with the interpretation of $|\psi|^2$ as the probability density of objective position, automatically entails the interpretation of $\vec{r}$ as the position operator for relativistic spin-1/2 particles too (which was also Dirac's original idea). The latter idea, however, was abandoned later on since it leads to serious set-backs in the conventional approach. Indeed, if $\vec{r}$ is the spin-1/2 position operator, then $d\vec{r}/dt$ should be the hermitean operator of velocity. But this operator possesses unacceptable properties : its only eigenvalues along a given axis are $\pm 1 (= \pm c)$ and its components are noncommutative. So numerous different proposals for presumably more correct position operators were made but all of them turned out to possess unacceptable properties as well [22].

However, the postulated objective character of particle position (entailing the interpretation of $\vec{r}$ as the QM position operator) has proved too useful in order to dismiss this conception easily. It was exactly this idea that made it possible to obtain the above results and clarifications and, in particular, to present a vivid picture of the macroscopic origin of the momentum operator $\hat{\vec{p}} = -i\partial/\partial\vec{r}$ by imparting a more concrete meaning to earlier assertions about the character of the velocity corresponding to $\hat{\vec{p}}$ [15] or the connection of $\hat{\vec{p}}$ with certain mean particle positions [23]. The probability density distribution $|a(\vec{p})|^2$ of the physical magnitude *macromomentum* whose QM image is operator $\hat{\vec{p}}$ yields, in agreement with special relativity, zero probability measure for velocities $|\vec{v}| \geq 1$ in the necessarily macroscopic experiments evincing 'uniform-velocity' particle motions of a classical type. But P2 entails zero probability measure as well for particle positions that might imply the existence of microvelocities $|\vec{v}_m| \geq 1$. Indeed, examine an initial Dirac state $\psi(\vec{r}, 0)$ with sharp boundaries (for which the Zitterbewegung that is believed to be performed with the velocity of light will always be conspicuous in the formulae of the theory). Eqs. (2.2-4) do not permit velocities > 1 of the boundaries themselves, hence we have a zero value at any $t > 0$ of the integral of $|\psi(\vec{r}, t)|^2$ over the region in space that lies outside the position of the boundaries at moment t, i.e. microvelocities $|\vec{v}_m| \geq 1$ really possess zero probability in the ensemble picture. In such a way the MSI resolves the difficulty by stating that $\vec{r}$ is a good position operator whereas operator $d\vec{r}/dt$, obtained via a simple classical analogy, is not a good velocity operator : it is neither proportional to the macroscopically observable momentum operator $-i\partial/\partial\vec{r}$, nor does Dirac's dynamics itself give any evidence about the actual existence of eigenvelocities $\pm 1 (\pm c)$ in any (macroscopic or microscopic) sense.

(One could preserve, at a will, the interpretation of $d\vec{r}/dt$ as the microvelocity operator only at the expense of revising other basic QM postulates, in particular the one stating that the eigenvalues of any hermitean operator are observable in physical experiment. Let us point out here that the existence of unobservable eigenvalues of QM operators is characteristic of Dirac's theory ; for instance, the existence of energies $\mathcal{E}_p < 0$ for free particles has never been experimentally confirmed. Note besides that Dirac's thought measurement procedure [16] employing two successive position measurements on the same particle and aimed at explaining why the Zitterbewegung velocity in state ψ should actually coincide with that of light is not correct from both the CI and SI viewpoints : his first $\vec{r}$-measurement would drastically modify ψ, so the eventual result would have no relevance to state ψ).

The above discussion removes as well (at least for the case of free motion) another difficulty : the possibility for causality violation discussed by Hegerfeldt [24,25]. Hegerfeldt adopts the usual idea that acceptable one-particle states are only those corresponding to positive energy quantum numbers, so our interpretation which rules out faster-than-light free motions resolves Hegerfeldt's alternative [24] by stating that states of the said kind cannot be strictly localized in a given finite region of space –a well known fact when $\vec{r}$ is treated (as we do) as the position operator.

(iv) Up to here we discussed local properties of particles in an ensemble picture of their motion. As well known and mentioned above, individual particles possess nonpoint-like properties too that determine regular diffraction patterns in the scattering of particles by crystal lattices, etc. These properties also need consideration within the MSI. We examine in a separate paper (submitted here) specific wave-like phenomena on the simple example of nonrelativistic QM. (Wave-like properties are of the same character in both relativistic and nonrelativistic QM). It is shown there that the idea of unrestricted applicability of the QM evolution equations entails the idea of an infinite range of the nonpointlike properties of individual particles. Physical intuition can hardly get reconciled with this, so de Broglie [2] proposed an ideology in which his well known wave attached to each particle is of a finite range and satisfies (by hypothesis) a nonlinear equation of motion. Interesting enough, analogous conclusions can be made on the basis of an additional locality postulate (Einstein's postulate) with the aid of which the present author arrived at certain interpretational results on measurement and time-irreversibility [26]. A more detailed account of such an enlarged MSI will be given in a future review paper. What should be emphasized here is that, even within the frame of the present MSI, one encounters essential differences in treating wave-like effects compared to the ideology of the CI.

The results discussed in the frame of the MSI, although numerous enough, are nevertheless confined by its very 'minimal' character (determined by the only postulates P1 and P2) to the position-momentum couple or the more general conclusions following from the consideration of the said couple. Other important problems as the physical essence of the indistinguishability of identical particles, the nature of spin, etc., are still waiting for an explanation that would be free of the unacceptable features of the CI or the present-day 'nonminimal' SIs. It is quite possible that interpretational postulates only would be insufficient for such a more profound description and that a logically consistent theory of a new kind might be necessary to this end. But physically consistent interpretations of QM may nevertheless have to play an important role in the meantime by not only helping the psychological revolution that seems to be taking place nowadays but also by indicating fields of physical experiment where present-day theory might be inadequate.

Appendix

We introduce the following notations. Let $\psi_L(\vec{r},0)$ be defined as $\psi_L(\vec{r},0) = \psi(\vec{r},0), |\vec{r}| \leq L$; $\psi_L(\vec{r},0) = 0, |\vec{r}| > L$, and let $\psi_L(\vec{r},t)$ be its time-evolved given by eq. (2.2) (in which ψ should be replaced by ψ_L). Denote by $a_L^{(i)}(\vec{p})$ and $a^{(i)}(\vec{p})$ the i-th momentum amplitudes ($i = 1, \ldots, 4$) of $\psi_L(\vec{r}, 0)$ and $\psi(\vec{r}, 0)$, respectively (or, which is the same for free action, of $\psi_L(\vec{r}, t)$ and $\psi(\vec{r}, t), t \geq 0$). Obviously,

$$\int |\psi(\vec{r}, 0) - \psi_L(\vec{r}, 0)|^2 d^3r = \Delta_L \stackrel{L\to\infty}{\longrightarrow} 0 \tag{A.1}$$

Probability conservation in position space entails

$$\Delta_L = \int |\psi(\vec{r}, t) - \psi_L(\vec{r}, t)|^2 d^3r \quad , \quad t \geq 0 \tag{A.2}$$

Besides, due to Parseval's relation,

$$\Delta_L = \sum_{i=1}^{4} \int |a^{(i)}(\vec{p}) - a_L^{(i)}(\vec{p})|^2 d^3p \tag{A.3}$$

Consequently, the mean-square limits of $\psi_L(\vec{r}, t), t \geq 0$, and of $a_L^{(i)}(\vec{p})$ exist at $L \to \infty$ and are almost everywhere equal to $\psi(\vec{r}, t)$ and $a^{(i)}(\vec{p})$, respectively. Hence

$$\lim_{L\to\infty} \int_{\Delta^3 v} |\psi_L(\vec{v}t, t)|^2 t^3 d^3v = \int_{\Delta^3 v} |\psi(\vec{v}t, t)|^2 t^3 d^3v \tag{A.4}$$

for *all* $t \geq 0$. One can, therefore, examine the limit at $t \to \infty$ of the integral at the left of (A.4) prior to taking the limit $L \to \infty$. Adding a subscript L to the densities in (2.14), we obtain that the said integral is equal to $R'_{QL}(v)\Delta^3 v = R_{QL}(v)\Delta^3 v$ in the limit of large t, so that (A.4) gives

$$\lim_{L\to\infty} R_{QL}(\vec{v}) = \lim_{t\to\infty} t^3 < |\psi(\vec{v}t, t)|^2 >_{\Delta^3 r_t} \tag{A.5}$$

But, as clear from the above, $\lim R_{QL}(\vec{v}), L \to \infty$, is equal to the density $R_Q(\vec{v})$ obtained with the help of $a^{(i)}(\vec{p})$ in the already described way. Consequently, $< |\psi(\vec{v}t, t)|^2 >_{\Delta^3 r_t}$ has the necessary form $R_Q(\vec{v})/t^3$ in the limit $t \to \infty$. Q.E.D.

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