A Relativistic Model of an Elementary Particle

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RESUME.

A partir de l'énergie électromagnétique, un modèle relativiste d'une particule élémentaire est construit. Dans ce modèle, des équations consistantes de l'énergie totale, de la longueur d'onde de de Broglie et de l'impulsion sont déduites et on retrouve l'égalité

$$E = pc + m_0c^2 \quad .$$

On montre que les les lois de conservation de l'énergie et de l'impulsion sont des expressions différentes d'une même loi, que la masse relativiste de la particule est déterminée par l'équation du mouvement et enfin que la structure interne de la particule est conforme aux équations de Maxwell.

Les résultats sont comparés en détails aux formules présentes. En particulier, l'énergie totale est analysée. La manière présentée de généraliser l'équivalence entre l'énergie et la masse, $E = m_0c^2$ en $E = mc^2$, est discutée du point de vue physique.

ABSTRACT. Starting with the electromagnetic energy, a relativistic model of an elementary particle is constructed. By means of this model, consistent equations of total energy, de Broglie wavelength and momentum are derived. As a result, the relation

$$E = pc + m_0c^2$$

is achieved. The conservation laws of energy and momentum are found to be different expressions of the same law. The relativistic

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mass of the particle is determined by the equation of motion. Further, the internal structure of the particle is according to Maxwell's equations.

Results are compared in detail to the present formulas. In particular, the total energy is analyzed. The present way to generalize the equivalence between energy and mass, $E = m_0c^2$ into $E = mc^2$, is discussed from the physical point of view.

1. Introduction.

One of the fundamental physical items is the concept of mass. When the behaviour of an atomistic particle is treated mathematically, normally the mass m and the spatial coordinates of its center of mass, are used. Further, if a physical model for an atom is considered, particles are the natural parts of the model. The particle itself has in fact no model. It is supposed to be constructed of smaller parts or particles, which again are more or less massive. So the solution of the problem of mass is usually sought by means of other masses. However, the physical origin of the concept of mass is macroscopic. From the microscopic point of view it is known, that the mass and energy are equivalent. Consequently the question arises, whether the model of a particle can be constructed by means of energy instead of using a mass point. Furthermore one may ask too, how the mass depends on the velocity. A mass point cannot characterize these essential features. The progress in physics has long ago reached the level, where the mass or its equivalent energy, is to be handled relativistic. It is of course straightforward, if nothing else is to be done, but if for example the relativistic Schrödinger equation is in question, the relations between such prime quantities as mass, energy, momentum and velocity, are to be considered. The linearization process of the energy equation illustrates some of these problems. To assume, as it is sometimes thought, that the origin of these problems is only of mathematical character, may emphasize too much these important analytical methods. Simultaneously the physically critical question, whether the foundations are sound in every respect, is totally avoided. Be it as it may, but too often a discrepancy or contradiction is tried to be solved by adding only mathematical methods and not by evaluating crititically enough the mixture of those relativistic and non relativistic quantities, which are used parallel.

In this paper the intention is to construct a model for the mass, which is based on energy. Further, that model is used to the analysis of the relations between the prime physical quantities of a particle. The principle of this approach is to apply only generally accepted and verified physical items.

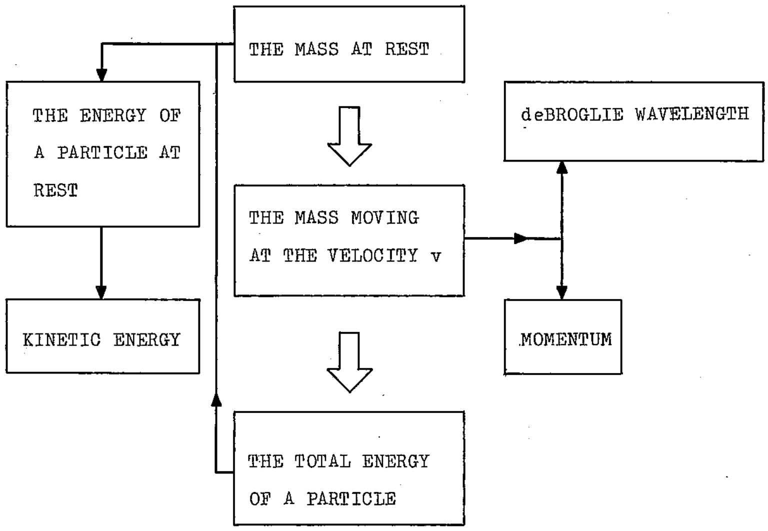

Figure 1. Relations between the essential physical quantities. The present energy equation is derived from the concept of mass. Momentum and kinetic energy have different physical basis.

The present situation is seen in Fig. 1. The concept of mass is found to be in a very central position. The other important quantities depend on it. Naturally all these relations are mathematically correct. However, from the physical point of view following remarks are made.

1. The total energy of a moving particle can easily be derived from the equations

$$E = mc^2 \tag{1}$$

$$m = \frac{m_0}{\sqrt{1-(v/c)^2}} \tag{2}$$

This mathematical operation has, in fact, no rigorous physical basis. If as it should be, the equation (1) is valid only, when $v = 0$, there is no other method to define the total energy E as

$$E = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}} \tag{3}$$

2. The momentum can be defined in two ways, classically by writing $p = mv$ or by means of de Broglie wavelength λ_{dB}, $p = h/\lambda_{dB}$. Neither of these has much to do with the kinetic energy. Physically this missing relation is in fact due to the same reason as is the case in the previous point.

These conclusions, which are of physical character, justify the different approach.

In the microworld the electromagnetic energy can be described by usual trigonometric functions. Its model can be said to be more informative than a mass point. On the contrary it has a geometry of its own. In the following a particle and also its mass is constructed by the electromagnetic radiation. By means of this model, which is called relativistic, first the total energy of the particle is derived. Then the de Broglie wavelength and momentum are determined. This is done without the concept of mass. However, it turns out, that when the equation of motion for the particle is calculated by derivating the momentum with respect to time, the relativistic mass is obtained as a part of that equation. The mass is no more a starting point, but a result of the equation of motion. This is in accordance with the classical concept of mass. Under these circumstances, when only generally accepted and verified physical items are applied, the interrelations between important quantities become consistent and consequent. The above described discrepancy between the momentum and kinetic energy is avoided.

As far as the priorities are concerned, it can be said, that the relativistic model prioritizes the total energy of the particle. All the other quantities are derived by means of that fundamental physical item. The model itself can explain qualitatively and quantitatively the behaviour of a particle under various conditions. A particle is, according to the model, no more just an object of calculations, a mass point with coordinates, but it itself can react to the influences.

The formal meaning of this approach is connected with the concept of mass. Instead of being a central part, now the mass is defined as a part of the equation of motion. It is clearly a result, not a fundamental starting point. It is well known, that the equation of motion can also be derived by means of the mass based system. However, in that case, the concept of mass is already needed, when the momentum is defined. Hence, it is not a result of the equation, where it, nevertheless can again be found, but it is a starting point. Some kind of classic analogy of this difference might be, if instead of writing

$$\frac{d(m \cdot v)}{dt}$$

one writes

$$m \cdot \frac{dv}{dt}.$$

Also it is to be emphasized, that based on Fig. 1, the kinetic energy $T \neq pc$. All this means, that the formula $p = mv$ is not valid in the relativistic region, as is also stated by Okun [1]. It is concluded, that the present method gives an equation of motion, which is based on mass, not on the momentum or its change, which is back to that motion. This is the principal difference between the mass based and energy based methods.

2. The structure of a particle.

Theoretically the relativistic model of a particle can be constructed by letting two identical electromagnetic radiation quanta resonate with each other in the following way. The first one moves along the positive direction of the x-axis, whereas the other moves to the negative direction. These two waves resonate with each other under such conditions, that a standing wave is formed. Denoting the electrical and magnetic vectors of the first wave by $\overline{E}_1$, $\overline{H}_1$ and of the second one $\overline{E}_2$, $\overline{H}_2$ respectively, they are expressed as follows

$$\overline{E}_1 = E_{10} \cdot \cos(\frac{2\pi}{\lambda}(x - c \cdot t)) \cdot \bar{j} \tag{4}$$

$$\overline{H}_1 = H_{10} \cdot \cos(\frac{2\pi}{\lambda}(x - c \cdot t)) \cdot \bar{k} \tag{5}$$

$$\overline{E}_2 = E_{20} \cdot \cos(\frac{2\pi}{\lambda}(x + c \cdot t)) \cdot \bar{j} \tag{6}$$

$$\overline{H}_2 = -H_{20} \cdot \cos(\frac{2\pi}{\lambda}(x + c \cdot t)) \cdot \bar{k} \tag{7}$$

where $\lambda =$ wavelength. Adding the components of the waves, it yields

$$\overline{E} = 2E_0 \cos \frac{2\pi x}{\lambda} \cdot \cos(-\frac{2\pi ct}{\lambda}) \cdot \bar{j} \tag{8}$$

$$\overline{H} = -2H_0 \sin \frac{2\pi x}{\lambda} \cdot \sin(-\frac{2\pi ct}{\lambda}) \cdot \bar{k} \tag{9}$$

Further the Poynting's vector is given by

$$\overline{P} = -E_0 H_0 \sin\frac{4\pi x}{\lambda}\cdot\sin(-\frac{4\pi ct}{\lambda})\cdot\bar{l} \tag{10}$$

The resulting wave has following properties.

1. It is a standing wave having a wavelength equal to λ. Effectively it seems as if the wave will reflect totally.
2. There is a $\pi/2$-phase shift between the electrical and magnetic vectors.
3. When $t = 0$, then

$$\begin{pmatrix} x & 0 & \lambda/4 & \lambda/2 \\ P & 0 & 0 & 0 \\ E & \neq 0 & 0 & \neq 0 \\ H & 0 & \neq 0 & 0 \end{pmatrix}$$

4. The equations fulfill the Maxwellian equations.

Depending on the way of reflection, this model can describe a particle, which is either positively or negatively charged. Further it can also be neutral, but then it has a magnetic moment. This kind of wave packet can also be compared with a mechanical situation, where two items are oscillating as mirror images. However, in this connection the use of the word mass is avoided. Internally this model is as accurate as the electromagnetic radiation can be determined by trigonometric functions. As far as the whole particle is concerned, the concept of mass is related to the model through energy. If the energy of the resonating wave packet is denoted by

$$E = h\nu_0 \tag{11}$$

where ν_0 is the resonance frequency equal to c/λ_0, then according to the special theory of relativity, also

$$E = m_0 c^2 \tag{12}$$

Now the mass m_0 is found to be

$$m_0 = h\nu_0/c^2. \tag{13}$$

The corresponding wavelength will be

$$\lambda_0 = \frac{c}{\nu_0} = \frac{h}{m_0 c}. \tag{14}$$

As a matter of fact the wavelength of this resonance frequency is of the order of magnitude as the known range of nuclear forces.

Apparently the relativistic model is in accordance with the equality of mass and energy. It is the starting point of the practical approach. Now the particle is no more a bare point, but it has a certain geometry. One of the spatial directions in this case the x-axis, has a particular meaning. When this particle or wave packet moves, that will be the direction ahead relative to the particle itself. In this way the particle and its internal geometry is in correlation and coordinated with the surrounding one.

3. The total energy E of a moving particle.

It is well known, that the special theory of relativity includes the statement, that the mass of a body varies with the velocity. This is not only true, but it is such a dominating discovery, that it almost prevents seeing another still more important relationship. It is to be emphasized, that the energy of the particle also varies as a function of velocity. That namely does not sound so mystic as the previous one. In principle there are three choices, the increase of the energy equals with that of mass (the present situation), or it is larger or smaller than that. Again the question arises, where the energy goes, when a certain amount of work is done on the particle. From the energy point of view it does not matter in which form the energy exists, important is only, that the conservation law is valid. A qualitative answer to the problem is given by the equation of motion. The equation

$$E = mc^2 \tag{15}$$

is always, whatever its form will be, the equivalence of mass and energy; it is not the equation of motion. If a particle moves, it certainly has a part of its energy related to the movement independently on what has happened within the particle. Why should it move, if the increase of the mass could be equivalent with the energy ? The relativistic model gives the following explanation to this question. When the wave packet, described above, moves, the frequency of the standing wave increases. This is the prime reason to the increase of energy, which also includes the increase of mass. There are two reasons for that. First the Doppler effect and then the relativistic correction. The difference between the frequency of the moving particle ν and the resonance frequency at rest ν_0, is the same as the de Broglie frequency ν_{dB},

$$\nu_{dB} = \nu - \nu_0 \tag{16}$$

According to this model, the de Broglie wavelength λ_{dB}, which is given by

$$\lambda_{dB} = \frac{c}{\nu_{dB}} \tag{17}$$

is a hypothetical quantity. When the particle interacts with, for example, other particles, it then behaves as if it had a wave character with a wavelength λ_{dB}. This is like "stored into the memory". The velocity of this "wave" can be said to be equal to c.

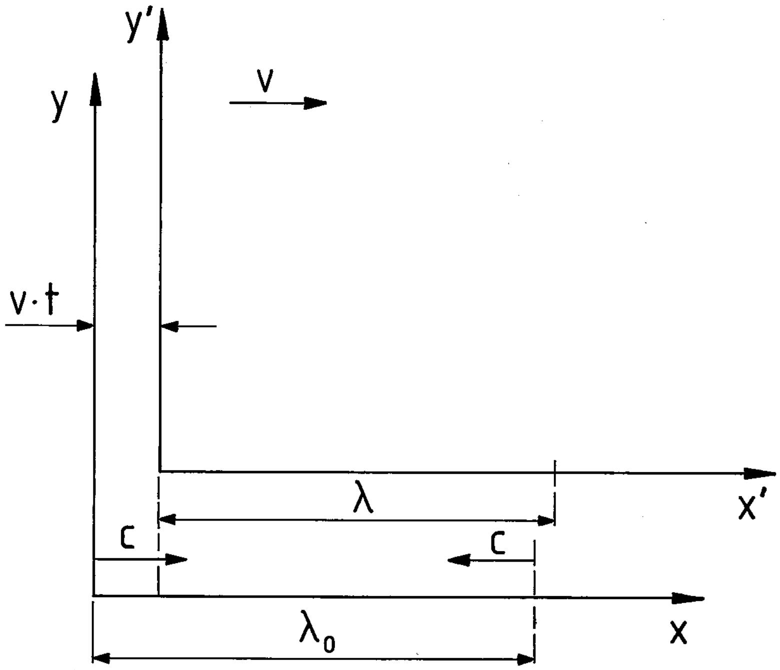

Figure 2. The determination of the resonance wavelength λ as a function of λ_0.

In Fig. 2 a particle is illustrated both at rest and also moving with a velocity v to the positive direction of the x-axis. The resonance wavelength λ of the moving particle is defined by the Lorentz transformation as follows :

$$\lambda = \frac{\lambda_0 - v \cdot t}{\sqrt{1-(v/c)^2}}, \tag{18}$$

The time t used in Fig. 2, is

$$t = \frac{\lambda_0}{c}. \tag{19}$$

The distance $v \cdot t$ is due to the movement of the particle. The resonating wavelength will be first reduced by that amount. It can also be called a Doppler effect within the particle. By means of the model, it is explained so, that the backward moving component, which leaves the point $x = \lambda_0$ at a time $t = 0$, will meet the other end, not at $x = 0$ but at $x = v \cdot t$. The time cannot be shorter, because the information within the particle cannot move faster. In this way the relativistic addition theorem is taken into account.

It is interesting to note, that if only the length contraction is applied as if this particle were a rigid object like a stick moving along the positive x-axis, the result will be

$$\lambda = \lambda_0 \sqrt{1 - (v/c)^2}. \tag{20}$$

In that case the influence of the internal structure is omitted. The Doppler effect belongs to this model, it is the essential factor in it. The derivation of the equation (20) is not in agreement with the special theory of relativity. There is no rigid connection between the both ends. Therefore the wavelength λ cannot be determined only by analyzing the movement of both ends. Also the flow of information between them is to be taken into account. The total energy E of the particle can be calculated on the basis of equations (11) and (16). This yields

$$E = h\nu = h \cdot \frac{c}{\lambda} = h\nu_0 \sqrt{\frac{c+v}{c-v}} \tag{21}$$

This equation includes both the influence of the Doppler effect and the Lorentz transformation. If in lieu of these both only the length contraction is applied, the result will be

$$E = \frac{m_0 c^2}{\sqrt{1 - (v/c)^2}} \tag{22}$$

In fact, this is in good accordance with the relativistic model. The mathematical derivation of this energy equation is based on the assumption, that all the energy of a moving particle is due to the increase of its mass.

By means of this model the length contraction does really give that portion of the change. These phenomena are in this respect equivalent. However the relativistic model takes also into account the influence of the movement by means of Doppler effect. This difference can be illustrated quantitatively by subtracting the former from the latter. Then the energy E given by the relativistic model, can be written

$$E = \frac{m_0 c^2}{\sqrt{1-(v/c)^2}}(1+v/c). \tag{23}$$

This clearly shows, that this energy is somewhat bigger than that in the equation (22). The difference is linearly proportional to the velocity of the particle. The asymptotic value ($v = c$) of the ratio is two. Similarly this energy becomes infinite, when $v = c$. It is to be emphasized, too, that because the difference between the two energies is related to the velocity, it inevitably represents a dynamical portion of the energy. As a matter of fact, through the relativistic model a situation arises, where the energy equation (22) does not belong to the theory of relativity, and it also disagrees with it.

4. The de Broglie wavelength and the momentum of a particle.

As it is earlier stated, the de Broglie wavelength of the particle is according to the relativistic model, determined by the frequency ν_{dB}. Based on the equation (16) it is written

$$E - E_0 = h(\nu - \nu_0) = h\nu_{dB} = \frac{hc}{\lambda_{dB}}. \tag{24}$$

For the wavelength λ_{dB} the expression

$$\lambda_{dB} = \frac{\lambda_0}{\sqrt{\frac{c+v}{c-v}} - 1} \tag{25}$$

is achieved. When this quantity is placed into the equation of the momentum p, it follows

$$p = \frac{h}{\lambda_{dB}} = \frac{h(\sqrt{\frac{c+v}{c-v}} - 1)}{\lambda_0} = \frac{E - E_0}{c} \tag{26}$$

With this model the wave/particle-dualism is avoided. Further the derivation of the momentum is based on the de Broglie wavelength. Apparently the equation (24) directly gives the kinetic energy in question. Consequently the momentum is directly proportional to that. This is an important and interesting result, because the derivative of p with respect to E becomes a constant:

$$\frac{dp}{dE} = \frac{1}{c} \tag{27}$$

That means, that the conservation law of energy and momentum are in fact the same law. As it can be seen, this model does not give any single particular mass, which could be multiplied by a velocity v in order to determine the corresponding momentum. On the contrary the logic tree in this case (Fig. 3) becomes different from the earlier one. In fact the important quantities, energy, de Broglie wavelength and momentum are interrelated with each other. They are almost like different expressions of the same thing.

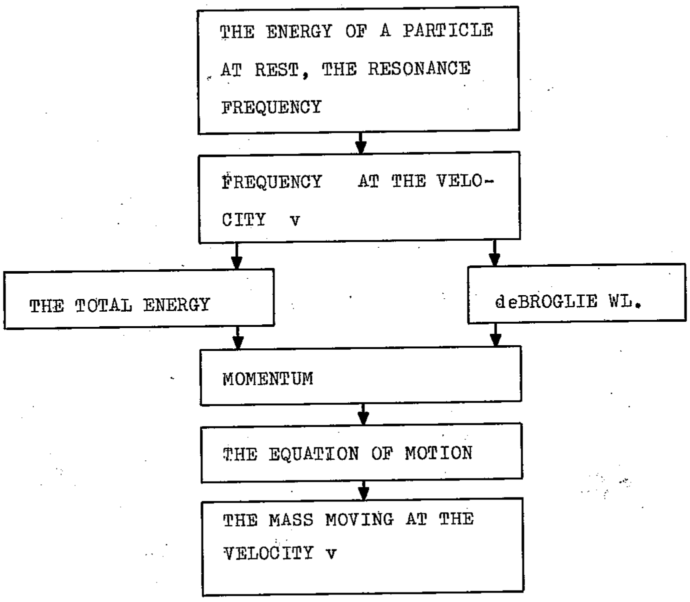

Figure 3. According to the relativistic model the logic tree is consistent. Momentum is linearly proportional to energy and inversely proportional to de Broglie wavelength. The relativistic mass is defined as a factor of the equation of motion.

5. The equation of motion.

Before it was concluded that the derivation of an expression for the relativistic mass is to be done by the equation of motion. The present situation means, that the equivalence between the energy and mass is used as an equation of motion. The motion is probably derived from the mass or at least from the increase of the mass due to the velocity. Now, when the first derivate of the momentum p is taken with respect to time, it yields

$$\frac{dp}{dt} = \frac{h\nu_0/c^2}{\sqrt{1-(v/c)^2}} \cdot \frac{dv/dt}{(1-v/c)} \tag{28}$$

This is the one dimensional equation of motion for a particle. The first term indeed is equal to the relativistic mass. Consequently, the second one is the acceleration given by this model. When v is small compared with to c, the equation becomes the classic

$$\frac{dp}{dt} = m_0 \cdot \frac{dv}{dt} \tag{29}$$

The evaluation of the acceleration is an experimental task. Because of the relation between the momentum and energy, it is found, that

$$\frac{dp}{dt} = \frac{1}{c} \cdot \frac{\partial E}{\partial t}. \tag{30}$$

If $v = x/t$ is placed into the energy equation and the partial differentials are formed, it follows

$$\frac{\partial E}{\partial t} = -\frac{x}{t} \cdot \frac{\partial E}{\partial x} \quad \text{or} \quad \frac{\partial E}{\partial x} = -\frac{1}{x/t}\frac{\partial E}{\partial t} \tag{31}$$

This relation is the same as that the power is velocity times the force. It is also valid only in one dimension at a time, because of its relativistic character. In Fig. 4 a comparison is made. This reveals the differences in momentum and kinetic energy. When writing the both equations as a function of momentum, respectively, it is given

$$E^2 = p^2c^2 + m_0^2c^4 \tag{32}$$

$$E = pc + m_0c^2. \tag{33}$$

These are the same, when $v = 0$ and $v = c$. In Fig. 4 both have been drawn at a value of $v = 0.6c$. From the experimental point of view, it is interesting to note, that pc in both cases is rather equal. That is the quantity, which is primarily measured. In any case the difference is much smaller than that between the total energies. As regards to the experimental results, one cannot avoid mentioning the role of neutriinos in this respect. Also the tunneling effect is to be considered, when the absolute values of energies are compared.

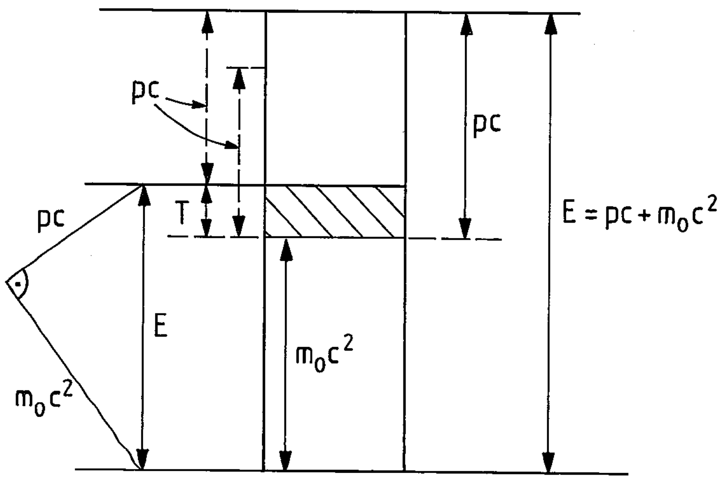

Figure 4. The total energy of a moving particle (at $v = 0.6c$) illustrated by the present equation (left side) and by the relativistic model (right side). Both pc's are rather equal, but the energies differ from each other more. If on the left, pc is added to E, the result is equal with E on the right side. The kinetic energy T does not correlate with pc's.

Some textbooks present the series expansion of the energy equation (22). It is

$$E = m_0c^2 + \frac{1}{2} \cdot m_0v^2 + \frac{3}{8}m_0\frac{v^4}{c^2} + \cdots \tag{34}$$

Based on this, the conclusion is drawn, that as a first approximation for the kinetic energy, the classical value is obtained. Of course it seems confirming, if the kinetic energy T is approximatively

$$T = \frac{m_0c^2}{1 - (v/c)^2} - m_0c^2 = \frac{1}{2}m_0v^2. \tag{35}$$

However, it is hard to explain, why $pc > T$, and that is not due to the series expansion, but it is true with the absolute values. In addition, the difference is clear, 0.75 versus 0.25 times m_0c^2.

If the series expansion of the derived energy equation is written, it is found to be

$$E = m_0c^2(1 + v/c) - \cdots \tag{36}$$

The same factor as in the equation (23) appears. The kinetic energy is ignored by this model, also in series expansions. If in this case exceptionally m_0v is denoted by p, then it becomes

$$E = m_0c^2 + pc \cdots \tag{37}$$

This is to be interpreted that according to the relativistic model the increase of energy due to the velocity is pc. This includes the contributions of both mass and the kinetic energy.

All the above derived formulas and their mathematical appearance can be simplified by denoting

$$n = \sqrt{\frac{c+v}{c-v}} \tag{38}$$

Then substitution yields

$$E = n \cdot E_0 \tag{39}$$

$$\lambda_{dB} = \frac{1}{n-1} \cdot \frac{hc}{E_0} \tag{40}$$

$$p = (n-1) \cdot \frac{E_0}{c} \tag{41}$$

In Figs. 5, 6 and 7 the graphical presentations of these quantities are seen. Because n can be also negative, the energy equation covers antiparticles too. If in the equations v would be greater than c, all the formulas will become imaginary. The physical reason, why it cannot be so, is according to this model the particle itself and its structure. The hindrance to achieve the velocity of light, is essentially the velocity of light.

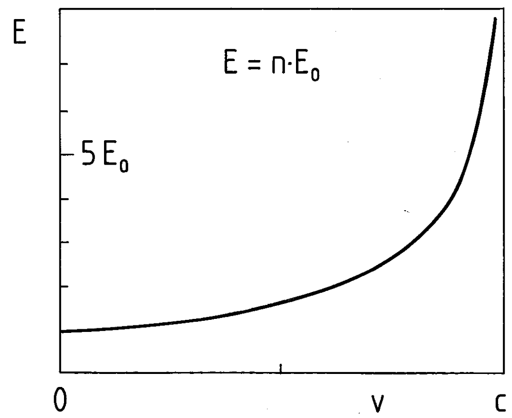

Figure 5. The total energy E as a function of v.

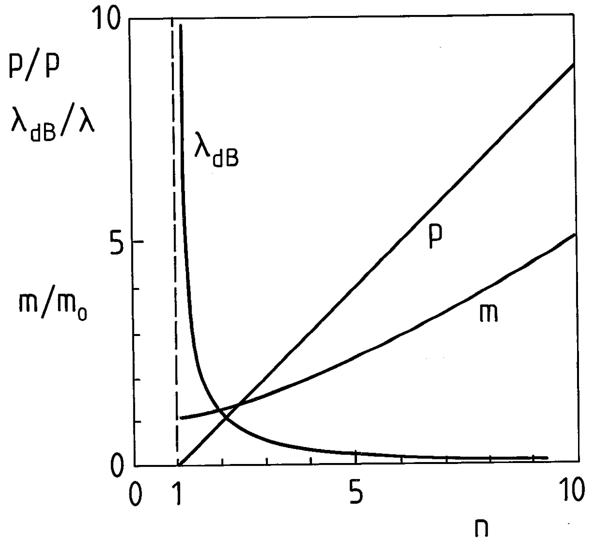

Figure 6. De Broglie wavelength λ_{dB}, momentum p and the relativistic mass m as a function of n.

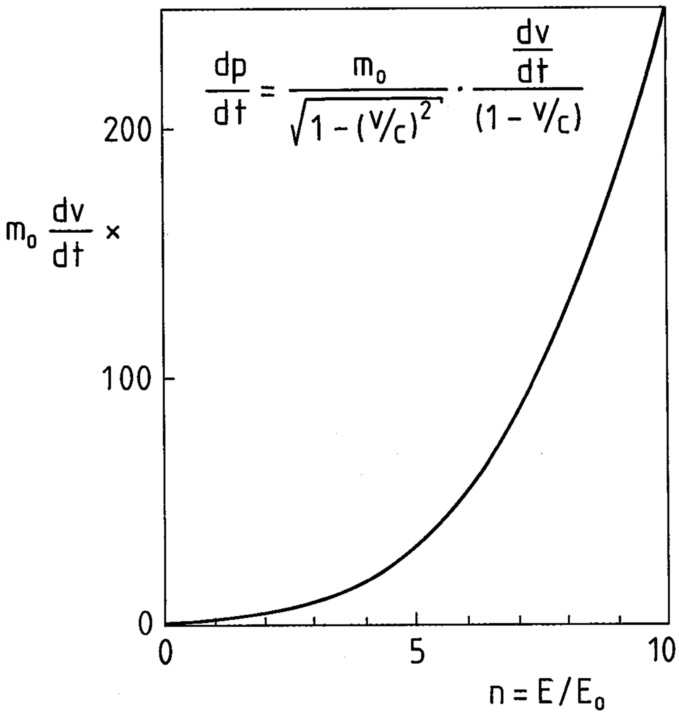

Figure 7. dp/dt as a function of n. For example, at $v = 0.6c$ this value is 1.6 times larger than the present one.

6. References.

The way, in which the idea of the relativistic model is connected with the present equations and concepts, can be analyzed in detail on the basis of L.B. Okun's article "The concept of mass" [1]. It summarizes by means of original sources both the early development as well as the essence of the present situation regarding the relations between fundamental quantities, energy, mass and momentum. Simultaneously it reveals also the rather careless way, how this equation is used. The author has tested opinions of colleagues on this subject too. Apparently the boundary between physically sound and proven results on the one hand and various mathematical combinations of relativistic and nonrelativistic equations on the other seem to be rather diffuse and unclear.

The very great discovery, $E = m_0c^2$, has a shadow $E = mc^2$, which follows it automatically, almost too automatically.

The relativistic model itself is based on generally accepted physical items and phenomena, each of which is presented in most physics textbooks. Therefore the following references are primarily illustrating the position and status of the discussed present energy equation in litterature.

Different cathegories of books and papers can be found as regards to this question. First there are sources, which use the equation for further analysis of experimental results and other similar purposes [2,3]. They need not raise the question, how the derivation of the equation is taken place. It is a starting point, which is characterized as "a very useful equation...". To enlarge the equation concerning the equivalence of energy and mass to the equation of total energy, does not seem to bother much. Once it is even mentioned, that the equation means, that the conservation laws of mass and energy are the one and same law. If the references are given, they are the originals, the same as treated in Okun's paper. Though the essential content of the special theory of relativity has been proved experimentally, not much can be found for the benefit for this particular equation. The exact proof seems to be missing [8].

Then there are such textbooks, which do not mention the energy equation at all, though they cover almost all other aspects of the theory of relativity. One of these [4], when describing the beetta-disintegration, presents the two logical conclusions; one has either to apply the concept of neutriino or analyze further the energy equation. As is known, the missing portion of energy is interpreted by means of neutriinos.

A very interesting and important document from the physical point of view is, no doubt the book "Non linear wave mechanics" by Louis de Broglie [7]. It is a detailed effort to change the prevailing situation and made by one of the original contributors. Also a model for a particle is sought. In the end of the book, it reads "Einstein has called these fields containing strong local condensations, which he thinks must be the true representation of particles 'bunch-like fields'. In our conception the u waves are indeed bunck-like wave-fields". However, neither that book goes into the derivation of the energy equation.

In a more recent paper by J.W.G. Wignall [6] a theory of macroscopically extended particles is used for the analysis and derivation of Maxwell's equations.

7. Acknowledgements.

The author likes to thank Professor Eino Tunkelo, the head of The Finnish Academy of Technology and Dr. Raimo Keskinen, University of Helsinki, Department of Theoretical Physics, for valuable comments and fruitful discussions.

Appendix: The relativistic model of an elementary particle and the energy conservation law.

In this appendix a collision of two elementary particles is analyzed with respect to the conservation of energy and momentum. Both the current formula and the derived formula for the total energy E are used.

Denoting first the total energy E of a particle as

$$E = m_0 c^2 \gamma \tag{1A}$$

where

$$\gamma = 1/\sqrt{1-(v/c)^2} \tag{2A}$$

and the momentum p

$$p = m_0 \cdot v \cdot \gamma \tag{3A}$$

the equations for the particles (1 and 2) can be written as follows

$$m_{01}c^2\gamma_1 + m_{02}c^2\gamma_2 = m'_{01}c^2\gamma'_1 + m'_{02}c^2\gamma'_2 \tag{4A}$$

$$m_{01} \cdot v_1\gamma_1 + m_{02} \cdot v_2\gamma_2 = m'_{01} \cdot v'_1 \cdot \gamma'_1 + m'_{02} \cdot v'_2\gamma'_2 \tag{5A}$$

The values of the entities after the collision are marked by $'$. In the case of an elastic collision the corresponding rest masses keep the same. Then the equations can be simplified in the following manner

$$m_{01}(\gamma_1 - \gamma'_1) = m_{02}(\gamma'_2 - \gamma_2) \tag{6A}$$

$$m_{01}(\gamma_1 \cdot v_1 - \gamma'_1 \cdot v'_1) = m_{02}(\gamma'_2 \cdot v'_2 - \gamma_2 \cdot v_2) \tag{7A}$$

However, it turns out, that these equations cannot be solved in a normal manner. If instead, the same procedure is made by the equations

$$E = h\nu_0\sqrt{\frac{c+v}{c-v}} = n \cdot E_0 \tag{8A}$$

$$p = \frac{h\nu_0}{c}(\sqrt{\frac{c+v}{c-v}} - 1) = \frac{E_0}{c}(n-1) \tag{9A}$$

the result can be achieved in this case. It is

$$n_1 E_{01} + n_2 E_{02} = n_1' \cdot E_{01}' + n_2' E_{02}' \tag{10A}$$

$$\nu_{01} + \nu_{02} = \nu_{01}' + \nu_{02}' \tag{11A}$$

Further, if in this case the collision is elastic, the former equation becomes

$$(n_1 - n_1') \cdot E_{01} = (n_2' - n_2) \cdot E_{02}. \tag{12A}$$

In fact this formula represents both the energy and the momentum equations in the elastic collision.

As a summary it is concluded that the present equations cannot be solved in a normal manner, whereas the suggested relativistic model fulfills the conservation laws of energy and momentum in an elastic collision of two particles.

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