On the true meaning of 'maximal parity violation': ordinary mirror symmetry regained from 'CP symmetry'

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ABSTRACT. The 'maximal parity violation' effect is revisited, in the light of a previously proposed Dirac-field-theory reformulation which naturally includes massive (besides massless) chiral fields, 'left-handed' for fermions and 'right-handed' for antifermions. A generalized chiral-field approach needing no *ad hoc* assumption (such as '$V - A$' prescription) is looked into, that further makes 'CP symmetry' simply reducible to P symmetry and automatically solves the question of the 'missing' (right-handed) neutrino. The meaning and the physical self-consistency of such *natural* approach are carefully examined in the general framework of a fermion model with two anticommuting (scalar and pseudoscalar) varieties of charges, in turn responsible for P-invariant and 'maximally P-violating' processes. It is stressed that the new scheme, besides clearing up the *origin* of the 'maximal parity violation' effect, is also able to predict in which kind of interactions such an effect should occur : this should in particular be the case of magnetic-monopole dynamics.

RESUME. *On réexamine l'effet de 'violation maximale de parité' à la lumière d'une reformulation, proposée précédemment, de la théorie du champ de Dirac, qui inclut naturellement les champs chiraux massifs (et sans masse) 'tournant à gauche' pour les fermions, et 'tournant à droite' pour les antifermions. On examine une approche à champ chiral généralisé qui n'a pas besoin d'hypothèse ad hoc et permet de réduire simplement la 'symétrie CP' à la symétrie P et résout automatiquement la question du neutrino 'manquant'. La signification et la cohérence d'une telle approche sont soigneusement examinées dans le cadre général d'un modèle de fermion à deux types de charges (scalaire et pseudo-scalaire) respectivement responsables de processus 'P-invariants' et 'à violation maximale de parité P'. On souligne que ce nouveau schéma non seulement éclaire l'origine de l'effet de 'violation maximale de parité', mais est aussi*

capable de prédire dans quel type d'interaction un tel effet peut survenir: ce serait en particulier le cas de la dynamique des monopôles magnétiques.

As is well-known, the 'maximal parity violation' ('MPV') effect [1,2] has actually no *true* account within the ordinary theoretical framework, in spite of its peculiar importance in weak-interaction phenomenology. The 'standard model' itself [3-5] can but make an *ad hoc* assumption as to the 'handedness' of fermions in their couplings to intermediate vector bosons, without providing at all any physical reason for such an 'odd' behaviour.

In a recent paper [6] we tried to fill such a gap, tracing the problem back to its origin. An improved basic field scheme was suggested that *naturally* predicts the 'MPV' effect and is further able to provide *a deep account* for its appearance. We in particular showed that even *massive* chiral fields, 'left-handed' for fermions and 'right-handed' for antifermions, can be rigorously defined within the theory of the Dirac free field, provided that charge conjugation C is given a new formal expression ensuing from *strictly* applying covariance under fermion-antifermion interchange. The clue starting-point was just the following basic prescription :

(i) The Dirac parity operator should always exhibit the *same* form, say $U_p = \eta\gamma^0 (\eta = \pm 1)$, whether it is defined in the fermion or antifermion four-spinor space.

Briefly, let $u_f(\vec{p})$ and $u_{\overline{f}}(\vec{p})$ denote, in principle, two (fermion and antifermion) mutually charge-conjugate free four-spinors with a positive energy eigenvalue. It follows from *(i)* that the *opposite*-intrinsic-parity condition to be fulfilled by such four-spinors [7] must be written down by setting, for a three-momentum $\vec{p} = 0$,

$$\gamma^0 u_f(0) = u_f(0) \quad , \quad \gamma^0 u_{\overline{f}}(0) = -u_{\overline{f}}(0). \tag{1}$$

Actually, requirement (1) *cannot* be satisfied by the usual formulation of Dirac's fermion-antifermion theory, where $u_f(\vec{p})$ and $u_{\overline{f}}(\vec{p})$ just enter as *coincident* four-spinors. In the light of *(i)*, $u_f(\vec{p})$ and $u_{\overline{f}}(\vec{p})$ should *no longer* be coincident : as can be easily inferred from (1), they should rather be eigenspinors of the *opposite*-proper-mass Dirac Hamiltonians

$$H_f \equiv H(\vec{p}, +m) \quad , \quad H_{\overline{f}} \equiv H(\vec{p}, -m) \tag{2}$$

where $H(\vec{p}, \pm m) = \vec{\alpha}.\vec{p} + \beta(\pm m)(c = 1 \quad , \quad \beta = \gamma^0 \quad , \quad m > 0)$. For an explicit check, it is sufficient to rewrite (1) as

$$\beta(+m)u_f(0) = mu_f(0) \quad , \quad \beta(-m)u_{\overline{f}}(0) = mu_{\overline{f}}(0) \qquad (1')$$

and to interpret m on the right side of such equations as the (positive) rest energy eigenvalue $E = m$ associated with both $u_f(0)$ and $u_{\overline{f}}(0)$. It is worth stressing, in this regard, that just owing to the well-known energy-momentum relation $E^2 = \vec{p}^2 + m^2$ a real antifermion at rest will not be bound at all to have a negative energy (in spite of its proper mass $-m$). Negative-mass eigenstates have here nothing to do with negative-energy eigenstates, and *no* contradiction can therefore arise either with QFT prescriptions or with the 'CPT' theorem itself [8] : what is really crucial in the standard analysis is the (positive) sign of rest energy, that should not be confused with the proper-mass sign. As can be seen by comparing (1) and (1'), the fact is that *the absolute sign of mass in the Dirac Hamiltonian is purely conventional, since it just corresponds to that one of intrinsic parity.* Since we shall anyhow have (in both c- and q-number approaches)

$$i(\partial/\partial t)\psi = H\psi$$

($\hbar = 1$) no matter which one of the two Hamiltonians (2) is concerned, we immediatly draw from (2) the *opposite-proper-mass* (fermion and antifermion) free Dirac equations

$$i\gamma^\mu \partial_\mu \psi_f = +m\psi_f \quad , \quad i\gamma^\mu \partial_\mu \psi_{\overline{f}} = -m\psi_{\overline{f}} \qquad (3)$$

($\mu = 0, 1, 2, 3 \quad , \quad \gamma^{k+} = -\gamma^k \quad , \quad k = 1, 2, 3$). Let $\psi_f(x^\mu)$ and $\psi_{\overline{f}}(x^\mu)$ stand in particular for the fermion and antifermion free fields. Since the matrix set $\{-\gamma^\mu\}$ obeys the same algebra as $\{\gamma^\mu\}$, Pauli's theorem on γ-matrices [9] ensures that the link of $\psi_{\overline{f}}$ to ψ_f be *unique* (apart from a phase factor). Charge conjugation C may now be formally identified, therefore, with the *chirality* operation

$$C : \psi_f(x^\mu) \to \psi_{\overline{f}}(x^\mu) = \gamma^5 \psi_f(x^\mu) \qquad (4)$$

($\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$) that primarily expresses *proper-mass conjugation* [10]. Relying upon the usual formalism and adopting the standard γ-matrix representation, we could also set, of course,

$$C : \psi_f(x^\mu) \to \psi_{\overline{f}}(x^\mu) = \gamma^5 \gamma^2 \psi_f^*(x^\mu) \qquad (4')$$

but *uniqueness* of definition (4) would imply (apart from a phase factor)[1]

$$\gamma^5\gamma^2\psi_f^* = \gamma^5\psi_f$$

Coincidence between (4) and (4') may hold for the free-particle case only : if ψ_f stands rather for a fermion field of electric charge $(-e)$ in the presence of an external four-potential A_μ,

$$i\gamma^\mu[\partial_\mu + i(-e)A_\mu]\psi_f = +m\psi_f,$$

then $\psi_{\overline{f}} = \gamma^5\psi_f$ will *quite symmetrically* stand for the antifermion field of charge e in the presence of the *charge-conjugate* four-potential $(-A_\mu)$,

$$i\gamma^\mu[\partial_\mu + ie(-A_\mu)]\psi_{\overline{f}} = -m\psi_{\overline{f}} \quad , \quad (\psi_{\overline{f}} = \gamma^5\psi_f \neq \gamma^5\gamma^2\psi_f^*). \qquad (5)$$

So, C as defined by (4) will act in general on the *whole* interacting system, and *not* on the mere fermion field, while C as defined by (4') will clearly leave (as usual) any external field *unchanged*. Definition (4) enables one to set down the orthogonal transformation

$$\psi_f = 2^{-1/2}(\psi_f^{ch} + \psi_{\overline{f}}^{ch}) \quad , \quad \psi_{\overline{f}} = 2^{-1/2}(-\psi_f^{ch} + \psi_{\overline{f}}^{ch}) \qquad (6)$$

that just defines the two (fermion and antifermion) *massive* chiral fields[2]

$$\psi_f^{ch} \equiv 2^{-1/2}(1-\gamma^5)\psi_f \quad , \quad \psi_{\overline{f}}^{ch} \equiv 2^{-1/2}(1+\gamma^5)\psi_{\overline{f}} \qquad (7)$$

as an *alternative coordinate system* in the (two-dimensional) internal space spanned by the opposite-mass "Dirac" field pair $(\psi_f \;\; , \;\; \psi_{\overline{f}})$. This actually leads to a *natural* theoretic interpretation of the 'MPV' effect, with no need of appealing to the *ad hoc* '$V-A$' prescription [11-13]. As a matter of fact, one can generally put (for either neutral or charged currents)

$$\overline{\psi}_f^{(a)}\gamma^\mu(1-\gamma^5)\psi_f^{(b)} \equiv \overline{\psi}_f^{ch(a)}\gamma^\mu\psi_f^{ch(b)}$$

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[1] Such a position – as we shall see better elsewhere (work in progress) – is actually meaningful within a *unique* (fermion-antifermion) state-vector space which is mapped by C into itself. This space includes also the freedom degrees connected with the two signs of proper mass, besides those ones related to four-momentum and helicity; so that C acts on it like a mere "coordinate" reversal $m \to -m$.

[2] Such fields are singly obeying the mere Klein-Gordon equation. In this regard, see ref. [11].

$$\overline{\psi}^{(a)}_{\overline{f}}\gamma^\mu(1+\gamma^5)\psi^{(b)}_{\overline{f}} \equiv \overline{\psi}^{ch(a)}_{\overline{f}}\gamma^\mu\psi^{ch(b)}_{\overline{f}} \tag{8}$$

($\overline{\psi}^{ch} = \psi^{ch+}\gamma^0$) so that the phenomenological '$V-A$' (fermion) and '$V+A$' (antifermion) currents may just take the natural form of *chiral-field 'V' currents.* Such an interpretation, furthermore, shows a quite peculiar feature : since currents (8) are C-invariant – recall (4)– it is now made possible even *regaining P symmetry from 'CP symmetry'* !

In the present paper we want to go deep into this basically new theoretical view of 'MPV', no longer compatible with a *pure* scalar-charge fermion model, and to discuss in detail the very subtle question of how (and in what sense) ordinary mirror symmetry may be in fact restored to 'maximally parity-violating' processes.

According to the usual formulation of fermion-antifermion field theory, *two distinct pairs* of '$V-A$' and '$V+A$' currents can actually be constructed, that are, on one hand, the ('$V-A$' fermion and '$V+A$' antifermion) currents

$$\overline{\psi}^{(a)}_{f}\gamma^\mu(1-\gamma^5)\psi^{(b)}_{f} \quad , \quad \overline{\psi}^{(a)}_{\overline{f}}\gamma^\mu(1+\gamma^5)\psi^{(b)}_{\overline{f}} \tag{9}$$

and on the other, the *specular* ('$V+A$' fermion and '$V-A$' antifermion) ones

$$\overline{\psi}^{(a)}_{f}\gamma^\mu(1+\gamma^5)\psi^{(b)}_{f} \quad , \quad \overline{\psi}^{(a)}_{\overline{f}}\gamma^\mu(1-\gamma^5)\psi^{(b)}_{\overline{f}}. \tag{10}$$

Only the *former* current pair, nevertheless, is happened to enter the 'MPV' effect (whence just the 'maximal' degree of 'P-breakdown'). That gives rise to the following basic question, for which no adequate answer can be provided in the ordinary theoretical framework: Why does weak interaction seem to ignore the further, equally sound, pair of '$V-A$' (antifermion) and '$V+A$' (fermion) currents ? Such a 'mystery' appears to be fully cleared up in our framework. Owing to the unusual definition (4) for C, the peculiar *identities* now hold

$$\overline{\psi}^{(a)}_{f}\gamma^\mu(1+\gamma^5)\psi^{(b)}_{f} = \overline{\psi}^{(a)}_{\overline{f}}\gamma^\mu(1+\gamma^5)\psi^{(b)}_{\overline{f}}$$

$$\overline{\psi}^{(a)}_{\overline{f}}\gamma^\mu(1-\gamma^5)\psi^{(b)}_{\overline{f}} = \overline{\psi}^{(a)}_{f}\gamma^\mu(1-\gamma^5)\psi^{(b)}_{f} \tag{11}$$

that make the above question *quite meaningless* : Within the alternative formalism proposed, merely *one* pair of '$V-A$' and '$V+A$' currents can be defined[3], that is the *actually existing* current pair (9) as rewritten in the more appropriate form (8) !

It is just identities (11) which afford the key to the new, *P-conserving*, interpretation of the 'MPV' effect. They show that the *ordinary* mirror image of the '$V - A$' fermion current is *not* 'missing' at all, being nothing but the *actual* '$V + A$' antifermion current, and vice versa. Hence, paradoxically, the usual '*maximal*' P-violation degree –due to the total absence of e.g. the '$V + A$' fermion current *as such*– is what now ensures a *full* respect of P symmetry itself[4]!

All that turns out to be physically meaningful in the light of a "*dual*" internal model for massive fermions [6] which is directly proceeding from transformation (6). More precisely, let the "fermion" and "antifermion" internal states associated with fields $\psi_f \quad , \quad \psi_{\overline{f}}$ be denoted by the two units kets $|f\rangle$ and $|\overline{f}\rangle(\equiv C|f\rangle)$ such that

$$M|f\rangle = +m|f\rangle \quad , \quad M|\overline{f}\rangle = -m|\overline{f}\rangle \tag{12}$$

M standing for a (covariant) one-particle mass operator. The opposite-intrinsic-parity condition will then be expressible (under the choice $\eta = 1$ for the parity phase factor) as

$$P|f\rangle = |f\rangle \quad , \quad P|\overline{f}\rangle = -|\overline{f}\rangle. \tag{13}$$

To the (internal) field transformation (6), moreover, there will correspond the state-vector transformation

$$|f\rangle = 2^{-1/2}(|f^{ch}\rangle + |\overline{f}^{ch}\rangle) \quad , \quad |\overline{f}\rangle = 2^{-1/2}(-|f^{ch}\rangle + |\overline{f}^{ch}\rangle) \tag{14}$$

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[3] At first sight identities (11) might seem to be unacceptable, since they are in particular implying the overall neutral-current identity

$$\bar{\psi}_f \gamma^\mu \psi_f = \bar{\psi}_{\bar{f}} \gamma^\mu \psi_{\bar{f}} \equiv J^\mu$$

which in the usual formalism would yield a null result for the fermion-antifermion Dirac-current operator. Actually, as will be stressed later on (in the text), any scalar charge (such as the electric one) should now be primarily represented by a one-particle operator, say Q, rather than by a c-number; so that the corresponding fermion-antifermion current will read QJ^μ and no formal inconsistency can arise.

[4] The only difference from the standard P-conserving processes would be that the "maximally P-violating" ones are clearly not invariant under P.

where $|f^{ch}\rangle$ and $|\overline{f}^{ch}\rangle$ denote two "chiral" states with opposite chirality eigenvalues :

$$C|f^{ch}\rangle = -|f^{ch}\rangle \quad , \quad C|\overline{f}^{ch}\rangle = |\overline{f}^{ch}\rangle. \tag{15}$$

From (13),(15) it follows, e.g.,

$$CP|f\rangle = C|f\rangle = |\overline{f}\rangle \quad , \quad CP|f^{ch}\rangle = P|f^{ch}\rangle = |\overline{f}^{ch}\rangle \tag{16}$$

(note that $CP = -PC$). So, with reference to ordinary space reflection, $|f\rangle(|\overline{f}\rangle)$ clearly looks like a *pure scalar-charge eigenstate*, whereas $|f^{ch}\rangle(|\overline{f}^{ch}\rangle)$, on the contrary, like a *pure pseudoscalar-charge eigenstate.* If the one-particle operators Q and Q^{ch} are just standing for any two such (scalar and pseudoscalar) charges with non-zero eigenvalues, it is immediate, moreover, to check the *anticommutivity* property

$$QQ^{ch} + Q^{ch}Q = 0. \tag{17}$$

Hence the following basic conclusion may be drawn : The *same* massive fermion carrying both scalar and pseudoscalar (conserved) charges should apparently be found in *either* one of the internal states $|f\rangle$ (associated with a field ψ_f) and $|f^{ch}\rangle$ (associated with a field ψ_f^{ch}) *according to whether "seen" through a pure scalar- or pseudoscalar-charge interaction, respectively.* This also means that $|f\rangle$ and $|f^{ch}\rangle$ can only give two *partial* internal pictures of such a fermion, which are merely concerning the single (scalar and pseudoscalar) aspects of its "dual" charge nature –the *true* particle $\rightarrow$ antiparticle conjugation is to be identified with the *whole* operation CP, and *not* with 'charge conjugation' C alone, that, according to (15), affects only *scalar* charges and leaves pseudoscalar charges *unvaried.*

In the light of this new fermion model, it may be argued that :

a) the 'weak' charge should just be an example of a *pseudoscalar* charge carried by all massive fermions ;

b) any massive fermion involved in a 'maximally P-violating' process would apparently be looking the same as a *pure pseudoscalar-charge object* (i.e., as if *no net* scalar charges were carried by it).

The latter statement is a direct consequence of property (17) and gives really an account of the clue formal identities (11) : as in particular shown by (16), the "chiral" fermions and antifermions associated with massive fields (7) should be straightforwardly interpreted as the *ordinary* specular images of each other.

As we already stressed in ref. [6], our new field description takes a peculiar meaning in the limit of zero mass. In that case, eqs.(3) coalesce, but another 'mystery' appears to be cleared up, namely the one concerning the 'missing' right-handed neutrino (and left-handed antineutrino). More precisely, let

$$\psi_v^{(L)} \equiv 2^{-1/2}(1-\gamma^5)\psi_v \quad , \quad \psi_{\overline{v}}^{(R)} \equiv 2^{-1/2}(1+\gamma^5)\psi_{\overline{v}} \tag{18}$$

stand for the (chiral) fields associated with the actually existing neutrino and antineutrino, and

$$\psi_v^{(R)} \equiv 2^{-1/2}(1+\gamma^5)\psi_v \quad , \quad \psi_{\overline{v}}^{(L)} \equiv 2^{-1/2}(1-\gamma^5)\psi_{\overline{v}} \tag{19}$$

for those ones associated with their 'missing' mirror images. In the ordinary scheme, as is well-known, (18) and (19) are two *distinct*, and *equally legitimate*, pairs of massless chiral-field solutions. Yet, just to match theory to experience, only the *former* field pair is to be retained [14-16] –even if admitting the existence of the further (specular) neutrino-antineutrino pair, it would be left to explain why such particles are completely ignored by weak interaction. In our formalism, we shall clearly have as a natural extension of (4),

$$\psi_{\overline{v}} = \gamma^5\psi_v, \tag{20}$$

and the following *link* will hold, on the contrary, between (18) and (19) :

$$\psi_v^{(R)} = \psi_{\overline{v}}^{(R)} \quad , \quad \psi_{\overline{v}}^{(L)} = -\psi_v^{(L)}. \tag{21}$$

So, by virtue of (20), merely *one* pair of neutrino and antineutrino chiral-field solutions can really be defined, just in line with experimental evidence !

Such alternative neutrino-antineutrino scheme, besides naturally fitting experience, does in fact restore P symmetry [6] : as follows from (21), the ordinary mirror image of the actual (negative helicity) neutrino would *not* be 'missing' at all, being nothing but the actual (positive helicity) antineutrino, and vice versa. The point is that *the peculiar 'screw' nature of a massless neutrino is now physically accounted for by its being a pure pseudoscalar-charge object.* This would be the reason why *only one* neutrino-spin polarization can be found in Nature.

To sum up, if the Dirac theory of the fermion and the antifermion is recast on the ground of the covariance prescription (i), a *natural* generalized chiral-field description of the 'MPV' effect can be gained, whose main peculiar features are the following ones :

1) it makes 'CP symmetry' itself straightforwardly reducible to P symmetry (with no change in the meaning of parity P).
2) It fully accounts for the 'chiral' behaviour shared by all (either massive or massless) fermions in weak interaction, without needing at all any *ad hoc* assumption to match theory to experience ('$V - A$' prescription) or any 'cut' to attain Weyl's two-component neutrino scheme [17].
3) It locates the *origin* of the 'MPV' effect in the intrinsic nature itself of fermions. The key-point is that any massive fermion would together bear two *anticommuting* general varieties of charges –namely, not only a *scalar* variety, underlying P-invariant processes, but also a *pseudoscalar* one, being responsible for 'maximally P-violating' processes. In the zero-mass limit, further, a *pure* pseudoscalar-charge fermion model would be left, giving rise just to Weyl's scheme.

All that might clearly act as a general background to an even more compact and well-founded formulation of the Glashow-Weinberg-Salam 'electroweak' scheme [3-5]. It is enough to think, e.g., that the (now) fully legitimate introduction of the *massive* conjugated chiral fields (7) –further such that

$$\begin{aligned} 2^{-1/2}(1+\gamma^5)\psi_f &= 2^{-1/2}(1+\gamma^5)\psi_{\overline{f}} \\ 2^{-1/2}(1-\gamma^5)\psi_{\overline{f}} &= -2^{-1/2}(1-\gamma^5)\psi_f \end{aligned} \tag{22}$$

in line with (6) and (11) –could just afford a *rigorous* theoretic support to the 'chiral' phenomenologic nature (even *after* spontaneous symmetry breaking!) of the fermion and antifermion weak-isospin groups $SU(2)_L$ and $SU(2)_R$. Identities (22) allow in particular to avoid the inconsistent standard prediction of, e.g., two *distinct*, 'left-handed' and 'right-handed', massive real fermions both called 'electron', that should have the same mass and electric charge, but should belong to *different* $SU(2)_L$-representations and exhibit *quite different* behaviours under weak interaction. Just as suggested by experience, only *one* "chiral" electron (and *one* "chiral" positron) is to be expected according to (22), the standard 'right-handed' electron iso-singlet being merely a $SU(2)_L$-representation for the *"chiral" positron* itself !

The new theoretical model here examined, besides giving a deep explanation of the 'MPV' effect, is also able to state which general kind of interactions should in principle display such an effect : those ones generated by *pseudoscalar* charges. This is a fundamental prediction of the model, that could be tested in the presence of a charge whose pseudoscalar nature is *already* well-known. The mere 'weak' charge does not seem to be useful to such a purpose, since its pseudoscalar behaviour is inferred just from the occurrence of the 'MPV' effect. A crucial test could be provided, on the contrary, by an eventual discovery of magnetic monopoles, whose pseudoscalar nature may directly be drawn from the actual pseudovector character of the magnetic field [18-20].

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