Magnetic Charge Screening in a $U(1) \times U(1)$ Gauge Theory

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ABSTRACT. By considering a $U(1) \times U(1)$ gauge theory with each gauge field strength represented by an electric and magnetic potential we discuss the screening and anti-screening properties of electric-like charges by magnetic charge distributions carried by both gauge groups.

RESUME. En considérant une théorie de jauge dans $U(1) \times U(1)$, où chaque champ de jauge est représenté par un potentiel électrique et magnétique, on discute les effets d'écran et d'anti-écran sur des charges de type électrique dus à des distributions de charges magnétiques portées par les deux groupes de jauge.

Introduction

The problem of charge screening in both Abelian [1] and non-Abelian gauge theory [2] has fundamental significance because it sheds light on the polarisible properties of the vacuum and the fundamental nature of the electromagnetic force and quark-anti-quark force. In the early universe where the energy scale of interactions is high, screening effects may influence reaction rates and may influence such phenomena as the onset of inflation [3], and the generation of the baryon asymmetry of the universe [4]. Screening effects are also of fundamental significance in calculating whether or not a theory is asympotically free and when infrared slavery becomes important in a theory [5]. There are numerous reasons that suggest that there may be additional $U(1)$ groups in nature other than the usual $U(1)$ of electromagnetism. Additional $U(1)$ factors arise in symmetry breaking schemes of G.U.T. theories [6,7,8] as well as left-right symmetric theories of elementric particles [9]. Recent studies involving gauged baryon and lepton number also lead to additional $U(1)$ gauge fields [10,11]. Also, it has been conjectured that the fifth force might be mediated by an additional $U(1)$ gauge field [12,13].

In fact Gasperini [14] and Gasperini et. al. [15] have elucidated on the implications of an additional $U(1)$ gauge field (baryonic photon) which includes the modification of photon propagation, modification of $\bar{n}n$ oscillation phenomena and the generation of magnetic fields by baryonic charge. Once additional $U(1)$ gauge fields are introduced, we must reckon with both the electric-like and magnetic-like charges that occur in such a theory. It has been noted by Vinciarelli that four dimensional space-time alone admits the dual symmetry of Maxwell's equations [16] and in non-Abelian gauge theory, the magnetic charge of a monopole or dyon has topological significance [17]. Of course long ago Dirac demonstrated that the existence of one unit of magnetic charge in the universe leads to electric charge quantization [18] and Schwinger demonstrated that two dyons admit the quantization condition [19]

$$\frac{e_1 q_2 - e_2 q_1}{\hbar c} = \frac{n}{2}.$$

(e_1, e_2 are electric charges, q_1, q_2 are magnetic charges of two dyons).

Remarkably Witten has demonstrated that CP violating interactions lead to the fact that the electric charge of a dyon need not be quantized [20]. Although monopoles would most likely not have survived inflation, they may have been produced around the time of helium synthesis by energetic collisions [21]. Recently Dimopoulous et. al. [22] and DeRujula et. al. [23] have pointed out that CHAMPS (charged massive particles between 20 and $1000 TeV$ predicted by particle theory) may be a component of the dark matter in galaxies, an accumulation of monopoles and CHAMPS would constitute a configuration of electric and magnetic charge that may give rise to a macroscopic dyon that may generate far ranging astrophysical consequences. In view of the fact that additional $U(1)$ factors may occur in nature and magnetic charge as well as electric charge may exist for each $U(1)$ group, we study the screening effects that magnetic charge generates for electric like charges. The model lagrangian that we employ is the Born-Infeld lagrangian [24] which also can be viewed as an effective action resulting from string theory [25]. By assuming a particle with electric-like charge values for each $U(1)$ group we study how the magnetic charge of each group screens the electric-like charge of the second group. We also employ a two potential theory for each Abelian gauge field which allows us to derive the field equations from a lagrangian in the presence of magnetic charge [26]. Following the discussion of screening we remark on the possible astrophysical consequences of our theory that may result from the screening phenomena.

2. Magnetic Charge Screening In a $U(1) \times U(1)$ Gauge Theory

We begin our analysis by writing the lagrangian for a $U(1) \times U(1)$ gauge theory coupled to a gravitation as

$$\begin{aligned} L = & \frac{c^4}{16\pi G} R\sqrt{-g} - \frac{b_1}{8\pi}\left(\sqrt{1+\frac{J_1}{b_1}}-1\right)\sqrt{-g} - \frac{b_2}{8\pi}\left(\sqrt{1+\frac{J_2}{b_2}}-1\right)\sqrt{-g} \\ & + \alpha\left(\frac{\epsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}F_{2\mu\nu}}{\sqrt{-g}}\right)\sqrt{-g} - J^{\mu}_{1E}A_{1\mu}\sqrt{-g} - J^{\mu}_{2E}A_{2\mu}\sqrt{-g} \\ & + J^{\mu}_{1M}B_{1\mu}\sqrt{-g} + J^{\mu}_{2M}B_{2\mu}\sqrt{-g} \end{aligned} \tag{2.1}$$

Here J^{μ}_{1E}, J^{μ}_{2E}, J^{μ}_{1M}, J^{μ}_{2M} are electric and magnetic four currents for groups 1 and 2 respectively. Also $J_1 = F_{1\mu\nu}F^{\mu\nu}_1$, $J_2 = F_{2\mu\nu}F^{\mu\nu}_2$, where

$$F_{1\mu\nu} = \frac{\partial A_{1\mu}}{\partial x^{\nu}} - \frac{\partial A_{1\nu}}{\partial x^{\mu}} - \frac{\epsilon^{\alpha\beta}_{\mu\nu}}{2\sqrt{-g}}\left(\frac{\partial B_{1\alpha}}{\partial x^{\beta}} - \frac{\partial B_{1\beta}}{\partial x^{\alpha}}\right) \tag{2.2}$$

$$F_{2\mu\nu} = \frac{\partial A_{2\mu}}{\partial x^{\nu}} - \frac{\partial A_{2\nu}}{\partial x^{\mu}} - \frac{\epsilon^{\alpha\beta}_{\mu\nu}}{2\sqrt{-g}}\left(\frac{\partial B_{2\alpha}}{\partial x^{\beta}} - \frac{\partial B_{2\beta}}{\partial x^{\alpha}}\right) \tag{2.3}$$

Here $F_{1\mu\nu}$, $F_{2\mu\nu}$ represent the gauge field strengths described by the electric-like, and magnetic-like potentials $A_{1\mu}$, $A_{2\mu}$, $B_{1\mu}$, $B_{2\mu}$ respectively. This description was originally studied by Cabibbo and Ferrari (Ref.26) allowing for a lagrangian variational approach in the presence of magnetic charge. The terms

$$-\frac{b_1}{8\pi}\left(\sqrt{1+\frac{J_1}{b_1}}-1\right) \quad , \quad -\frac{b_2}{8\pi}\left(\sqrt{1+\frac{J_2}{b_2}}-1\right)$$

represent Born-Infeld terms with constants b_1, b_2 of dimensions erg/cm^3. The coupling between the two fields is

$$\alpha\left(\frac{\epsilon^{\mu\nu\alpha\beta}F_{1\mu\nu}F_{2\alpha\beta}}{\sqrt{-g}}\right)\sqrt{-g} \qquad (\alpha = \text{coupling constant})$$

this represents a parity violating coupling term that generates the screening or anti-screening effects of one gauge field on the other [27].

Varying Eq.(1) with respect to $A_{1\mu}$, $B_{1\mu}$, $A_{2\mu}$, $B_{2\mu}$ gives

$$\frac{\partial}{\partial x^\nu}(\frac{F_1^{\mu\nu}\sqrt{-g}}{4\pi\sqrt{1+\frac{J_1}{b_1}}}) - \frac{\partial}{\partial x^\nu}(2\alpha\epsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}) - J_{1E}^\mu\sqrt{-g} = 0 \quad (2.4)$$

$$\frac{\partial}{\partial x^\nu}(\frac{\tilde{F}_1^{\mu\nu}}{4\pi\sqrt{1+\frac{J_1}{b_1}}}) - \frac{\partial}{\partial x^\nu}(\frac{2\alpha\epsilon^{\mu\nu\alpha\beta}\tilde{F}_{2\alpha\beta}}{\sqrt{-g}}) - J_{1M}^\mu\sqrt{-g} = 0 \quad (2.5)$$

$$\frac{\partial}{\partial x^\nu}(\frac{F_2^{\mu\nu}\sqrt{-g}}{4\pi\sqrt{1+\frac{J_2}{b_2}}}) - \frac{\partial}{\partial x^\nu}(2\alpha\epsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}) - J_{2E}^\mu\sqrt{-g} = 0 \quad (2.6)$$

$$\frac{\partial}{\partial x^\nu}(\frac{\tilde{F}_2^{\mu\nu}}{4\pi\sqrt{1+\frac{J_2}{b_2}}}) - \frac{\partial}{\partial x^\nu}(\frac{2\alpha\epsilon^{\mu\nu\alpha\beta}\tilde{F}_{1\alpha\beta})}{\sqrt{-g}} - J_{2M}^\mu\sqrt{-g} = 0 \quad (2.7)$$

For the field strengths we have

$$F_{14}^1 = E_1(r) \quad , \quad F_{14}^2 = E_2(r)$$

$$F_{23}^1 = r^2\sin\theta B_1(r) \quad , \quad F_{23}^2 = r^2\sin\theta B_2(r) \quad ,$$

$$\tilde{F}_1^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}}{2} \quad , \quad \tilde{F}_2^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}}{2} \quad ,$$

Here E_1, E_2, B_1, B_2 represent the radial electric and magnetic fields of the two groups. We employ a spherically symmetric metric of the form

$$(ds)^2 = e^\nu(dx^4)^2 - e^\lambda(dr)^2 - r^2(d\theta)^2 - r^2\sin^2\theta(d\phi)^2,$$

the energy momentum tensor components for this metric obey $T_1^1 = T_4^4$ which implies $\lambda + \nu = 0$ for $r < R$ from the Einstein equation. For the magnetic four currents we have

$$J_{1M}^4 = \rho_{01}e^{-\nu/2}J_{2M}^4 = \rho_{02}e^{-\nu/2} \quad (2.8)$$

for $r < R$, $R =$ radius of configuration, ($\rho_{01}, \rho_{02}, =$ proper magnetic charge densities).

If we set $\rho_{01} = \overline{\rho}_{01} e^{-\lambda/2}$, $\rho_{02} = \overline{\rho}_{02} e^{-\lambda/2}$ ($\overline{\rho}_{01}, \overline{\rho}_{02}$ = constant) Eq.(2.4), Eq.(2.5), Eq.(2.6) and Eq.(2.7) become

$$\frac{\partial}{\partial r}\left(\frac{r^2 E_1}{4\pi\sqrt{1+\frac{2B_1^2}{b_1}-\frac{2E_1^2}{b_1}}}\right) - \frac{\partial}{\partial r}(4\alpha r^2 B_2(r)) = 0$$

$$\frac{\partial}{\partial r}\left(\frac{r^2 B_1}{4\pi\sqrt{1+\frac{2B_1^2}{b_1}-\frac{2E_1^2}{b_1}}}\right) + \frac{\partial}{\partial r}(4\alpha r^2 E_2(r)) = r^2 \overline{\rho}_{01}$$

$$\frac{\partial}{\partial r}\left(\frac{r^2 E_2}{4\pi\sqrt{1+\frac{2B_2^2}{b_2}-\frac{2E_2^2}{b_2}}}\right) - \frac{\partial}{\partial r}(4\alpha r^2 B_1) = 0$$

$$\frac{\partial}{\partial r}\left(\frac{r^2 B_2}{4\pi\sqrt{1+\frac{2B_2^2}{b_2}-\frac{2E_2^2}{b_2}}}\right) + \frac{\partial}{\partial r}(4\alpha r^2 E_1(r)) = r^2 \overline{\rho}_{02} \tag{2.9}$$

Integrating the four equations in Eq.(2.9) gives

$$\frac{r^2 E_1}{4\pi\sqrt{1+\frac{2B_1^2}{b_1}-\frac{2E_1^2}{b_1}}} - 4\alpha r^2 B_2 = \frac{e_1}{4\pi} \tag{2.10}$$

$$\frac{r^2 B_1}{4\pi\sqrt{1+\frac{2B_1^2}{b_1}-\frac{2E_1^2}{b_1}}} + 4\alpha r^2 E_2 = \frac{r^3 \overline{\rho}_{01}}{3} \tag{2.11}$$

$$\frac{r^2 E_2}{4\pi\sqrt{1+\frac{2B_2^2}{b_2}-\frac{2E_2^2}{b_2}}} - 4\alpha r^2 B_1 = \frac{e_2}{4\pi} \tag{2.12}$$

$$\frac{r^2 B_2}{4\pi\sqrt{1+\frac{2B_2^2}{b_2}-\frac{2E_2^2}{b_2}}} + 4\alpha r^2 E_1 = \frac{r^3 \overline{\rho}_{02}}{3} \tag{2.13}$$

Here e_1, e_2 represent the electric-like charges of a particle at $r = 0$, corresponding to the two $U(1)$ groups.

We solve for E_1, B_1 to first order and α. To accomplish this we first solve Eq.(2.12) and Eq.(2.13) to zeroth order in α

$$E_2 = \frac{e_2}{[r^4 + \frac{2e_2^2}{b_2} - \frac{2(\overline{q}_{02})^2}{b_2}]^{1/2}} \tag{2.14}$$

$$B_2 = \frac{\overline{q}_{02}}{[r^4 + \frac{2e_2^2}{b_2} - \frac{2(\overline{q}_{02})^2}{b_2}]^{1/2}} \tag{2.15}$$

where $\overline{q}_{02} = \frac{4}{3}\pi r^3 \overline{\rho}_{02}$.

Now upon inserting the expressions in Eq.(2.14) and Eq.(2.15) into Eq.(2.10) and Eq.(2.11) we have

$$E_1 = \frac{\overline{e}_1}{[r^4 + \frac{2\overline{e}_1^2}{b_1} - \frac{2(\overline{q}_1)^2}{b_1}]^{1/2}} \tag{2.16}$$

$$B_1 = \frac{\overline{q}_1}{[r^4 + \frac{2\overline{e}_1^2}{b_1} - \frac{2(\overline{q}_1^2}{b_1}]^{1/2}} \tag{2.17}$$

where

$$\overline{e}_1 = e_1 + \frac{16\pi\alpha r^2 \overline{q}_{02}}{(r^4 + \frac{2e_2^2}{b_2} - \frac{2\overline{q}_{02}^2}{b_2})^{1/2}} \tag{2.18}$$

$$\overline{q}_1 = \frac{4}{3}\pi r^3 \overline{\rho}_{01} - \frac{16\pi\alpha r^2 e_2}{(r^4 + \frac{2e_2^2}{b_2} - \frac{2\overline{q}_{02}^2}{b_2})^{1/2}} \tag{2.19}$$

here

$$\overline{q}_{01} = \frac{4}{3}\pi r^3 \overline{\rho}_{01}$$

from the above formula for E_1 we have to first order in α

$$\begin{aligned} E_1 =& \frac{e_1}{[r^4 + \frac{2e_1^2}{b_1} - \frac{2}{b_1}(\frac{4}{3}\pi r^3 \overline{\rho}_{01})^2]^{1/2}} \\ &- \frac{\alpha S(r) e_1}{2[r^4 + \frac{2e_1^2}{b_1} - \frac{2}{b_1}(\frac{4}{3}\pi r^3 \overline{\rho}_{01})^2]^{3/2}} \\ &+ \frac{16\pi\alpha r^2 \overline{q}_{02}}{[(r^4 + \frac{2e_2^2}{b_2} - \frac{2(\overline{q}_{02})^2}{b_2})]^{1/2}[r^4 + \frac{2e_1^2}{b_1} - \frac{2}{b_1}(\frac{4}{3}\pi r^3 \overline{\rho}_{01})^2]^{1/2}} \end{aligned} \tag{2.20}$$

where

$$S(r) = \frac{2}{b_1}[\frac{32\pi r^2 \overline{q}_{02} e_1}{(r^4 + \frac{2e_2^2}{b_2} - \frac{2\overline{q}_{02}^2}{b_2})^{1/2}} + \frac{128\pi^2 r^5 \overline{\rho}_{01} e_2}{3(r^4 + \frac{2e_2^2}{b_2} - \frac{2\overline{q}_{02}^2}{b_2})^{1/2}}] \tag{2.21}$$

we see that the domainant correction term for large r is the third term in Eq.(2.20) and the electric charge e_1 will be anti-screened by the second field since E_1 increases over and above its value for the non-interacting case for large r. Also from Eq.(2.17) and Eq.(2.19) B_1 will be screened by the second gauge field for large r since the field B_1 will decrease relative to its value when interactions are not present. If we solve Eq.(2.10) through Eq.(2.13) to second order in α for E_1 and B_1 we find that the next order terms screens the electric charge of e_1 and anti-screens the magnetic charge distribution of q_{01}. Thus the screening or anti-screening property is a function of the order of α and depends on the strength of the coupling constant α. This reversal of sign of the screening properties for increasing powers of α for coupled Abelian gauge fields suggests that if cosmological conditions determine the strength of α then the attraction or repulsion of fundamental dyons can be altered with the evolution of the universe if α is determined by cosmological factors.

This in turn would effect such processes as proton decay [28] in the presence of dyons as well as baryogenesis and might provide an alternate explanation of corrections to the neutron-proton mass ratio that otherwise would be attributed to extra dimensions that might effect the fine structure constant that determines the electromagnetic mass difference of m_p and m_n [29].

To calculate the exterior charges in terms of the magnetic charge distributions and the fundamental electric charges (e_1, e_2) of the particle we would have to solve Eq.(2.10) through Eq.(2.13) for $r > R$ ($R =$ radius of configuration). Integrating Eq.(2.10), Eq.(2.11), Eq.(2.12), Eq.(2.13) for $r > R$ gives in the empty space surrounding the configuration.

$$\frac{r^2 E_1}{4\pi(1 + \frac{2B_1^2}{b_1} - \frac{2E_1^2}{b_1})^{1/2}} - 4\alpha r^2 B_2 = \frac{e_{1E}}{4\pi} \tag{2.22}$$

$$\frac{r^2 B_1}{4\pi(1 + \frac{2B_1^2}{b_1} - \frac{2E_1^2}{b_1})^{1/2}} + 4\alpha r^2 E_2 = \frac{q_{1E}}{4\pi} \tag{2.23}$$

$$\frac{r^2 E_2}{4\pi(1 + \frac{2B_2^2}{b_2} - \frac{2E_2^2}{b_2})^{1/2}} - 4\alpha r^2 B_1 = \frac{e_{2E}}{4\pi} \tag{2.24}$$

$$\frac{r^2 B_2}{4\pi(1 + \frac{2B_2^2}{b_2} - \frac{2E_2^2}{b_2})^{1/2}} + 4\alpha r^2 E_1 = \frac{q_{2E}}{4\pi} \tag{2.25}$$

where we have used $\lambda + \nu = 0$ for $r > R$ which follows from the equality $T_1^1 = T_4^4$ for the energy momentum component calculated from Eq.(2.1) and the Einstein equations.

Here $e_{1E}, q_{1E}, e_{2E}, q_{2E}$ are effective electric and magnetic charges as seen for $r > R$. If we solve Eq.(2.22) through Eq.(2.25) and match the solutions of Eq.(2.10), Eq.(2.11), Eq.(2.12) and Eq.(2.13) at $r = R$ we may obtain expressions for the effective charges seen for $r > R$ in terms of the electric charges of the particle at $r = 0(e_1, e_2)$ and the magnetic charge densities $\overline{\rho}_{01}, \overline{\rho}_{02}$ for $R > r$.

To obtain an expression for the mass of the configuration we first assume that there is no non-electro-magnetic energy density and pressure in the interior of the configuration. From Eq.(2.1) the energy momentum tensor for both $U(1)$ fields is

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial L}{\partial g^{\mu\nu}} = \frac{g_{\mu\nu} b_1}{8\pi}(\sqrt{1+\frac{J_1}{b_1}} - 1) + \frac{g_{\mu\nu} b_2}{8\pi}(\sqrt{1+\frac{J_2}{b_2}} - 1) - \frac{1}{4\pi}\frac{F_{1\mu\alpha}F^{\alpha}_{1\nu}}{\sqrt{1+\frac{J_1}{b_1}}} - \frac{1}{4\pi}\frac{F_{2\mu\alpha}F^{\alpha}_{2\nu}}{\sqrt{1+\frac{J_2}{b_2}}}. \tag{2.26}$$

Note in Eq.(2.26) we do not have a contribution from the coupling term since it does not contain $g_{\mu\nu}$. From the $\binom{4}{4}$ Einstein equation and T^4_4 from Eq.(2.26) we have for $r < R$

$$\frac{d}{dr}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 T^4_4 \tag{2.27}$$

giving

$$e^{-\lambda} = 1 - \frac{8\pi G}{c^4 R}\int_0^R r^2 T^4_4 dr \tag{2.28}$$

In Eq.(2.28) we use E_1, B_1, E_2, B_2 calculated from Eq.(2.10), Eq.(2.11), Eq.(2.12) and Eq.(2.13) for $r < R$.

Also for $r > R$ the same expression for the energy momentum tensor is applicable from Eq.(2.26). From the solutions for the fields in Eq.(2.22), Eq.(2.23), Eq.(2.24) and Eq.(2.25) we may obtain the energy momentum components T^1_1 and T^4_4 for $r > R$ from Eq.(2.26). From Eq.(2.27) we may obtain the metric for $r > R$ as

$$e^{-\lambda} = 1 - \frac{2GM}{rc^2} - \frac{8\pi G}{c^4 r}\int^r T^4_4 r^2 dr \tag{2.29}$$

(M = mass of configuration).

In Eq.(2.29) M appears as an integration constant and the term containing T_4^4 in Eq.(2.29) will yield powers of $1/r^\alpha$ with $\alpha > 2$ providing the radius of the charge configuration is large enough. When Eq.(2.28) is matched to Eq.(2.29) a value of the total mass of the configuration can be obtained in terms of e_1, e_2, the radius R, and the magnetic charge densities $\overline{\rho}_{01}, \overline{\rho}_{02}$.

Conclusion

Our classically motivated calculation has suggested that the coupling term in Eq.(2.1) generates anti-screening of electric charge by the second Abelian gauge field to first order in α and screening of the B_1 magnetic field by the second Abelian gauge field due to the coupling term. The situation reverses itself to next highest order in α which suggests that screening and anti-screening are dependent on the coupling strength. If the early universe passed through epochs when the coupling constants changed, the universe might experience a phase transition due to enhancement of elementary processes involving dyons that may radiatively correct the Higgs potential that to date had not been taken into account. Also the screening of a dyon as mentioned might alter the catalysis of proton decay since the interaction of the proton with dyons or monopoles would be electro-magnetic prior to the time that the strong G.U.T. forces take over. The asymptotic freedom of gauge theories has suggested the existence of a quark-gluon plasma at high temperature and the present result for the screening effects in a $U(1) \times U(1)$ gauge theory might suggest certain critical phenomena in the early universe brought about by cosmological variations of the coupling constant α. Also if additional $Z's$ are found in accelerator experiments, studies of $U(1) \times U(1)$ screening effects due to mixing would shed light on electroweak phenomena at high energy (Ref.6) for both gauge boson-gauge boson interactions and interquark forces. In closing our classical analysis might suggest a deeper analysis of the quantum field theory problem involving $U(1) \times U(1)$ interactions and the corresponding screening effects generated.

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