A Model for a Quasi-ergodic Interpretation of Quantum Mechanics

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RESUME. Un système formé d'une particule soumise, d'une part, à une force déterministe et, d'autre part, à la force électromagnétique de freinage et à une série de très brèves impulsions électromagnétiques provenant du rayonnement des autres systèmes de l'univers, a un comportement quasi-ergodique, très analogue à celui prévu par la mécanique quantique lorsque la force déterministe agit seule. En particulier, on retrouve la relation de Heisenberg, l'équation de Schrödinger. Mais les valeurs des divers observables fluctuent, ce qui correspond à un formalisme de type quantique où les opérateurs sont complètement symétrisés par rapport à q et à p, les valeurs obtenues étant les moyennes temporelles sur des intervalles de temps suffisamment longs. Le spin apparaît au niveau relativiste comme un moment cinétique propre attaché à la particule. L'inégalité de Bell est remplacée par une plus faible, jamais violée, de sorte que le caractère non local attribué à la mécanique quantique orthodoxe disparaît. Le problème de la dualité onde-corpuscule est discuté.

ABSTRACT. A system constituted by a particle submitted, on the one hand, to a deterministic force, on the other, to the electromagnetic damping force and to very brief electromagnetic bursts arising from the radiation of the other systems of the universe, exhibits a quasi-ergodic behavior, very analogous to that which quantum mechanics foresees when the deterministic force acts alone. In particular, the Heisenberg relationship and the Schrödinger equation are found again. But the values of the various observables fluctuate, which corresponds to a quantum-like formalism where the operators are completely symmetrized with respect to q and p, the obtained values being the average values on sufficiently long time intervals. Spin appears at the relativistic level as an eigen kinetic momentum attached to the particle. The Bell inequality is replaced by a weaker one which is never violated, so that the non local character assigned *to the orthodox quantum mechanics is vanishing. The problem of the dual nature of particles is discussed.*

Introduction

If quantum mechanics is a remarkable formalism whose power cannot be reasonably contested, we must, nevertheless, recognize that its physical meaning remains rather obscure in spite of the various interpretations which have been proposed. Quantum mechanics, indeed, leads us to a description of the world completely different from that proposed by classical physics based on the macroscopic experiment. In particular, quantum mechanics imposes boundaries to our knowledge of the system, excluding the existence of hidden variables which would permit to know the behavior of the system with all the details, as classical physics does [1]. Furthermore, it questions the locality principle which is the basis of this latter [2].

The situation, after all, would not be very disturbing if, in practice, according to the problem under consideration, we were not constrained to use either quantum mechanics or classical physics, whereas, obviously, physics must be unitary. In order to conciliate both these concepts, we could guess that classical physics is a particular case of quantum mechanics when $\hbar$ (considered as a parameter) formally tends to zero, in the same way as newtonian mechanics corresponds to $v/c \to 0$ limit in the relativistic equations. Unfortunately, the problem is more complex. Independently of the fact that the $\hbar \to 0$ limit is not always very clear, the question of the hidden variables and that of the locality remain unsolved in such an approach. In an opposite way, we can try to reconstruct quantum mechanics from classical concepts. That is the case of various stochastic models which implicitely reject an a priori completeness of quantum mechanics, i.e. accept hidden variables, imputing the so-called quantum character to a more or less random cause [3]. Among these models, stochastic electrodynamics (SED) is certainly the most popular one [4]. This latter is based on the hypothesis that the system is submitted, independently of the deterministic forces which are acting on the particles which constitute it, to the classical electromagnetic damping force, and to a random omnipresent electromagnetic field.

Historically, this field has been introduced by Braffort, Spighel and Tzara [5]. For these authors this field would originate from the highly irregular motion of the atoms which, according to the absorbers theory of

Wheeler and Feynman, would create a remnant fluctuation field. This hypothesis has been subsequently adopted by several authors [6]. For his part, Marshall suggests that the field which is acting on a harmonic oscillator arises from all the other oscillators of the universe [7]. Nevertheless, in the majority of the works, the origin of this field is passed over in silence and even deliberately eliminated, as being "a teleological question comparable to inquiring after the origin of the matter in the universe"[Boyer, Ref.4].

Although the "double solution" theory of L. de Broglie [8] is slightly different, it can be considered as a stochastic model since it introduces the interaction with a quantum medium which affords the random element which is necessary to reproduce the quantum results. More precisely, this interaction would be responsible for the jumps between the "guidage" trajectories of the particles. But the physical nature of this medium remains well mysterious.

For our part, we have proposed to interpret the stability of atoms and molecules by the balance between the average energy radiated by the electrons in their motion around the nuclei, and the average energy arising from all the other systems of the universe [9]. In such a model, the universe is filled by an electromagnetic field whose physical nature is very clear and which would act on all the systems. Moreover, it has to be emphasize that the field and the organized matter (atoms and molecules) are strongly connected since, if the field is created by matter, this latter can exist through the field only.

Whatever the formal differences between these models may be, a common point has to be noted : The system is not isolated but unceasingly interacts with the surrounding medium. It is interesting to recall that in 1924, Slater wrote : "Any atom may, in fact, be supposed to communicate with the other atoms all the time it is in a stationary state, by means of a virtual field radiation"[10].

In the framework of these stochastic theories, in this paper, we will thoroughly study our assumption concerning the field which would fill the universe in order to see how it can introduce the random element which seems to be necessary, and whether it permits effectively to interpret the quantum formalism and to clear up the thorny question of the hidden variables and that of the locality.

Whatever that may be, before tackling the discussion, we would make our intent precise to rule out any misunderstanding. We do not search for an alternative theory to quantum mechanics, but only try to throw a bridge over the gap which separates quantum mechanics from classical concepts.

The model

Let us consider a particle of charge q and of mass m, submitted

i. to the deterministic force $\mathbf{F}(r)$,

ii. to the electromagnetic damping force $-\mathbf{f}$,

iii. to an electromagnetic field of components $\mathbf{E}$ and $\mathbf{H}$, designed to simulate the field created by all the other systems of the universe. (We will call this field, the universe field).

The general equation which governs the motion of the particle is the following

$$-\mathbf{f} + \dot{\mathbf{p}} = \mathbf{F}(\mathbf{r}) + q\mathbf{E} + q\frac{\dot{\mathbf{r}}}{c} \wedge \mathbf{H} \tag{1}$$

where $\mathbf{p}$ is the momentum of the particle.

Given the postulated origin of the universe field, on the analogy of the background noise in radioelectricity, we will assume this field as being constituted by a sequence of non-correlated very sudden and very brief *bursts*, travelling at the speed of light c, and oriented, in an isotropic manner (the universe is isotropic on a large scale) so that the average value on a sufficiently long time of this field along an arbitrary direction is equal to zero. (N.B.: The time-average value of the field within the duration of a burst is not assumed to be equal to zero).

On the other hand, in the general case, the expression of the damping force is complex, including both the derivates of $\mathbf{E}$ and $\mathbf{H}$, and terms of second degree in $\mathbf{E}$ and $\mathbf{H}$. Given the sudden variations of the field during the bursts, we can neglect these terms of the second degree with respect to those containing the derivatives so that the damping force reduces as follows [11]

$$\mathbf{f} = \frac{2q^3}{3mc^3}(1 - \frac{v^2}{c^2})^{-1/2}\{[\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla)]\mathbf{E} + \frac{\mathbf{v}}{c} \wedge [\frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla)]\mathbf{H}\} \tag{2}$$

Within the non relativistic approximation ($v \ll c$) it reduces to

$$\mathbf{f} = \frac{2q^3}{3mc^3}\,\dddot{\mathbf{r}} \tag{3}$$

At this level, the magnetic term can be neglected, as well as the $\mathbf{r}$ dependence of $\mathbf{E}$, so that Eq(1) reduces to

$$-\tau \dddot{\mathbf{r}} + \ddot{\mathbf{r}} = \frac{\mathbf{F}(r)}{m} + \frac{q}{m}\mathbf{E}(t) \tag{4}$$

with $\tau = 2q^2/3mc^3$, which appears as being a time characteristic of the particle ($\sim 6 \times 10^{-24} s$ for electron).

Study of the motion

The hypothesis we have made concerning the variation of the electric field, leads us to separate the contribution arising from $\mathbf{F}(r)$ from that arising from $\mathbf{E}$.

Let us assume that at $t = 0$, the particle is located at the point $M_0(r_0)$ with the speed $\dot{\mathbf{r}}_0$ and the acceleration $\ddot{\mathbf{r}}_0$. In the absence of the electric field, from M_0, the particle draws the arc $\mathcal{T}$ corresponding to the following equation

$$-\tau \dddot{\mathcal{T}} + \ddot{\mathcal{T}} = \frac{\mathbf{F}(r_0 + \mathcal{T})}{m} \tag{5}$$

with $\mathcal{T}_0 = 0$, $\dot{\mathcal{T}}_0 = \dot{\mathbf{r}}_0$, ...

Let us assume that a burst begins precisely at $t = 0$. The actual motion is governed by the following equation

$$-\tau(\dddot{\mathcal{T}} + \dddot{\mathcal{S}}) + (\ddot{\mathcal{T}} + \ddot{\mathcal{S}}) = \frac{\mathbf{F}(r_0 + \mathcal{T} + \mathcal{S})}{m} + \frac{q}{m}\mathbf{E} \tag{6}$$

($r(t) = r_0 + \mathcal{T} + \mathcal{S}$); $\mathcal{S}$ and its derivatives being equal to zero at $t = 0$. Given that the duration of the burst is very brief, to the second order in $\mathcal{T}$ and $\mathcal{S}$, according to (5), Eq.(6) reduces to

$$-\tau \dddot{\mathcal{S}} + \ddot{\mathcal{S}} - \mathcal{S}\frac{F'(r_0)}{m} = \frac{q}{m}\mathbf{E} \tag{7}$$

At the end of the burst, the particle is located at $M_1(r_1 = r_0 + \delta\mathcal{T} + \delta\mathcal{S})$ with the speed $\dot{\mathbf{r}}_0 + \delta\dot{\mathcal{T}} + \delta\dot{\mathcal{S}}$, and with the acceleration $\ddot{\mathbf{r}}_0 + \delta\ddot{\mathcal{T}} + \delta\ddot{\mathcal{S}}$, $\delta\mathcal{T}$, $\delta\mathcal{S}$ and its derivatives being the corresponding variations given by Eqs(5) and (7). From this point M_1 and the new initial conditions, the process starts again with the following burst. Consequently, under the conjugated effect of the force F and the electric field, the particle draws a trajectory Γ, defined by Eq(4) and which can be reconstructed, burst by burst, from *jumps* $\mathcal{S}$ (7) between arcs $\mathcal{T}$ (5).

The equation (7) which governs a given jump can be written as follows

$$-\tau \dddot{\mathcal{S}} + \ddot{\mathcal{S}} + \omega^2 \mathcal{S} = \frac{q}{m}\mathbf{E} \quad , \quad (\omega^2 >< 0) \tag{8}$$

ω being a suitable "frequency" depending on the jump under consideration. Let us assume that the duration τ_0 of the burst is small with respect to the characteristic time τ, and that, after a variation very sudden with respect to the duration of the burst, the field $\mathbf{E}$ is very quickly damped. For instance, we can imagine the following variation which can be considered as the limit of an extremely sudden one

$$\mathbf{E}\begin{cases} 0 & t < 0 \\ \mathbf{E}_0 & t = 0 \\ \mathbf{E}_0 \exp(-\alpha t) & t > 0 \quad \text{with} \quad \alpha\tau_0 \gg 1 \end{cases} \tag{9}$$

Given these hypotheses, we will use the Heaviside method, based on the Laplace transformation. As it is easy to verify, neglecting the terms in $\omega\tau$ on and after the second order ($\omega\tau$, indeed, is very small given the value of τ and the order of magnitude of ω in the physical systems) we obtain the displacement $\delta\mathcal{S}$ and the variation $\delta\dot{\mathcal{S}}$ of the speed for the *whole burst* under consideration

$$\begin{cases} \delta\mathcal{S} = -\dfrac{q\tau_0}{m}\displaystyle\int_0^{\tau_0} \mathbf{E}dt \\ \delta\dot{\mathcal{S}} = -\dfrac{q\tau_0}{m\tau}\displaystyle\int_0^{\tau_0} (1 - \dfrac{t}{\tau_0})\mathbf{E}dt \sim -\dfrac{q\tau_0}{m\tau}\displaystyle\int_0^{\tau_0} \mathbf{E}dt \end{cases} \tag{10}$$

From Eqs(10) it results that the product $\delta\mathcal{S}_u.\delta(m\dot{\mathcal{S}}_u)$ for each component ($u = x, y, z$) is *independent* of the nature of the particle ($\tau \approx q^2/m$). On the other hand, in the average, on a sufficiently long time, we will put

$$\overline{\delta\mathcal{S}_x}.\overline{\delta(m\dot{\mathcal{S}}_x)} = \ldots = K \tag{11}$$

At last, we will remark that on each burst, we have

$$\delta\mathcal{S} = \tau\delta\dot{\mathcal{S}} \tag{12}$$

First consequences

It results from what precedes that, starting from an arbitrary point M_0 of the space $\{\mathbf{r}\}$, the particle, under the effect of the successive bursts, draws a very complicate trajectory Γ which, under the condition (we assume to be realized) that the system does not disintegrate, passes in the neighborhood of its starting point again. Given this latter is arbitrary, the trajectory Γ passes in the vicinity of all (or almost all) points of the space whatever the initial conditions may be. Hence the trajectory Γ fills the space completely and can be considered as being closed.

The loss of memory of the initial conditions which results from the geometrical structure of Γ entails that, after a sufficiently long time, all the possible trajectories Γ are equivalent. Consequently, to obtain the average value of a given dynamical property G, it is sufficient to consider *one* arbitrary trajectory Γ, the average value $\overline{G}$ becoming stable from a sufficiently long time. This is typically the character of a *quasi-ergodic* process [12] which involves the existence of a probability density $\rho(\mathbf{r})$ in the space $\{\mathbf{r}\}$.

Another consequence is the absence of *first integrals*. Indeed, if a property G was constant on a trajectory Γ, it would be constant at all the points of space, which would be absurd. Consequently, all the dynamical properties do fluctuate versus time around a well determined average value $\overline{G}$. This is true for energy, in particular.

At last, as it is easy to see

$$\overline{G(-t)} = \overline{G(t)} \tag{13}$$

so that the average value of an odd function of t is equal to zero (e.g. $\dot{\mathbf{r}}$, $\dddot{\mathbf{r}}$).

Energy balance

Let us multiply Eq(4) by $\dot{\mathbf{r}}$ and calculate the corresponding average value on an arbitrary trajectory Γ. We obtain

$$-m\tau\overline{\dddot{\mathbf{r}}\;\dot{\mathbf{r}}} + m\overline{\ddot{\mathbf{r}}\dot{\mathbf{r}}} = \overline{\mathbf{F}\dot{\mathbf{r}}} + q\overline{\dot{\mathbf{r}}\mathbf{E}} \tag{14}$$

i.e.

$$m\tau\overline{\ddot{\mathbf{r}}^2} = q\overline{\dot{\mathbf{r}}\mathbf{E}} \quad \text{or} \quad 2q\overline{\ddot{\mathbf{r}}^2} = 3c^3\overline{\dot{\mathbf{r}}\mathbf{E}} \tag{15}$$

The quantity $m\tau\overline{\ddot{\mathbf{r}}^2} = -m\tau\overline{\dddot{\mathbf{r}}\ \dot{\mathbf{r}}}$ is the average radiated power and $q\overline{\dot{\mathbf{r}}\mathbf{E}}$, the average absorbed power. This relationship (14) shows the correlation which exists between $\dot{\mathbf{r}}$ and $\mathbf{E}$ through the damping force. This result is general. In particular, it is valid in SED [13].

The virial theorem

Let us multiply Eq(4) by r and calculate the corresponding average value. We obtain

$$-m\tau\overline{\dddot{\mathbf{r}}\ \mathbf{r}} + m\overline{\ddot{\mathbf{r}}\mathbf{r}} = \overline{\mathbf{r}\mathbf{F}} + q\overline{\mathbf{r}\mathbf{E}} \tag{16}$$

On the one hand, $\overline{\dddot{\mathbf{r}}\ \mathbf{r}} = 0$ and $\overline{\ddot{\mathbf{r}}\mathbf{r}} = -\overline{\dot{\mathbf{r}}^2}$. On the other, along Γ, $\oint \mathbf{r}\mathbf{E}dt$ is equal to the flux of $\mathbf{E}$ through a surface delimited by Γ, i.e. to zero given the geometrical structure of Γ. From which it results the well-known virial relationship

$$m\overline{\dot{\mathbf{r}}^2} + \overline{\mathbf{r}\mathbf{F}} = 0 \tag{17}$$

Effect of a virtual deformation of the trajectory

Let us consider a virtual trajectory $\tilde{\Gamma}$ obtained from an actual trajectory Γ by a very small arbitrary modification of the position and of the speed

$$\tilde{\mathbf{r}} = \mathbf{r} + \eta \tag{18}$$

with $|\eta| < \epsilon\sqrt{\overline{\mathbf{r}^2}}$ and $|\dot{\eta}| < \epsilon\sqrt{\overline{\dot{\mathbf{r}}^2}}$, ϵ being an infinitely small of the first order.

The average energy on Γ is the following

$$\overline{E} = \frac{1}{2}m\overline{\dot{\mathbf{r}}^2} + \overline{U(\mathbf{r})} \tag{19}$$

$U(\mathbf{r})$ being the potential energy. On the perturbed trajectory, this energy is equal to

$$\tilde{E} = \frac{1}{2}m\overline{(\dot{\mathbf{r}} + \dot{\eta})^2} + \overline{U(\mathbf{r} + \eta)} \tag{20}$$

Thus, to the second order of the perturbation

$$\tilde{E} - \overline{E} = \frac{1}{2}m\overline{\dot{\eta}^2} + \frac{1}{2}\sum_u\sum_v \overline{\frac{\partial^2 U}{\partial u \partial v}} \cdot \overline{\eta_u \eta_v} \tag{21}$$

($u = x, y, z$), which shows that the average energy is stationary with respect to any variation of the trajectory.

The free particle

Let us consider a free particle being at rest at $t = 0$, and compelled to moving on a staight line x. Such a particle begins to move under the effect of the electric field. As it is easy to see, after the n^{th} burst, the particle is at the point

$$X_n = \delta x_1 + \ldots \delta x_n + \tau_0[(n-1)\delta\dot{x}_1 + (n-2)\delta\dot{x}_2 + \ldots + \delta\dot{x}_{n-1}] \quad (22)$$

(δx and $\delta\dot{x}$ being the variations on the burst under consideration).

The bursts being not correlated with one another, for a sufficiently long time $(n \to \infty) X_n^2$ behaves as

$$\tau_0 n^2 \overline{\delta x \delta\dot{x}} = \tau_0 n^2 K/m \quad (23)$$

Owing to the isotropic character of the electric field, the average value of X_n is equal to zero, so that, taking Eq(11) into account, the quadratic dispersion is the following

$$(\Delta X)^2 = \frac{t^2 K}{3m\tau_0} \quad (24)$$

with $t = n\tau_0$. This dispersion tends to infinity when $t \to \infty$. This result is not surprising. Indeed, burst by burst, the particle will reach any point of x with a probability tending to the same value for each point, so that the dispersion becomes infinite.

The charged harmonic oscillator

We can write the corresponding equation as follows

$$-\frac{d^3x}{d\theta^3} + \frac{d^2x}{d\theta^2} + \omega^2\tau^2 x = \frac{q}{m}\tau^2 \mathbf{E}_x \quad (25)$$

with $\theta = t/\tau$. This equation shows that the solution is a function of the product $\omega\tau$. On the other hand, the displacement on each burst is proportional to δx (9), so that we have

$$\overline{x^2} = \overline{\delta x^2} \sum_{-\infty}^{+\infty} a_n (\omega\tau)^{-n} \quad (26)$$

the coefficients a_n being independent of m and q owing to the fact that they are dimensionless.

According to relationship (11), and neglecting the terms in $(\omega\tau)^k$ when $k \geq 0(\omega\tau \ll 1)$, this dispersion reduces as follows

$$\overline{x^2} = \frac{K}{m\omega}[a_1 + \frac{a_2}{\omega\tau} + \ldots] \tag{27}$$

Thus, according to the virial theorem, the average energy is the following

$$\overline{E} = m\omega^2\overline{x^2} = K\omega[a_1 + \frac{a_2}{\omega\tau} + \ldots] \tag{28}$$

If $m \to \infty$, $\tau \to 0$: The oscillator is at rest, its energy is equal to zero. This involves that $a_2 = a_3 = \ldots = 0$, i.e.

$$\overline{E} = Ka_1\omega = K'\omega \tag{29}$$

At last, according to the virial theorem between the quadratic dispersions Δx and Δp_x, we have the following relationship

$$(\Delta x)^2(\Delta p_x)^2 = \overline{x^2}.\overline{p_x^2} = {K'}^2 \tag{30}$$

Remark on the nature of the particles

Up to now we have considered structureless point charges, e.g. electrons, muons, quarks. Let us examine the case of complex "particles", constituted by a rigid set of point charges, e.g. neutrons, nuclei or molecules. For each constituent particles i we have

$$-\tau_i \dddot{\mathbf{r}}_i + \ddot{\mathbf{r}}_i = \frac{\mathbf{F}(\mathbf{r}_i)}{m_i} + \frac{q_i}{m_i}\mathbf{E} + \phi_i \tag{31}$$

ϕ_i being the internal force which is acting on i.

The whole of the particles being rigid, by adding we obtain

$$-\overline{\tau} \dddot{\mathbf{r}} + \ddot{\mathbf{r}} = \frac{\mathbf{F}(r)}{\mu} + \overline{(\frac{q}{m})}\mathbf{E} \tag{32}$$

where $\overline{\tau}$ is the average value of the τ_i's and μ^{-1} that of the m_i^{-1}'s. This equation typically corresponds to the motion of a *fictitious particle* of mass m' and of charge q', such as $\overline{\tau} \sim {q'}^2/m'$, i.e.

$$q' = \frac{\sum q_i^2/m_i}{\sum q_i/m_i} \tag{33}$$

Consequently, even if $\sum q_i = 0$, in the general case, q' is different from zero, so that, formally, our model can be applied to any "particle", with the same constant K (12). In particular, the expression of the average energy of any harmonic oscillator (29) remains unchanged.

Transcription into an operator formalism

Given the complexity of the dynamics of the model, we can search for a global formalism which would allow us to obtain the average values G directly, renouncing the detailed knowledge of the motion.

In this view, by definition, we will put

$$\overline{G} = \int \psi^* \hat{G} \psi dv = < \psi \hat{G} \psi > \tag{34}$$

where $\hat{G}$ is a linear operator associated with the property G, ψ a space function such as the square of its modulus is equal to the density $\rho(\mathbf{r})$ in the space $\{\mathbf{r}\}$

$$\psi^* \psi = \rho(\mathbf{r}) \tag{35}$$

This involves that ψ is normalized.

$\overline{G}$ being real, it is necessary and sufficient that $\hat{G}$ be *hermitian.* For a property G depending on $\mathbf{r}$ only, we can put

$$\hat{G} = G \tag{36}$$

Indeed,

$$\overline{G} = \int G(\mathbf{r})\rho(\mathbf{r})dv = \int \psi^* G(\mathbf{r})\psi dv$$

More generally, we will put

$$\hat{G}(x, p_x, \ldots) = G(x, \hat{p}_x, \ldots) \tag{37}$$

$\hat{p}_x, \ldots$, being the operators associated with $p_x, \ldots$, respectively. These operators must be invariant under a translation (in particular the speed and the kinetic energy must be unchanged), so that they are of the following form

$$\hat{p}_u = \sum_{(k>0)} C_k \frac{\partial^k}{\partial u^k} \quad , \quad (u = x, y, z) \tag{38}$$

On the other hand, given $p_u(-u) = -p_u(u)$ for the classical properties as well as for the corresponding operators, k must be odd. Practically, we will limit the development (38) to the first order. Operator $\hat{p}_u$ being hermitian, we will put

$$\hat{p}_u = \frac{C}{i}\frac{\partial}{\partial u} \tag{39}$$

Thus, the average energy appears as follows

$$\overline{E} =< \psi| - \frac{C^2}{2m}\nabla^2 + U|\psi >\equiv< \psi\hat{H}\psi > \tag{40}$$

The energy must be stationary with respect to any variation of the trajectory Γ, therefore to that of ψ. This entails that ψ is an eigenfunction of the operator $\hat{H}$ defined by Eq(40), i.e. that we have

$$\hat{H}\psi = \overline{E}\psi \tag{41}$$

By integrating this equation for the harmonic oscillator, we see that

$$C = 2K' \tag{42}$$

The excited states

Let us assume that besides the deterministic force $\mathbf{F}$, the system is submitted to a monochromatic radiation. Owing to the energy exchange between the system and the rest of the universe, the system reaches an equilibrium state which –a priori– must be different from that in the absence of the radiation. As it is easy to verify, for the currently used radiations the contribution of the radiation to the product $\Delta\mathcal{S}_x\Delta m\dot{\mathcal{S}}_x$ (11) is completely negligible with respect to that which arises from the universe field. Consequently, the constants K and C we have introduced

(11,39) are unchanged. Moreover, the general results we have previously obtained remain valid, in particular the energy is stationary with respect to any deformation of the new trajectory Γ', so that Eq(41) applies to the new state of the system. In other words, the average energy of a system submitted to a monochromatic radiation cannot exhibit an arbitrary value but it must necessarily be equal to one of the eigenvalues of the operator $\hat{H}$ (40).

An important point has to be emphasized. An excited state is stable only in the presence of the radiation. When this latter is switched off, the system loses the supplementary energy which the radiation brought to it and falls down again into its fundamental state.

Relationship between the transition energy and the absorbed frequency

The passing from the ground state to an excited state physically corresponds to the absorption of a certain quantity of energy ΔE which is given up by the absorbed radiation. When the frequency ω of this latter tends to zero, the energy carried by the radiation tends to zero, so that ΔE tends also to zero. From which it results that

$$\Delta E = f(\omega) = a_1\omega + a_2\omega^2 + \ldots \tag{43}$$

Intuitively, it is reasonable to think that two systems for which the ΔE's are the same, absorb the same frequency. Under this condition, $f(\omega)$ appears as being a universal function. In order to determine this latter we will use the result concerning the harmonic oscillator, which can be obtained by integrating (41)

$$\Delta E = 2K'\omega \tag{44}$$

Wave associated to a particle

Let us consider a particle of mass m. Let $V(\ll c)$ be its speed at $t = 0$. Owing to the universe field, its actual motion is uniform in the average only :

$$x = Vt + \xi \tag{45}$$

ξ being the fluctuation arising from the universe field around the average trajectory defined by $x = Vt$.

For small displacements ξ, the force acting on the particle can be assimilated to that of an harmonic oscillator of frequency ω. According to (29), the *total* energy of the particle is equal to $(mV^2)/2 + K'\omega$.

On the other hand, let us consider an electromagnetic radiation of the same frequency ω. This radiation carries a "usable" energy equal to $2K'\omega$ (44). (This energy, for instance, can be absorbed by an atom).

By identifying this energy with that carried by the particle, we obtain

$$\lambda = \frac{4\pi K'}{mV} \tag{46}$$

This relationship shows that, energetically speaking, a particle of mass m and of speed V, is equivalent to a wave whose wave-length λ is given by (46). It is to be emphasized that λ is independent of the charge of the particle.

Very recently, Surdin [6] has proposed a proof of this relationship based on the fluctuating zero point field of SED, which, in spite of certain analogies with our calculation, is different in the principle.

Let us now consider a diffraction experiment, say the Young holes to fix the ideas. The presence of the screen and of its holes provoks a local perturbation for the universe field. Owing to the reflections of the electromagnetic waves which constitute this field an interference system appears for *each* frequency of the field. Let us assume that the particle passes through *one* of the holes. By resonance, the energy exchanged with the universe field is maximal when the frequency of the fluctuation ξ is equal to that of the field. Consequently, given that the wave-length associated to the particle is well determined, this latter will be moving along a trajectory corresponding to a maximal amplitude of the interference system of wave-length equal to λ. Owing to the fluctuations of the universe field, a set of a large number of particles, randomly passing through the one *or* the other of the holes, after a sufficiently long time, will give the same interference pattern as a radiation of wave-length equal to λ (46).

This example clearly shows the purely mathematical nature of the wave associated to the particles, these latters, all things considered, playing the role of detectors of a pre-existent field.

The eigen kinetic momentum of the electron

The results we have up to now obtained have been deduced from the nonrelativistic Eq(4). Let us examine what happens when the relativistic effects are taken into account. In this view, we will write this equation over again without neglecting the $v/c = \beta$ terms, as follows

$$-\mathbf{f} + \dot{\mathbf{p}} = \mathbf{f}(\mathbf{r}) + q\mathbf{E}(t - \frac{\mathbf{n}\cdot\mathbf{r}}{c}) + q\frac{\dot{\mathbf{r}}}{c} \wedge \mathbf{H}(t - \frac{\mathbf{n}\cdot\mathbf{r}}{c}) \tag{47}$$

with

$$\mathbf{f} = \tau(1 - \frac{\dot{\mathbf{r}}\cdot\mathbf{n}}{c})(1-\beta^2)^{-1/2}[\ddot{p} - q\frac{\ddot{\mathbf{r}}}{c} \wedge \mathbf{H}]$$

$$\ddot{p} = m(1-\beta^2)^{-3/2}[\dddot{\mathbf{r}} + 3\frac{\dot{\mathbf{r}}}{c}\ddot{\mathbf{r}}^2]$$

Let us put

$$\mathbf{r} = \rho + \mathbf{R} \tag{48}$$

ρ corresponding to the trajectory Γ, i.e. to the following equation

$$-\tau\dddot{\rho} + \ddot{\rho} = \mathbf{F}(\rho) + \frac{q}{m}\mathbf{E}(t - \frac{\mathbf{n}\cdot\rho}{c}) \tag{49}$$

By substracting Eq(49) from Eq(47), to the first order in τ, and by neglecting R^2 terms and $(\mathbf{R}-\rho)$ coupling terms whose average values are equal to zero, we obtain

$$\begin{aligned} -\tau(1 - \frac{\dot{R}_n}{c} - \frac{\dot{R}_n^2}{c^2}) + \ldots)\,\dddot{\mathbf{R}} + (1 + \frac{3}{2}\frac{\dot{R}^2}{c^2} + \ldots)\ddot{\mathbf{R}} \\ = \frac{q}{mc}R_n\dot{\mathbf{E}}(t) + \frac{q}{m}\frac{\dot{\mathbf{R}}}{c} \wedge \mathbf{H}(t) + \frac{q}{m}\frac{R_n}{c^2}\ddot{\mathbf{R}} \wedge \dot{\mathbf{H}}(t) \quad , \end{aligned} \tag{50}$$

$(\dot{R}_n = \mathbf{R}\cdot\mathbf{n})$.

Owing to the fact that $\dot{\rho} \ll c$, this equation describes the motion of the particle within a frame attached to it on the average trajectory Γ, under the condition to choose the origin such as $\overline{R} = 0$. If the fields $\mathbf{H}$ and $\mathbf{E}$ were constant during the burst, the particle would drawn a helix whose axis is parallel to $\mathbf{H}$. In the actual case, the fields varying both in direction and in modulus, the motion is more complex. The particle draws a trajectory Z around its centroid ($\overline{R} = 0$), similar to a tangled ball of string.

In order to make the corresponding kinetic momentum σ explicit, let us multiply Eq(50) by $\mathbf{R}\wedge$. To the second order in $\dot{R}/c$ and neglecting the coupling terms between σ and $\mathbf{R}$ (i.e. assuming that the direction of σ varies very slowly with respect to that of $\mathbf{R}$), we obtain

$$-\tau\ddot{\sigma} + \dot{\sigma} + \omega^2\tau\sigma = \frac{q}{mc}\sigma \wedge \mathbf{H} \tag{51}$$

with $\omega = q\mathbf{H}/mc$. The terms in $\dot{\mathbf{E}}$ and in $\dot{\mathbf{H}}$ being of the second order in $\mathbf{R}$ are negligible.

Given that $\dot{\rho} \ll c$, this equation is valid both within the at rest frame and the moving one, so that everything occurs as if the particle carried an eigen kinetic momentum, independently of its orbital momentum corresponding to the motion on Γ.

The electromagnetic field of the universe varying both in modulus and direction, this momentum appears as exhibiting the following form

$$\sigma = \mathbf{N}f(t) \tag{52}$$

$\mathbf{N}$ being a unitary vector whose direction varies in a random manner versus time, and $f(t)$ a function of time (not correlated with the direction of $\mathbf{N}$).

Given the isotropic properties of the universe field, the component of σ along an arbitrary direction z is equal to zero

$$\overline{\sigma_z} = \overline{f(t)}.\overline{\cos\theta} = 0 \tag{53}$$

with $\theta = (\sigma, z)$. Moreover

$$\overline{\sigma_z^2} = \frac{1}{3}\overline{f^2} \quad \text{and} \quad \overline{\sigma^2} = \overline{f^2} \tag{54}$$

The equation (46) which governs the motion on Z presents strong analogies with Eq(8), so that the displacement is proportional to q/m and the momentum to q/τ. On the other hand, the moduli of the electric and magnetic fields are equal, so that the eigen kinetic momentum σ is proportional to the constant K of Eq(11). This conclusion remains valid if we take the complete development of Eq(50) with respect to β into account. Consequently, the function $f(t)$ depends on the average speed of the particle. From Eq(9) it results that the kinetic energy exhibits the following form : $(m/q^2) \cdot (1 +$ function of $\overline{\beta})$. Now this energy can be written $m\overline{\beta^2}(1 +$ function of $\overline{\beta})$. So that β appears as being a function of q^2 only. In other words, $f(t)$ depends on the square of the charge and not on the mass. Electron and muon exhibit the same eigen kinetic momentum [14].

Effect of a magnetic field

Let us assume that the system is immersed within a constant magnetic field $\mathbf{H}_0$ whose modulus is much greater than that of $\mathbf{H}$. At the nonrelativistic approximation, the motion on Γ is unchanged. The effect of this field appears only at the relativistic level, i.e. on the trajectory Z.

Let us multiply Eq(51) in which $\mathbf{H}$ is replaced by $\mathbf{H}+\mathbf{H}_0$, on the one hand, by $\mathbf{N}$ (52), and, on the other, by a unitary vector oriented along $\mathbf{H}_0$. By difference between the two obtained equations, we get

$$-\tau\ddot{\theta} + \theta(1 - 2\tau\frac{\dot{f}}{f}) + \tau\dot{\phi}^2 \sin\theta\cos\theta = \omega_y \cos\phi - \omega_x \sin\phi \tag{55}$$

where θ and ϕ are the polar angles defined with respect to $\mathbf{H}_0$ (oriented along the z-axis), and $\omega_u = q\mathbf{H}_u/mc(u = x, y, z)$, $\dot{\phi}^2 = (\omega_0 + \omega_z)^2$ with $\omega_0 = q\mathbf{H}_0/mc$.

Neglecting the terms in $\omega_0\tau$ on the one hand, and the ratios ω_x/ω and $\omega_y/\omega(\mathbf{H}_0 \gg \mathbf{H})$ on the other, it follows

$$-\tau\ddot{\theta} + \dot{\theta}(1 - 2\tau\dot{f}/f) + \tau\omega_0^2 \sin\theta\cos\theta = 0 \tag{56}$$

The fact that $\dot{\theta} \sim \tau\omega_0^2 \sin\theta\cos\theta$ entails $\ddot{\theta} \sim \tau\omega_0^2\dot{\theta}$, so that $\tau|\ddot{\theta}| \ll |\dot{\theta}|$. Finally, Eq(56) reduces to

$$\dot{\theta}(1 - 2\tau\frac{\dot{f}}{f}) + \tau\omega_0^2 \sin\theta\cos\theta = 0 \tag{57}$$

i.e.

$$\operatorname{tg}\theta = \operatorname{tg}\theta_0 \exp[-\omega_0^2\tau \int_0^t \frac{dt}{1 - 2\tau\dot{f}/f}] \tag{58}$$

Now we have seen that f fluctuates. In order to obtain the order of magnitude of the variation rate of θ, we will assume that $f \sim \sin^2 \alpha t$, α being a certain frequency such as $\alpha\tau \ll 1$. Hence

$$\operatorname{tg}\theta \sim \operatorname{tg}\theta_0 \exp(-\omega_0 t/4\alpha) \tag{59}$$

This relationship shows that σ orients itself along $\mathbf{H}_0$ if $0 < \theta_0 < \pi/2$ and along $-\mathbf{H}_0$ if $\pi/2 < \theta_0 < \pi$. On the other hand, the alignment is very quickly reached : In a 10^4 gauss field, if $\alpha \sim 10^{16} s^{-1}$ (i.e. $\alpha\tau \sim 10^{-7}$), $10^{-5}s$ is sufficient to pass from $\theta = 85°$ to $1°$. If f was constant, $70s$ would be necessary! These results explain the phenomenon observed by Stern and Gerlach. Indeed, according to the value of $\theta_0 (>< \pi/2)$, the magnetic momentum carried by the valence electron of Ag atoms (arising from the eigen kinetic momentum) provoks the splitting of the Ag-beam into two ones in the non-homogeneous field whose gradient is perpendicular to the direction of the beam. Given that the alignment time is smaller than the fly-time of the atoms in the Stern and Gerlach device ($\sim 10^{-3}s$) two spots are observed.

We can guess that the taking into account of the superior β-terms keeps this conclusion unchanged.

Comparison with quantum mechanics

The model we propose leads us to results which are both consistent and inconsistent with the orthodox quantum theory.

On the one hand, indeed, the system exhibits a behavior quite analogous to that which quantum mechanics foresees for a particle submitted to the deterministic force only. Among the convergence points we will remind the density of probability, the Heisenberg relationship (30), the energy of the harmonic oscillator (29), the de Broglie relationship (46) concerning the dual nature of particles (if we put $K' = \hbar/2$), the spreading of a wave packet, proportional to t (24), the existence of discrete excited states, and an equation which coincides with that of Schrödinger. At last the spin directly appears as an eigen kinetic momentum (proportional to K') attached to the particle (with $\overline{f} = \hbar/2$ and $\overline{f^2} = 3\hbar^2/4$).

But, on the other hand, a fundamental difference between quantum mechanics and our model appears concerning the meaning of the values obtained for the dynamical properties of the system. In our model only average values are obtained. No property can be measured without dispersion. This results from the fact that the system unceasingly exchanges energy with the rest of the universe, while quantum mechanics considers the system as being isolated. Consequently, strictly speaking, the model we propose is not equivalent to quantum mechanics.

In fact, the situation is more complex. It is, indeed, well-known that the construction rule of quantum operators is equivocal when products

$x^n p_x^n$ appear, e.g. in the calculation of the quadratic dispersion of the various properties G (energy in particular). The conventional quantum formalism based on the axiom $op(A^2) = (opA)^2$ leads in these cases to difficulties, in particular more than one operator can correspond to the same property (Temple's paradox [15]). On the contrary, the complete symmetrization of A^2 with respect to $x, p_x, \ldots$ affords a unique operator for the corresponding property. But the values of the various dynamical properties exhibit dispersions different from zero, i.e. fluctuate. In other terms, our model would correspond to a non-conventional quantum formalism, based on the complete symmetrization of the operators.

In a previous paper [16], we have discussed this problem and shown that, finally, no chief difficulty exists to accept this new formalism from the moment when we assume that the stability of the average values $\overline{G}$ of the various observables is reached after a time inferior to that of the measurement. In electron systems, this minimal time (ergodicity time) is too small to be measurable ($ca.10^{-17}s$) so that experiment gives $\overline{G}$ directly. But this time can become much greater and the dispersion of G can be detected, e.g. for the inversion of pyramidal molecules where it can reach one year as in arsines [17]. Moreover, the fluctuation in energy allows the system to *jump* over potential barrier (whatever its height may be) according to a classical process without being necessary to invok the tunnelling effect whose quantum character appears as a pure artefact of the formalism.

The problem of the hidden variables

An objection, nevertheless, can be made to our interpretation. Our model, indeed, is a hidden variable one. Now von Neumann's theorem and the works of Bell [1] show that such a theory cannot reproduce the quantum results. In fact, the proofs of these theorems are explicitely based on the measurement axiom according to which we obtain dispersionfree eigenvalues only. This constraint does not appear in our model, so that these theorems cannot be objected to it.

In order to see how the problem occurs in our model, we will go back to the proof of the Bell inequality to make apparent the difference between the orthodox formalism and our interpretation.

Let us consider two particles A and B whose spins are opposite (singulet state). Let $P(a, b)$ be the correlation coefficient corresponding to the measurements of the spins of A and B on the directions a

and b respectively, the measurement results being governed by hidden variables.

Given that the spins are opposite, we have

$$P(a,b) = \overline{A_a B_b} = -\overline{A_a A_b} \tag{60}$$

Quantum mechanics foresees that

$$P(a,b) = -\cos(a,b) \tag{61}$$

Let us introduce another direction c. We can write

$$P(a,b) - P(a,c) = -\overline{A_a A_b} + \overline{A_a A_c} = -\overline{A_a A_b[1 - \frac{A_c A_b}{A_b^2}]} \tag{62}$$

Within the orthodox quantum formalism, A_a and A_b are equal to ± 1, so that, using the relationship $|\overline{xy}| \leq |x|_{max}\overline{|y|}$, we obtain

$$|P(a,b) - P(a,c)| \leq |A_a A_b|_{max}\overline{|1 + A_b B_c|} = 1 + P(b,c) \tag{63}$$

i.e.

$$|\cos(a,b) - \cos(a,c)| \leq 1 - \cos(b,c) \tag{64}$$

That is the Bell inequality which is violated for certain directions a, b, c. From which it is concluded that a hidden variable model cannot reproduce quantum formalism. A corollary of this violating is that quantum mechanics does exhibit a non local character. This latter point is very disturbing because it is inconsistent with the ground hypothesis of relativity, namely that no signal does travel with a speed greater than that of light.

In our model, to keep the values normalized, we will put

$$A_\alpha = \frac{f \cos\alpha}{\tilde{s}} \tag{65}$$

with $\alpha = (\sigma, a)$, and the analogous β and γ for the directions b and c, and with

$$\tilde{s}^2 = \overline{f^2}.\overline{\cos^2\alpha} = \frac{1}{3}\overline{f^2} \tag{66}$$

From which it results

$$|A_a A_b|_{max} = \frac{f^2_{max}}{\tilde{s}^2} > \frac{\overline{f^2}}{\tilde{s}^2} = 3 \tag{67}$$

Moreover

$$\overline{|1-\frac{A_c}{A_b}|}=\overline{|1-\frac{f_c}{f_b}\frac{\cos\gamma}{\cos\beta}|}>|1-(\frac{\overline{f_c}}{f_b})(\frac{\overline{\cos\gamma}}{\cos\beta})|=1 \tag{68}$$

f_c and f_b being the values of f at the time of the measurements along c and b respectively. Hence

$$|A_aA_b|_{max}\overline{|1-A_c/A_b|}<3 \tag{69}$$

From which it results the following inequality

$$|P(a,b)-P(a,c)|<3 \tag{70}$$

which is obviously never violated. Indeed, as it is easy to verify, relationship (61) remains valid.

Consequently, the non local character assigned to quantum mechanics appears as being the simple consequence of the too restrictive measurement axiom rather than the reflect of any physical reality.

To conclude, we will say that the introduction of an unceasing energy exchange between all the systems of the universe is a plausible hypothesis which permits to construct a classical-like interpretation of quantum mechanics, although, in the practice, the quantum formalism remains an irreplaceable mathematical tool... at the present time, at least.

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