Lagrange function, de Broglie's model and relativity

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ABSTRACT. In this article the relativistic Lagrange function and de Broglie's model are discussed. An hypothesis is postulated which suggests another fundamental energy equation of special relativity. It leads to another relativistic variance and a new energy concept. Some interesting and enlightening phenomena are presented.

RESUME. Cet article concerne la fonction de Lagrange et le modèle de de Broglie. Il postule une hypothèse suggérant une équation alternative de relativité spéciale qui amène notamment à une autre variation relativiste et à un tout nouveau concept d'énergie. L'article promet d'être intéressant sinon instructif.

Introduction

In classical dynamics, Lagrange function L is expressed as

$$L = T - V \tag{1}$$

where T is kinetic energy, V is potential energy. For a free particle, one has

$$L = T \tag{2}$$

So the physical interpretation of Lagrange function for classical dynamics is the difference between kinetic and potential energies. For a free particle, Lagrange function is equal to kinetic energy. However, in relativistic dynamics, Lagrange function for a free particle becomes[1]

$$L = -m_0c^2(1 - \beta)^{1/2} \tag{3}$$

where m_0 is rest mass, $\beta = (v/c)^2$. The question which has never been raised and discussed is : what is the physical interpretation of the relativistic Lagrange function ?

Let us consider another phenomenon. In 1924 Louis de Broglie[2,3] proposed that matter possesses wave as well as particle characteristics. He suggested for a particle at rest ($v = 0$) having relation

$$E_0 = h\mu_0 = m_0 c^2 \tag{4}$$

where frequency μ_0 is described by de Broglie as a certain internal, clock-like periodic process. When $v \neq 0$, according to relativistic variance for clock frequencies, μ_0 becomes

$$\mu' = \mu_0(1 - \beta)^{1/2} \tag{5}$$

Equations (4) and (5) yield

$$E' = h\mu' = m_0 c^2 (1 - \beta)^{1/2} \tag{6}$$

However de Broglie pointed out that the frequency μ of the wave associated with a moving particle is characterized by another relativistic variance of the mass ($m = m_0/(1 - \beta)^{1/2}$) as following

$$E = h\mu = mc^2 \tag{7}$$

therefore

$$\mu = \mu_0/(1 - \beta)^{1/2} \tag{8}$$

So μ' and μ are two phenomena of essentially different nature distingwished from one another by their relativistic variance. What is interested in is equation (6). If $h\mu'$ is certain internal periodic process, then what is the interpretation for energy term $m_0 c^2 (1 - \beta)^{1/2}$?

Now it seems that function $m_0 c^2 (1 - \beta)^{1/2}$ plays an unusual role in relativistic dynamics. In what follows we attempt to propose an hypothesis which might give certain physical understanding of this function.

Hypothesis

In relativistic dynamics, a free particle with velocity v has kinetic energy

$$E_k = \int v d(mv) = \int v d[m_0 v/(1 - \beta)^{1/2}] \tag{9}$$

Integration by parts from 0 to v yields

$$
\begin{aligned}
E_k &= m_0 v^2/(1-\beta)^{1/2} - \int m_0 v dv/(1-\beta)^{1/2} \\
&= mv^2 + m_0 c^2 (1-\beta)^{1/2} - m_0 c^2 \\
&= mc^2 - m_0 c^2
\end{aligned}
\tag{10}
$$

This is one of the fundamental equations of special relativity. However, the most interesting point to note is that from equation (10) one can obtain following simple, beautiful equation

$$
mc^2 = m_0 c^2 (1-\beta)^{1/2} + mv^2 \tag{11}
$$

We propose that equation (11) is another fundamental equation of special relativity. Here, $m_0 c^2 (1-\beta)^{1/2}$ is certain internal energy of a moving particle. Let us call it "moving mass-energy". Since $m_0 c^2$ is rest mass-energy of the particle at rest, we obtain another relativistic variance : mass m_0 of a rest particle will decreased to $m_0(1-\beta)^{1/2}$ when it is in motion, although the total apparent mass of the particle is increased to $m_0/(1-\beta)^{1/2}$.

Next, we suggest that mv^2 is "total kinetic energy". From equations (10) and (11), one gets

$$
mv^2 = E_k + \emptyset \tag{12}
$$

where

$$
\emptyset = m_0 c^2 - m_0 c^2 (1-\beta)^{1/2} \tag{13}
$$

We call E_k "apparent kinetic energy" and $\emptyset$ "hidden kinetic energy". Of course, the "apparent kinetic energy" is just another terminology of kinetic energy. However, the "hidden kinetic energy" is something new. It suggests that part of the mass-energy can be changed into some other forms of energy or movement. It tells us that characteristics of a moving particle might be determined not only by external energy such as E_k, but also by its inherent energy $\emptyset$. Notice that when $1 \gg \beta$, one has

$$
E_k = \emptyset = m_0 v^2/2 \tag{14}
$$

They are numerically equal.

Lastly, mc^2 is the total energy. More precisely, it is "total potential energy" for a free particle.

Discussion

By considering equation (11), for a free particle relativistic Lagrange function is

$$L = -m_0 c^2 (1-\beta)^{1/2} = mv^2 - mc^2 \tag{15}$$

The term corresponds to the "total kinetic energy" minus the "total potential energy". It means that the definition of Lagrange function by equation (1) is eventually satisfied for relativistic dynamics too.

Let us look back at equation (6) again. It is clear that it describes that for a moving particle certain internal periodic process $h\mu'$ corresponds to certain internal energy $m_0 c^2 (1-\beta)^{1/2}$. Both of them are consequence of relativistic variance.

Now let us consider equation (11) in the extreme case which $v = c$ – the velocity of a particle is equal to that of light. In this case, the internal energy $m_0 c^2 (1-\beta)^{1/2}$ is equal to zero. It means that the particle has no mass-energy, its total energy is purely kinetic. Thus, equation (11) describes the kinematics of a free particle for all velocities from O to c.

Conclusion

An hypothesis is proposed which suggests another fundamental energy equation of special relativity. Another relativistic variance is described which gives self-consistent definition of Lagrange function for both classical and relativistic dynamics. Its relation with de Broglie's model is also discussed. The hypothesis leads to a new energy concept : the "hidden kinetic energy" $\emptyset$. Some interesting and enlightening phenomena are presented. If the hypothesis is confirmed, it might give us a much deeper understanding of relativity and microphysics.

References

[1] J.D. Jackson, *Classical Electrodynamics* (New York, 1975).
[2] L. de Broglie, Thesis, Paris 1924.
[3] M. Mugur-Schachter, Found. Phys., **2**, 261 (1989).

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