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Quantum effects of matter fields, causality and thermodynamics in an isotropic and an anisotropic Universe*

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Introduction

As pointed out in I. Prigogine and I. Stengers' s book "Order out of Chaos" the problem of irreversibility is a cosmological problem[1]. The main idea is to show that the growth of entropy takes place in the Universe due to some reasons. The first problem we shall discuss is the problem of the definition of entropy for the whole Universe. The second question is how to describe the process of coming to thermodynamical equilibrium in an evolving Universe. Usually we define an entropy as the entropy of matter. That's why the production of entropy in the Universe is considered in connection with the effects of particle creation from vacuum in an external gravitational field. Some efforts have been undertaken recently to give a thermodynamical meaning to the effects of particle creation from vacuum and to generalize the second law of thermodynamics in the external gravitational field of the Universe [2, 3, 4].

Besides that, beginning with the longstanding paper of R. Penrose[5], there were some efforts to interpret matter creation from the vacuum in an external gravitational field as a thermodynamical effect of transition of the so called "entropy of gravitation" into the entropy of created matter. It is easy to see that only those configurations of the gravitational field possess "entropy", which can create particles, and

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the number of these particles will be a measure of the "entropy of gravitation". Applied to cosmology this means that there is no "entropy of gravitation" for conformal matter fields in conformally flat Universe since there is no massless particle production in a homogeneous and isotropic Universe, because the field equations are conformally invariant and there is no particle production in conformally flat space-time. The situation changes when we have a nonconformal scalar field (minimally coupled with gravity), or gravitons, the equations of which are not invariant under conformal transformations. In this case the effect of particle creation takes place and the creation rate is proportional to the square of the scalar curvature of the metric. In this case the scalar curvature of the metric can be taken as a measure of the "entropy of gravitation". Anisotropic space-time is not conformally flat and massless particle creation is possible here. As it was shown in a number of articles and books[2,6] the effect is determined by the square of the Weyl tensor, so in accordance with R. Penrose's proposal the Weyl curvature can be taken as a measure of the "entropy of gravitation". But there are some problems in this identification, as it is not clear how to come to the expression of the entropy from the formula for the particle creation rate

$$\frac{dN}{dt} = \frac{V(t)}{80\pi\beta}[C_{\alpha\beta\mu\nu}(t)C^{\alpha\beta\mu\nu}(t) + 60\gamma(\xi - \frac{1}{6})^2 R^2(t)], \tag{1}$$

$$N(t) = n(t)V(t),$$

where $C_{\alpha\beta\mu\nu}$ is the Weyl curvature, R is the scalar Ricci curvature, ξ is the parameter of coupling of the scalar field with gravity (when $\xi = 1/6$ we have conformal coupling), $\beta = 24, \gamma = 1$ for scalar particles, $\beta = 4$ for neutrino, $\beta = 1$ for photons and $\beta = 1, \gamma = 1/6, \xi = 0$ for gravitons[2,6,7]; n is the number density of particles created, $V(t) = vA^3(t)$ is the comoving volume. Hu[2] for example proposed that the hydrodynamical approximation can be applied and the entropy is simply proportional to N . But it is not easy to understand why in a strongly nonequilibrium situation of particle creation this approximation is valid for an arbitrary instant of time on a whole hypersurface $t = const$ in regions which have never been causally connected. Therefore one should develop a more accurate approach to the calculation of the entropy, generated in the process of particle creation in an anisotropic Universe . This will be the aim of our paper.

1. Entropy production in an isotropic Universe.

Let us consider background matter and matter created due to quantum effects in a homogeneous and isotropic cosmological model with metric

$$ds^2 = dt^2 - A^2(t)\sum_{i=1}^{3}(dx^i)^2 \tag{2}$$

where $A(t)$ is the scale factor of the model. The matter is described by Stress-Energy-Tensor (SET)

$$T_{\mu\nu}^{tot} = T_{\mu\nu}^{backgr} + T_{\mu\nu}^{creat} \tag{3}$$

both parts of which are conservative

$$\nabla_\mu T_\nu^{\mu\ backgr} = 0, \quad \nabla_\mu T_\nu^{\mu\ creat} = 0 \tag{4}$$

The last equation reflects the general property of SET in curved space- time. In the metric (2) the SET (3) can be presented in the form

$$T_0^{0\ tot}(t) = \rho_{tot}(t) = \rho_{backgr}(t) + \rho_{creat}(t)$$

$$T_{(i)}^{(i)\ tot}(t) = -p_{i\ tot}(t) = -p_{i\ backgr}(t) - p_{i\ creat}(t)$$

and equations (4) will have the well-known form

$$\frac{d}{dt}(\rho_{backgr}(t)V(t)) + p_{backgr}(t)\frac{dV(t)}{dt} = 0 \tag{5a}$$

$$\frac{d}{dt}(\rho_{creat}(t)V(t)) + p_{creat}(t)\frac{dV(t)}{dt} = 0 \tag{5b}$$

By analogy with the first law of thermodynamics in the static case, the total differential of the entropy for systems with a variable number of particles, has the form

$$T(t)dS(t) = d(\rho_{tot}(t)V(t)) + p_{tot}(t)dV(t) - \mu(t)dN_{tot}(t)$$

Here $T(t)$, $S(t)$, $\mu(t)$ are only symbols for temperature, entropy and chemical potential, which ought to be defined precisely. From equations (5) we come to the formula

$$T(t)dS(t) = -\mu(t)(d(n_{backgr}(t)V(t)) + d(n_{creat}(t)V(t)))$$

which means that the only way for the entropy to be changed is due to changes in the number of particles. Taking into account the fact that

$$n_{backgr}(t) \sim V^{-1}(t)$$

we come to the conclusion that any change in the entropy of matter in an homogeneous and isotropic Universe is possible only due to the effect of particle creation from the vacuum in an external gravitational field:

$$T(t)dS(t) = -\mu(t)d(n_{creat}(t)V(t)) \tag{6}$$

Since we consider fields minimally coupled with gravity and without any interaction with other matter fields, one can say that the entropy $S(t)$, the temperature $T(t)$ and the chemical potential $\mu(t)$ are the characteristics of particles created on the whole hypersurface $t = const$ and are constant values for all points of this hypersurface not because created particles have come somehow to the thermodynamical equilibrium but rather by definition of $S(t)$, $T(t)$ and $\mu(t)$ as global values. In principle we can integrate the formula (6) and obtain the total amount of entropy, produced in the isotropic Universe from the initial vacuum state at t_{in} to the arbitrary time t

$$S(t) = -\int_{t_{in}}^{t} \frac{\mu(t')}{T(t')} \frac{d}{dt'}(n_{creat}(t')V(t'))dt'$$

Using the isotropic part of the formula (1)

$$\frac{dN}{dt} = \frac{3V(t)}{4\pi\beta}\gamma(\xi - \frac{1}{6})^2 R^2,$$

we get an expression for the entropy of scalar nonconformal particles or for gravitons , created in an isotropical Universe

$$S(t) = -\frac{3}{4\pi\beta}\gamma(\xi - \frac{1}{6})^2 \int_{t_{in}}^{t} \frac{\mu(t')}{T(t')} V(t')R^2(t')dt',$$

but there still remains the question of how to calculate $T(t)$ and $\mu(t)$.

2. Production of entropy in an anisotropic Universe.

Let us consider the case of an anisotropic space-time with metric

$$ds^2 = dt^2 - \sum_{i=1}^{3} A_i^2(t)(dx^i)^2, \tag{7}$$

where

$$A_i(t) = A(t)exp[\lambda_i \int A^{-3}(t)dt]$$

and $A(t)$ is the mean scale factor, corresponding to the isotropic contribution to the Universal expansion;

$$\prod_{i=1}^{3} A_i(t) \equiv A^3(t) \equiv V(t), \quad \sum_{i=1}^{3} \lambda_i = 0$$

It is seen from formula (7) that the type of anisotropic expansion is determined by A as a function of t and can be found as a solution of the Einstein equations

$$3H^2 - 3\frac{\Lambda^2}{A^6} = 8\pi G \rho_{backgr}$$

$$\frac{d}{dt}H(t) + 3\frac{\Lambda^2}{A^6} = -8\pi G\frac{\rho_{backgr} + p_{backgr}}{2},$$

where

$$\Lambda^2 = 1/6\sum_{i=1}^{3} \lambda_i^2.$$

Here, $E_{isot} = H^2/8\pi G$ gives the energy density of the total isotropic expansion of the Universe and $E_{anis} = (3/8\pi G)\Lambda^2/A^6$ by analogy with Ref. [8] may be considered as the energy density of anisotropy for the gravitational field. For the parameters of the anisotropy we have the following expressions

$$H_i = \frac{1}{A_i}\frac{dA_i}{dt},$$

$$H = \frac{1}{3}\sum_{i=1}^{3} H_i$$

$$\Delta H = H_3 - \frac{1}{2}(H_1 + H_2) = \frac{1}{A^3}(\lambda_3 - \frac{1}{2}(\lambda_1 + \lambda_2)) \equiv \frac{\Delta\lambda}{A^3},$$

$$\overline{\Delta H} = H_1 - H_2 = \frac{1}{A^3}(\lambda_1 - \lambda_2) \equiv \frac{\overline{\Delta\lambda}}{A^3}.$$

Supposing that the SET of background matter has a form corresponding to the Kasner vacuum case we come to an equation similar to (6). As we shall show later the SET of created particles has a more complicated structure in anisotropic homogeneous space-time. This SET is conserved and, in accordance with (4), we have

$$\frac{d}{dt}(\rho_{creat}(t)V(t)) + \overline{p_{creat}(t)}\frac{d}{dt}V(t) + \frac{2}{3}\Delta\lambda\Delta p_{creat}(t) + \frac{1}{2}\overline{\Delta\lambda}\overline{\Delta p_{creat}(t)} = 0, \quad (8)$$

where

$$\overline{p} = \frac{1}{3}\sum_{i=3}^{3} p_i, \Delta p = p_3 - \frac{1}{2}(p_1 + p_2), \overline{\Delta p} = p_1 - p_2.$$

Then for the total differential of entropy one can obtain

$$T(t)dS(t) = -(\frac{2}{3}\Delta\lambda\Delta p_{creat}(t) + \frac{1}{2}\overline{\Delta\lambda}\overline{\Delta p_{creat}(t)})dt - \mu(t)d(n_{creat}(t)V(t)). \quad (9)$$

It is seen from here that the production of entropy in an anisotropic Universe takes place due to two factors: a) the anisotropic structure of the SET, i.e. the pressure gradients of the created matter; b) the changes in the number of created particles . As in the isotropic case we can integrate formula (9) using the expression (1) for the creation rate of particles in anisotropic space-time:

$$S(t) = -\int_{t_{in}}^{t}(\frac{1}{T(t')}(\frac{2}{3}\Delta\lambda\Delta p_{creat}(t') + \frac{1}{2}\overline{\Delta\lambda}\ \overline{\Delta p_{creat}(t')}) + \frac{\mu(t')}{T(t')}\frac{V(t')}{80\pi\beta}[C_{\alpha\beta\mu\nu}(t')C^{\alpha\beta\mu\nu}(t') + 60\gamma(\xi - \frac{1}{6})^2 R^2(t')])dt'$$

But then we see that interpreting $S(t)$ as the thermodynamical entropy of a system of particles, created at the arbitrary moment of time t , is not possible. This is because particles created in non-causally connected regions of space would not have enough time to exchange any information and to come to a state, which we could call the state of thermodynamical equilibrium at the moment of time t (this is similar to the so-called horizon problem in inflationary cosmology). The value S has thermodynamical meaning only when $t \to \infty$; only in this case S contains the contribution from all field modes beginning from momentum $K = \infty$ and finishing at $K = O$; all these modes are inside the infinite horizon and we can talk about S as a classical thermodynamical quantity. One should mention also that the expression (1) was obtained under the assumption that the Universe was isotropic at $t = -\infty$ and $t = +\infty$; that is why the results given by the integration of (9) for the calculation of the entropy of particles produced from t_{in} until t with $\Delta H(t_{in}) \neq 0$, $\overline{\Delta H(t_{in})} \neq 0$, $\Delta H(t) \neq 0$, $\overline{\Delta H(t)} \neq 0$ will not be correct. That is why we shall calculate the entropy and the temperature of created particles in another way. It corresponds to the idea that we may correctly define the concept of particle in the asymptotic regions where there is no interaction with field and vacuum is stable. At intermediate moments of time between t_{in} and t we can speak only about quasiparticles which are created and are annihilated due to strong gravitational fields. That's why one should be very careful using the formulas (6) and (9) for the definition of the entropy production. One should be sure that N indeed increases. In other words one should show that $dN > 0$ and $\mu/T < 0$.

3. The problem of isotropization and the production of entropy. The process of coming to thermodynamical equilibrium.

Let us consider in details equation (9) for the entropy production and the equation of SET's conservation (8). If we present $n_{creat}(t)$ and $\rho_{creat}(t)$ in the form

$$n_c(t) = A^{-3}(t)f(t), \;\; \rho_c(t) = A^{-4}F(t)$$

and take into account the equation of state for created massless particles

$$\overline{p} = \frac{\rho}{3},$$

we can rewrite (8) as

$$\frac{1}{A}\frac{d}{dt}(\rho_c A^4) + \frac{2}{3}\Delta\lambda\Delta p_{creat}(t) + \frac{1}{2}\overline{\Delta\lambda}\;\overline{\Delta p_{creat}(t)} = 0$$

or

$$\frac{1}{A}\frac{d}{dt}F(t)+\frac{2}{3}\Delta\lambda\Delta p_{creat}(t)+\frac{1}{2}\overline{\Delta\lambda}\ \overline{\Delta p_{creat}(t)}=0, \tag{10}$$

while for equation (9) we have

$$T(t)dS(t)=\frac{1}{A}dF(t)-\mu(t)df(t).$$

It is easy to understand now that if in the process of evolution inside a fixed volume of space $F(t)$ is going to a constant value ($F(t)\to const$) and the same takes place for $f(t)$ ($f(t)\to const$), then from equation (10) we see that $\Delta p\to 0$, $\overline{\Delta p}\to 0$, which means that the isotropisation of matter has taken place inside the same volume. From the second equation it follows that while the isotropisation of matter takes place, the process of entropy production in this volume also finishes. This gives us a strong evidence that the anisotropy of space-time is essential for entropy production and particle creation. It is useful here to recall the long-standing statement of Hu that the anisotropy acts as a "tranducer" of gravitational entropy (anisotropic space-time) to matter entropy (field dynamics)[2]. If it were possible to point out the time interval during which the functions $F(t)$ and $f(t)$ become constants, i.e. the distribution of matter in the fixed volume of the Universe becomes like in the isotropic Universe, we would be able to calculate the duration of time for the process of matter isotropisation in this volume. Due to back-reaction the matter isotropization naturally leads to isotropization of metric inside this volume.

The most striking feature of massless particle creation in anisotropic space-time is that for small scales in momentum space , i.e. for wavelength less than the horizon length, it is possible to speak about an effective isotropic space time for them, but in isotropic space-time massless particle are not created due to conformal flatness. That is why one has a good definition of particles as in S-matrix theory. Massless particles not only are not created by an isotropic metric but also are not annihilated by it. This leads to an important effect of "accumulation" of particles which are created outside or near the horizon of anisotropic space-time, then due to expansion they go inside the horizon of the effective isotropic space-time and are accumulated there. This is the reason for fundamental irreversibility. One has always $dN>0$ inside the horizon.(As an example we show how this mechanism of separation of field modes works in isotropic case for massless nonconformal particles. See Appendix 1.)

Now we propose the following concept of thermodynamical equilibrium in an expanding anisotropic Universe for massless particles without any interaction with other matter fields. We will say that only those particles have reached "thermodynamical equilibrium" , if their wavelength is less than the size of the particle horizon for moment t' . The growth of the entropy and the process of coming to "thermodynamical equilibrium" shall be described as a flow of the long field modes of field, which were created at $t < t'$ through the surface of the sphere with radius of the order of horizon.

We shall show below that the process of coming to thermodynamical equilibrium, corresponding to the flow of field modes through the surface of the horizon sphere, is "automatically" accompanied by the fact that the functions $F(t)$ and $f(t)$ become constant and by the end of entropy production inside the horizon region. That is why only for these regions we can say that a definite amount of equilibrium entropy and temperature can be calculated.

4. Entropy of photons created in an anisotropic Universe.

We begin our calculation with the number of real vector massless particles and the energy density of photons, which were created at earlier times $t' < t$ by a quantum process as quasiparticles, and the wavelength of which became less than the horizon. Due to the expansion of the Universe their energy density at time t is given by the sum of all previous increments $\delta\rho$ at earlier times arising from the change of the momentum $K(t)$ with time, and being red-shifted by a factor $A(t')/A(t)$. Thus we have

$$n_c = \int_{t_{in}}^{t} \left(\frac{A(t')}{A(t)}\right)^3 \left(-\frac{\partial n_q(K_h(t'),t')}{\partial K_h}\right) \frac{\partial K_h(t')}{\partial t'} dt' \equiv A^{-3}(t) f(t) \tag{11a}$$

$$\rho_c = \int_{t_{in}}^{t} \left(\frac{A(t')}{A(t)}\right)^4 \left(-\frac{\partial \rho_q(K_h(t'),t')}{\partial K_h}\right) \frac{\partial K_h(t')}{\partial t'} dt' \equiv A^{-4}(t) F(t) \tag{11b}$$

$$\overline{p} = \frac{\rho}{3}$$

(in Appendix 2 a justification for this equation of state is given) where the time $t = t_{in}$ corresponds to the initial vacuum state of field, $K_h(t) = A/t$ - is the momentum, corresponding to the size of the horizon at time t ; n_q and ρ_q in general are defined as the vacuum mean values of the particle number operators and the operator of SET of an electromagnetic field obeying the Maxwell equations $\nabla_\mu F^{\mu\nu} = O$

$$n_q(t) = \frac{1}{V(t)v} < O_{t_{in}} \mid \hat{N}(t) \mid O_{t_{in}} >,$$

$$\rho(t) = T_0^0(t),$$

$$T_{\alpha\mu}(t) =< O_{t_{in}} \mid N_t \hat{T}_{\alpha\mu}(t, \overline{x}) \mid O_{t_{in}} >,$$

$$N_t \hat{T}_{\alpha\mu} = \hat{T}_{\alpha\mu} - < 0_t \mid \hat{T}_{\alpha\mu} \mid O_t >,$$

$$T_{\alpha\mu} = -\frac{g^{\nu\rho}}{2}\{F_{\mu\rho}, F_{\alpha\nu}\} + \frac{1}{8} g_{\alpha\mu}\{F^{\beta\nu}, F_{\beta\nu}\}$$

$$\{A, B\} = AB + BA.$$

The initial vacuum state $\mid O_{t_{in}} >$ is determined by means of diagonalizing the Hamiltonian of the electromagnetic field at $t = t_{in}$. The transition to the vacuum state $\mid 0_t >$ at t is done by Bogolubov's transformations. So we can present the vacuum mean values of $T_{\alpha\mu}$ in the form :

$$T^\mu_\nu = \int d\varphi d\theta sin\theta \int dK_0(t, K, \theta, \varphi) \overline{T}^\mu_\nu(t, K, \theta, \varphi)$$

where $K_0(t, K, \theta, \varphi) = K g^{-1}(t, \theta, \varphi)$ is the physical frequency of a photon, K, θ, φ are spherical coordinates in momentum space, and

$$g^{-1}(t, \theta, \varphi) = [\frac{sin^2\theta cos^2\varphi}{A_1^2} + \frac{sin^2\theta sin^2\varphi}{A_2^2} + \frac{cos^2\theta}{A_3^2}]^{\frac{1}{2}} \qquad (12)$$

The non-zero components of T^μ_ν can be expressed in the following way:

$$\overline{T}^0_O = \frac{K^3}{V} \sum_r 2S^r,$$

$$\overline{T}_1^1 = \frac{K^3}{V} \sum_r \left(-cos2\varphi X^r + sin2\varphi Y^r - \frac{sin^2\theta}{2}(2S^r + U^r) \right),$$

$$\overline{T}_2^2 = \frac{K^3}{V} \sum_r \left(cos2\varphi X^r - sin2\varphi Y^r - \frac{sin^2\theta}{2}(2S^r + U^r) \right),$$

$$\overline{T}_3^3 = \frac{K^3}{V} \sum_r \left(-2S^r cos^2\theta + U^r sin^2\theta \right)$$

$$\overline{T}^{12} = \frac{K^3}{A_1 A_2 V} \sum_r \left(sin2\varphi X^r + cos2\varphi Y^r \right),$$

$$\overline{T}^{13} = \frac{K^3}{A_1 A_3 V} \sum_r \left(cos\varphi \frac{sin2\theta}{2}(2S^r + U^r) + sin\varphi tg\theta Y^r \right),$$

$$\overline{T}^{23} = \frac{K^3}{A_2 A_3 V} \sum_r \left(sin\varphi \frac{sin2\theta}{2}(2S^r + U^r) - cos\varphi tg\theta Y^r \right).$$

The fact that $T^{0i} = 0$ is due to the metric (7) being homogeneous; the trace of the SET, $T_\mu^\mu = 0$ is due to the maslessness of the field and the field equations being conformally invariant . For the values (11) we are interested in one can obtain

$$n_q(K_h(t'), t') = \frac{1}{(2\pi)^3 A^3(t')} \sum_r \int_0^{K_h} K^2 dK \int_0^{\pi}\int_0^{2\pi} sin\theta d\theta d\varphi S^r(t', K, \theta, \varphi) \tag{13a}$$

$$\rho_q(K_h(t'), t') = \frac{1}{(2\pi)^3 A^4(t')} \sum_r \int_0^{K_h} K^3 dK \int_0^{\pi}\int_0^{2\pi} sin\theta d\theta d\varphi S^r(t', K, \theta, \varphi) \tag{13b}$$

where $r = +1, -1$ corresponds to the two states of photon polarization. Futher we will consider expressions (13) in second order of the anisotropy parameter ΔH, restricting ourselves only to the axial-symmetric case ($A_1 = A_2$). In this case $S^{+1} = S^{-1}$ and the functions $S(t, k, \theta)$ obey the set of equations

$$\frac{dS}{dt} = \frac{1}{2} W U$$

$$\frac{dU}{dt} = W(2S + 1) - 2(\frac{K}{A})V$$

$$\frac{dV}{dt} = 2(\frac{K}{A})U,$$

where $W = -\Delta H(cos^2\theta - 1)$. In second order approximation one can obtain

$$S = (\frac{1}{4})\left(\int_{t_{in}}^{t} W(t')dt'\right)^2$$

Now we suggest that A_i has the form

$$A_i = A_{io}t^{\lambda_i}(t+t_{rec})^{2/3-\lambda_i}, A = (\prod_{i=1}^{3} A_i)^{1/3} = (\prod_{i=1}^{3} A_{io})^{1/3} t^{1/3}(t+t_{rec})^{1/3}, \tag{14}$$

where t_{rec} is the time of recombination. It easy to see that if $t << t_{rec}$ (14) has the Kasner form; in the case $t >> t_{rec}$ it corresponds to an isotropic metric with dust matter. Then we will use the new variable $X = 1 + Z$, where Z is the red shift. In terms of X we have

$$H(X) = H_o B(X^3 + A^3)^{1/2} X^{3/2}; \Delta H(X) = \Delta H_o X^3; A(X) = A_o X^{-1},$$

$$dt = -\frac{dX}{H_o B X^{5/2}(X^3 + A^3)^{1/2}}$$

where $A = 2(1 + Z_{rec}), B = (1 + A^3)^{-1/2}, H_o, \Delta H_o$ and A_o correspond to the modern epoch. Now we are going to calculate n_c and ρ_c for the period of time $X_{in} > X > X_{rec}$. After a rather long calculation one can get[9]

$$n_c = (\frac{9}{8\pi^2})C_\theta^2 B H_o^3 (\frac{\Delta H_o}{H_o})^2 X^3 f(X),$$

$$\rho_c = (\frac{81}{32\pi^2})C_\theta^2 B^2 H_o^4 (\frac{\Delta H_o}{H_o})^2 X^4 F(X),$$

where

$$f(X) = (1/8)(X_{in}^6 - X^6) - X^6 ln^2(X/X_{in}) + (1/3)X^6 ln(X/X_{in}), \quad (15a)$$

$$F(X) = (1/32)(X_{in}^8 - X^8) - X^8 ln^2(X/X_{in}) + (1/4)X^8 ln(X/X_{in}) \quad (15b)$$

It is seen that $n_c = 0, \rho_c = 0$ at $X = X_{in}$, because at that moment we have the vacuum state without any particles. We suggest that X_{in} has a great value $\approx 10^{30}$ corresponding to the Planckian time $t_{pl} \approx 10^{-44}s$. Because we try to calculate n_c and ρ_c for the evolution period $X_{in} >> X$ we can neglect all terms, contained X after a short period of the Universe evolution beginning from X_{in} . Thus the main contribution to n_c and ρ_c comes from particles created near $X = X_{in}$. Afterwards $n_c X^{-3}$ and $\rho_c X^{-4}$ become constants and there will no more particle production and entropy production. That is why if we assume that after X becomes of order $0.01 X_{in}$ we neglect all terms in (15) containing X; one can say that the isotropization of matter has taken place during the period of time when the relation of scale factors become equal

$$\frac{A(t_{in})}{A(t_{is})} = \frac{(1 + Z_{is})}{(1 + Z_{in})} = \frac{X_{is}}{X_{in}} = 0,01$$

Beginning from the moment of time $X \approx 0,01 X_{in}$ the particle density number and energy density have the thermodynamical form and are changed in value only due to the total expansion of the Universe. The contribution to the integrals n_c and ρ_c from the epoch of time $t_{rec} < t < t_o$ ($X_{rec} > X > 1$) is negligible in comparison with the previous one .

Then for the $X << X_{in}$ we can write

$$n_c = \frac{9}{144\pi^2} C_\theta^2 B H_o^3 (\frac{\Delta H_o}{H_o})^2 X_{in}^6 X^3,$$

$$\rho_c = \frac{81}{1024\pi^2} C_\theta^2 B^2 H_o^4 (\frac{\Delta H_o}{H_o})^2 X_{in}^8 X^4.$$

By analogy with the thermodynamics of black body radiation let us define the temperature as

$$\frac{\rho_c}{n_c} = c T_{ph},$$

$$T_{ph} = c' H_o X_{in}^2 X \equiv \frac{T}{A} \tag{16}$$

This expression is independent of the anisotropy parameter ΔH_o in spite of the fact that this result was obtained for $\Delta H \neq 0$. The effective thermodynamical temperature T_{ph} doesn't contain any information about the anisotropy of the background space-time and type of matter field. Then we can write

$$n_c = c_n (\frac{\Delta H_o}{H_o})^2 T_{ph}^3, \tag{17a}$$

$$\rho_c = c_\rho (\frac{\Delta H_o}{H_o})^2 T_{ph}^4. \tag{17b}$$

The expressions (17) have a quasi-thermodynamical form, which differs from the standard black body radiation thermodynamics by a multiplier

$$(\frac{\Delta H_o}{H_o})^2 = (\frac{\Delta H(X)}{H(X)})^2.$$

To give it strongly thermodynamical meaning let us suggest that we can represent n_c and ρ_c as integrals over the whole momentum space, as in the case of the Bose-gas:

$$n = (\frac{1}{\pi^2 A^3}) \int_0^\infty \frac{dK K^2}{exp(\frac{(K-\mu)}{T}) - 1} \tag{18a}$$

$$\rho = (\frac{1}{\pi^2 A^4}) \int_0^\infty \frac{dK K^3}{exp(\frac{(K-\mu)}{T}) - 1} \tag{18b}$$

where K, μ, T are related to the corresponding physical values as it was for temperature (16).

$$K_o(t) = K/A(t), \mu_{ph}(t) = \mu/A(t)$$

Comparing the formulas (17) and (18) we can calculate the value of the chemical potential. To do this let us turn to the new variable

$$z = K/T,$$

in the integrals; then we will have

$$n = \frac{1}{\pi^2}(\frac{T}{A})^3 \int_0^\infty \frac{dzz^2}{exp(z - \frac{\mu}{T}) - 1}$$

$$\rho = \frac{1}{\pi^2}(\frac{T}{A})^4 \int_0^\infty \frac{dzz^3}{exp(z - \frac{\mu}{T}) - 1}$$

Let us suggest now that $exp(\mu/T) << 1$ (we will verify this fact below) then in the main order

$$\int_0^\infty \frac{dzz^n}{exp(z - \frac{\mu}{T}) - 1} \simeq \chi_n exp(\frac{\mu}{T})$$

where $\chi_2 = 2, \chi_3 = 3$ and for n_c and ρ we have

$$n_c = \frac{2}{\pi^2}(\frac{T}{A})^3 exp(\frac{\mu}{T}),$$

$$\rho_c = \frac{6}{\pi^2}(\frac{T}{A})^4 exp(\frac{\mu}{T})$$

Comparing these formulas with those previously obtained (17) we get the relation

$$\alpha(\frac{\Delta H_o}{H_o})^2 = exp(\frac{\mu}{T})$$

or

$$\frac{\mu}{T} = ln(\alpha(\frac{\Delta H_o}{H_o})^2) = ln(\alpha(\frac{E_{anis}}{E_{isot}})) \tag{19}$$

From here one can write the effective energy distribution of photons created in anisotropic space-time the wavelength of which is less than horizon length at moment t ; this distribution is constant in time and the only reason for the particle density to change in time is the Universal expansion:

$$n_k = \frac{1}{exp\left(\frac{K/A}{T/A} - ln\alpha(\frac{\Delta H_o}{H_o})^2\right) - 1} \tag{20}$$

We see then the meaning of the chemical potential in the particle creation effect in anisotropic Universe : it is related to the property of the gravitational field represented by the degree of anisotropy. From the expression of the chemical potential (19) one can come to the conclusion that the growth of entropy takes place in an anisotropic Universe indeed, because in the formula

$$dS(t) = -\frac{\mu(t)}{T(t)} dN(t) = -\frac{\mu}{T} dN(t)$$

$\mu/T < 0$ (because $\Delta H_o/H_o << 1$) while the number of particles increases. In conclusion we calculate the entropy of photons created in an anisotropic Universe beginning from the moment of time t_{in} . By the definition of the entropy through the thermodynamical potential

$$S = -\left(\frac{\partial \Omega}{\partial T_{ph}}\right)_{V,\mu}$$

For a Bose-gas with distribution (20) one can use the following relation

$$\Omega = -\frac{1}{3}\rho,$$

for the density of entropy one obtains that

$$S = \frac{1}{3}\left(\frac{\partial \rho}{\partial (T/A)}\right)_{V,\mu} = \frac{4}{(2\pi)^3}(\frac{T}{A})^3 exp(\frac{\mu}{T})(4 - \frac{\mu}{T})$$

where μ/T is given by the formula (19).

On page 3 we called $S(t), T(t)$ and $\mu(t)$ symbols. But here we show that these symbolical values really correspond to some entropy, temperature and chemical potential, because they coincide with their values at times when thermodynamical equilibrium has already taken place.

Let us summarize the results of our paper.

1. We proposed a way of interpreting the process of coming to thermodynamical equilibrium in the system of particles, created in an anisotropic Universe without any interaction with other matter fields inside the region restricted by the horizon size.
2. We have shown that in this region the process of entropy production takes place only during a short period of time after the initial vacuum state of the field was chosen. During the same period of

time the isotropization of matter created for the above mentioned region takes place.

3. After the entropy production is finished photons behave like a Bose-particle gas with negative chemical potential (which is determined by the degree of anisotropy of the external gravitational field or by the ratio of the energy of anisotropy to the energy of the total isotropic expansion), and with a temperature which is independent of the degree of anisotropy. The equilibrium value of the entropy of created particles was calculated.

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Appendix 1. The separation of field modes for a massless non-conformal scalar field in isotropic space-time.

Let's consider an equation for a massless nonconformal scalar field

$$\nabla_\mu \nabla^\mu \varphi = 0$$

in the metric (2) in terms of the conformal time η

$$dt = A(\eta) d\eta$$

The time-dependent part of this equation has the form

$$\frac{d^2}{d\eta^2}\varphi(\eta,k) + 2\frac{1}{A}\frac{dA}{d\eta}\frac{d}{d\eta}\varphi(\eta,k) + k^2\varphi(\eta,k) = O \qquad \text{(A1)}$$

After changing the variable to $\varphi = \phi/A$ we come to the equation

$$\frac{d^2}{d\eta^2}\phi + \left(k^2 - \frac{1}{A}\frac{d^2A}{d\eta^2}\right)\phi = 0 \qquad \text{(A2)}$$

We consider separately the two cases of power law and inflationary expansion 1) In the case of power law expansion $A(\eta) = \eta^p$,(where p=1 corresponds to the radiatiuon-dominated stage of evolution, p=2 - to the dust matter) equation $(A2)$ will have the form

$$\frac{d^2}{d\eta^2}\phi + (k^2 - p(p-1)\frac{1}{\eta^2})\phi = 0$$

It is seen, that if $k >> 1/\eta$ or in terms of synchronous time $k/A >> 1/t$ we have the usual equation for a massless conformal field in conformally-flat space-time. Now we can separate those field modes with momentum larger than the "horizon momentum"

$$k >> K_h = \frac{A}{t}$$

for which there is no parametric amplification by the external gravitational field. Thios means that there is no creation or annihilation of quasiparticles for these modes in the isotropic metric (2). That's why for these field modes one has a good definition of particles in the external gravitational field like in S-matrix theory.

For field modes with

$$k << K_h$$

we have an essentially quantum regime of behaviour. Since the Universe expands we can say, that those modes of the field which were created by quantum process with a wavelength larger than the size of the horizon, then go inside horizon region and are accumulated there, become "frozen" (in the sense that they are not affected there by quantum effects). That's why in this case we can talk about growth of particles inside the horizon region.

2) Inflationary Universe. Let us consider a model of inflationary expansion with scale factor

$$A(\eta) = (2 - \frac{\eta}{\eta_1})^{-1} A(\eta_1)$$

$$\eta < \eta_1$$

corresponding to the de-Sitter stage of evolution, matched with the metric

$$A(\eta) = (\frac{\eta}{\eta_1}) A(\eta_1)$$

corresponding to the radiation-dominated stage.

During the de-Sitter stage, as follows from Einstein equations

$$H^2 = \frac{8\pi G}{3c^2}\rho = A(\eta_1)^{-2}{\eta_1}^{-2} = const.$$

If we take into account now that

$$\frac{d^2A}{d\eta^2} = 2A^2H^2$$

one can rewrite equation $(A1)$ as

$$\frac{d^2}{d\eta^2}\phi + (k^2 - 2A^2H^2)\phi = 0$$

and we again can conclude that if

$$K_{phys}^2 = \left(\frac{k}{A}\right)^2 >> H^2 = const$$

equation $(A1)$ becomes the usual equation for a massless conformal field in the isotropic metric (2), and there are no quantum effects inside the region with constant horizon $\sim H^{-1}$.

It can be seen also from the solution of equation $(A1)$ representing a de-Sitter-invariant field vacuum state that

$$\varphi(\eta, k) = \frac{A(\eta_1)}{A(\eta)}\left(1 + i\frac{A(\eta)}{k}H\right) exp(-ik(\eta - \eta_1)) \tag{A3}$$

which corresponds to the Hadamard form of two-point function or vacuum state with a finite SET[10]. If $AH/k << 1$ then we can neglect the second term in the brackets and we will have the usual solution for a massless field equation in a conformally-flat metric.

If $AH/k >> 1$ then we can write $(A3)$ in the form

$$\varphi(\eta, k) = i\frac{A(\eta_1)}{k}Hexp(-ik(\eta - \eta_1)).$$

The amplitude of φ is

$$\mid \varphi \mid = \frac{A(\eta_1)}{k}H = const.$$

In this case field modes are "frozen" (in the classical sense) outside the horizon in the de-Sitter space-time.

During the radiation-dominated stage

$$\frac{d^2A}{d\eta^2} = 0$$

we have a stable vacuum, there is no quasiparticle production and annihilation there. One has again a good definition of real particles. After the radiation-dominated era begins the "long", "frozen" modes of the field in the de-Sitter invariant vacuum are going inside the expanding horizon and are accumulated there so that we can speak again about the growth of particle number.

Appendix 2. An exclusion of the local part and the conformal anomaly of SET as a justification for the equation of state for real particles.

To justify using in our considerations the equation of state

$$\overline{p} = \frac{\rho}{3}$$

for real particles , which in "classical" theory takes place due to tracelessness of SET

$$T^{\mu}_{\mu} = 0$$

we have to prove that in a approach of quantum field theory approach it is also valid. In other words we have to show that there is no so called "conformal anomaly" of the renormalized SET (RSET) for field modes, corresponding to the particles, accumulated inside the horizon region.

Usually this conformal anomaly is connected with the local (in time) part of RSET

$$T^{ren}_{\mu\nu} = T^{nonloc}_{\mu\nu} + T^{loc}_{\mu\nu}$$

and

$$T^{\mu anom}_{\mu} \equiv \lim_{m\to 0} T^{\mu loc}_{\mu}(m)$$

Let's take, for example, the $T^{0\ loc}_{0}$ component of RSET. It is known that the expression for it has the form[7]

$$T^{0\ loc}_{0} = \lim_{m\to 0}\left(-\frac{1}{(2\pi)^3 A^4}\int d^3k K_0 (S^f_2 + S^f_4)\right), \tag{A4}$$

where S_2^f and S_4^f are the corresponding terms in the sum

$$S = \sum_{n=2}^{\infty} h^{-n}(S_n^d + S_n^f)$$

when $h \to \infty$ (the parameter h is defined in the standard way: $m \to mh$, $k \to kh$, $K_0 \to K_0 h$) and initially the theory is supposed to be massive. The superscript "d" denotes the terms in S_2 and S_4 having divergent integrals in the ultraviolet limit ($k \to \infty$). According to the generally accepted point of view the expression $T_0^{0\ loc}$ describes the energy density of vacuum polarization.

Functions S_2 and S_4 arise in the expression for $T_{\mu\nu}^{ren}$ because of the regularization in the ultraviolet part of the spectrum. This part of the spectrum corresponds to those field modes with high momentum k, which in our approach correspond to the real particles accumulated inside the region of the size of the particle horizon. That's why if we represent $(A4)$ in the form

$$T_0^{0\ loc} = \lim_{m\to 0}\left(-\frac{1}{(2\pi)^3 A^4}\left(\int_0^{\lambda_i} + \int_{\lambda_i}^{\infty}\right)d^3k K_0(S_2^f + S_4^f)\right)$$

where λ_i is a parameter in momentum space of the order of the horizon ($\lambda_{iphys} = \lambda_i/A \sim 1/t$), the integral $\int_0^{\lambda_i}$ will correspond to vacuum polarization connected with quantum effects outside the horizon and with creation and annihilation of quasiparticles.

The integral $\int_{\lambda_i}^{\infty}$ will correspond to the contribution in RSET from field modes inside the horizon region. This part of the spectrum is associated with real particles accumulated inside horizon. We show now that this contribution to the RSET is equal to zero.

To do this, consider the structure of the local part of RSET. Using the results of Ref.7 for S_2^f and S_4^f we have

$$S_2^f = Y(g)\frac{(mg)^4}{K_0^6},$$

$$S_4^f = \sum_{n=3}^{6} \frac{((mg)^2)^{n-2}}{(K_0^2)^n} Y_{2n}(g),$$

$$K_0^2 = k^2 + m^2 g^2$$

where the expression for g is given by formula (12). Then

$$T_0^{0\ loc}(\lambda_i;\infty) = \lim_{m\to 0}\left(-\frac{1}{(2\pi)^3 A^4}\int d\varphi d\theta sin\theta\left(Y(g)\int_{\lambda_i}^{\infty} k^2 dk\frac{(mg)^4}{K_0^5} + \sum_{n=3}^{6} Y_{2n}(g)\int_{\lambda_i}^{\infty} k^2 dk\frac{((mg)^2)^{n-2}}{K_0^{2n-1}}\right)\right),$$

Consider the expression within the internal round brackets. The first term is of no interest for our investigation because after integration it goes to zero for $m \to 0$. Consider another term. Let's take, for example the term with $n = 3$. If just for the moment we take $\lambda_i = 0$, then we will have

$$\int_0^{\infty} k^2 dk\frac{(mg)^2}{K_0^5} = \frac{1}{3}$$

and there is no mass in the final result. The same fact is true for $n = 4, 5, 6$ and as a result

$$\int_0^{\infty} k^2 dk\frac{((mg)^2)^{n-2}}{K_0^{2n-1}} = C_n.$$

Then

$$T_0^{0\ loc} = \lim_{m\to 0}\left(-\frac{1}{\pi^2 A^4}\sum_{n=3}^{6} Y_{2n}(g)C_n\right) = -\frac{1}{\pi^2 A^4}\sum_{n=3}^{6} Y_{2n}(g)C_n,$$

gives a contribution to the vacuum polarization and for the trace $T_\mu^{\mu loc}$ we will have the conformal anomaly. But since we consider the contribution into RSET only from those field modes with $k > \lambda_i = A/t$, i.e. in our case $\lambda_i \neq 0$, we have

$$\lim_{m\to 0}\int_{\lambda_i}^{\infty} k^2 dk\frac{(mg)^2}{K_0^5} = \lim_{m\to 0}\frac{1}{3}\left(1 - \frac{\lambda_i^3}{(m^2 g^2 + \lambda_i^2)^{3/2}}\right) = 0.$$

The same result holds for $n = 4, 5, 6$. In the long run we have[7]:

$$T_0^{0\ loc}(k > \lambda_i) = 0,$$

$$T_{\mu}^{\mu loc}(k > \lambda_i) = 0.$$

This fact proves our choice for the equation of state

$$\overline{p} = \frac{\rho}{3}$$

for real particles, accumulated inside horizon region.

References

[1] Prigogine I., Stengers I. Order out of chaos. Heineman, London, 1984.

[2] Hu B.L. Physics Letters, v. 97A, p.368, 1983.

[3] Gunzig E., Geheniau J., Prigogine I. Nature, v.330, p.621, 1987.

[4] Prigogine I., Geheniau J., Gunzig E., Nardone P. Proc. Natl. Acad. Sci. USA,v.85, p. 7428, 1988.

[5] Penrose R., in : General Relativity: An Einstein centenary, eds. S.W.Hawking and W.Israel Cambridge University Press, London, 1979.

[6] Birrell N.D., Davies P.C.W. Quantum fields in curved space.Oxford.,1979.

[7] Grib A.A., Nesteruk A.V.,Pritomanov S.A. Classical and Quantum Gravity, v.5,p.1053,1988.

[8] Misner C.W., Thorn K.S., Wheeler J.A., Gravitation (San Francisco:Freeman), 1973.

[9] Nesteruk A.V., Pritomanov S.A. Classical and Quantum Gravity, v.7, p.L155,1990.

[10] Allen B. Physical Review, v.37, N 8, p.2078-2085,1988

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