Conceptual Foundations of Quantum Theory: A Map of the Land

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ABSTRACT. This is *not* a review article on the conceptual foundations of quantum theory. Rather, it should be considered an "overview paper", or better, as claimed in the title, a "map" whose aim is to help one in orienting through the subject. The material is organized as follows. After a brief exposition of the three classic paradoxes of quantum mechanics—the measurement, Zeno, and EPR paradoxes—the main interpretations of the theory, as well as the major alternative theories, are outlined. Some puzzling topics of conceptual interest are also presented and briefly discussed. Particular attention has been devoted to the preparation of the bibliography.

RESUME. Cet article n'est pas *une revue des fondements conceptuels de la théorie quantique. Il faudrait plutôt le considérer comme un "article de survol" ou encore mieux, comme l'affirme le titre, une "carte" dont le but est de permettre de s'orienter dans le sujet. Après un bref exposé des trois paradoxes classiques de la mécanique quantique, les paradoxes de la mesure, de Zénon et E.P.R., on esquisse les principales interprétations de la théorie ainsi que les théories alternatives. Quelques sujets énigmatiques d'intérêt conceptuel sont aussi présentés et briévement discutés. Un soin particulier a été apporté à la réalisation de la bibliographie.*

Contents.

1. Introduction.

Almost seventy years after de Broglie's discovery of the so-called matter waves (de Broglie 1923, 1992), the debate about their nature is still open. There is in fact no general agreement about the implications of the conceptual framework of quantum theory, the "orthodox" interpretation being questioned by a growing number of researchers (see, e.g.: de Broglie 1953, 1963, 1982; Bohm 1952; d'Espagnat 1971; Flato et al. 1976; Diner et al. 1984; Kamefuchi et al. 1984; Lahti and Mittelstaedt 1985, 1991; Davies and Brown 1986; Gorini and Frigerio 1986; Greenberger 1986; van der Merwe et al. 1988; Cini and Lévy-Leblond 1989; Miller 1990). The possibility of performing experiments devoted to test fundamental aspects of quantum mechanics (see, e.g.: Leggett

1980, 1984; Aspect et al. 1981, 1982; Greenberger 1983; Kamefuchi et al. 1984; Gould et al. 1986; Shimony 1988; Bollinger et al. 1989; Horne et al. 1989, 1990; Zajonk 1989; Matteucci 1990; Itano et al. 1990; Majumder et al. 1990; Rarity and Topster 1990; Carnal and Mlynek 1991; Gähler and Zeilinger 1991; Keith et al. 1991; Scully et al. 1991; Tan et al. 1991) has recently led to an increase of interest in this subject, for a long time regarded by the majority of physicists as part of speculative philosophy rather than of science. On the theoretical side, several "heretic" theories have been formulated, differing from each other in the physical picture of quantum phenomena (and sometimes even in the general philosophical view of the world!), though all leading to conclusions compatible with the available experimental data.

The present article has not been conceived as a review paper on these topics; the size of such a work would be very different. Our aim is rather to present an overview of the current status of the subject, that could be used as a map in moving through such a wild and complex territory. Like any map, it shows only the general features, whereas the details are not represented. For more informations about any particular subject the reader can consult the sources listed in the bibliography. The choice of the references contained in the latter reflects unavoidably the author's tastes; nevertheless, it should be considered as a reasonably representative sample, although it is not supposed to be exhaustive. Other very useful works of bibliographical character are: De Witt and Graham 1971; Nilson 1976; Ballentine 1987a. The collections edited by Wheeler and Zurek (1983) and Ballentine (1988b) contain reprints of important articles. Sections concerned with conceptual problems of quantum theory are present in the textbooks by Sudbery (1986), Bohm (1989), Ballentine (1990b) and Rae (1992).

The perspectives offered by the various viewpoints appear most clearly when considering how they behave in extreme situations: For this reason Sec. 2 contains a brief exposition of the classic paradoxes of quantum mechanics. Since any physical theory consists roughly of a *mathematical formalism* and an *interpretation*, we find convenient to distinguish between theories differing from conventional quantum mechanics only by the latter, and alternative theories in the proper sense, that entail also a modification of the formalism: They are presented, respectively, in Secs. 3 and 4. Sec. 5 is a review of some important (and difficult) open problems. Section 6 contains some final remarks.

The notations adopted in the few equations displayed through the text are standard in quantum mechanics (see, e.g.: Dirac 1958; Messiah

1966; Sudbery 1986; Ballentine 1990b). Operators in the Hilbert space are always denoted by a caret; the operator associated to an observable A is therefore represented by $\hat{A}$.

2. Paradoxes.

The formulation of paradoxes is always a philosophically healthy activity, as it sharpens one's understanding of concepts and helps to recognize possible limitations in their domain of applicability. Within quantum mechanics, three paradoxes have become classic (Peres 1984b; Selleri 1990); each of them leads to ask a very basic question concerning the foundations of the theory.

2.1 Quantum Measurement: What Is a State Vector?

The description of the measurement process is perhaps the main fundamental problem of quantum theory. Its paradoxical features were first pointed out by Von Neumann (1932) and in a more spectacular version (cat's paradox) by Schrödinger (1935a). These seminal works started a debate which has not yet settled down (see, e.g.: de Broglie 1957, 1982; Jammer 1974; Wheeler and Zurek 1983; Lochak 1984; Busch et al. 1991). The vitality of such a controversy can well be understood, since the very *meaning* of the quantum theoretical description is at stake.

In the measurement process a microscopic system $\mathcal{S}$ is coupled to a macroscopic measuring device $\mathcal{M}$ in such a way that a one-to-one correspondence is established between the spectrum of an observable A (the measured quantity) of $\mathcal{S}$ and that of the "pointer position" P of $\mathcal{M}$. More precisely, let us suppose that, before the coupling is switched on, the system $\mathcal{S}$ is in the eigenstate $|a_r\rangle_{\mathcal{S}}$ of $\hat{A}$, corresponding to the eigenvalue a_r (for sake of simplicity, we assume that the spectrum of $\hat{A}$ is nondegenerate and discrete, labeled by $r \in \mathbb{N}$) and the device $\mathcal{M}$ is in the eigenstate $|p_0, s\rangle_{\mathcal{M}}$ of $\hat{P}$, where s labels the many other degrees of freedom of $\mathcal{M}$. The initial state of $\mathcal{S} + \mathcal{M}$ is therefore

$$|i\rangle = |a_r\rangle_{\mathcal{S}} \otimes |p_0, s\rangle_{\mathcal{M}} \ . \tag{2.1.1}$$

The interaction between $\mathcal{S}$ and $\mathcal{M}$ produces, after some time, a final state

$$|f\rangle = \hat{U}|i\rangle \ , \tag{2.1.2}$$

where $\hat{U}$ is the unitary evolution operator for $\mathcal{S}+\mathcal{M}$. In order to describe a measurement of A, $\hat{U}$ must lead to a state $|f\rangle$ in which the position of the pointer is correlated to the initial value a_r of the observed quantity. The most general form of such an $|f\rangle$ is (Ballentine 1988a, 1990b)

$$|f\rangle = \sum_{r's'} u_{rs}^{r's'} |a_{r'}\rangle_{\mathcal{S}} \otimes |p_r, s'\rangle_{\mathcal{M}} \equiv |p_r; (r, s; u)\rangle \ , \tag{2.1.3}$$

where $u_{rs}^{r's'}$ are suitable complex coefficients. The value p_r of the pointer's position in the state (2.1.3) allows an observer to infer the value a_r which the quantity A possessed before the measurement. In general, however, the system $\mathcal{S}$ is not initially in an eigenstate of A, but rather in the superposition

$$\sum_r c_r |a_r\rangle_{\mathcal{S}} \ , \tag{2.1.4}$$

where c_r are complex numbers whose square moduli sum up to one. The initial state of $\mathcal{S}+\mathcal{M}$ is thus

$$|i\rangle = \sum_r c_r |a_r\rangle_{\mathcal{S}} \otimes |p_0, s\rangle_{\mathcal{M}} \ , \tag{2.1.5}$$

and the final state is, by linearity of $\hat{U}$,

$$|f\rangle = \sum_r c_r |p_r; (r, s, u)\rangle \ . \tag{2.1.6}$$

Equation (2.1.6) describes a coherent superposition of eigenstates of the pointer's position. It is an empirical fact, however, that macroscopic observables (of which P is an example) are always found in only one of their macroscopically distinguishable states. The analysis of the measurement process leads therefore to the problem of interpreting the state (2.1.6) in a way which is consistent with experience. It is not difficult to realize that this problem is really present only if the state vector is supposed to provide a *complete* description of an *individual* copy of the system $\mathcal{S}+\mathcal{M}$ (Ballentine 1988a, 1990b), copy whose *actual* state should be correctly represented by an eigenstate of P,

$$|p_r; (r, s, \alpha)\rangle \equiv \sum_{r's'} \alpha_{rs}^{r's'} |a_{r'}\rangle_{\mathcal{S}} \otimes |p_r, s'\rangle_{\mathcal{M}} \ , \tag{2.1.7}$$

rather than by the superposition (2.1.6). The task of passing from (2.1.6) to (2.1.7), which these interpretations must accomplish if they have to provide a satisfactory theory of measurement, has been called the *objectification problem* (Busch et al. 1991).

Let us be more precise about this important point. Since the pointer's position is observed to be p_r with probability $|c_r|^2$, the objectification problem consists in finding a prescription which transforms the pure state (2.1.6) into the mixture

$$\hat{\rho}_m = \sum_r |c_r|^2 |p_r;(r,s,\alpha)\rangle\langle p_r;(r,s,\alpha)| \ . \tag{2.1.8}$$

However, (2.1.8) cannot derive from (2.1.6) by the usual quantum evolution, because (for nontrivial c_r's)

$$\hat{\rho}_m^2 \neq \hat{\rho}_m \ , \tag{2.1.9}$$

whereas the density operator $\hat{\rho} \equiv |f\rangle\langle f|$ enjoys the property

$$\hat{\rho}^2 = \hat{\rho} \ , \tag{2.1.10}$$

which is preserved by linear unitary evolution (Von Neumann 1932). To introduce larger portions of the system's environment, as well as the human observer, in the description cannot change this result, insofar as environment and observer are supposed to obey the quantum mechanical laws. Even assuming that the initial state of $\mathcal{S}+\mathcal{M}$ is not pure one cannot recover (2.1.8) (Ballentine 1988a, 1990b).[1] It is therefore unavoidable to conclude that the objectification—hence the measurement—problem is insoluble within the context of standard quantum formalism (Fine 1970, 1972; Moldauer 1972; Brown 1986; Ballentine 1988a, 1990b).

To avoid this unpleasant conclusion without giving up the idea that the state vector describes completely an individual physical system, a *collapse postulate* has been proposed (Von Neumann 1932; Dirac 1958), according to which the linear unitary evolution is replaced, whenever a measurement is performed, by the prescription

$$\hat{\rho} \longrightarrow \hat{\rho}_m \ . \tag{2.1.11}$$

[1] Random phase hypothesis (Gottfried 1966, 1990, 1991) implicitly assume that one deals with an ensemble of systems (Landsberg and Home 1987), in which case there is no objectification problem.

This additional hypothesis has been strongly criticized both on aesthetic and ethical grounds (Bell and Nauenberg 1966; Bell 1986, 1987b, 1990), as well as from more technical points of view (Cini 1983; Ballentine 1990a). It still finds, nevertheless, several supporters.

Experiments devoted to investigate whether macroscopic systems can actually be prepared in superpositions of macroscopically distinguishable states such as (2.1.6), have been proposed (Leggett 1980, 1984; Leggett and Garg 1985). However, their theoretical grounds and implications are not free from controversies (Ballentine 1987b; Peres 1988; Leggett and Garg 1987, 1989).

2.2 Zeno Effect: Do State Vectors Collapse?

This section deals with the most spectacular consequence of the hypothesis that during a measurement the linear unitary evolution is replaced by the collapse law (2.1.11). Let $\mathcal{S}$ be a quantum system and A an observable of $\mathcal{S}$. Let us assume, for sake of simplicity, that the spectrum of A is nondegenerate; then we can write

$$\hat{A}|r\rangle = a_r|r\rangle \ , \tag{2.2.1}$$

with a_r a real number. If the state of $\mathcal{S}$ at time $t = 0$ is $|\psi(0)\rangle = |\bar{r}\rangle$ the Schrödinger equation gives for the state at a time t

$$|\psi(t)\rangle = \exp\left(-\frac{it}{\hbar}\hat{H}\right)|\bar{r}\rangle \ , \tag{2.2.2}$$

where $\hat{H}$ is the Hamiltonian of $\mathcal{S}$, that we assume time independent. The probability that a measurement of A performed on $\mathcal{S}$ at time t give again the result $a_{\bar{r}}$ is

$$P(\bar{r}|t) = |\langle\bar{r}|\exp\left(-\frac{it}{\hbar}\hat{H}\right)|\bar{r}\rangle|^2 \ . \tag{2.2.3}$$

For small finite times $P(\bar{r}|t)$ is smaller than one, unless $|\bar{r}\rangle$ is an eigenstate of energy.

We can also ask what is the probability $P'(\bar{r}|t)$ that a measurement of A give the result $a_{\bar{r}}$ at time t *if* $\mathcal{S}$ *is continuously observed.* This problem can be seen as the limit of a corresponding one in which $\mathcal{S}$ is subjected to repeated measurements of A at time intervals Δt. Applying the law of reduction of the state vector (2.1.11) after each interval Δt one gets

$$P'(\bar{r}|t) = \lim_{\Delta t \to 0} \sum_{r_1 \cdots r_N} |\langle \bar{r}| \exp\left(-\frac{i\Delta t}{\hbar}\hat{H}\right)|r_N\rangle|^2 \cdots \\ |\langle r_1| \exp\left(-\frac{i\Delta t}{\hbar}\hat{H}\right)|\bar{r}\rangle|^2 \;, \tag{2.2.4}$$

where $t = (N+1)\Delta t$. It is not difficult to see, on expanding the exponentials in Eq. (2.2.4), that for each value of t

$$P'(\bar{r}|t) = 1 - \lim_{N \to +\infty} \frac{t^2}{(N+1)\hbar^2}(\langle \bar{r}|\hat{H}^2|\bar{r}\rangle - \langle \bar{r}|\hat{H}|\bar{r}\rangle^2) = 1 \;. \tag{2.2.5}$$

The meaning of Eq. (2.2.5) is that the continuous observation of $\mathcal{S}$ inhibits its evolution (Misra and Sudarshan 1977; Chiu et al. 1977; Peres 1980a; Kraus 1981). The paradoxical character of this conclusion, known as "quantum Zeno effect", can be realized by thinking that a radioactive atom decays even if continuously "monitored" by surrounding detectors.

Since in the derivation of Eq. (2.2.5) the only non-standard hypothesis which has been used is the reduction postulate, it appears very likely that the latter is responsible for the paradox (Bunge and Kálnay 1983a). It has also been argued (Bunge and Kálnay 1983b) that it is very dangerous to idealize a measurement simply replacing it by an instantaneous collapse and that a less unrealistic treatment should get rid of the paradox. A more detailed analysis, performed within the context of the studies about the influence of the environment on the behaviour of a quantum system (see Sec. 5.1), shows indeed that if the apparatus $\mathcal{M}$ is included in the quantum description, then the Zeno effect occurs only in the limit of strong coupling between $\mathcal{S}$ and $\mathcal{M}$ (Joos 1984). This result leads one to think of the reduction postulate as a mere phenomenological description of no general validity and is in agreement with a recent claim that Zeno-like effects can be predicted without invoking collapse at all (Home and Whitaker 1992a).

A recently performed experiment was interpreted as a manifestation of the quantum Zeno effect (Itano et al. 1990; Knight 1990). It has been pointed out, however, that the results obtained can be explained by conventional quantum theory without invoking the reduction postulate at all (Peres and Ron 1990; Petrowsky et al. 1990; Ballentine 1990a, 1991b; Itano et al. 1991). It is also curious to mention that the suggestion has been advanced to regard the failure in detecting the decay of protons bound within nuclei as experimental evidence for the Zeno effect (Horwitz and Katznelson 1983a). Such a bold proposal has, of course, provoked strong reactions in the community of physicists (Lepage et al. 1983; Cahill 1983; Wheather and Peierls 1983; Horwitz and Katznelson 1983b; Maddox 1983; Peres 1984b).

2.3 EPR: Is Quantum Theory Complete?

This was conceived (Einstein et al. 1935) as an argument in support of the opinion that quantum theory does not provide a complete description of physical reality (see Fine 1986, Deltete and Guy 1990, 1991 for detailed analysis of Einstein's viewpoint on this subject, and Jammer 1974, Howard 1990 for the historical evolution of the idea). In Bohm's formulation (Bohm 1951, 1989; Clauser et al. 1978), which is closer to a description of a real experimental situation (see, e.g., Aspect et al. 1981, 1982) than the original one, the reasoning goes as follows. A pair of identical particles $\mathcal{A}$ and $\mathcal{B}$ with spin 1/2 is prepared in the singlet state in such a way that each particle occupies a different region $\mathcal{R}_1$ or $\mathcal{R}_2$, with $\mathcal{R}_1$ and $\mathcal{R}_2$ non-overlapping. The state vector appropriate to describe this situation is

$$|\Psi\rangle = |\phi\rangle \otimes |\psi\rangle \ , \tag{2.3.1}$$

where

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|L\rangle_{\mathcal{A}} \otimes |R\rangle_{\mathcal{B}} + |R\rangle_{\mathcal{A}} \otimes |L\rangle_{\mathcal{B}}) \tag{2.3.2}$$

and

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle_{\mathcal{A}} \otimes |-\rangle_{\mathcal{B}} - |-\rangle_{\mathcal{A}} \otimes |+\rangle_{\mathcal{B}}) \tag{2.3.3}$$

are, respectively, the orbital and the spin components; $|L\rangle$ and $|R\rangle$ are mutually orthogonal vectors representing a particle in the region $\mathcal{R}_1$ and $\mathcal{R}_2$, respectively; $|+\rangle$ and $|-\rangle$ are eigenstates of an arbitrary component of the spin. The vector (2.3.3) is rotationally symmetric.

The state (2.3.1) contains remarkable correlations between the two particles. By introducing the two orthonormal vectors

$$|L,\pm;R,\mp\rangle \equiv \frac{1}{\sqrt{2}}(|L,\pm\rangle_{\mathcal{A}} \otimes |R,\mp\rangle_{\mathcal{B}} - |R,\mp\rangle_{\mathcal{A}} \otimes |L,\pm\rangle_{\mathcal{B}}) \qquad (2.3.4)$$

(where $|X,r\rangle \equiv |X\rangle \otimes |r\rangle$, with $X \in \{L,R\}$ and $r \in \{+,-\}$) $|\Psi\rangle$ can be rewritten as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|L,+;R,-\rangle - |L,-;R,+\rangle) \; , \qquad (2.3.5)$$

from which it is evident that a measurement of the spin component of both particles along the same direction will always give opposite results. More precisely, the joint probability that a meaurement of the same component of spin give result r in $\mathcal{R}_1$ and s in $\mathcal{R}_2$ is

$$P(r,\mathcal{L};s,\mathcal{R}) = |\langle L,r;R,s|\Psi\rangle|^2 = \frac{1}{2}(1-\delta_{rs}) \; , \qquad (2.3.6)$$

which gives, for the conditional probabilities,

$$P(r,\mathcal{L}|s,\mathcal{R}) = P(s,\mathcal{R}|r,\mathcal{L}) = 1-\delta_{rs} \; . \qquad (2.3.7)$$

Equation (2.3.7) allows an observer in one of the two regions $\mathcal{R}_1$ and $\mathcal{R}_2$ to infer with certainty, from a measurement of the component of spin along some direction, what the result of a corresponding measurement performed in the other region would be, even if $\mathcal{R}_1$ and $\mathcal{R}_2$ are so far from each other that no signal could have been exchanged among them during the time in which the measurement has been performed. It follows that an observer in $\mathcal{R}_1$ can predict the result of a measurement of the component of spin of the particle in $\mathcal{R}_2$ along an arbitrary direction, without in any way perturbing it, and vice versa. According to Einstein, Podolsky and Rosen, this is a sufficient criterion for such a component to be an element of physical reality. But the arbitrariness in the choice of the direction entails that *all* the components of the spin are elements of physical reality, whereas quantum theory allows to consider at most *one* of them as having a definite value. Therefore Einstein, Podolsky and Rosen conclude that quantum theory must be incomplete.

The puzzling features of "entangled" states such as $|\Psi\rangle$ above have also been noticed and stressed by Schrödinger (1935b, 1936). It took however almost thirty years to turn generic qualitative arguments into a well defined quantitative statement expressing the incompatibility between quantum theory and some additional hypothesis (Bell 1964). These developments of the subject will be discussed in Sec. 5.2.

3. Interpretations.

In discussing the various alternative interpretations of quantum theory we shall adopt a terminology introduced by R.G. Newton (1980), who classifies an interpretation as *realistic*, *nonrealistic subjective*, or *nonrealistic objective*, according to the answer it provides to the crucial question: "What does the state vector describe?". This tri-partition can be roughly justified as follows. Although nobody objects to the empirical validity of Born's rule (Born 1926; Jordan 1926), some physicists argue that the connection

$$|\psi|^2 \longleftrightarrow \text{probability} \tag{3.0.1}$$

should not be actually regarded as primitive, but rather as derived from a more basic interpretation of ψ: This leads essentially to the realistic viewpoint. The other possibility, i.e., to consider the relationship (3.0.1) as fundamental (nonrealistic interpretation of ψ), finds people divided as to whether *probability* requires a subjective or an objective interpretation.[2]

3.1 Realistic Interpretations.

The most immediate and intuitively appealing interpretations of quantum mechanics are perhaps those assuming that the wave function ψ is ontological, i.e., that it represents elements of the physical reality. The wide variety of ways in which this idea can be understood is exemplified by the following three possibilities.

3.1.1 Schrödinger's Interpretation

The first realistic interpretation of the wave function was proposed by Schrödinger himself (Schrödinger 1927; Barut 1987; Lochak 1987), and originally consisted in regarding $|\psi|^2$ as proportional to the density of charge of the electron. In a modern version, $|\psi|^2$ represents the density

[2] Other surveys of the interpretations of quantum theory can be found in: Margenau 1954; Bunge 1956; Jammer 1974; Sudbery 1986; Davies and Brown 1986; Rae 1986, 1992.

of the "stuff" of which the system is made (Bell 1990), i.e., of its physical properties. The fact that the Schrödinger equation for a particle of mass m is equivalent to the continuity and Euler equations for a fluid of density $m|\psi|^2$ and suitable velocity and pressure (Madelung 1926; de Broglie 1926, 1927a; Takabayasi 1953; Fer 1964, 1977; Harvey 1966; Fargue et al. 1976; Ghosh and Deb 1982; Paillère 1991; Sonego 1991a) appears at first to provide support to this idea. However, a considerable difficulty arises immediately when considering the many particle case, for which the function ψ is not defined in the physical space, but rather in an abstract higher dimensional configuration space.

Further problems (see, e.g., Sonego 1991a) are connected with the so-called wave-particle dualism (see, e.g., Selleri 1992), expressed by the fact that in the course of detection particles exhibit corpuscular aspects, whereas the ψ-description has a continuous character. In particular, during a measurement of position the particle is always found to be in a well defined place. In the interpretation that we are now considering this necessarily requires to accept that the "stuff" could be subjected to a tremendously rapid localization, since a very short time before the detection take place it might well happen that ψ is nonvanishing over a very large region of space. Moreover, such a process must "select" a point $\mathbf{x}$ with probability density $|\psi(\mathbf{x},t)|^2$, in order not to contradict Born's rule, which is supported by an impressive body of experimental evidence. Within the context of the usual quantum mechanical formalism these features look very unnatural and contribute to make Schrödinger's interpretation hard to accept.

In spite of these difficulties there has been recently a resurrection of Schrödinger's ideas, in connection with the development of two different theories that involve modifications of quantum mechanics. One of them (quantum mechanics with spontaneous localization) assumes that the localization process described above takes actually place, whereas the other (self-field QED) is a nonlinear theory which relies on the hypothesis that to a particle with electric charge e is associated a density of charge $e|\psi|^2$. Since they are not interpretations in a strict sense, their description is postponed to Secs. 4.3 and 4.2.2, respectively.

3.1.2 Pilot Wave.

If one accepts the conclusion of the EPR argument (Sec. 2.3), the obvious thing to do is to try to complete quantum mechanics, either replacing it by another theory or by supplementing the state vector description with "hidden variables" which should account for all the elements of reality of physical systems (de Broglie 1926, 1927b; Belinfante 1973). The pilot wave theory is an example of this second attitude, although it was still considered "too cheap" by Einstein (Born 1971), who did not believe that quantum mechanics could provide an adequate starting point for further developments (Deltete and Guy 1990). Strictly speaking, it should be regarded as a different *theory*, rather than a mere *interpretation*. However, since its predictions coincide with those of quantum theory (see Bohm 1953, and Valentini 1991b,c, 1992 for possible explanations of this fact) we have included it among the interpretations.

The key idea of the pilot wave theory (de Broglie 1927b, ; Bohm 1952; Bohm and Hiley 1985; Bohm et al. 1987; Bell 1980, 1981, 1982, 1986; Valentini 1992) is to suppose that a physical system follows a well defined trajectory $\{\mathbf{x}_i(t)\}$ in the configuration space, determined by the guidance law (de Broglie 1927b)

$$m_i \dot{\mathbf{x}}_i = \boldsymbol{\nabla}_i S \ , \tag{3.1.1}$$

where m_i is the mass of the i-th particle in the system and S is the phase of the wave function ψ, obeying the Schrödinger equation. ψ is hence considered as a physical field "guiding" the particles motion along trajectories which differ from the classical ones by corrections that can be traced to the action of a "quantum potential"

$$Q = -\frac{\hbar^2}{2} \sum_i \frac{1}{m_i} \frac{\nabla_i^2 |\psi|}{|\psi|} \ , \tag{3.1.2}$$

which is responsible for typical quantum effects (Madelung 1926; de Broglie 1930, 1956; Philippidis et al. 1979; Dewdney and Hiley 1982; Bohm et al. 1985; Dewdney et al. 1987). As for the Schrödinger interpretation, an obvious drawback of this picture is the fact that ψ is defined in the configuration space and not in the physical one. This difficulty can be overcome by considering ψ more alike a "field of active information" than, e.g., an electromagnetic field (Bohm and Hiley 1984; Bohm et al. 1987; Valentini 1992). It has been pointed out, however, the possibility that the pilot field ψ have only a statistical meaning, i.e., that it might account only for an *average* motion of particles (Kyprianidis 1988; Sonego 1991a; Garbaczewski 1991). In this case the theory should be modified in the direction of stochastic mechanics (Bohm and Vigier 1954; see also Sec. 4.1.1).

It is possible to extend straightforwardly the theory to treat a generic field Φ rather than a system of particles, simply replacing the wave function $\psi(x)$ by a wave *functional* $\Psi[\Phi]$ (Bohm and Hiley 1984; Bohm et al. 1987; Valentini 1992). One can realize that this necessarily requires a $3+1$ splitting of spacetime to be performed, thus breaking explicit covariance. In the spirit of relativity such a procedure is rather unpalatable, but since the very structure of the theory appears to be nonlocal (Bohm and Hiley 1975), in agreement with Bell's theorem (Sec. 5.2), it is not obvious whether this should be regarded as a defect. Indeed, it has been argued (Bohm and Hiley 1984; J.S. Bell, in Davies and Brown 1986; Valentini 1991b,c, 1992; Hardy 1992b; Hardy and Squires 1992) that the natural framework for the pilot wave theory is a pre-relativistic one, consisting of an absolute space and an absolute time, and that Lorentz invariance might not have a fundamental character. In spite of these speculations, relativistic versions of the theory can be constructed (de Broglie 1927b, 1956, 1971b; Fer 1966; Kyprianidis 1985, 1987), although their validity is, of course, far from being established.

Since in the realistic interpretations ψ is logically independent of the probability of presence for the particle, it is necessary to show that Born's rule (3.0.1) follows as a necessary consequence, in order to guarantee agreement with the experiments (Pauli 1953). For the pilot wave theory, this was first done by Bohm (Bohm 1953), who showed that an arbitrary probability density eventually decays into $|\psi|^2$ as a result of random collisions. This result has been recently generalized by proving a "subquantum" H-theorem (Valentini 1991a,e, 1992).

There is no measurement paradox in this interpretation, because the observables of a physical system have always well defined values, and superposition in the wave function does not entail "superposition" of physical properties. The collapse of the state vector corresponds only to an arbitrary choice of neglecting the components of ψ which do not contain the system (Bohm and Hiley 1984); however, these "empty" waves remain present and several experiments have been suggested (Croca 1987; Croca et al. 1988; Schmidt and Selleri 1991; Hardy 1992a), and one has been performed[3] (Wang et al. 1991a), that aim to detect them in order to provide a crucial test for the pilot wave interpretation of ψ.

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[3] Although its interpretation is controversial (Holland and Vigier 1991; Wang et al 1991b).

3.1.3 Relative State and Many-Worlds Interpretations.

The use of the collapse postulate (2.1.11) presupposes a decomposition of the physical world into a system and a measuring device (or observer). However, nowhere in the axioms of quantum theory can one find any precise indication about the criteria to follow in deciding when and how should such a decomposition be performed (Bell 1990). This problem becomes particularly serious if one wants to describe the whole universe in quantum mechanical terms. This is the main reason why the interpretations that we shall discuss in the present section—which accept only the standard formalism of quantum mechanics, without collapse postulate—are so popular among relativists and cosmologists in particular (see, e.g., Tipler 1986).

Usually no distinction is made between "relative state" and "many-worlds" interpretations. But since it has been argued recently (Whitaker 1985; Ben-Dov 1990a) that they actually correspond to different physical pictures and even present different problems,[4] we shall discuss them separately. The ideas common to both interpretations are to reject the collapse postulate and to assume that the entire state vector represents physical reality. The difference resides in the meaning given to the latter concept. Their mathematical structure is nevertheless the same—that of standard quantum theory.

Let us consider a simple universe $\mathcal{U}$ composed exclusively of a system $\mathcal{S}$ and a measuring device $\mathcal{M}$, as in Sec. 2.1. We remember that, if the initial state of $\mathcal{U} \equiv \mathcal{S} + \mathcal{M}$ is

$$|i\rangle_{\mathcal{U}} = \sum_r c_r |a_r\rangle_{\mathcal{S}} \otimes |p_0, s\rangle_{\mathcal{M}} \ , \tag{3.1.3}$$

then the linear unitary evolution leads to (2.1.6), which we write explicitly as

$$|f\rangle_{\mathcal{U}} = \sum_{rr's'} c_r u_{rs}^{r's'} |a_{r'}\rangle_{\mathcal{S}} \otimes |p_r, s'\rangle_{\mathcal{M}} \ . \tag{3.1.4}$$

The meaning of the state vector (3.1.4) is the issue about which the two interpretations here considered diverge, though both of them consider $|f\rangle_{\mathcal{U}}$ to faithfully represent the physical reality of $\mathcal{U}$.

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[4] For example, is seems (Whitaker 1985) that the EPR paradox can be solved in the relative state interpretation (Page 1982) but not in the many-worlds one (Kunstatter and Trainor 1984b).

The relative state interpretation (Everett 1957; Wheeler 1957) focusses on the notion of *correlations*. Its underlying philosophy is that the purpose of physics is only to describe and predict correlations between different systems. Pushing this idea to its logical limit, one might say that physical reality is *nothing but* correlations—a viewpoint which is not entirely new in the history of thought. The state vector (3.1.4) is clearly a very good description of this notion of physical reality, since its components represent all the possible correlations between the subsystems $\mathcal{S}$ and $\mathcal{M}$ of the universe $\mathcal{U}$. Moreover, by extending the universe to contain other measuring devices, and possibly even human observers with their minds (assumed to obey the laws of quantum theory), one would obtain a state vector describing all the correlations among these systems, each component representing only consistent observations (Everett 1957; Cooper and Van Vechten 1969). The collapse postulate was motivated by a different notion of physical reality, based on the concept of unique and absolute values for the macroscopic pointer's position P. The relative state interpretation admits that, within each single component of $|f\rangle_{\mathcal{U}}$, a state of $\mathcal{M}$ can be defined which corresponds to a precise value of P, but this state is *relative* to that particular component and to the decomposition of $\mathcal{U}$ into $\mathcal{S}$ and $\mathcal{M}$, and has no fundamental significance. It is the whole state vector which thoroughly describes $\mathcal{U}$ and represents reality.

This picture, though logically consistent, appears hard to reconcile with common experience. When looking at a pointer, we are aware of a single value of its position, even in circumstances in which the wave function of the universe contains components corresponding to ours being aware of *other* positions. There are essentially three ways out of this problem. One of them consists in accepting a special role for consciousness, which would somehow "choose" a unique world among all the possibilities represented in the state vector (Squires 1987, 1988; Sudbery 1988; Ben-Dov 1990b). In the second the theory is supplemented with extra "coordinates" labeling the *actual* copy of the universe, and a suitable law is postulated which describes the evolution of these coordinates once the state vector is known. One ends up, therefore, with a structure which is strongly reminiscent of the pilot wave theory presented in Sec. 3.1.2 (Bell 1976, 1981, 1986; Ben-Dov 1990c). Both these suggestions entail the failure of the idea that the state vector should provide a complete description of reality. The last possibility is to accept the many-worlds version of the interpretation.

The many-worlds interpretation (De Witt 1970; Ballentine et al. 1971; De Witt and Graham 1973) is more conservative about the concept of physical reality, and still considers the notion of values of observables as central. Since it assumes that the state vector is in a one-to-one correspondence with such a reality, it necessarily requires that all the possibilities represented in (3.1.4) are simultaneously present, i.e., that the pointer's position has the value p_1 *and* the value p_2, and so on. The fact that in common experience the position is observed to be *either* p_1 *or* p_2, etc., leads therefore one to conclude that in going from (3.1.3) to (3.1.4) the universe $\mathcal{U}$ has branched into many copies of itself, each corresponding to one of the components of (3.1.4). If other instruments were contained in $\mathcal{U}$, as well as human observers, they would also be splitted in many copies together with $\mathcal{U}$, but they would not register, or feel, anything that might allow one to infer that the splitting has taken place (De Witt 1970). This viewpoint, though bizarre, offers a straightforward resolution of the measurement paradox: Our experience of a single value of P cannot be taken as evidence for (2.1.8) instead of (2.1.6), because it concerns only with one "branch" of the universe. The state vector (2.1.6) *does* represent physical reality; rather, it is our observations which have access only to a limited portion of the latter.

This interpretation has been often criticized for being tremendously antieconomic (d'Espagnat 1976; Bell 1981; see also J.A. Wheeler, and J. Taylor, in Davies and Brown 1986), as it introduces a huge amount of non-testable elements.[5] Other criticisms are concerned with the arbitrariness and vagueness of the notion of "branching". In fact, if branching of the universe is supposed to be a physically real phenomenon, it ought to find a precise description within the theory. However, not only there is no indication about the time and the way in which splitting occurs (Ballentine 1973; d'Espagnat 1976), but Eq. (3.1.4) seems also to be ambiguous about the so-called "splitting basis", i.e., the basis of the Hilbert space that diagonalize the observables which have definite values in each branch (Ballentine 1973; d'Espagnat 1976; Bell 1981). More precisely, we can expand each vector $|p_r, s'\rangle_{\mathcal{M}}$ into eigenstates of some pointer's observable Q different from P, as

$$|p_r, s'\rangle_{\mathcal{M}} = \sum_{k\sigma} {}_{\mathcal{M}}\langle q_k, \sigma | p_r, s'\rangle_{\mathcal{M}} |q_k, \sigma\rangle_{\mathcal{M}} \ , \tag{3.1.5}$$

[5] It has however been argued that a slightly modified many-worlds theory might admit an experimental test (Deutsch 1985b, 1986; see also D. Deutsch, in Davies and Brown 1986).

to get

$$|f\rangle_{\mathcal{U}} = \sum_{rr's'k\sigma} c_r\, u_{rs}^{r's'}{}_{\mathcal{M}}\langle q_k,\sigma|p_r,s'\rangle_{\mathcal{M}} |a_{r'}\rangle_{\mathcal{S}} \otimes |q_k,\sigma\rangle_{\mathcal{M}}\ . \tag{3.1.6}$$

In spite of being the same vector, (3.1.4) and (3.1.6) suggest different splitting processes. According to (3.1.4), in each branch of the universe the pointer has a definite value of P, whereas according to (3.1.6) it has a definite value of Q. But since P and Q might well be incompatible observables, it follows that simple expressions like (3.1.4) or (3.1.6) are not sufficient to determine uniquely the nature of the splitting: A supplementary notion of preferred basis is needed to this purpose. A plausible candidate seems to be the "pointer basis" emerging in the study of environment-induced superselection rules (Zurek 1981, 1991; see also Sec. 5.1).

These interpretations would have probably not received much attention if it were not for the claim that they are able to produce Born's rule (3.0.1) *as a theorem*, without the need to postulate it from the outset (Everett 1957; De Witt 1970; De Witt and Graham 1973).[6] However, it was soon realized (Ballentine 1973; Clarke 1974) that the proof of such a theorem is rather independent of the specific assumptions characterizing the relative state and many-worlds interpretations (actually, the same theorem was also proved within a rather different context (Hartle 1968)). Furthermore, the proof appears suspiciously circular (Clarke 1974; Deutsch 1985a; Squires 1990) and cannot thus be regarded as truly satisfactory.

3.2 Nonrealistic Subjective Interpretations.

At the opposite extreme, with respect to the realistic interpretations, are the nonrealistic subjective ones, according to which ψ is associated to knowledge about the state of a system. Since the very idea of "knowledge" assumes the notion of somebody—or something—who *knows*, these interpretations attribute a fundamental importance to the special role played by the observer. However, up to now nobody has succeeded in giving a precise definition of what an observer is.

[6] We remember that this is a *necessary* requirement which every realistic interpretation must fulfill.

Due to their very speculative character, one can count almost as many versions of these interpretations as there are authors of papers on the subject. The various nuances of mixing between subjective and objective elements that can be identified are consequently numerous, so that the corresponding interpretations range from "almost completely subjective" to "almost objective". Rather than making pedantic and fruitless distinctions, we shall present only a general discussion.

If the state vector is associated to knowledge of the observer, it is perfectly reasonable to assume that it can change as soon as such knowledge changes. The process of state vector collapse should therefore be understood in these terms.

Let us reconsider the measurement process of Sec. 2.1, focussing on its description as given by an observer $\mathcal{O}$. After the interaction between $\mathcal{S}$ and $\mathcal{M}$ has taken place, the state of $\mathcal{S}+\mathcal{M}$ is given by the coherent superposition (2.1.6). This cannot, however, represent the knowledge of $\mathcal{O}$ after he/she has observed the pointer's position P because we know, by our own experience, that a human observer is only aware of a single value of P. Such a knowledge should rather be represented by one of the vectors $|p_r;(r,s;\alpha)\rangle$ given by (2.1.7). It appears therefore that the idea that a state vector represents knowledge leads to a natural justification for the law of collapse (Von Neumann 1932; London and Bauer 1939).

That this resolution of the objectification problem is not free from difficulties can be realized by asking the obvious question: "Does the process

$$|f\rangle \longrightarrow |p_r;(r,s,\alpha)\rangle \tag{3.2.1}$$

only represent new acquisition of information by the observer, or does it correspond *also* to some change in the objective state of the system $\mathcal{S}+\mathcal{M}$?". The first option seems hard to maintain without changing radically the content of quantum theory, because it implies that the pointer's position is well defined *already in the state* $|f\rangle$, i.e., that the presence of components with different values of P in (2.1.6) should be attributed only to ignorance of the observer. But since (2.1.6) expresses the best description that quantum theory can give of the system $\mathcal{S}+\mathcal{M}$, one is led to conclude that such a description is incomplete. The only justification for retaining standard quantum mechanics could thus be a sort of belief that our description of nature is *doomed to be* incomplete, for example on the basis of ideas akin to Bohr's complementarity (see Sec. 3.3.1).

Those who are unhappy with this viewpoint, and yet would like to continue to think of (3.2.1) as corresponding to a change in the knowledge of the observer, are left with the second possibility, i.e., to suppose that such a change is accompanied also by a physical change in $\mathcal{S}+\mathcal{M}$. This viewpoint amounts to interpreting the state vector as a mixture of subjective and objective descriptions (Heisenberg 1959). Although presenting all the advantages due to the cloudiness of its formulation, this interpretation runs into severe troubles, too. In fact, it implicitly assumes that the observer has not simply a passive role in the measurement process, but rather that he/she participates to it, by exerting a physical influence on $\mathcal{S}$ and $\mathcal{M}$. This influence cannot be however described by quantum laws: On trying to apply quantum mechanics to the new system $\mathcal{S}+\mathcal{M}+\mathcal{O}$, one would end up with a coherent superposition of components involving different states of knowledge for $\mathcal{O}$ (Von Neumann 1932). By following this line of reasoning, therefore, one concludes that the observer—his/her counsciousness in particular—cannot obey quantum theory. It is this difference between $\mathcal{O}$ and $\mathcal{S}+\mathcal{M}$ which is responsible for the state vector collapse (Von Neumann 1932; Wigner 1961; Peierls 1979).

Strictly speaking, this final picture corresponds to an objective, though very unconventional, description. One may think that concepts like consciousness, knowledge, and state vector collapse, will be regarded one day as belonging to the phenomenology of a more general theory, including quantum mechanics as an approximation and capable to treat mind in precise terms. Similar ideas have been expressed, for example, by Penrose (Penrose 1989), who argues that new physics is required in order to discuss mind, and suggests that such a physics might be the same which is necessary to describe the state vector reduction and to formulate a satisfactory quantum theory of gravity.

The latter views are clearly highly speculative, but there is a sense in which they are completely traditional: They tacitly accept that, sooner or later, even the concepts of mind and observer will fall under the description of physics. It is interesting to notice that the opposite viewpoint—that observers might have a fundamental role not only in formulating, but even in giving meaning to, the physical laws—has sometimes been expressed (Peres 1980b; Peres and Zurek 1982; J.A. Wheeler, in Davies and Brown 1986).

3.3 Nonrealistic Objective Interpretations.

The assumption that $|\psi|^2$ represents an objective probability, though apparently sober and precise, is not sufficient to determine a unique interpretation of quantum theory, unless the meaning of "objective probability" has been priorly established. Two such meanings exist, which refer to individual events and to ensembles, respectively. The corresponding interpretations of quantum mechanics can be called "minimal", in the sense that they make the smallest possible number of assumptions in addition to the chosen characterization of probability.

3.3.1 Copenhagen Interpretation

This interpretation (Bohr et al. 1928; Rosenfeld 1958, 1961; Bohr 1959; Hall 1965) is inspired to instrumentalism and is therefore fundamentally pragmatic (Stapp 1972). Its basic concepts do *not* belong to the microscopic domain, but rather to that of everyday's experience. Consequently, it does *not* pretend to give a representation of the microphysical processes, its aim being only to establish correlations among macroscopic events, which can be described using the unambiguous language of classical physics. The formalism of quantum theory is considered as a set of rules that enable one to make predictions within this framework.

The typical problem addressed in this context is to correlate the outcome of a measurement to the *preparation* of the experimental setup. Since it is empirically observed that the same preparation can lead to different outcomes, such a correlation cannot be of a deterministic kind, contrarily to what happens in classical physics. Borrowing few concepts from Aristotelian philosophy one can think of the preparation as determining only a set of "potentialities" for the outcome, one of which is actualized in the measurement.

These ideas can be formalized by introducing the notion of *propensity* (Popper 1957), which is an interpretation of probability that is supposed to apply to single events (Home and Whitaker 1992b). If $P(A|B)$ is the conditional probability for the event A given the condition B, then $P(A|B)$ is interpreted as the propensity that B produce A, meaning by this that it measures the "strength" with which A is actualized among the potentialities determined by B. As well known, expressions like $P(A|B)$ are computed in quantum theory by using the concept of state vector. For example (Heisenberg W. 1930; critics in Fer F. 1956), one can say that the propensity that a definite preparation lead, after a time t, to the formation of a bubble at the point $\mathbf{x}$ in a Wilson chamber is $|\langle \mathbf{x}|\hat{U}(t)|\psi\rangle|^2$. The state vector $|\psi\rangle$ can thus be interpreted as encoding the process of preparation (Peres 1984a, 1986; Lamb 1969); in a sense, it "lists" the propensities for the given preparation to produce any possible outcome of a measurement. The process of actualization of the outcome, which can sometimes be regarded as a re-preparation, corresponds to the state vector reduction, which is not described by quantum theory since it involves the measuring device, assumed to be classical.

Within this set of ideas, concepts like "electron" are mere shorthands for some of the rules of correlation between preparations and measurement outcomes, and no objective reality should be ascribed to them. Paradoxes arise when one asks a "foolish question" about these phantomatic "objects" (Feshbach and Weisskopf 1988; Brown et al. 1989), question which do not admit any concrete representation in terms of macroscopic apparatuses. Foolish questions often derive from applying the classical prejudice that systems should have physical properties, or from extrapolating the rules of quantum mechanics to the classical world, to which they are not supposed to apply. The EPR argument and the measurement paradox are, respectively, very representative examples of these mistakes (Bohr 1935). According to the Copenhagen interpretation, one should instead characterize physical properties only in terms of responses of the apparatus under well defined circumstances; from this point of view, it is neither astonishing nor mysterious that incompatible properties—hence observables—exist, as they correspond to experiments whose simultaneous realization is impossible. This justifies the so-called complementarity principle (Bohr 1959; Rosenfeld 1961).

It must be pointed out that the fundamental role played by the macroscopic devices, which are supposed to have a classical nature and not to admit a quantum mechanical description, has been regarded by several people as representing a serious limit of applicability of the theory (Bell and Nauenberg 1966; Stapp 1972, 1991; Bell 1990). Another defect is the vagueness of the criteria which one should use in deciding *which* objects are classical (Bell and Nauenberg 1966; Bell 1990).

The Copenhagen interpretation is usually presented as the orthodox point of view (see, e.g., Messiah 1966). This privilege is often justified by reminding that it is battle-tested (J.A. Wheeler, in Davies and Brown 1986): It has resisted to any attack for more than sixty years, and it even survived to several strong criticisms advanced by Einstein (Bohr 1949; Wheeler and Zurek 1983). Three objections—of epistemological, historical, and sociological character, respectively—can be raised against this attitude.

First, one may notice that since the philosophy underlying the Copenhagen interpretation is essentially pragmatic (Stapp 1972), much care is adopted in avoiding to speak about "reality", and only statements concerning classical macroscopic measuring devices are, in principle, admitted. A theory which refrains from describing everything except measurement configurations is certainly on a safer ground than others that make more hazardous hypothesis. But the so achieved logical coherence might require a very high price to pay: The risk of blocking future developments. It is good to remind, in this context, that a reasonable requirement for an idea to have scientific value is that it be falsifiable (Popper 1959). Actually, the unshakability of the Copenhagen interpretation strongly reminds that of solipsism, which is also a form of pragmatism, though an extreme one, and which can hardly be considered as a scientifically stimulating position.

On the historical side, it has to be remarked that the acceptance of the Copenhagen interpretation has been much a byproduct of the cultural environment which existed in Germany in the period between the First World War and the rise of Nazism, and was strongly influenced by the popularity of the idealist philosophy (Forman 1971; Selleri 1990; Valentini 1992).[7] It thus appears that the historical circumstances have played a nonnegligible role in determining its predominant position, perhaps more than what criteria based on objective judgement could allow.

As third and last remark, we feel necessary to notice that only few textbooks give a decent account of what they present as the correct interpretation of quantum theory, most of them limiting the discussion to few disconnected comments about uncertainty and complementarity. Moreover, not very many "supporters" of the Copenhagen interpretation seem prepared to defend it. This situation appears to be a consequence of what has been defined as a "brainwashing" (Gell-Mann 1979), especially in consideration of the fact that the founders themselves did not fully agree on several points (compare, for example, the ideas of Pauli as expressed in Laurikainen 1985, with those of Bohr (1949, 1959) and Heisenberg (1959)).

3.3.2 Statistical Interpretation.

The concept of propensity conveys the intuitive feeling of what a probability is. However, it does not provide by itself an operational criterion for checking predictions expressed in terms of probabilities. A

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[7] The author is deeply indebted to A. Valentini for many enlightening discussions concerning this point.

check cannot obviously be performed by analizing a single event, because of the indeterministic character of quantum theory. What one can do (and what is done in practice) is to prepare in the same way many copies of an experimental arrangement (or to re-prepare many times the same one) and then to compare the statistics of the outcomes with the probabilistic predictions of quantum theory. This amounts to adopting a statistical interpretation of probability (Brody 1989; Ballentine 1990b). In a sense, the theory of probability is regarded also as a *physical theory*, with an interpretation which connects the mathematical formalism to the world of experience.

If Born's law (3.0.1) is assumed to hold, and if probability is interpreted as above, then we cannot say any more that $|\psi\rangle$ accounts for the properties of individuals, or for an individual preparation. Since the predictions of quantum theory require, in general, to be tested on ensembles, the interpretation itself must refer to ensembles. Unless we are so pragmatic to exclude words such as "particle" from our dictionary, we can say that $|\psi\rangle$ describes the statistical properties of the outcomes of measurements performed on a conceptual ensemble of equally prepared copies of a quantum system (Einstein 1936; A. Einstein, in Schilpp 1949; A. Einstein, in Born 1971; Blokhintsev 1964, 1968; Pearle 1967; Ballentine 1970, 1972, 1990b; Belinfante 1975; Newton 1980; Brody 1989; Home and Whitaker 1992b; Park and Band 1992). Consequently, quantum mechanics does not describe individuals or single events any more than kinetic theory describes the behaviour of a single molecule (Sonego 1991a).

The consequences of quantum theory lose their paradoxical character if this interpretation is accepted (Ballentine 1970). Since the state vector is not assumed to characterize a single system (see, e.g.: van Heerden 1975; Peres 1975), the objectification problem of Sec. 2.1 can be dispensed with as ill-posed, and there is no justification for the claim that (2.1.7), rather than (2.1.6), represents the state of $\mathcal{S}+\mathcal{M}$. Actually, the whole issue of collapse of the state vector as a physical process taking place at the level of a single system turns out to be completely meaningless; in the statistical interpretation, collapse only represents shifting the description from an ensemble $\mathcal{E}$ to a subensemble $\mathcal{E}' \subset \mathcal{E}$ (Newton 1980). There is no reason why such a process, which corresponds to a change in the object of description, should be described by quantum theory; indeed, it is perfectly well described by general probability theory (Keller 1990). Anyway, it is not a necessity of principle and all the quantum predictions can be obtained without its use (Ballentine 1990a).

The EPR argument loses also much of its strength, as quantum theory is not supposed to be a complete description of physical reality for individuals (see also Whitaker and Singh 1982). The separate issue as to whether such a description could be given is clearly beyond the scope of the statistical interpretation. It is nevertheless perfectly natural to pose the question in the following terms: "Is the statistical character of quantum mechanics fundamental, or is it a consequence of an incomplete description?". The statistical interpretation remains neutral about the answer, but one must admit that it is difficult to resist to the temptation of trying to construct a theory of individuals of which quantum theory would represent, somehow, the "statistical mechanics".

If such a theory exists, it is certainly not trivial (Gillespie 1986; Home and Whitaker 1986, 1992b). Attempts of this kind are usually frustrated by the apparent need to introduce negative probabilities for individual events (Moyal 1949; Feynman 1982; Sonego 1990, 1991a). It can be argued, however, that this implausible feature is only due to the unmotivated assumption that the physical properties of the individuals are straightforwardly related to the possible outcomes of measurements (Valentini 1991d, 1992). The no-hidden-variables theorems (Kochen and Specker 1967; Redhead 1987; Mermin 1990b; Peres 1990a, 1992; Pagonis et al. 1991; Żukowski 1991) seem to enforce this idea—that, if physical properties can be defined for individuals, they cannot be the same physical properties which are explored[8] through usual measurements. This interpretation is, however, not universally accepted (Barut 1992). The construction and the features of a "prequantum" theory of individuals remain therefore open problems.

4. Alternative Theories.

According to some physicists, the conceptual problems of quantum theory cannot be solved simply by adopting a suitable interpretation; instead, they suggest that the mathematical formalism itself should be modified. The motivations for facing such an ambitious programme range from the try-and-see-what-happens philosophy to the claim that the measurement paradox provides experimental evidence for deviations from the quantum theoretical framework.

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[8] Perhaps "defined" would be a more adequate term.

4.1 Stochastic Theories.

These theories regard the quantum behaviour of a system as the effect of a stochastic process to which the system is subject. When one tries to justify such an idea on physical grounds, two classes of theories can be distinguished:

1. Theories that assume the stochasticity as a *primitive* and *fundamental* property of *all* the physical systems;
2. Theories that reduce the stochastic behaviour of a system to some uncontrolled random influence of the environment.

Stochastic mechanics and stochastic electrodynamics are, respectively, good representatives of these two classes.

4.1.1 Stochastic Mechanics

In 1966 Nelson showed that the Schrödinger equation is mathematically equivalent to a stochastic process (Nelson 1966, 1985; de la Peña-Auerbach 1967) by proving that, if $\psi(\mathbf{x}, t_0)$ is the wave function for a particle[9] with mass m at a time t_0, there exist a Wiener process in the configuration space with diffusion constant $\hbar/2m$ which evolves the probability density $|\psi(\mathbf{x}, t_0)|^2$ into $|\psi(\mathbf{x}, t)|^2$. Similar considerations had been made earlier by Fényes (1952). This result has important consequences for the problems of interpretation, since it provides quantum mechanics with a formalism that contains explicitly the concept of *particle trajectory* (see, e.g.: Bozić and Marić 1991; Cufaro Petroni and Vigier 1992), and is therefore suitable for extending the theory in a realistic spirit. Since 1966, a large number of papers have been published on the subject, devoted to enrich the formal apparatus of the theory and to discuss its implications (see, e.g.: de la Peña-Auerbach 1969, 1971; Davidson 1979; Guerra 1981; Guerra and Marra 1984; Wang 1988; Cufaro Petroni 1989; Garbaczewski 1990; Jibu et al. 1990; Sanz 1990; Hajra and Bandyopadhyay 1991) as well as to extend its applicability to the relativistic domain (Lehr and Park 1977; Morato 1991) and to field theory (Guerra 1981). It remains still unclear, however, whether stochastic mechanics should be regarded as a purely abstract reformulation of quantum theory, or whether the diffusion process that it postulates might actually represent a physically real phenomenon.

Stochastic mechanics has a precursor in the theory of Bohm and Vigier (Bohm and Vigier 1954), which is a hybrid between the pilot wave

[9] For sake of simplicity, we consider only the one particle case.

theory (see Sec. 3.1.2) and the hydrodynamical models (Sec. 3.1.1). If one takes seriously the idea of a subquantum fluid whose streamlines are determined by the guidance law (3.1.1) it is natural to think, by analogy with Brownian motion (Kershaw 1964; Comisar 1965; de la Peña-Auerbach, Braun and Garcia Colin 1968; de la Peña-Auerbach and Garcia Colin 1968), that a particle follows these trajectories only on the average, but in reality it is subjected to much more complex forces than those derived by the simple quantum potential (3.1.2). Accordingly the pilot wave formulation would correspond to a "macroscopic" hydrodynamical description of a subquantum diffusion process which is more properly accounted for by stochastic mechanics (Bohm and Hiley 1989); the physical origin of such a process would be the subquantum medium with which the particle is assumed to be in permanent contact. As a concrete model for this misterious medium, Dirac's quantum aether (Dirac 1951; Sinha et al. 1976) has been sometimes proposed (Vigier 1980; Cufaro Petroni et al. 1981).

Though attractive, this intuitive picture cannot unfortunately be maintained in any simple and natural way. The formalism of stochastic mechanics requires that the features of the diffusion process depend on $\psi(\mathbf{x}, t_0)$. Hence, the preparation of the particle strongly influences the physical properties of the medium (Ghirardi et al. 1978)—a circumstance which has no counterpart in the classical examples of Brownian motion. Similar difficulties emerge when considering systems of identical particles, whose diffusion processes turn out to be strongly correlated (Ghirardi et al. 1978) in a way that cannot be explained by classical models of the subquantum fluid. It must also be pointed out that, although stochastic mechanics allows one to think of particles as following definite trajectories, these trajectories are not, even in principle, observable (Ghirardi et al. 1978); alternatively, one may distinguish between *real* and *virtual* trajectories (Cufaro Petroni 1991; Cufaro Petroni and Vigier 1992), but this does not seem to improve much their operational status. On considering these conceptual difficulties it is probably safer, at the moment, to regard stochastic mechanics as a mere simulation of quantum theory (Kracklauer 1974) and not to ascribe a physical reality to the hypothetical subquantum medium.[10]

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[10] Further comments about stochastic mechanics can be found in: Mielnik and Tengstrand 1980; Wallstrom 1989.

4.1.2 Stochastic Electrodynamics.

The idea that a classical particle interacting with a suitable medium might exhibit quantum behaviour finds a concrete realization in stochastic electrodynamics. In this theory it is assumed that each region of space contains a background classical electromagnetic field, randomly fluctuating and with energy density spectrum (at zero temperature)

$$\rho(\omega) = A\,\omega^3 \;, \tag{4.1.1}$$

where A is a constant. It can be shown (Marshall 1965a; Boyer 1980, 1984; Eriksen and Grøn 1987) that (4.1.1) is the only Lorentz invariant expression for $\rho(\omega)$, i.e., that it represents the only electromagnetic background which cannot be directly detected by inertial observers. The value of the constant A is not determined by this requirement but can be fixed by comparing the results of the theory with those of quantum mechanics. It turns out that

$$A = \frac{\hbar}{2\pi^2 c^3} \;, \tag{4.1.2}$$

where c is the speed of light. Within this context $\hbar$ has the meaning of a measure of the intensity of the background electromagnetic field, rather than the usual one related to noncommutativity of canonically conjugate variables.

A particle with mass m and charge e moving in the zero-point field obeys a Lorentz-Dirac equation which, in the nonrelativistic limit, becomes the so-called Braffort-Marshall equation (Braffort et al. 1954, 1965; Braffort and Tzara 1954; Marshall 1963, 1965; Claverie and Diner 1978)

$$m\ddot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t), t) + \frac{2}{3}\frac{e^2}{c^3}\,\dddot{\mathbf{x}}\,(t) + e\,\mathbf{E}(t) \;, \tag{4.1.3}$$

where $\mathbf{F}$ represents an external force acting on the particle and $\mathbf{E}$ is the background random electric field, whose space dependence has been neglected, as well as the effects of the magnetic field.

The theory based on Eqs. (4.1.1)–(4.1.3) applied to linear systems has produced a number of remarkable results in agreement with experiments and quantum mechanical predictions (Boyer 1970, 1975, 1980, 1984; de la Peña-Auerbach and Cetto 1971, 1972, 1978, 1979; Jáuregni and de la Peña-Auerbach 1981). However, it appears not to work very well when *nonlinear* systems are considered (Pesquera and Claverie 1982). To quote a concrete result, one finds that the equilibrium state of the hydrogen atom is the auto-ionized one (Claverie et al. 1980; Claverie and Soto 1982)! The total disagreement of this prediction both with quantum mechanics and experiments has been regarded as a serious difficulty of stochastic electrodynamics, and even as a sign of its failure (Diner 1984); nevertheless, some authors still maintain that the theory, once fully exploited in its possibilities, can still make sense (de la Peña-Auerbach and Cetto 1984; Puthoff 1987; Brody 1988; Cetto and de la Peña-Auerbach 1991). The debate seems therefore not yet closed.

This is not the only problem of stochastic electrodynamics. Besides various difficulties shared with stochastic mechanics (locality, influence of the preparation process on the zero-point field), the universality of quantum behaviour creates serious troubles to any theory which is supposed to derive it from the interaction with an electromagnetic field; we must remind, in fact, that neutral particles exhibit the same quantum effects as charged ones, though not feeling the zero point field (Brody 1988). Attempts at solving this problem lead either to the formulation of a general stochastic theory or to postulate a stochastic gravitational background.[11]

The task of explaining the puzzling presence of a classical stochastic background has been recently undertaken by Puthoff (Puthoff 1989, 1991; Wesson 1991; Santos 1991b). He suggests to regard the zero-point field as due to the radiation emitted by all the charges in the universe under their mutual scattering. According to this idea, the background field would continuously regenerate itself by causing the charges to radiate; on requiring self-consistency of the process, one can establish a relationship between $\hbar$, c, e and some cosmological quantities. It might be interesting to pursue further these investigations, in order to understand whether they can shed some light on the origin of several numerical "coincidences" which remain still unexplained (Davies 1982; Barrow and Tipler 1986).

4.2 Nonlinear Theories.

Although there is no experimental evidence for deviations from the linear evolution of quantum mechanics (see, e.g., Majumder et al. 1990),

[11] Stimulating considerations about the common feature of quantum theory and gravitation of being universal, have been made by Smolin (1986).

theories have been suggested that contemplate possible nonlinear corrections. They can be roughly partitioned into two classes, but some phenomena of osmosis might be expected to occur.

4.2.1 Nonlinear Quantum Mechanics.

The oldest and perhaps stronger motivation, within the context of pure quantum theory, for considering nonlinear versions of wave mechanics is due to de Broglie (1956, 1964, 1987). Pilot wave theory consists essentially of the Schrödinger equation *and* the guidance law (3.1.1); these two fundamental prescriptions are, however, logically independent of each other and must be postulated separately. This is similar to what happens in electromagnetism, where Lorentz equation of motion for charges is independent of Maxwell field equations. General relativity is much more satisfactory under this respect: Einstein field equation *implies* the geodesic equation for the world-line of a test particle, by considering the latter as a suitable limit of a "lump" in the gravitational field. de Broglie's idea is that Eq. (3.1.1) should have a similar origin. Regarding the particle as a small region in which the ψ field has a very high intensity, one might hope that the equation for ψ could entail the guidance law. It seems that this goal can be successfully achieved only if the field equation for ψ is nonlinear. So far, no criterion has been formulated which allows one to guess the correct form of such equation (for an example of work on this subject, see Vigier 1991).

Other theories, mainly motivated by generic considerations about the anomalous status of linearity in physics, deal essentially with nonlinear modifications of the Schrödinger equation, of the kind (Mielnik 1974; Bialynicki-Birula and Mycielski 1976)

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + C[\psi] \; , \tag{4.2.1}$$

where the form of the correction $C[\psi]$ is strongly constrained by requirements of locality, as well as of phase and Galilean invariance. Relativistic extensions of these theories based on quantum fields have been also studied (Kibble 1978). Recently, nonlinear evolution equations have been considered (Weinberg 1989) in order to provide concrete models for the investigation of possible new experimental effects; however, such a programme has been criticized even on theoretical grounds, since the proposed modifications would lead to violations of the second law of thermodynamics (Peres 1990b; Weinberg 1990) and to the possibility of arbitrarily fast communication (Gisin 1990; Polchinski 1991). As far as the experiments are concerned, linearity is confirmed to a very high degree of accuracy (Bollinger et al. 1989; Majumder et al. 1990; Gähler and Zeilinger 1991).

4.2.2 Semiclassical Theories.

Newtonian physics is marked by a dichotomy between the concepts of particles and interactions. One of the most remarkable consequences of Maxwell's formulation of a dynamical theory for the electromagnetic field was that of reducing the gap separating these two entities, by attributing to the interactions some physical properties which usually characterize matter. Nowadays the general view is that there is really no fundamental difference between the two concepts, both being aspects of a more general one—the quantum field. Not everybody, however, accepts this opinion and there are suggestions that a distinction in the behaviour of matter and fields should be maintained. In particular, we shall now present two theories in which quantum matter is coupled, respectively, to a *classical* gravitational and electromagnetic field. It is important to remark that, although sometimes these are considered as *effective* approximations of a full quantum theory, we shall here stick to the viewpoint that they are instead *exact* descriptions.

Semiclassical gravity is based on the field equation (Møller 1962; Rosenfeld 1963; Kibble and Randjbar-Daemi 1980; Kibble 1981)

$$G_{ab}(x) = \frac{8\pi G}{c^4} \langle\psi|\hat{T}_{ab}(x)|\psi\rangle \ , \tag{4.2.2}$$

$$i\hbar \frac{d|\tau\rangle}{d\tau} = \hat{H}(\tau)|\tau\rangle \ , \tag{4.2.3}$$

$$\hat{H}(\tau) = \int_{\Sigma(\tau)} d\Sigma(x)\hat{T}_{ab}(x)u^a(x)u^b(x) \ . \tag{4.2.4}$$

where G_{ab} and G are the Einstein tensor and Newton's gravitational constant, and $|\psi\rangle$ and $\hat{T}_{ab}$ represent, respectively, the state vector and the stress-energy-momentum tensor operator of quantum matter. Equation (4.2.2) is actually incomplete, since it must be supplemented by a theory which allows to express $\hat{T}_{ab}(x)$ in terms of more fundamental operators acting on the state $|\psi\rangle$. This is, however, a completely different subject, and we can simply assume that such a treatment is given. The proponents of this theory motivate it essentially by reminding the tremendous difficulties encountered in trying to carry on the usual quantization programme for gravity. We are here interested in the fact that Eq. (4.2.2) is *nonlinear in the state vector* $|\psi\rangle$ (Mielnik 1974; Kibble 1981); the coupling to classical gravity induces therefore a nonlinear behaviour of quantum matter. Curiously enough, Eq. (4.2.2) has been tested experimentally using a device in which a Cavendish torsion balance was responding to the gravitational field produced by a lead ball whose position was triggered by the decay of a radioactive substance (Page and Geilker 1981). The unsurprising negative result, interpreted by the authors as indirect evidence for quantum gravity, should rather be regarded as direct refutation of a semiclassical theory based on Eq. (4.2.2) (see also: Hawkins 1982; Ballentine 1982; Page and Geilker 1982). Other arguments against this theory come from the fact that apparently none of the interpretations of the state vector presented in Sec. 3 allows Eq. (4.2.2) to make sense conceptually and to be mathematically consistent at the same time.

A more founded theory than semiclassical gravity is, is self-field QED, which has been developed essentially by Barut and his collaborators (see Barut 1988c for a short review, and Barut and Dowling 1990b for a simplified treatment of some applications).[12] The starting point consists in trying to account for the radiation reaction of a quantum particle with charge e in a nonperturbative way. This task is accomplished through the physical hypothesis that the charge responds to its own electromagnetic field, assumed *classical* and due to the charge density and current

$$\rho = e\,|\psi|^2\;, \tag{4.2.5}$$

$$\mathbf{j} = \frac{e\hbar}{2mi}\,\psi^*\overleftrightarrow{\boldsymbol{\nabla}}\psi - \frac{e}{m}\rho\mathbf{A}\;. \tag{4.2.6}$$

Given the boundary conditions, the electromagnetic potentials $\mathbf{A}$ and φ can be eliminated by using the appropriate Green's function, and the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{1}{2m}(\hbar\boldsymbol{\nabla} - ie\mathbf{A}[\psi])^2\psi + e\,\varphi[\psi]\,\psi \tag{4.2.7}$$

[12] Earlier theories treating the electromagnetic field as classical turned out to be unsuccessful (see, e.g.: Crisp and Jaynes 1969; Leiter 1970; Jaynes 1970; Clauser 1972, 1974; Scully and Sargent III 1972; Walls 1979).

becomes then a complicated integro-differential nonlinear equation for ψ. It is truly remarkable that such a simple idea, conveniently generalized to the relativistic domain, leads to the correct predictions[13] for the Lamb shift (Barut and Kraus 1983; Barut and van Huele 1985), the anomalous magnetic moment of the electron (Barut et al. 1988) and the spontaneous emission (Barut and Salamin 1988; Barut and Dowling 1990b), as well as for more exotic effects due to boundaries (Barut and Dowling 1987), apparatus corrections (Barut and Dowling 1989) and acceleration (Barut and Dowling 1990a). Overall, this is an impressive body of results, since such phenomena are usually taken as providing unambiguous evidence for the need to second quantize both the particle and the electromagnetic field. The theory should thus be considered as a serious antagonist of standard QED, and it is worth analyzing its implications upon the foundations of quantum mechanics.

In a sense, this theory represents a viewpoint opposite to that of stochastic electrodynamics. In fact, while the latter traces the quantum behaviour of particles to the presence of a random background field, in self-field QED one *assumes* that particles behave quantum mechanically and that there is *no* field except the particle's one. That such opposite extreme attitudes could be maintained, can to some extent be understood on the basis of the fluctuation-dissipation theorem (Senitzky 1960), which entails that it is impossible to make a clearcut distinction between the contributions of zero-point fluctuations and of radiation reaction to such effects as spontaneous emission and Lamb-shift (Milonni 1976, 1981, 1982, 1984, 1988; Cohen-Tannoudji 1986).

It is clear from Eqs. (4.2.5) and (4.2.6) that the interpretation of ψ adopted in self-field QED is Schrödinger's (Barut 1988a), in which the particle is identified with a fluid whose physical densities (charge, mass, etc.) are proportional to $|\psi|^2$. As discussed in Sec. 3.1.1, this idea runs against two main difficulties, namely its apparent incompatibility with Born's rule and the fact that it sounds unnatural in the many particles case. Within the context of self-field QED one might further remark that it is not clear how the high nonlinearity of Eq. (4.2.7) could show up only through small corrections without affecting, for example, interference phenomena. Consistent answers to these objections can be given by following the suggestion that a distinction should be made between two different notions of the wave function (Barut 1988b).

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[13] At least to first order in e^2.

According to this idea, which is essentially a development of de Broglie's double solution theory (de Broglie 1927b, 1956, 1964, 1971a, 1987), a single particle is described by a sharply localized wave function ψ obeying the nonlinear equation (4.2.7) and to be interpreted *à la* Schrödinger, whereas an ensemble of similarly prepared copies of the particle is described by a conceptually different wave function Ψ, obtained from the various individual ψ's through an averaging process and satisfying the linear Schrödinger equation *without self-field corrections.* Born's rule holds for Ψ but, of course, not for ψ. The viability of this interesting interpretation of the wave function(s) is conditioned on the possibility of finding sharply localized non-spreading solutions of Eq. (4.2.7) (or, better, of its relativistic versions) corresponding to "lumps of ψ" and representing almost pointlike particles. Some preliminary work in this direction seems promising (Barut 1990; Barut and Grant 1990).

It is worth remarking, in closing this section, that semiclassical gravity and self-field QED are not disconnected theories. Should the latter succeed in affirming itself, then it would be very natural to construct a semiclassical theory of gravity along similar lines. The fundamental equations of this theory would be essentially similar to Eq. (4.2.2), but they would require a first quantization treatment of matter based on the *individual* wave function ψ.

4.3 Dynamical Reduction Models and Quantum Theory with Spontaneous Localization.

Among the various proposed resolutions to the measurement paradox (Sec. 2.1) one has been to consider *dynamical reduction models*, i.e., modifications of the evolution law for the state vector leading to a suppression of the unwanted interference terms (see Pearle 1986 for a review). One way to understand this idea is to think that the probability is continuously exchanged among the various components of the state vector following rules analogous to those of the "gambler's ruin" game (Pearle 1982, 1986), until eventually it remains definitely associated to a single component. A fundamental difficulty of these models is their dependence on a specific choice of basis in the Hilbert space, choice which is left essentially undetermined. Significant progress in removing this unpleasant feature has been recently performed (Pearle 1989, Ghirardi, Pearle and Rimini 1990) by embodying in the model the main ideas of quantum mechanics with spontaneous localization.

The latter is a modification of quantum theory that attempts at solving the measurement paradox by postulating a non-unitary evolution law which is supposed to hold both at the microscopic and at the macroscopic level (Ghirardi et al. 1986, 1987, 1988; Benatti et al. 1987). The main idea consists essentially in accepting that a localization process of the type described in Sec. 3.1.1 takes place for each constituent of any physical system. More precisely, one assumes that, at random times with a mean frequency λ, the linear unitary Schrödinger evolution of a particle is perturbed by a jump (Bell 1987a)

$$|t^-\rangle \longrightarrow |t^+\rangle = \hat{L}_{\mathbf{x}}\,|t^-\rangle \ , \tag{4.3.1}$$

where $\hat{L}_{\mathbf{x}}$ is a norm-reducing, positive, self-adjoint linear operator such that

$$\int d^3x\,\hat{L}^2_{\mathbf{x}} = \hat{1} \ , \tag{4.3.2}$$

corresponding to a localization process about the point $\mathbf{x}$ whose spatial width we denote by $\alpha^{-1/2}$. This modified evolution law is no more unitary, though still linear; the non-unitarity is exploited in order to assign a prescription concerning the probability that the localization take place around a particular point $\mathbf{x}$: This is assumed to be equal to $\langle t^+|t^+\rangle$. It is not difficult to realize that such a law implies that the localization process is more likely to occur where $|\psi|^2$ is bigger, so that it leads to an exponential damping with characteristic time λ^{-1} of the off-diagonal terms of the density matrix $\rho(\mathbf{x},\mathbf{x}')$ corresponding to $|\mathbf{x}-\mathbf{x}'| \gg \alpha^{-1/2}$. Furthermore, one can show that for a macroscopic body composed of N elementary constituents, the localization frequency for the center of mass is essentially $N\lambda$: One can thus choose values for λ and α such that the behaviour of systems with few components is unaffected by the modifications of the dynamics, whereas macrosystems are prevented from being in superpositions of macroscopically distinguishable states (Ghirardi et al. 1986).

A continuum limit of the mechanism described above leads essentially (Nicrosini and Rimini 1990) to consider a Markov process in the Hilbert space, and to the modified Schrödinger equation (Pearle 1989; Ghirardi, Pearle and Rimini 1990; Ghirardi and Pearle 1990)

$$\frac{d|\psi(t)\rangle}{dt} = -\frac{i}{\hbar}\hat{H}(t)|\psi(t)\rangle - \left[\lambda\,\hat{1} - \int d^3x\,w(\mathbf{x},t)\,\hat{L}_{\mathbf{x}}\right]|\psi(t)\rangle \ . \tag{4.3.3}$$

This is the fundamental equation of the continuous spontaneous localization theory and contains a stochastic field $w(\mathbf{x},t)$ which "drives" the reduction of the state vector towards one of the eigenvectors of $\hat{L}_{\mathbf{x}}$.

This theory offers a straightforward resolution of the measurement problem within a Schrödinger-like interpretation of ψ, in which the state vector is supposed to represent the physical attributes of an individual system. However, it is far from being satisfactory, as it presents severe difficulties. The spontaneous localization process, in fact, prohibits the existence of steady states and leads unavoidably to energy nonconservation (Ballentine 1991a; Squires 1991)—a very unpleasant feature for a theory that should have a fundamental character. Other problems arise when one tries to generalize the theory to the relativistic domain; the models constructed so far (Pearle 1990, 1991; Ghirardi and Pearle 1990; Ghirardi, Grassi and Pearle 1990) appear rather messy and involved. On the conceptual side, not only the theory inherits the difficulties of Schrödinger's interpretation connected with the association of $|\psi|^2$ to a density of "stuff", but also it introduces indeterminism in the dynamics with all the implications that follow from it. Besides the obvious question as to whether such indeterminism is reducible, i.e., whether the stochastic field $w(\mathbf{x},t)$ can be linked to some other physical process,[14] one might remark that the theory *describes* individuals, but nevertheless *cannot make any prediction concerning their specific behaviour*; it may be a matter of debate whether this can be considered as a satisfactory achievement.

5. Other Issues.

The paradoxes presented in Sec. 2 are only the tip of the iceberg. Deeper analysis reveals a tangle of difficult issues. Here is an unavoidably biased selection of topics of conceptual interest, some of which are so puzzling that not only the answers, but even the correct questions are still unknown.

5.1 Transition from Quantum to Classical.

The issue of performing the classical limit of quantum mechanics is of course a very old one; however, in the last ten years there has

[14] For example, gravity is often invoked as a possible responsible for state vector reduction (see, e.g.: Károlyházy et al. 1986; Károlyházy 1990; Diosi 1989; Ghirardi, Grassi and Rimini 1990).

been a renewal of interest in the subject, finalized to understanding the notion of "classical behaviour" within the framework of quantum theory, without postulating it from the outset—as done instead in the Copenhagen interpretation (Sec. 3.3.1). The resolution of this problem is crucial in determining whether quantum mechanics can be applied to the whole universe; in fact, one of the goals of quantum cosmology is to account for the emergence of the present classical properties of spacetime (see, e.g.: Halliwell 1989; Morikawa 1990).

We want to stress that, contrarily to a popular misconception, the classical limit of quantum theory is not classical mechanics, but classical *statistical* mechanics (Ballentine 1990b; Cini and Serva 1990, 1992). This is almost obvious if one thinks that classical and quantum mechanics are formulated in terms of essentially different concepts. Of course, nobody forbids one to perform, in a further step, the limit

$$\text{classical statistical mechanics} \longrightarrow \text{classical mechanics}\ ,$$

but this requires additional (and independent) assumptions and is a completely different problem. The task of studies about the quantum-to-classical transition is thus not to show that, in some limit, a notion of "single trajectory" emerges, but rather to explain how *decoherence* takes place—i.e., how interference effects become negligible.

Formally this looks as an approximate version of the objectification problem in the quantum theory of measurement (Sec. 2.1). Actually, some of the investigations concerning classical behaviour have been performed in the course of attempts at solving this problem purely at the level of the quantum formalism (Daneri et al. 1962; Rosenfeld 1965; van Zandt 1977; van Kampen 1988) without invoking any particular interpretation except for Born's rule. It is therefore important to recognize that any such attempt is doomed to fail, since the measurement paradox can be solved only by adopting a suitable interpretation of the state vector (which is in fact the main issue involved in it). Hence, the studies about decoherence do not provide by themselves an interpretation, but can be usefully appended to those interpretations which do not assume a breakdown of quantum theory at the macroscopic level (Zurek 1986, 1991).

The key concept of the decoherence programme is that of *environment*. The environment $\mathcal{E}$ of a system $\mathcal{S}$ consists of the degrees of freedom which are coupled to $\mathcal{S}$ but whose state can be regarded as irrelevant.

This notion derives not only from recognizing that it is very difficult to isolate a system (Zeh 1970) but also from the fact that, if the system is sufficiently complicated, a complete specification of its state would not only be tremendously difficult, but also of little practical use. It is therefore more appropriate to think of $\mathcal{S}$ and $\mathcal{E}$ as, respectively, the relevant and irrelevant degrees of freedom of a complex system.

The interaction between $\mathcal{S}$ and $\mathcal{E}$ induces correlations between the relevant and the irrelevant degrees of freedom. One can show (Zurek 1981, 1982) that when only the state of $\mathcal{S}$ is considered by writing

$$\hat{\rho}_{\mathcal{S}} = \mathrm{tr}_{\mathcal{E}}\, \hat{\rho} \,, \tag{5.1.1}$$

the magnitude of the off-diagonal components of $\hat{\rho}_{\mathcal{S}}$ decreases in time, and $\hat{\rho}_{\mathcal{S}}$ decays into

$$\sum_n \pi_n \, \hat{\Pi}_n \,, \tag{5.1.2}$$

where $\hat{\Pi}_n$ are projection operators onto linear subspaces $\mathcal{H}_n$ of the Hilbert space $\mathcal{H}_{\mathcal{S}}$, and π_n are non-negative real numbers which sum up to one. The decay is quite effective even for relatively simple systems (Joos and Zeh 1985; Caldeira and Leggett 1985; Joos 1986; Unruh and Zurek 1989). This corresponds to the establishment of an environment-induced superselection rule (Kübler and Zeh 1973; Zurek 1982): The possible states of $\mathcal{S}$ alone belong to one of the $\mathcal{H}_n$'s rather than to $\mathcal{H}_{\mathcal{S}} = \bigoplus_n \mathcal{H}_n$.

This result does not solve the objectification problem of quantum measurement (which requires—we repeate—a choice of *interpretation*) because the state of $\mathcal{S} + \mathcal{E}$ is still a superposition. However, it explains why one cannot detect interference between different states of the pointer's position P. In fact, by considering the pointer as part of $\mathcal{S}$, it is evident that most of the information that would be necessary in order to reveal interference effects is hidden in the irrelevant degrees of freedom $\mathcal{E}$; as far as the experimenter does not—or cannot—consider the detailed state of $\mathcal{E}$, he/she will be unable to measure these effects.[15] Still within the context of the measurement problem, the analysis outlined

[15] Even having access to the whole $\mathcal{S} + \mathcal{E}$ system, the task of detecting interference would be a terribly difficult one, since it would require to prepare a large number of copies of $\mathcal{S} + \mathcal{E}$ in the same quantum state. Even for systems which are not very large this may turn out to be practically impossible.

above provides a way to determine *which* observables of the measuring apparatus can be regarded as pointer's position for a specified type of measurement. It turns out (Zurek 1981, 1982) that they are the observables P such that

$$[\hat{P}, \hat{H}_i] = \hat{0} \, , \tag{5.1.3}$$

where $\hat{H}_i$ is the interaction Hamiltonian between $\mathcal{S}$ and $\mathcal{E}$. It is remarkable to notice (Zurek 1982) that these results might allow to give a rigorous justification of Bohr's idea of complementarity of measurements (Bohr 1959; Wootters and Zurek 1979).

5.2 Bell's Theorem.

The formulation of Bell's theorem (Bell 1964, 1971; Clauser et al. 1969; Herbert 1975; Garuccio and Selleri 1980; Harrison 1982; Bertlmann 1990; Home and Selleri 1991) was motivated by the nonlocal features of the pilot wave theory (see Sec. 3.1.2). The original idea was to show that the quantum mechanical predictions for EPR-like experiments are incompatible with any deterministic subquantum theory that assumes local hidden variables (see also: Mermin 1981, 1985; Aspect et al. 1985). The hypothesis of determinism was later on removed (Clauser and Horne 1974; Eberhard 1982). Common to these proofs were some assumptions about the statistical distribution of the hidden variables and about the relationship between the latter and the measured values of quantum observables, that lead straightforwardly to define joint probabilities for incompatible quantities (Lochak 1976; Fine 1982). Since this fact alone would contradict well established consequences of quantum mechanics (see, e.g., de Broglie et al. 1976), such derivations of Bell's theorem have been criticized as confusing between *actual* and *measured* values of physical quantities (Lochak 1976). As a matter of fact, however, the hypothesis of hidden variables is also superfluous, as several authors have pointed out (Stapp 1971, 1985; Eberhard 1977; Peres 1978; Corwin 1984): The theorem appears thus to exhibit an incompatibility between some quantum mechanical predictions and a notion of locality (Eberhard 1978). The latter is not, however, the *weak locality* requirement that arbitrarily fast signalling should be forbidden—which is satisfied by quantum theory—but a stronger condition whose physical meaning is not yet completely clear. Whether Bell's theorem entails nonlocal connections (Stapp 1982, 1988, 1989, 1991; Kraus 1989), violation of *predictive* *completeness* (Jarrett 1984, 1986; Ballentine and Jarrett 1987), violation of *counterfactual definiteness* (Peres 1978), or other consequences (d'Espagnat 1979, 1980, 1983, 1984, 1989, 1990; Weisskopf 1980; Brody and de la Peña-Auerbach 1979; Berthelot 1980; Barut and Meystre 1984; Paul 1985; Dotson 1986; Seipp 1986; Brans 1988; Selleri 1988; Sudarshan and Rothman 1989; Santos 1991a) is therefore still matter of debate.

It is worth remarking that the quantum mechanical correlation formula seems to agree with the experimental results (Clauser and Shimony 1978; Aspect et al. 1981, 1982; Reid and Walls 1984; Rarity and Topster 1990). The violation of Bell's inequality, with the consequences mentioned above, appears therefore to be a property of nature rather than merely of quantum theory. This fact rules out refutations based on claims of inadequacy of the quantum formalism (see, e.g., de Broglie 1974). Actually, due to the low efficiency of the detectors used in the experiments (less than 20%), it is still possible to devise local realistic models which fit the available data (Pearle 1970; Marshall et al. 1983; Ferrero et al. 1990; Santos 1991c). The issue can be definitely resolved either by increasing the efficiency of the detectors to more than 83% (Mermin 1986), or by realizing a new generation of experiments designed to distinguish between quantum mechanics and local realistic theories in a single run (Greenberger et al. 1990; Mermin 1990a,c). Alternatively, the principles of democracy allow to recur to a referendum among physicists, in order to settle the problem once and for all (Duch and Aerts 1986).

5.3 Quantum Theory and Relativity.

The relationship between quantum mechanics and the theory of relativity has never been completely clear. In his work on wave mechanics, de Broglie made a considerable use of relativistic considerations (de Broglie 1923, 1927b, 1987, 1992). Nevertheless, Schrödinger's theory was nonrelativistic and only with Dirac an acceptable relativistic quantum theory began to take shape. The unification of the two theories, although leading to highly successful results in the form of quantum electrodynamics (see, e.g., Schwinger 1958) and, later on, of gauge theories, cannot however be considered as fully satisfactory for several reasons:

1. Relativistic quantum field theory is technically very "dirty". The numerical results are obtained by employing renormalization techniques, which consist in *ad hoc* prescriptions for removing the divergences that unavoidably show up in the course of the calculations. No fundamental theory should be allowed to include such a clumsy mathematics in its formulation.

2. In relativity time is merely one of the four coordinates labeling the events on the spacetime manifold, whereas in quantum theory it plays the very special role of "evolutionary parameter". This circumstance leads to the unpleasant feature of the relativistic quantum formalism of not being *explicitly* covariant. The Tomonaga-Schwinger formulation (see, e.g.: Tomonaga 1946; Schwinger 1948; Ghose and Home 1991) is coordinates-independent, but it still relies on the choice of an arbitrary foliation of spacetime which is, in the spirit of relativity, not very attractive.

3. Relativity and quantum theory are based on very different, perhaps not fully compatible, sets of concepts.[16] The key ideas of relativity are *locality* and *separability*, from which the model of spacetime as a Lorentzian differentiable manifold arises by extrapolation. On the other hand, these notions do not appear to be natural in quantum theory, in which "entangled" states like (2.3.1) are perfectly normal and the concept of *correlations* (see, e.g.: Zurek 1982; Page and Wootters 1983; Wootters 1984) seems more fundamental. Moreover, locality and separability are in the list of the ideas threatened by the experiments about Bell's inequality (Sec. 5.2).

Point 1 represents a very difficult technical issue; apparently, it is not directly linked to any of the conceptual problems discussed so far. It is nevertheless a symptome of unhealthiness of the theory which might be dangerous to neglect as irrelevant. Point 2 seems to admit two logically possible solutions. One is to modify the formalism of relativistic quantum theory by making it explicitly covariant; this can be done by distinguishing the coordinate time from an evolutionary parameter, the latter reducing to the proper time in the classical limit (Nambu 1950; Cooke 1968; Fanchi 1981, 1993; Kyprianidis 1987; Droz-Vincent 1988; Sonego 1991b). The other is to regard Lorentz invariance as non-fundamental and to assume that spacetime has essentially a prerelativistic structure distinguishing between space and time (Bohm and Hiley 1984; J.S. Bell, in Davies and Brown 1986; Valentini 1991b,c, 1992; Hardy 1992b; Hardy and Squires 1992; see also Clifton and Niemann 1992). In the light of the considerations of point 3 this second option might appear more justified.

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[16] Many difficulties of relativistic quantum theory (Bloch 1967; Aharonov and Albert 1981, 1984; Malin 1982) can be traced to this conceptual gap.

However, we want to point out that there is actually a third possibility, which we consider more plausible and appealing than the previous two, though perhaps not so easy to pursue.

The peaceful coexistence of quantum mechanics and relativity is of a very peculiar kind. There is no evidence of direct contradiction among them (Ballentine and Jarrett 1987), but one feels that quantum theory is, in a sense, only enough relativistic as it is necessary in order to avoid such a contradiction. In other words, we might say that the conceptual overlap between quantum mechanics and relativity is just as little as possible compatibly with the need to lead to experimentally consistent results. From this viewpoint, their unification should not consist in trying to cast the concepts of both theories into a coherent scheme. Rather, we suggest to regard them as limits of a more general theory $\mathcal{T}$ *corresponding to different, and incompatible, approximations.* They would be "complementary" theories dealing essentially with different classes of natural phenomena and mutually non-contradictory, though each formulated within its own set of concepts, which may well be unnatural for the other one. The independence of the historical developments of quantum mechanics and relativity and the fact that their conceptual and formal structures do not seem to match properly, suggest that this idea might not be unreasonable. The technical difficulties mentioned in point 1 could also find an explanation, because relativistic quantum mechanics would turn out to be an unhappy hybrid creature obtained by extrapolating the ideas of relativity and quantum theory into a domain to which they are not supposed to apply.

It is very difficult to say what the theory $\mathcal{T}$ would look like. Presumably, it should not embody the notion of spacetime as a Lorentzian differentiable manifold, the latter being a model very useful in relativity but unnatural in quantum theory. Obviously, hypothesis about this subject are necessarily speculative, but they can always be selected on the basis of their simplicity and efficiency by adopting the criterion of Occam's razor. From this point of view, an idea which is certainly worth considering is to replace spacetime by a much simpler pregeometric and pretopologic structure. A plausible suggestion for the latter is that of a *causal set* (Bombelli et al. 1987). A causal set C is a locally finite partially ordered set; the elements of C generalize, somehow, the concept of spacetime events, and the partial order among them corresponds to the causal relation "before". The theory of causal sets is still at a very rudimentary level, but it appears nevertheless to admit a limit in which

C can be approximated by a Lorentzian differentiable manifold. In this scenario the concepts of distance and locality would arise thus only as suitable approximations at some "macroscopic" level. Bohr's idea that microphysical phenomena do not fully admit a representation in space and time would turn out to be correct, but in a very different sense from what he meant.

At the level of the theory $\mathcal{T}$, one might expect that also quantum mechanics should be replaced by a conceptually simpler structure. This issue is, however, even less clear and definite than the previous one and we shall limit ourselves to mention the possibilities of generalizing quantum theory on the basis of its geometrical properties (Kibble 1979; Wootters 1981; Anandan 1991) or of the sum-over-histories formulation (Feynman 1948; Feynman and Hibbs 1965; Schulman 1981; Caves 1986; Hartle 1988, 1991; Sinha and Sorkin 1991).

5.4 Quantum Probability.

The prominent role played by probability within quantum theory has obviously captured the attention of many physicists and a consistent amount of work has been devoted to clarify the relationship between the probabilistic behaviours characteristic of classical and quantum systems (Accardi 1981, 1984; Accardi and von Waldenfels 1985; Cohen 1988; Bitsakis and Nicolaides 1989; van den Berg et al. 1990; Home et al. 1991; Cufaro Petroni 1991, 1992). The typical feature exhibited by probabilities in quantum mechanics, which has no counterpart in classical physics, is interference. The phenomenon is so striking (Feynman has defined it "the only mistery" of quantum theory) that it is often presented as evidence for a violation of the Kolmogorov axioms by quantum probability (Feynman and Hibbs 1965). This inference is however unjustified, since it has been shown that it arises from an incorrect use of conditioning, and that quantum probabilities do satisfy the Kolmogorov axioms (Ballentine 1986). The lesson that one learns from considering this issue is, rather, that probabilities in quantum mechanics are more sensitive to the process of conditioning and that differences in the conditions, that can often be neglected in classical circumstances, may be relevant when quantum phenomena are involved; this feature is an aspect of the qualitative notion of "wholeness" (Bohm 1980).

In quantum theory probabilities are usually derived by squaring amplitudes. One of the advantages of this procedure is that the, sometimes involved, dependence of quantum probabilities on conditions, simplifies considerably due to the linear structure of the amplitude calculus. For example, the relation

$$P(A = a|C = c) = \sum_b P(A = a|B = b \,\&\, C = c)\, P(B = b|C = c) \tag{5.4.1}$$

can be replaced by the simpler[17]

$$\langle a|c\rangle = \sum_b \langle a|b\rangle\langle b|c\rangle \ , \tag{5.4.2}$$

together with Born's rule

$$P(A = a|C = c) = |\langle a|c\rangle|^2 \ . \tag{5.4.3}$$

The fundamental reason for this gain in simplicity, as well as the very meaning of the concept of amplitude, are nevertheless not yet clear (Wootters 1981; Hilgewoord and Uffink 1991).

Full appreciation of the crucial role played by conditions in quantum theory helps also to understand the appearances of negative values for formal expressions which should represent probabilities of incompatible observables (Wigner 1932; Dirac 1945; Moyal 1949; Margenau and Hill 1961; Cohen 1966; Wigner 1970; Feynman 1982; Tatarskiĭ 1983; Hillery et al. 1984; Sonego 1990, 1991b). Although these "quasiprobabilities" can be justified as referring to unobservable virtual phenomena (Bartlett 1945; Mückenheim 1986; Feynman 1987), it is more likely that they are a consequence of the illecit assumption that microscopic variables should have the same statistical distribution as their measured values (de Broglie 1956, 1982; de Broglie et al. 1976; Valentini 1991d, 1992). In the spirit of Bohr's complementarity one may also say that, if A and B are incompatible observables, no joint probability exists for their values simply because there are no conditions which allow to measure A and B simultaneously.

It is interesting to notice that the validity of classical probability calculus in quantum theory is not the only achievement of a correct use of conditioning, another consequence being that many arguments which are often presented as evidence for the need to replace classical logic by a

[17] We assume that the spectrum of B is complete.

nonstandard quantum logic (see, e.g.: Birkhoff and Von Neumann 1936; Reichenbach 1944; Jammer 1974; Suppes 1976; Hughes 1981; Adler and Wirth 1983; Wallace Garden 1984) lose much of their appeal and turn out to be not compelling.

6. Perspectives.

A general, and unpleasant, feature of the material presented in this article is the lack of testable predictions beyond those of standard quantum theory. This claim applies not only to interpretations (Sec. 3), but also to the heretic theories which have not yet been ruled out (Sec. 4). In judging which are the merits and the defects of each of them, we are thus left essentially with criteria of logical consistency and of personal taste. One might therefore ask whether this subject should not be considered more appropriately as belonging to metaphysics.

The answer depends clearly on whether one believes or not that the quantum formalism represents a definitive description of natural phenomena. Personally we think that this is very unlikely and that the validity of any investigation about the conceptual foundations of quantum theory should be evaluated not only by considering the possibility that it could lead to new predictions, but also by the insight it provides about the structure of physical laws. If quantum mechanics is not the last word, the nature of the next step to perform represents a very important problem, and *this* can be strongly influenced by the way in which one presently looks at quantum theory.

Of course, there is always the possibility that the quantum theoretical description could be, in a sense, a “logical necessity” (see, e.g., Peres and Zurek 1982, Deutsch 1985, for considerations of this kind). In this case, however, a thorough and appropriate understanding of its significance and relationship with the world of experience is even more crucial and indispensable.

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