An extension of wave mechanics to the rest frame

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ABSTRACT. An extended form of the de Broglie hypthesis is proposed, as the basis for a new particle concept. The extended hypothesis identifies a massive particle in its rest frame with a stable standing-wave packet that can be decomposed into travelling waves counter-propagating along the particle trajectory. This packet transforms, in frames moving with respect to the particle, to produce a stable packet composed of de Broglie waves with the correct properties, combined with waves of very short wavelength, that would generally not be detectable in experiments involving objects with atomic dimensions. A consequence of the proposed hypothesis is that the description of events in the history of a particle must be strictly symmetrical with respect to time.

RESUME. On propose une forme étendue de l'hypothèse de de Broglie comme base d'un nouveau concept de particule. Cette hypothèse identifie une particule massive dans son référentiel propre à un paquet d'ondes stationnaires pouvant se décomposer en ondes se propageant en sens contraires sur la trajectoire de la particule. Ce paquet se transforme dans les systèmes de référence se déplaçant avec la particule et produit un ensemble stable composé d'ondes de de Broglie, de propriétés convenables, qui sont associées à des ondes de faibles longueurs d'onde généralement non décelables lors d'expériences sur des objets de dimensions atomiques. Une conséquence de l'hypothèse proposée est que l'historique d'une particule doit être décrit sans faire de distinction dans la chronologie des événements passés et futurs.

1. Introduction.

This work shows how some changes to the basic assumptions of wave mechanics can lead to a new conception of a quantum particle.

The changes include the creation of a stable wave packet to represent the particle in all inertial frames, including the rest frame, and the assumption that the trajectory of a quantum particle is fixed in space-time. In some simple examples, the resulting kinetics are shown to agree with experiment, and to produce the correct quantum mechanical results, including the selection of the appropriate portions of the waveforms arising from interaction with, for example, a double slit system, or a partially reflecting barrier.

2. Historical background.

We begin by recalling the background of current particle concepts [1]. Wave mechanics received a major impetus from de Broglie's hypothesis, which stated that with a free particle of energy γmc^2 and momentum γmv, there is associated a travelling wave with frequency $\gamma mc^2/h$, and wavelength $h/\gamma mv$ (h is Planck's Constant and γ is $1/\sqrt{\{1-(v/c)^2\}}$). Moving-frame (or moving-particle) wave properties were associated with the energy and momentum of the particle. The work of Schrödinger, and later Dirac and many others, modified and extended the concept of de Broglie, introducing the concept of a wave packet, and identifying the particle energy and momentum with differential operators. Wave equations were developed for describing moving particles as waves (herein generically referred to as de Broglie waves). Current wave mechanics thus describes the behaviour of a particle that is in motion with respect to the observer. The de Broglie wavelength of a particle becomes infinitely large as the particle comes to rest, and there is essentially no information available from quantum mechanics on the particle location, and very little on other rest-frame properties.

3. Rest frame properties.

There seems, however, to be no *a priori* reason to preclude a more informative rest-frame description of a particle. This does not imply a violation of the uncertainty principle, since, as we shall see, the principle is not directly applicable to particles in the rest frame.

Based on well-established principles of relativity theory, one would expect all inertial frames with $v < c$ to be equivalent, accessible in the physical description of a massive particle, and to contain exactly the same information concerning the intrinsic properties of the particle. It therefore seems anomalous that this is not the case for quantum mechanics.

These considerations, and the hope for further insight into the nature of de Broglie waves, have led to a search for an extended hypothesis that permits the particle description to include the rest frame. Such a hypothesis must, of course, produce a kinetic theory that agrees with observation, and should have a logical connection with the wave equations. The hypothesis should also include massless particles in a consistent manner.

4. Extended hypothesis.

An extended hypothesis analogous to that of de Broglie (that is, without reference to fields and spin), is presented here and used to illustrate the main features of the concept. The hypothesis would, of course, have to be augmented to accomodate fields and spin, in developing a comprehensive theory. The hypothesis is in three parts :

(i) With a massive free particle in its rest frame can be associated a stable standing wave packet, with central wavelength $\lambda = h/mc$, and frequency $f = mc^2/h$. The standing waves of the rest-frame packet transform, in a frame moving at velocity $-v$, to travelling waves with two sinusoidal components. One component is the de Broglie wave; the other has a wavelength equal to $h/\gamma mc$, and moves with velocity v. These travelling waves combine to produce a stable packet in all frames.

(ii) With a massless free particle moving at velocity c can be associated a (dispersionless) wave packet with central wavelength $\lambda = hc/E$, and frequency $f = E/h$.

(iii) The wave packet of a massive or massless free particle can be decomposed into counter-propagating travelling waves moving at velocity c along the particle trajectory, which is fixed in space-time.

To illustrate the consequences of the hypothesis, we will discuss some simple examples, and we assume that the rest-frame waveforms introduced are acceptable if they transform to moving-frame waveforms containing familiar de Broglie wavefunctions. The examples here are taken from the low velocity region, $v << c$, to allow comparison with results from the Schrödinger equation. But it should be noted that the treatment is relativistically covariant up to where approximations are made to bring the waveforms to resemble Schrödinger wavefunctions. Other wave functions will be included in a future study.

5. Example 1 : plane standing waves.

In the first example, we represent the particle in its rest frame not as a packet, but as a plane-standing-wave pattern, and transform it to a frame moving with velocity $-v$. The standing wave will later be localized to form a packet. The plane-standing-wave pattern is represented by:

$$F(z,t) = Z\cos(kz)\cos(2\pi ft), \tag{1}$$

where $k = 2\pi f/c$, and Z is, in general, a complex number.

To transform $F(z,t)$ to an inertial frame moving in the negative-z direction at speed v, we first decompose F into its component counter-propagating travelling waves:

$$F(z,t) = (Z/2)[\cos(2\pi ft - kz) + \cos(2\pi ft + kz)]. \tag{2}$$

The travelling wave components are then Doppler-shifted appropriately to transform to the moving frame. The wave travelling in the positive-z direction has the new frequency:

$$f_{\rightarrow} = \gamma f(1+\beta),$$

while that of the wave moving in the negative-z direction is:

$$f_{\leftarrow} = \gamma f(1-\beta) \quad , \qquad \text{with} \quad \beta = v/c.$$

Substituting these frequencies and the corresponding wavenumbers into (2) gives:

$$F'(z,t) = Z'\cos 2\pi(\gamma ft - \gamma\beta fz/c)\cos 2\pi(\gamma\beta ft - \gamma fz/c), \tag{3}$$

where the coordinates now refer to the moving frame.

The first cosine term in (3) is the de Broglie wave, with wavelength $\lambda' = h/p$, frequency $f' = \gamma mc^2/h$ and velocity c^2/v. The second cosine term has wavelength $\beta\lambda'$, frequency $\beta f'$ and velocity v.

If the wavelengths of these two terms differ greatly, i.e., if $\beta << 1$, the de Broglie wave becomes a long-wavelength modulation of the "carrier wave" denoted by the second cosine term. In this case, the presence of the rapid spatial oscillations of the latter term are undetectable in most experiments (see the discussion of the next example); for instance, the electron carrier wavelength is nearly constant at about

$2.43 \times 10^{-10}cm$. The long-wavelength de Broglie component can, however, result in observable interference and diffraction effects on interacting with structures having atomic dimensions. As v/c approaches 1, the de Broglie and carrier wavelengths, frequencies and velocities respectively, approach equality.

The de Broglie wave in this picture is a modulation that appears when there is relative motion of the reference frame and the particle. A natural consequence is that a momentum change of the particle causes the de Broglie wave modulation in the original moving frame to disappear, to be replaced by one corresponding to the new particle momentum. When the particle comes to rest, the de Broglie wave increases indefinitely in wavelength, and vanishes when $v = 0$.

6. Localization in the rest frame.

A packet can be constructed using a standing wave similar to that of (1), and modulating its space and time dependences with any smooth, symmetric function of position and time, having a single well-behaved maximum. For a stable particle, we use a time-independent Gaussian function, setting k_0 to be the rest-frame central wave number of the particle:

$$G(z,t) = Z \exp(-z^2 a^2) \cos(k_0 z) \cos(k_0 ct). \tag{4}$$

The choice of modulating function might be made with some particle model in mind, for example the electron model of Barut and Zanghi [2]. Representing (4) by a Fourier integral in z reveals that the spectrum of travelling waves that contribute to the function ranges in k from 0 to ∞, with the greatest weights given to values near k_0, as indicated by the weighting function for the integrals:

$$\phi(k) = \exp\Big[-\frac{(k+k_0)^2}{4a^2}\Big] + \exp\Big[-\frac{(k-k_0)^2}{4a^2}\Big] \quad . \tag{5}$$

7. Transformation of a wave packet.

Selecting the weighted central waves in G represented by k_0, we can write them as:

$$g(z,t) = z'[1 + \exp(-\frac{k_0^2}{a^2})][\cos k_0(ct - z) + \cos k_0(ct + z)]. \tag{6}$$

This pair of waves will thus be added together in the Fourier integral in phase and with equal weights, so when transformed to a moving frame, their sum produces a product of sinusoidal functions like [3]. All the travelling waves contributing to (4) can be paired in this way. The relative weights of the wave pairs is also preserved; thus counter-propagating waves are produced in every moving frame. Again viewing the packet in its form (4), the packet is narrower in a moving frame, since $1/a$ transforms to $1/a' = 1/\gamma a$.

If we express the rest frame packet in the form:

$$h(z,t) = Z \exp -(a^2 z^2) \cos(k_0 z) \cos(k_0 ct) = f(z) g(z,t), \tag{7}$$

where

$$g(z,t) = 1/2 \cos k_0(ct - z) + 1/2 \cos k_0(ct + z) = g_{\rightarrow}(z,t) + g_{\leftarrow}(z,t),$$

we can expand the factors in (7) as Fourier integrals at some time t. We do this by finding the Fourier transforms of the factors in

$$h(z,t) = f(z) g_{\rightarrow}(z + z_0) + f(z) g_{\leftarrow}(z - z_0), \tag{8}$$

where $z_0 = ct$. The Fourier transform, $H(k)$, of $h(z,t)$ is then

$$H(k) = (1/\pi)[F(k) \otimes G_{\rightarrow}(k) + F(k) \otimes G_{\leftarrow}(k)],$$

where the symbol $\otimes$ denotes the convolution, and $F(k)$, $G_{\rightarrow}(k)$, $G_{\leftarrow}(k)$ are the Fourier transforms of $f(z)$, $g_{\rightarrow}(z,t)$ and $g_{\leftarrow}(z,t)$ at time t. Note also that:

$$G_{\rightarrow}(k) = \exp(+ikct) G(k) \quad \text{and} \quad G_{\leftarrow}(k) = \exp(-ikct) G(k),$$

where $G(k)$ is the transform of $g(z,0)$. Evaluating the integrals and convolutions results in:

$$h(z,t) = (1/a\sqrt{\pi}) \int_0^\infty \phi(k)[\cos(k_0 ct - kz) + \cos(k_0 ct + kz)] dk, \tag{9}$$

where

$$\phi(k) = \exp -\frac{(k - k_0)^2}{4a^2} + \exp -\frac{(k + k_0)^2}{4a^2},$$

as discussed above (equation 5).

To study transformations, we consider a standing wave packet in the rest frame and transform it to a frame moving at velocity $-v$. The counter-propagating pairs of waves in the packet transform to Doppler-shifted pairs, as previously described. The resulting wave packet can be obtained by applying a Lorentz transformation to the coordinates in the function modulating the Doppler-shifted waves:

$$h'(z,t) \sim \exp -\gamma^2 a^2 (z-vt)^2 \{\cos\gamma k_0(1+\beta)(ct-z)+\cos\gamma k_0(1-\beta)(ct+z)\}.$$

The transformed packet must, of course, remain stable, but its observable width in the moving frame would be $1/\gamma a$, narrower than that observed in the rest frame by the factor γ.

The effect of the Doppler shift is to produce the carrier and de Broglie components of the packet. If a standing-wave packet exists in the rest frame, each of its component waves transforms in this way. The Fourier integral in the rest frame,

$$2\pi \int_0^\infty \phi(k)\{\cos(k_0 ct - kz) + \cos(k_0 ct + kz)\} dk, \tag{10}$$

transforms to :

$$2\pi \int_0^\infty \phi'(k) \exp -ik(\beta ct - z) dk \tag{11}$$

in the moving frame, in which the weighting function is now:

$$\begin{aligned} &\phi'(k) \\ &\sim \exp i\frac{k_0 ct}{\gamma} \cdot \exp -\frac{(k+k_\rightarrow)^2}{4\gamma^2 a^2} + \exp -i\frac{k_0 ct}{\gamma} \cdot \exp -\frac{(k-k_\rightarrow)^2}{4\gamma^2 a^2} \\ &+ \exp -i\frac{k_0 ct}{\gamma} \cdot \exp -\frac{(k+k_\leftarrow)^2}{4\gamma^2 a^2} + \exp i\frac{k_0 ct}{\gamma} \cdot \exp -\frac{(k-k_\leftarrow)^2}{4\gamma^2 a^2} \end{aligned} \tag{12}$$

where $k_\rightarrow$ and $k_\leftarrow$ are $\gamma k_0(1+\beta)$ and $\gamma k_0(1-\beta)$. The form of the integral (11) shows that the waves contributing to the moving packet are all of the "carrier-wave" type, with the same velocity, v. The time-varying modulation of these waves generates the de Broglie component.

The stability of this packet in all frames is at variance with the standard treatment of wave packet spreading, because the latter includes only the de Broglie components of the waves [3].

It is also clear from the relationship of the de Broglie waves to the rest-frame wavepacket, that the uncertainty principle cannot be directly applied to position and momentum in the rest frame, because the de Broglie waves to which the uncertainty principle refers do not exist in this frame.

It is important to note that the packet is composed of a spectrum of counter-propagating pairs of travelling waves in all reference frames. These waves traverse the particle trajectory in both the negative-z and positive-z directions.

9. Time symmetry and fixed space-time trajectory.

In the present conception, the particle description is made symmetric and equivalent in the space and time co-ordinates by requiring that the (relativistic) wave packet of a free particle decompose in all frames into counter-propagating plane waves moving along a fixed trajectory. This is in contrast to the special status of the time in relativistic wave mechanics, as embodied in the Klein-Gordon and Dirac wave equations [4], but parallels the co-ordinate equivalence incorporated in the path integral formulation of relativistic quantum mechanics [5,6], and a similar approach based on the notion of a gauge transformation [7].

The time symmetry and fixed trajectory of the particle will be shown to imply that the correct quantum-mechanical results can be obtained without recourse to "collapse" of unwanted components of the wavefunction.

10. Massless particles.

An analogous picture can be developed for a massless particle. It is considered to be a stable wave packet that is composed of travelling waves traversing the trajectory of the particle in a manner similar to that described for massive particles.

11. Example 2: interaction with a partially reflecting barrier.

We next analyse the interaction of a free particle incident normally at speed $v \ll c$, on a partially reflecting potential barrier.

We will assume that the barrier potential is slightly greater than the particle kinetic energy, and that the barrier width is comparable to the de Broglie wavelength of the incident particle. These conditions lead, according to the Schrödinger equation, to both transmission and reflection of components of the packet at the barrier.

We start by representing the incident free particle in its rest frame by a plane standing wave packet, that can be decomposed into travelling waves counter-propagating along the line of relative motion of the barrier and the incoming particle. As described previously, these waves combine, in the frame in which the barrier is stationary, to produce a packet containing trains of travelling carrier waves moving toward the barrier at speed v, modulated by a packet of de Broglie waves with central wavelength h/p.

In determining their behaviour at the potential barrier, we use an exponential form for the waves in the moving frame:

$$F(z,t) = \exp(i/\hbar)[\gamma mc^2 t - pz]\exp(i/\hbar)[cpt - \gamma mcz] = F_{db}F_c \qquad (13)$$

We first put F_{db} into the Schrödinger form for $\beta \ll 1$, and set $\gamma mc^2 = mc^2 + p^2/2m$. Following Schrödinger, we extract the rapidly time-varying component:

$$F_{db} = [\exp(i/\hbar)mc^2 t]\psi_s \quad , \text{where} \quad \psi_s = \exp(i/\hbar)[p^2 t/2m - pz],$$

is the Schrödinger wavefunction. Now comparing time derivatives,

$$(dF_c/dt)/(d\psi_s/dt) = 2mc/p \gg 1,$$

we are justified in extracting the rapidly time-varying component from F_c, as we did the mc^2 term from F_{db}. Comparing the space derivatives,

$$(dF_c/dz)/(d\psi_s/dz) = \gamma mc/p \gg 1,$$

we extend the extraction process to rapidly space-varying terms, and remove the corresponding mc term in the approximation $\gamma mc = mc + p^2/2mc$:

$$F(z,t) = \{\exp(i/\hbar)[(mc^2 + cp)t - mcz]\exp(i/\hbar)[p^2 z/2mc]\}\psi_s. \qquad (14)$$

We note that $(p^2/2mc)/(d\psi_s/dz) = p/2mc \ll 1$, so the term $p^2 z/2mc$ can be neglected in comparison to pz in ψ_s. We therefore write, for $\beta \ll 1$:

$$F(z,t) = \{\exp(i/\hbar)[(mc^2 + cp)t - mcz]\}\psi_s(z,t) \quad . \qquad (15)$$

We then note that the rapid time and space oscillations of the first exponential term will average out over intervals of the coordinates observed in experiments, so the low-velocity variation of F is dominated by ψ_s, the Schrödinger wavefunction.

It is thus plausible that the particle in this picture behaves like a Schrödinger wave packet when interacting with the barrier, except for the consequences of time symmetry, as we now illustrate.

It is suggested that the behaviour of the packet on encountering the barrier is not entirely what would be predicted by the Schrödinger equation for de Broglie waves alone. The incoming packet can interact with the barrier and divide to produce a reflected packet and a transmitted packet, as usual, but to exist, each packet should be composed of travelling waves arriving from opposite directions along its trajectory. The particle is assumed to be a stable object, with a fixed, though not necessarily single, trajectory in space-time.

The set of travelling waves arriving from one direction should therefore follow the past trajectory of the incident packet and, subsequently, that of the reflected and/or transmitted packets. The waves arriving from the opposite direction should trace, in reverse, the future trajectories of the packets, connecting them with the future particle waveforms.

Thus the only packets, reflected or transmitted, that should be generated at the barrier are those connected with the particle waveform following the interaction with the barrier.

For example, if an absorber is placed in the path of either the transmitted or reflected packet, and results in an energy change of the particle, the corresponding packet would automatically be present, and the other absent, since the latter would not be connected with the new waveform resulting from the energy change. If the two packets are brought together so they first interfere before an energy change, then both packets, composed of past– and future– going travelling waves, would traverse their respective paths prior to the interaction.

Note that the Schrödinger equation describes quantitatively the splitting of the incoming packet into two packets at the barrier, but provides no logical way to eliminate one or the other when it takes no part in the future history of the particle. This results in the requirement that the unwanted components "collapse" at the time of an energy change, in the usual representation of wave mechanics.

The exact description of the experiment would follow if time symmetry is maintained. This implies that a particular outcome must be included in the *a priori* specification of an experiment; this is automatic in the proposed conception, by virtue of the requirement for both past– and future–directed travelling waves, and is also automatic in the Feynman integral formulation [5,6], and similar approaches [7].

12. Example 3: double slit interference experiment.

We next consider a double-slit interference experiment, consisting of a massive free particle incident with velocity $v \ll c$ on a barrier with two slits, and a screen that is many de Broglie wavelengths away from the slits.

We again represent the particle by a plane standing wave packet in its rest frame, then transform to the frame in which the slits and screen are stationary and the particle is approaching the slits at speed v. If the resulting de Broglie modulation has a wavelength commensurate with the slit spacing, then the wave passing through the slits would experience interference and diffraction as would be predicted for de Broglie waves alone, since the carrier wavelength is too short to affect these processes significantly.

Experimental results suggest that the spread in k values in the Fourier expansion of free particle wave packets must be very small; the observed interference patterns are relatively sharp, so the only wavenumbers contributing significantly are close to the central value. This suggests the possibility of experimental verification of the existence of the carrier waves in an interference experiment, but the angular resolution required is equivalent to a spatial resolution of $10^{-10}cm$, and would be very difficult to achieve. The more serious problem with such attempts to observe the carrier wave would be that normal matter does not exhibit sufficiently rapid spatial transitions on the scale of the carrier wavelength to provide sharp interference patterns or other phenomena with optical counterparts.

An experiment capable of detecting the electron carrier wave would require, for example, a crystalline solid whose lattice spacing in at least one direction has been reduced by a factor of over 100. This might be achievable using inertial compression techniques similar to those employed in fusion experiments.

The incident packet in this conception is connected to waves passing through the slits, which there form packets that interfere and converge to the small region on the screen where the particle is absorbed. As in the previous example, only those portions of the interference pattern are produced that are in packets connected with the trajectory of the particle preceding the interaction with the slits, and following the interaction with the screen.

13. Time symmetry and determinism.

In this picture, quantum wave mechanics would rigourously adhere to the principle of time symmetry, that no distinction is allowed between past-directed and future-directed sequences of events.

This time-symmetric approach would maintain the single-particle nature of the energy exchanges with the environment, and would remove the requirement for collapse of extraneous components of the final wavefunction.

It is interesting to note that this conception of a particle involves near-classical determinism, similar to that identified previously by Bell as removing the apparent non-locality from quantum-mechanics [8]. All events in the history of a quantum particle would be determined. There is an important difference, however, between this picture and classical determinism: given an outcome, a particle interaction would be exactly describable (using time symmetry), but the desired outcome would have to be specified in advance, and is not predictable. The existing wave equations would only identify the possible outcomes and estimate their probabilities.

14. Conclusion.

An extended form of the de Broglie hypothesis is proposed as the basis for a new conception of a particle. The extended hypothesis represents a massive particle in its rest frame as a stable standing-wave packet that can be decomposed into travelling waves counter-propagating along the fixed particle trajectory. This waveform transforms, in frames moving with respect to the particle, to produce a stable packet comprising de Broglie waves with the correct properties, combined with waves having a very short wavelength, that are generally not detectable in experiments involving objects with atomic dimensions.

In this picture, the particle is thus treated as a stable wave packet propagating in space along a fixed trajectory.

A consequence of the proposed hypothesis is that the description of events in the history of a particle must be strictly symmetrical with respect to time. This should lead to the correct quantum-mechanical description of these events, using the proposed picture and stable wave packets.

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