On the impossibility of existence of an absolute, unambiguous interpretation of quantum mechanics and physical reality

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ABSTRACT. In this paper it has been shown that there *does not exist* a unique, a priori interpretation of quantum mechanics and physical reality and that different, *relatively consistent* and *relatively complete* interpretations, Copenhagen interpretation and the interpretations in terms (non-contextual) hidden variables (whose existence is also here proved), for example, can be considered *meta-theoretically complementary* in the sense of *meta-epistemologically generalized* Bohr's *principle of complementarity. Meta-theoretical complementarity* of physics and mathematics has also been discussed.

RESUME. On montre dans cet article qu'il n'existe pas d'interprétation unique, a priori, de la mécanique quantique et de la réalité physique et qu'on peut envisager des interprétations relativement cohérentes et relativement complètes, par exemple l'interprétation de Copenhague et les interprétations en termes de variables cachées (non-contextuelles et dont on prouve ici l'existence). Elles sont complémentaires en méta-théorie au sens d'un principe de complémentarité de Bohr généralisé en méta-épistémologie. On discute aussi la complémentarité méta-théorique de la physique et des mathématiques.

1. Introduction

Since the very occurence of the standard Copenhagen interpretation of quantum mechanics (CQ) the question of its logical consistency and completeness has been raised. It has, most often, been considered that the question can be answered by means of a physical theory either *absolutely positively* (absolute confirming of the consistency and completeness of CQ and absolute denying the consistency and completeness of hidden variables (HV) theories) or *absolutely negatively* (absolute negating of the consistency and completeness of CQ and absolute confirming of the consistency and completeness of HV). Anyhow, it has been considered that the answer is *completely determined.* In fact, more that has been considered, i.e. that it is possible by means of a physical theory, to given an *absolute determined* answer to the question on the ontological and epistemological content of quantum mechanics and physical reality.

Beginning from the famous von Neumann's theorem [1], many theorems have been formulated, e.g. Jauch-Piron's [2], Gleason's [3] Kochen-Specker's [4], etc., which how it seems, were proving the absolute inconsistency of HV theories. However, Bell [5], Bohm [6], Gudder [7], etc., proved *the limitation of the validity of these theorems on the absolute inconsistency* of HV theories. The fact, von Neumann, Jauch-Piron, Gleason, Kochen-Specker and the others theorems were able to prove, *was not the absolute inconsistency* of CQ, i.e. the consistency of CQ on the quantum level of the preciseness of a theoretical analysis, without extending the analysis to the eventual subquantum domain of hidden variables. On the other hand *neither* Bohm's *nor* Gudder's *model of* HV *could not prove the absolute inconsistency* of CQ (i.e. *could not negate the relative consistency* of CQ), than *could prove only relative consistency of* HV and *the impossibility of the absolute completeness* of CQ. It was shown, however, [8], that, starting from a relatively consistent statistical theory, i.e. from CQ, and from a relatively consistent deterministic HV theory which is the extension of CQ, it was possible theoretically to continue the process of completing any statistical theory to a deterministic one, and the process of completing any deterministic theory to a statistical, *ad infinitum*, by which *the impossibility of absolute completeness* of any statistical or deterministic theory was proved theoretically, i.e. was proved theoretically *the relative character of completeness* of both statistical and deterministic theories. It was also shown theoretically [9] that a relatively consistent and complete HV theory *on the condition of absolute applicability of* CQ *to all experiments on the quantum level of preciseness* of experimental analysis, can not be verified in the sense of experimental dustinguishing HV from CQ, so that, with the assumption, HV can be considered an *absolutely meta-physical theory.* However, by means of theory *can not be decided*, i.e. *can not be determined* whether CQ is really absolutely applicable to all experiments of the quantum level of the preciseness of an analysis (experiments in physics of "elementary" particles or "high" energies, for example), thus, according to that, HV *can not be considered* an absolutely meta-physical theory. The choice of CQ and not HV, for describing the existing experimental results, which is accepted by most physicist, then, is not dictated by the absolute meta-physicality of HV, but by the relative minimality of CQ, i.e. by the ablity of CQ to describe the existing experimental results by a minimal number of theoretical constructs. [1] But it should be noted that it is *not possible to determine absolutely* even the notion of minimality of theory using only theoretical formalism. The main reason of this is the fact that the theory is made up of both a set of measuring $\mathcal{M}$ and a set $\mathcal{R}$ of relations defined consistently on $\mathcal{M}$, and *the whole complexity* of theory implies both the degree of complexity (or simplicity) of $\mathcal{M}$ and the degree of complexity (or simplicity) of the set $\mathcal{R}$. It is true that in CQ we have a minimal degree of complexity of $\mathcal{M}$ (we do not take into account hidden variables), but it is also true that the degree of complexity of $\mathcal{R}$ of CQ (here $\mathcal{R}$ includes the relation of non-commutativity of complementary observables) is greater than degree of complexity of $\mathcal{R}$ of classical physics, i.e. of HV. And vice versa, in HV, $\mathcal{M}$ of CQ is extended by potentially measurable hidden variables to $\mathcal{M}$ of HV, but $\mathcal{R}$ of CQ is reduced to a kind of classical physics $\mathcal{R}$. But, the total degree of complexity for CQ and for HV is basicaly *the same* (QED), so that both theories are equal in that respect. The choice of a minimal theory according to the degree of complexity of $\mathcal{M}$, which prefers CQ compared with HV, is, therefore, not a complete demand, but it can be justified by the minimality of time intervals needed for the theoretical calculation of the observable quantities. But the lenght of the time intervals is largerly an experimental, not only a theoretical quantity. In the rest of the paper a relative minimal theory will stand for theory of the minimal degree of complexity of $\mathcal{M}$, i.e. CQ.

So, theoretical considerations do not produce an absolutely, but only a relatively definite answer to the question on the theoretical consistency, completeness, applicability and minimality of CQ or HV. This reminds one very much on the situation of *theoretical relativization* of the ontological schemes, particles and waves, or of the epistemological description

[1] Instead of the expression "minimal theory", i.e. "minimal theoretical system" used in this paper, Hans Reichenbach uses expression "normal system" (see ref.[10]). Besides having some definite similarities, the notions "minimal system" and "normal system", still differ, partially, even in content, since, according to Reichenbach "normal system" does not exist in atomic, i.e. in quantum domains. Besides, among the attitudes given here, that are meant to be meta-theoretical generalization of CQ, and Reichenbach's empiric-positivistic attitudes there are significant similarties but there are also distinct differences between CQ and empiric-positivistic phylosophy (see ref.[11], for example).

of the change of the state of quantum system, evolution and collapse, in CQ, conveyed through Bohr's *principle of complementarity* [12] and further *theoretical generalization* of the principle of complementarity in the framework of Herbut's relative collapse theory of measuring (RC) [13]. The aim of this paper is to show that it is possible to understand the principle of complementarity in a wider sense, like a *meta-theoretical* one, i.e. like a *meta-epistemological* principle, meaning that CQ and HV can be treated as *being meta-theoretically complementary*, by which *the absolute unambiguosity of theoretical interpretations of quantum mechanics and physical reality would be negated* at the same time.

2. Proof of the existence of a type of non-contextual HV theory

Bell's proof [2] of the limited validity of von Neumann's, Jauch's and Piron's, Gleason's, Kochen's and Specker's theorems, [3] was not, at the same time, a rigorously positive proof of the possibility of a theoretical existence of HV. Bohm's HV theory, [4] on the other hand, was intented to be a simple positive example of HV theory (that would, in an obvious way, negate absolute validity of von Neumann's theorem), but was the simplification of HV model that caused many difficulties at the attempt of its generalization to relativistic quantum phenomenons. [5] Gudder's HV theory, [6] on the other hand, was constructed as an abstract theory of *exclusively contextual* HV (i.e. HV which depend on the context of the measuring procedure, similar as in CQ, but which differently from CQ, in the given context and at the sub-quantum level of preciseness of analysis, describe the measuring process in a deterministic way, i.e. through dispersion free state). All that shows that the proof of existence, i.e. of relative consistency of HV theories, is not entirely deprived of, at least, technical difficulties, the degree of which we shall try to diminish by a new theoretical proof of the existence of HV theories.

Let $\hat{A} = \sum_n A_n \hat{P}_n(\hat{A})$ and $\hat{B} = \sum_m B_m \hat{P}_m(\hat{B})$ be two non-commutative, i.e. incompatibile or complementary observables of a discrete, nondegenerated spectrum in Hilbert's space $\mathcal{H}$, at which A_n are eigenvalues and $\hat{P}_n(\hat{A})$ are eigenprojectors of observable $\hat{A}$, for

[2] See ref.[5])
[3] See ref.[1]-[4]
[4] See ref.[6]
[5] Many other concrete models of HV theories have the same problem.
[6] See ref.[7]

$n = 1, 2, \ldots$, while B_m are eigenvalues and $\hat{P_m}(\hat{B})$ are eigenprojectors of observable $\hat{B}$, for $m = 1, 2, \ldots$. We denote by a_n the event that the value A_n, for $n = 1, 2, \ldots$, is defined on the physical system, and by b_m the event that value B_m, for $m = 1, 2, \ldots$, is defined on the same physical system, not assuming a priori that the words "event" and "defined" necessarily must have the meaning which is attributed to them in CQ. Let us assume that the words "event" and "defined" have somewhat an *abstract meaning*, like abstract state vector in $\mathcal{H}$. Let us further assume that, as exact quantum mechanical calculation the aim of which is getting numerous values requires a representation of an abstract state vector in some chosen basis in $\mathcal{H}$, so a theoretical analysis of a physical system requires a "representation" of abstract notions "event" and "definition" in some chosen algebraic structure, where different "representations", like different basis in in $\mathcal{H}$, will be regarded *meta-theoretically complementary.*

Regarding the relative consistency of CQ we know that at least one such, CQ "representation", a non-komutative quantum algebra of an event, exist, and that, according to the theorem of Gleason, probability defined on such a set of events must satisfy quantum mechanics propositions, on the base of which it is possible, let us say, to calculate the probabilities $p(a_n)$ and $p(b_m)$ of the events a_n and b_m respectively, for $n = 1, 2, \ldots$, and $m = 1, 2, \ldots$. Let us now show that there exist such a "representation" that might be regarded a "representation" of hidden variables. Such a HV "representation", should satisfy the condition of existence of product (cut) definition of any two events, for example events a_n and b_m, where their product, which implies a simultaneous happening, i.e. simultaneous definition of both events a_n and b_m, will be denoted by $a_n \cap b_m$, for $n = 1, 2, \ldots$, and, $m = 1, 2, \ldots$. Or, precisely, we shall considered that in the HV "representation" the family $\mathcal{F}$ of accidental events, such as the events a_n and b_m, for $n = 1, 2, \ldots$, and, $m = 1, 2, \ldots$, for example, makes σ-field. That means that: 1) a certain event Ω belongs to $\mathcal{F}$, i.e. $\Omega \in \mathcal{F}$, 2) for each $a \in \mathcal{F}$ it follows that for its complementary event $\overline{a} = \Omega \setminus a$ (where we have in mind mathematical not Bohr's complementarity) $\overline{a} \in \mathcal{F}$ is valid, 3) if $a_i \in \mathcal{F}$ for $i = 1, 2, \ldots$, then $(\cup_{i=1}^{\infty} a_i) \in \mathcal{F}$, where $\cup_{i=1}^{\infty} a_i$ is a sum (union) of a series of events a_i, where sum is defined or realized iff at least a event a_i is defined or realized. From the definition of σ-field it also follows: 4) if $a_i \in \mathcal{F}$, $i = 1, 2, \ldots$, then, $(\cap_{i=1}^{\infty} a_i)$, where $\cap_{i=1}^{\infty} a_i$ is product of a series of events a_i. Further on, over the family of accidental events $\mathcal{F}$ which has the structure of σ-field, we shall define probability according the standard

Kolmogorov's axiomatics [14][15]: K_1 –for each $a \in \mathcal{F}$, probability $p(a)$ is a real non-negative number ; K_2 –$p(\Omega) = 1$; K_3 –for a series of events $a_i \in \mathcal{F}$ for $i = 1, 2, \ldots$, and which are mutually disjunctive, i.e. $a_i \cap a_j = \emptyset$ for $i \neq j$, where $\emptyset$ is an impossible event, $p(\cup_{i=1}^{\infty} a_i) = \sum_{i=1}^{\infty} p(a_i)$ is valid. Finally, let us assume that a is an abstract physical event, which has its "representative" in the CQ "representation". As in quantum mechanics the final result of calculation is not dependend by the choice of representation, although a suitable choice can simplify, i.e. minimalize the calculating procedure a great deal, so here we are to set a requirement that the probability of the event a in HV "representation" be equal to the probability of the same event in CQ "representation".

We shall consider that above mentioned conditions define an HV "representation" of physical events, i.e. a HV theory. Since at any moment of time before measurement on any physical system, according to the given HV representation, as well as according to classical physics, all physical observables exist simultaneously "defined" where distribution of theirs values on the quantum ensembles is given in accordance with predictions of quantum mechanics, this theory *is not contextual*. But, in context of some observable measurement, values of all complementary observables on the single quantum object will be changed, in a uncontrolable way at the quantum level of analysis preciseness. This change can be treated as non-local [17], [18], [19], or, meta-theoretically complementary, as local. [7]

Let us now prove that the given HV "representation" is consistent, i.e. that it does not contradict the results of CQ "representation" on the quantum level of analysis preciseness. Let us denote by $\mathcal{S}$ the set of all HV "represented" events which also have their CQ "representation" so that their probability in HV "representation", taking into account its definition, is defined and equivalent to a quantum mechanical one. Taking into consideration the well-known theorem of *the extension (prolonging) of probability*, [8] according to which for probability p given on the set $\mathcal{S}$ there exist a *unambiguos* extension of the minimal σ-field [9]

[7] It can be shown (see ref.[16]) that in non-Kolmogorov's axiomatics of probability HV theories can be local, so that the analysis of the locality of HV theories depend, among other things, on selection of axiomatic of probability, which may be conveyed on the many meta-theoretically complementary ways.

[8] See ref.[14],[15], for example.

[9] It is interesting to note that Gudder's theory of contextual HV also gives a unambiguosly extension of CQ to minimal HV. This *minimal extension of CQ theory*, however, should be distinguished from CQ as a *minimal theory.*

$\mathcal{F}(\mathcal{S})$ which contains $\mathcal{S}$, so that there is probability p' on the $\mathcal{F}(\mathcal{S})$ and that $p'(a) = p(a)$ for any $a \in \mathcal{S}$, we can say that the mentioned HV "representation" is consistent, in the sense that the probability of physical events which have not got CQ "representatives" do not contradict the probabilities of physical events which have CQ "representatives". Of course, and in the pure mathematical sense, HV "representation" is consistent as a mathematical theory of probability, and the problems of inconsistency of theory which may occur on the base of Gödel's theorems of incompleteness [20], in the extent in which they are important for HV "representation" (including the one mentioned above), they are also important for all other "representations" (including CQ "representation").

So, it is possible to construct a relatively consistent and relatively complete HV theory, which is in accordance with CQ in respect of determining probability of the events which have both HV and CQ "representatives", but not in the sense that the HV and CQ "representatives" of events belong to equivalent algebraic structures. Moreover, from the mentioned theorems of von Neumann, Jauch-Piron, Gleason, Kochen-Specker, follows, as Gudder [10] remarks, that a HV "representation" can not be realized in the framework of CQ "representation", and vice versa, so that these two "representations" are mutually different and non-reducable in the usual sense, although the CQ "representation", being relatively minimal, is in a way "placed" in a relatively non-minimal HV "representation". This situation is, to some extent, similar to the situation in differential geometry where every M dimensional Rieman's space of positively definite metrics can be "placed" in $N = \binom{M+1}{2}$ dimensional Euclid's space, although, as it is well known, Rieman and Euclid geometry are relatively consistent, but they are not isomorphyc. Thus, it is obvious, that the relation between HV and CQ "representations" can be completely adequately characterized, as a complementar in the sense of meta-epistemological generalized principle of complementarity of Bohr.

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[10] Gudder speaks: "It is felt by some that the paper of Zierler and Schlessinger, which considers imbedding of quantum proposition system into Boolean algebras, gives a proof of the impossibility of HV theories. However, HV theories do not suggest that quantum mechanics be imbedded in a classical structure. Similary, Kochen and Specker define an HV theory as an imbeding of the set of quantum observables into the set of dynamical variables on a phase space. Again, this is a claim that one can imbed quantum mechanics into a classical system, which is not what the HV proponents mean an HV theory to be." (See ref.[7], p. 432.)

3. Discussion

Immediate results of experimental empiry at the present level of exacteness of experimental analysis, are, by classical instrument measured values of observable physical quantities and probability characteristics of their occurance on the ensembles of the measured systems. Any relatively consistent and relatively complete correlation scheme of experimental results which determines even the "representation" of the notion of a physical event and its definition, represent a physical theory. [11] There are various meta-epistemologically complementary physical theories and accompaned to them interpretations of physical reality, [12] and the theory itself can not decide whether any of them is absolutely consistent and complete, and it is just *the unresoluteness* or *uncertainity* that represents the main characteristic of the meta-theoretical generalized principle of complementarity.

At the existing level of exacteness of experimental analysis, a *relatively minimal theory*, i.e. theory which correlates experimental data by a minimal number of theoretical constructs for minimal calculation time, has special importance for the analysis of correlations among empiric results. In this sense, an *actual physical theory* can, *conditionally*, mean *only a minimal* physical theory. Conditioned by experimental results and determined *unambiguosly* (without consideration the isomorphisms), a relatively minimal theory has the status of a *relatively objective theory of physical reality* i.e. a theory which is not an absolute "economy of thinking". This property of a minimal theory is best characterized by Bohr's metaphoric replica to Einstein at the Solvay's congres in 1927, that it is true that the good Lord (Nature) throws dice, but it is not our tash to tell the good Lord (Nature) how to rule the world. The importance of a minimal theory does not mean that non-minimal theories are not important.

[11] Such a determination of the notion of physical theory is met in many lectures of Bohr. See ref.[12], [21], [24], for example. Yet, Bohr did not think that a further theoretical or meta-theoretical generalization of the principle of complementarity in physics which are discussed in Herbut's *RC* or this one work respectively, was necessary, although he set a base for using of the principle of complementarity in many other fields of human knowledge and experience: biology, physiology, psychology, sociology, culture, etc. Bohr used the notion of a minimal theory only intuitively.

[12] Meta-theoretically interpreted from V. Pankovic Herbut's *RC* permits that, after rejecting absolute statements on collapse, i.e. measuring, Copenhagen, von Neumann's ortodox (see ref.[1], [27]), Everett's many world (see ref.[22]), Gudder's and the other relatively consistent "representations" be regarded meta-theoretically complementary.

From the experimental analysis point of view their importance is a *potential* one (in the same sense in which the observable properties of physical objects, when being measured, i.e. actualized of the complementary observables, in CQ "representation", regarded potential), while *from the point of view of further development of the theory itself* the existence of non-minimal theories is of *primary* importance. Namely, restriction to a minimal theory would be *too-restrictive* for development of a theory, so that we may even introduce the following important definition: *a mathematical theory represents a non-minimal physical theory.* From this definition, taking into consideration the obvious meta-theoretical complementarity of minimal and non-minimal physical theories, follows *the complementarity* of physics and mathematics, respectively. By William Occam "razor" (i.e. principle "entia non sunt multiplicanda preater necessitatem") defined and determined boundary between them, as well as between the classical measuring instrument and the quantum object in CQ "representation", *is not strict and not sharp, i.e. it is not absolute*, and *can be moved relatively*, but any discussion of both physical reality and mathematical theory would require its being placed somewhere and only after that one can speak approximatively about "pure" mathematics, or "pure" physics. In accordance with the mentioned definitions, it *can not be decided* theoretically whether the mathematical constructions have an absolute physical meaning, by which the fundamental premises of Plato's idealistic phylosophy are invalidated, while the mathematical constructions without the potential, i.e. non-minimal physical meaning, would be out of knowledge. But, even if the question of physical reality of mathematical constructions is put but the side for a moment, on the base of Gödel's theorems of incompleteness which have, in "pure" mathematics, *completely the same meaning* as Bohr's principle of complementarity (or Heisenberg's uncertainity relations) in "pure" physics, [13] it can be concluded that it is impossible to prove the absolute consistency or the absolute inconsistency of the mathematics using the means of mathematics itself. It is true that Gödel's theorems refer *only* to standard algebra (and to the isomorphic mathematical structures), but the

[13] In ref.[23], [8] are mentioned some works (which had the greatest influence on the author when he writing this paper) which emphasize the equivalency of Gödel's theorems of incompleteness and Bohr's complementarity principle (or Heisenberg's uncertainity relations). From the historical point of view it should be pointed out that the uncertainity relations and complementarity principle had been formulated in 1927, which means *before* Gödel's theorems which were formulated in the course of 1931 and later.

importance of standard algebra for the understanding of the results of mathematical theories is equivalent to the importance of classical physics for the understanding of the results of quantum mechanics in CQ "representation", so, based on this fact, the importance of Gödel's theorems can be extrapolated to the complete mathematics.

As a conclusion one can quote the words of Niels Bohr from his essay on the discussion with Albert Einstein: "The lesson we have hereby received would seem to have brough us a decesive step further in the neverending struggle for harmony between content and form, and taught us once again that no content can be grasped without a formal frame and that any form, however useful it has hitherto proved, may be found to be too narrow to comprehend new experience." [14] So, it is not possible to eliminate entirely "deep truth" (we consider here to Bohr's expression) [15] from the given minimal theory, what does not mean that it is impossible to construct such a non-minimal theory, meta-theoretically complementary to the first one, in which only "clear and simple truths" exist relatively. But, meta-theoretical complementarity of the two theories essentially means that the "deep truths" are actualized relatively at the level of a minimal meta-theory, what means not that it is not possible to construct such a non-minimal meta-theory, meta-meta-theoretically complementary to the first one, in which only "clear and simple meta-theoretical truths" exist relatively. But the meta-meta-theoretical complementarity of the above mentioned meta-theories, essentially means, that the "deep truths" are now relatively actualized at the level of minimal meta-meta-theory,. . . , and so ad infinitum [16] (see fig. 1).

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[14] See ref.[24], p.239.

[15] Ibid. The expression "deep truth" should be understood as unresolute, or uncertain statement, complementary to the expression "clear and simple truth" which denotes resolute or definite statements. The principle of complementarity is equivalent to the claim of the impossibility of absolute reducibility of "deep truths" to "clear and simple truths".

[16] For comparison with mathematics see ref.[25].

↑	⤧	↑
minimal meta-meta-theory which considers that different meta-theories are mutually complementary		non-minimal meta-meta-theories which prefer only one meta-theory as primary
↑	⤧	↑
minimal meta-theory which considers that different theories are mutually complementary		non-minimal meta-theories which prefer only one theory as primary
↑	⤧	↑
minimal theory which considers that different ontological concepts are mutually complementary		non-minimal theories which prefer only one ontological concept as primary

Figure 1. The infinite translation of complementarity from the level of the theory to level of meta-meta-. . . -meta-theory.

In that way the intention of absolute and unambiguosly formalization of quantum mechanics and physics has been disputed; the intention which originated from von Neumann's ortodox interpretation of quantum mechanics, and was taken uncritically over in many other quantum mechanics interpretations. It is interesting to remark, that von Neumann's attempt of absolute canonization of quantum mechanics is conveyed before the appearance of Gödel's incompleteness theorems, at the time when von Neumann believed firmly in absolute possibility of conveying of Hilbert's program of absolute formalization of mathematics. [17] Nowadays we know that it is impossible to convey Hilbert's program in mathematics absolutely and the same goes for von Neumann's program of absolute formalization of physics (neither is it possible to negate absolutely the existence of HV, nor is von Neumann-London-Bauer's absolute collapse theory of measurement [18] a minimal theory

[17] Compare von Neumann's work on mathematics, ref.[26], and quantum mechanics, ref.[1], in the period 1927-1931.
[18] See ref.[1], [27]

of quantum measurement). [19] Also von Neumann's foundating of quantum mechanics upon the theory of linear operators of infinite norm and Stieltje's integrals, is, for calculation application "more non-minimal" than Dirac's approach [28] accros δ function (von Neumann considers this Dirac's approach mathematical inadmissable). We, by no means, want to negate the importance of von Neumann's work for the proof of relative consistency of quantum mechanics in CQ "representation", but we are only pointing out the fact that, as Feynman [29] and many other physicist said, physics is not mathematics, and mathematics is not physics (and neither of them allows absolute formalization). They are complementary.

As the main point of the discussion let us mention the extensified paraphrase of the well-known thoughts of Bohr and Socratus; *in science one always aspiries for certainity, but is not certain that, what we certainly consider certain, is certain.*

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