Amplitude and frequency of de Broglie wave with Bohr hydrogen atomic model

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ABSTRACT. In this paper de Broglie's original concept of associating a wave motion to a moving material particle, say an electron, is extended to electron moving in Bohr atomic orbit. To obtain the probable amplitude of this motion Heisenberg's uncertainty relation is taken into consideration. The concept of inherent potential energy change for a moving material particle and the consequent total change of energy E_{ch}, given in a previous paper by the present author is also incorporated. We may call, de Broglie's wave motion of electron superimposed on Bohr's "orbital motion" a sort of "circuital motion". Circuital motion gives not only the amplitude but also the frequency of de Broglie wave. The energy of the emitted radiation when an electron transits, say, from nth to mth orbit is related to $E_{ch(n)}$ and $E_{ch(m)}$ and the frequency of the emitted radiation is related to de Broglie circuital frequency of nth and mth orbit. Calculation of emitted frequency of radiation from hydrogen atom, for electron transition between any two states, by Bohr energy method and by our de Broglie's circuital frequency method gives same value.

Further, the transition probability for hydrogen atom between two consecutive states calculated by considering amplitude of de Broglie's wave compares fairly well with that calculated by quantum mechanical method. This possibly points towards the existence of de Broglie matter wave with proper amplitude and frequency in the atomic shell. Einstein, de Broglie even Dirac and Born believed that to describe an individual system we may have to leave behind our present prejudice and try to unearth a hidden concept not justified by experience. Fundamental reason for above de Broglie motion may be due to the change of potential energy of a moving electron, a hidden variable given by the author in a previous paper. This we feel, will probably have far reaching implications in physics including the interpretation of uncertainty principle, wave particle duality aspect, spin, close spin-orbit coupling etc.

RESUME. Dans cet article, on étend l'idée originelle de De Broglie – qui consiste à associer un mouvement ondulatoire à une particule matérielle en mouvement – à un électron en mouvement dans le modèle atomique de Bohr. La relation d'incertitude d'Heisenberg est prise en compte pour obtenir l'amplitude probable de ce mouvement. Nous pouvons dire que le mouvement ondulatoire de l'électron est une sorte de "mouvement circulatoire" superposé au mouvement orbital de Bohr. Le concept de changement d'énergie potentielle propre pour une particule matérielle et la variation totale consécutive d'énergie E_{ch} donnée dans un article précédent est incorporée ici. L'énergie du rayonnement émis quand un électron transite de l'orbite n à l'orbite m est reliée à $E_{ch(n)}$ et à $E_{ch(m)}$, et sa fréquence est reliée à la fréquence circulatoire des orbites m et n. Le calcul de l'énergie et de la fréquence d'une telle transition, par la méthode de l'énergie de Bohr, et par notre méthode de fréquence circulatoire de De Broglie, donne exactement la même valeur.

D'autre part, la "probabilité de transition" entre deux états consécutifs de l'atome d'hydrogène, calculée en considérant l'amplitude de l'onde de De Broglie s'accorde assez bien avec celle qui résulte du calcul de mécanique quantique.

Einstein, de Broglie, et même Dirac et Born défenseurs de la mécanique quantique probabiliste, pensaient que, pour décrire un système individuel, nous pouvons être amenés à abandonner nos préjugés actuels et à essayer d'exhumer un concept de variables cachées, non justifié par l'expérience. La raison fondamentale en faveur du mouvement ondulatoire de De Broglie, évoqué ci-dessus, peut être liée au changement d'énergie potentielle d'une particule matérielle, une sorte de "variable cachée" donnée par l'auteur dans un article précédent. Il est fort possible qu'il y ait un échange continuel entre l'augmentation d'énergie cinétique "$E_2 - E_1$" et la diminution d'énergie potentielle "$E_1 - E_3$". Nous avons l'impression que les quelques concepts développés ici et dans les articles précédents auront probablement des implications importantes en physique, en particulier sur l'interprétation du spin, le principe d'incertitude, la dualité onde-corpuscule, le couplage spin orbite etc.

1. INTRODUCTION

We know, de Broglie [1] gave the bold concept of matter wave very difficult to believe at that time. Only a few years later experiments of Davisson and Germer [2], Thomson [3] confirmed the existence of wavelength of de Broglie wave. Probably people still disbelieve in matter wave, and hence no body talks about its amplitude and frequency. Even

de Broglie [1] initially wrote about velocity of phase wave, group wave and particle. The phase wave has an imaginary velocity more than c and imaginary high frequency. He gave the correct expression of wave length for his matter wave. We further know from Poincaré [4] and Einstein [5] that the rest or inherent energy of a material particle, say an electron, of rest mass m_0 is

$$E_0 = m_0 c^2 = E_1 \tag{1.1}$$

In a recent paper present author Kundu [6] has shown that the above particle when moving with relativistic velocity v will have its energy split into two levels E_2 and E_3.

$$\text{Increased Kinetic Energy} \quad E_2 = m_0 c^2 (1 - \beta^2)^{-1/2} \tag{1.2}$$

$$\text{Decreased Potential Energy} \quad E_3 = m_0 c^2 (1 - \beta^2)^{1/2} \tag{1.3}$$

where $\beta = v/c$.

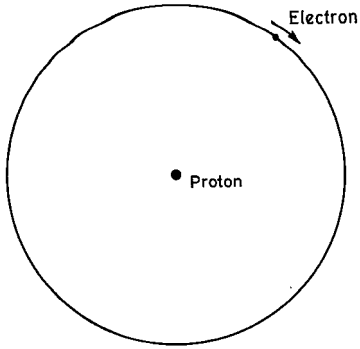

Figure 1. Bohr hydrogen atom, nth orbit

The difference $(E_2 - E_1)$ is the increase of kinetic energy, given in scientific literature. The difference $(E_1 - E_3)$ is the decrease of inherent rest energy or potential energy, not given anywhere. This is a "Velocity Dependent Potential Energy" and seems to be a "Hidden Variable". It has further been shown there that in the nonrelativistic approximation i.e. when $c \gg v$ both the difference energy become

$$(E_2 - E_1) = (1/2) m_0 v^2 \tag{1.4}$$

$$(E_1 - E_3) = (1/2) m_0 v^2 \tag{1.5}$$

and the total change of Energy

$$E_{ch} = m_0 v^2 \tag{1.6}$$

With the above knowledge in mind, we intend to restudy Rutherford [7], Bohr [8] hydrogen atomic model in the light of de Broglie's [1] postulates of wave character of a travelling material particle. In this case, the electron travels in a closed Bohr orbit. Figure 1 shows Bohr hydrogen atom with an electron moving, say, in the nth orbit. According to de Broglie the electron moving with velocity v in Bohr orbit should also have a wavelength λ according to his equation

$$\lambda = \frac{h}{mv} = \frac{h}{m_0(1-\beta^2)^{-1/2}v} \tag{1.7}$$

Its frequency f should be

$$f = \frac{v}{\lambda} = \frac{m_0(1-\beta^2)^{-1/2}v^2}{h} \tag{1.8}$$

where h = Planck constant.

Because the orbit is stationary and stable the number of wavelengths in a particular orbit should be constant. Figure 2 should then represent approximately de Broglie electron wave motion in Bohr orbit.

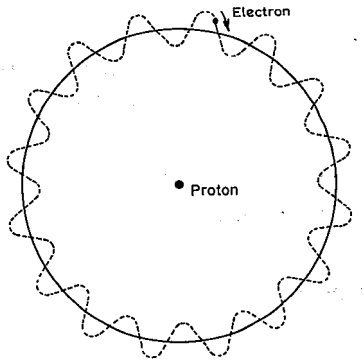

Figure 2. De Broglie electron wave in Bohr nth orbit

As our knowledge has increased step by step over the ages, we feel that a brief historical account is required to understand the need for the above restudy.

We know that Bohr [8] considered Rutherford [7] planetary model, incorporated the quantum hypothesis of Planck [9] and brilliantly explained the atomic spectra of hydrogen and helium atoms. He deduced Rydberg constant in terms of fundamental constants. Further, Franck and Hertz [10] experiment proved beyond doubt that Bohr stationary energy orbits do exist. De Broglie's [1] wave motion of a travelling material particle i.e. electron has also been experimentally proved by Davisson and Germer [2], Thomson [3].

We know that Bohr model faced difficulty to explain complex spectra. So this model was first replaced by matrix model of Heisenberg [11]. Matrix model through the quantum mechanics of Born et al [12,13] does provide solution to complex spectra. Abstract matrix model, we know, discards unobservable quantities like orbits, orbital frequencies, positions, momentum etc. It considers only observable quantities like emitted energies, frequencies, intensities etc, of the spectral lines. Although Heisenberg-Born matrix model would like to deny the existence of Rutherford-Bohr planetary model, but would like to accept the stationary states or energy levels of Bohr as they quantitavely predicted the emitted radiation energies. The wave mechanics of de Broglie [1], Schrödinger [14] and the probabilistic interpretation assigned to it by Born [15] explain many atomic and nuclear phenomena. Both de Broglie [16] and Schrödinger, see Jammer [17], we know were not happy with the way probabilistic interpretation was assigned to the wave motion they conceived. Also, Einstein [18] and many others had some reservation on solely depending upon the probabilistic interpretation. Einstein [18] wrote in reply to Born, Bohr and others that if the quantum theory didn't pretend to describe the individual system and its development in time completely, it appeared unavoidable to look elsewhere for a complete description of the system. Dirac [19] also believed that we had to leave behind a prejudice which we hold very strongly to-day. Even Born [20] creator and staunch supporter of the probabilistic quantum mechanics expressed in his Nobel Lecture that the lesson to be learnt from what he told of the origin of quantum mechanics was that the probable refinements of mathematical methods would not suffice to produce a satisfactory theory. But somewhere in our doctrine was hidden a concept unjustified by experience which we must eliminate to open up the road.

The concept of change of inherent potential energy and change of total energy in a moving material particle has been developed in an

earlier paper, Kundu [6]. Some of the effects of these energy changes are studied in this paper.

In Section 2 the probable amplitude of de Broglie wave is obtained from Heisenberg [21] uncertainty relation. The probable amplitude is a new concept. Section 3 outlines hydrogen atom model considering de Broglie wave motion with wave amplitude. Cause of this amplitude may be the change of kinetic energy and inherent potential energy Kundu [6]. In Section 4 the total energy change E_{ch} as given by Kundu [6], is considered for the Bohr orbit. Emitted radiation energy is obtained from $E_{ch(n)}$ to $E_{ch(n-1)}$. It is further shown that the emitted radiation frequency is related to the circuital frequency $f_{(n)}$ and $f_{(n-1)}$ of de Broglie wave. This is a new deduction and experimental verification is available. In Section 5 the concept of amplitude of de Broglie wave developed in Section 2 is utilized to estimate the transition probability of hydrogen atomic spectra between consecutive orbits. It compares well with values calculated by other method.

2. PROBABLE AMPLITUDE OF DE BROGLIE WAVE

The success of de Broglie's [1] hypothesis that a moving material particle with momentum p is to be associated with a wavelength λ through Planck constant h as

$$\lambda = h/p \tag{2.1}$$

leaves one in a very uncomfortable situation. This led Heisenberg [21] to postulate his famous "Uncertainty Principle" between two canonically conjugate quantities, position x and momentum p, to retrieve the situation. We know, it says, "It is not possible to measure simultaneously and precisely the position and momentum of a particle". For de Broglie waves making a packet for the particle, the size (Δx) of the packet and spread (Δp) in momentum have to be such that

$$(\Delta x)(\Delta p) \geq \hbar \tag{2.2}$$

We know that quantum mechanics deduced the above uncertainty as

$$(\Delta x)(\Delta p) \geq \frac{1}{2}\hbar \tag{2.3}$$

where $\hbar = h/2\pi$ is known as Dirac constant. In hydrogen atomic model Bohr [8] predicted step increase of angular momentum for the electron

in successive orbit by $h/(2\pi)$. For the nth orbit the angular momentum will be

$$n\hbar = m_0 v_n R_n \tag{2.4}$$

where $v_n =$ velocity of electron in the nth orbit

$R_n =$ radius of the nth orbit (2.5)

$\hbar = m_0 v_1 R_1$

Let the uncertainty in the position of electron in the nth orbit be r_n and for the first orbit r_1. We would like to assume from the relations (2.2) and (2.3) the uncertainty in position r_1 nearly equals to $\hbar/p_1$ i.e.

$$r_1 = \frac{\hbar}{p_1} = \frac{\hbar}{m_0 v_1} = \lambda\!\!\!^{-} \tag{2.6}$$

The uncertainty in position of electron in Bohr orbit is the probable amplitude of de Broglie wave for the moving electron. This is a reasonable assumption, we feel, within the limit of uncertainty principle. Present author Kundu [22] has also shown from heuristic point of view that an electron with rest mass m_0 and travelling with velocity v the probable amplitude r of motion is

$$r = \frac{\hbar}{m_0 v} = \lambda\!\!\!^{-} \tag{2.7}$$

The change of inherent potential energy for a travelling material particle, Kundu [6], may possibly be the cause for the above. From relations (2.6) and (2.7) we may write

$$\hbar = m_0 v_1 r_1 \tag{2.8}$$

From eqns. (2.5) and (2.8) we get

$$r_1 = R_1 \tag{2.9}$$

It is an interesting result. It means that uncertainty in the position of the electron in the first orbit is of the order of radius of the orbit itself. Also the uncertainty in position of the electron in the nth orbit i.e. r_n will be

$$r_n = \frac{\hbar}{m_0 v_n} = \lambda\!\!\!^{-}_n \tag{2.10}$$

or

$$\hbar = m_0 v_n r_n \tag{2.11}$$

Combining eqns. (2.4) and (2.11) we obtain

$$r_n = R_n/n \tag{2.12}$$

3. ATOMIC MODEL WITH PROBABLE ELECTRON WAVE AMPLITUDE

A model of hydrogen atom upto $4-f$ circular orbit is shown in Fig. 3, which was earlier presented by Kundu [23]. The electron following de Broglie wave motion, say in Bohr nth orbit may or may not have actual spiral motion. But, in line with present thinking we may say that it will have the probability amplitude r_n which will be changing its sign with respect to main Bohr orbit with radius R_n over a de Broglie wave length λ_n. A sort of vibrational motion is also possible.

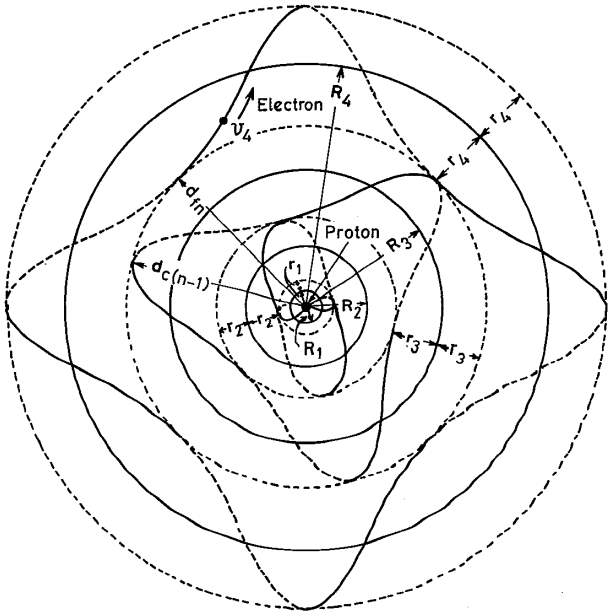

Figure 3. Bohr hydrogen atomic model incorporating de Broglie's electron wave up to $4f$ circular orbital and circuital motion.

We may call the electron motion in Bohr orbit as the "Orbital Motion" and the superimposed de Broglie's probable wave motion as the

"Circuital Motion". Let the trough of the nth circuital wave lies at a distance d_{tn} from the nucleus and the crest of the $(n-1)th$ circuital wave lies at a distance $d_{c(n-1)}$ from the nucleus. Then we get

$$d_{tn} = R_n - r_n = n^2 R_1 - nR_1 = n(n-1)R_1 \tag{3.1}$$

$$d_{c(n-1)} = R_{(n-1)} + r_{(n-1)} = (n-1)^2 R_1 + (n-1)R_1 = n(n-1)R_1 \tag{3.2}$$

Equations (3.1) and (3.2) show a remarkable result. Electrons in consecutive orbits touch each other through de Broglie wave motion. Transition of electrons take place between consecutive orbits through the touching of circuits. Hence, for the transition of electron between nth and $(n-1)$th Bohr orbits for the emission of radiation, the electron need not make the unknown jump as envisaged in the old quantum theory of Bohr [8]. Orbit to orbit, through the touching of circuits there will be smooth transition.Instantaneous transition takes place on the dotted circle. For the emission of spectra, Sommerfeld [24] calculated for elliptic orbits considering relativistic effects. We expect the touching of de Broglie's circuital motions of electrons will hold good for relativistic motion considering elliptic orbits also.

4. EXPERIMENTAL SUPPORT FOR EMITTED RADIATION FREQUENCY FROM TWO CIRCUITAL FREQUENCIES

It is a well established fact that for the electron transition between nth and (n-1)th orbit the emitted energy of the radiation is given by

$$h\nu_{n(n-1)} = E_{n(n-1)} = E_n - E_{(n-1)} \tag{4.1}$$

where E_n = kinetic plus potential energy of the electron in nth orbit

$$E_n = \frac{1}{2} m_0 v_n^2 + (-\frac{e^2}{R_n}) = -\frac{1}{2}(m_0 v_n^2) \tag{4.2}$$

But we know from our recent paper Kundu [6] and eqn. (1.6)

$$E_{ch(n)} = m_0 v_n^2 = \frac{4\pi^2 e^4 m_0}{h^2 n^2} \tag{4.3}$$

So actually energy of radiation

$$h\nu_{n(n-1)} = \frac{1}{2}[E_{ch(n-1)} - E_{ch(n)}] = \frac{1}{2}[\frac{4\pi^2 e^4 m_0}{h^2(n-1)^2} - \frac{4\pi^2 e^4 m_0}{h^2 n^2}] \tag{4.4}$$

The frequency of radiation

$$\nu_{n(n-1)} = \frac{1}{2}[\frac{4\pi^2 e^4 m_0}{h^3(n-1)^2} - \frac{4\pi^2 e^4 m_0}{h^3 n^2}] \tag{4.5}$$

We note that both eqns. (4.4) and (4.5) for the emitted energy and frequency are "difference equation" with a factor of 1/2. It has been shown, Kundu [6], that the energy $m_0 v^2$ is more fundamental and comes from the relativistic change of energy for a uniformly moving relativistic particle.

If F_n denote the mechanical frequency of the orbital rotation of the electron, then for the hydrogen it is given by

$$F_n = \frac{4\pi^2 e^4 m_0}{h^3 n^3} \tag{4.6}$$

From the frequency eqn. (4.6) we do not get the frequency of the emitted photon following the "difference equation" (4.5) i.e.

$$\frac{1}{2}[F_{n-1} - F_n] = \frac{1}{2}[\frac{4\pi^2 e^4 m_0}{h^3(n-1)^3} - \frac{4\pi^2 e^4 m_0}{h^3 n^3}] \neq \nu_{n(n-1)} \tag{4.7}$$

So, we say –the frequency of the emitted radiation does not depend upon the mechanical frequency of the orbital rotation of the electron. We know that only through Bohr's "Correspondence Principle" [25] the frequency of emitted radiation i.e. eqn. (4.5) will approach the frequency of the orbit i.e. eqn. (4.6) when both the Principal Quantum Numbers n and $(n-1)$ are very high.

But, de Broglie's hypothesis says that there are n de Broglie waves in the nth orbit. If f_n denote the circuital frequency of the electron, then

$$f_n = nF_n = \frac{4\pi^2 e^4 m_0}{h^3 n^2} \tag{4.8}$$

So a "Difference equation" constructed from eqn. (4.8) with a factor of 1/2 gives

$$\nu_{n(n-1)} = \frac{1}{2}[f_{n-1} - f_n] = \frac{1}{2}[\frac{4\pi^2 e^4 m_0}{h^3(n-1)^2} - \frac{4\pi^2 e^4 m_0}{h^3 n^2}] \tag{4.9}$$

Equation (4.9) is exactly same as eqn. (4.5) deduced from Bohr energy eqn. (4.4). This means that de Broglie's circuital frequencies are directly

related to the frequency of emitted radiation. Table-1 shows no correlation between Bohr's orbital and emitted radiation frequencies, column 2 and 3 respectively.

Table-1. Correspondence between Bohr's radiation frequency and de Broglie's circuital frequency of electron in hydrogen atom.

Principal quantum number n	Bohr's orbital frequency of electron F_n	Bohr's emitted radiation frequency $\nu_{n(n-\tau)}$	de Broglie's circuital frequency of electron f_n	Related radiation frequency from circuital frequency $(1/2)(f_{n-\tau} - f_n)$
1	6.5798×10^{15}		6.5798×10^{15}	
$2 \to 1$		2.4674×10^{15}		2.4674×10^{15}
2	8.2248×10^{14}		1.6450×10^{15}	
$50 \to 1$		3.2886×10^{15}		3.2886×10^{15}
49	5.5927×10^{10}		2.7404×10^{12}	
$50 \to 49$		5.4260×10^{10}		5.4260×10^{10}
50	5.2638×10^{10}		2.6319×10^{12}	

But there is definite correlation between de Broglie's circuital and emitted radiation frequencies column 4 and 5 respectively. There is remarkable agreement between column 3 and 5, emitted radiation frequency calculated by Bohr method and our method. The agreement exists, as shown in 4th row, even for orbit change from $n = 50$ to $n = 1$.

5. TRANSITION PROBABILITY

We know that the "Transition Probability" A_{nm} between two states $n \to m$ is defined as power divided by energy of a "photon". The transition probability is also reciprocal of mean life τ.

$$A_{nm} = \frac{S}{E} = \frac{1}{E}\frac{dE}{dt} = \frac{1}{\tau} \tag{5.1}$$

where the power is represented by S, energy by E.

Again, it is our expectation that the transition probability of a dipole radiation should contain dipole moment associated with two circuital motion and comes as a "Difference Equation" in line with the emitted energy equation and emitted frequency equation, (4.4) and (4.9). Equation (5.1) has two parts $1/E$ and dE/dt. They individually have different associated frequencies. The "circuital motion" of electron over Bohr "orbital motion" also creates a steady magnetic field – a sort of magnetic dipole. The dipole moment changes when an electron transition takes place between nth and (n-1)th orbit. The power S to be radiated from the above dipole is

$$S = \frac{dE}{dt} = \frac{4}{3}\frac{\omega^4}{c^3}|\mathbf{P}|^2 = \frac{4}{3}\frac{\omega^4}{c^3}|e\mathbf{X}|^2 \tag{5.2}$$

where $\omega =$ angular velocity
$\mathbf{P} =$ dipole moment, a vector quantity
$\mathbf{X} =$ distance for dipole moment, a vector quantity.

Hence, the transition probability for the transition $(n) \to (n-1)$ will be given by

$$A_{n(n-1)} = \frac{1}{E_{n(n-1)}}\frac{1}{2}(S_{n-1} - S_n) \tag{5.3}$$

The calculation process for $E_{n(n-1)}$ is well known. To calculate S_n from eqn. (5.2) we may note that the distance X for the dipole moment should come from the effective radius of the nth circuital motion. It is reasonable to take $X = r_n/\sqrt{2}$, as rms of ac quantity is maximum$/\sqrt{2}$. We know $\omega = 2\pi f$. We further noted from eqn. (4.9), that the emitted radiation frequency came from the circuital frequency f as

$$\nu_{n(n-1)} = \frac{1}{2}[f_{n-1} - f_n] \tag{5.4}$$

We expect the effective angular velocity to come as $\omega = \omega_n/2$, similar to difference equation (5.4), for the emission process. So the transition probability, combining eqns. (5.4) and (5.3) with eqn. (5.2) we get

$$\begin{aligned} A_{n(n-1)} &= -\frac{1}{h\nu_{n(n-1)}}\frac{1}{2}[\frac{4}{3c^3}(\frac{\omega_n}{2})^4(\frac{er_n}{\sqrt{2}})^2 - \frac{4}{3c^3}(\frac{\omega_{n-1}}{2})^4(\frac{er_{n-1}}{\sqrt{2}})^2] \\ &= \frac{1}{h\nu_{n(n-1)}}\frac{1}{2}[\frac{4e^2}{3c^3}\{\frac{\omega_1}{2(n-1)^2}\}^4\{\frac{(n-1)r_1}{\sqrt{2}}\}^2 - \frac{4e^2}{3c^3}(\frac{\omega_1}{2n^2})^4(\frac{nr_1}{\sqrt{2}})^2] \end{aligned} \tag{5.5}$$

as $\omega_n = \omega_1/n^2$ and $r_n = nr_1^2$.

Table-2. Transition probability of hydrogen atomic spectra between consecutive circular orbits (unit 10^8 per second).

Transition between quantum levels	$6h \to 5g$	$5g \to 4f$	$4f \to 3d$	$3d \to 2p$	$2p \to 1s$
By our method	0.0231	0.0536	0.1553	0.6871	8.7902
By conventional quantum mechanical method	0.0164	0.0420	0.1370	0.6400	6.2500

Table-2 gives the transition probability of the hydrogen atomic spectra between two consecutive circular orbits, calculated by eqn. (5.5) and by quantum mechanical method Condon and Shortley [26]. The agreement between the two results is quite encouraging.

6. DISCUSSION AND CONCLUSION

We know that wave length λ of de Broglie wave for a moving electron exists and has been experimentally verified. Theoretical, experimental and circumstancial evidences show – that de Broglie wave with amplitude and frequency for electron in Bohr orbit probably also exists. It has been shown in this paper that the amplitude of the wave lies within Heisenberg uncertainty relation. De Broglie wave motion for the orbital electron and its probable amplitude makes the transition between orbits say nth and (n-1)th easier, eqns. (3.1) and (3.2). Emitted radiation energy comes as a difference of energy change E_{ch} in two orbits with a factor of 1/2, eqn. (4.4). The factor 1/2 is a fundamental requirement. This is because $E_{ch} = m_0v^2$, eqn. (1.6) is more fundamental and the available energy to external world is only $(1/2)m_0v^2$. Moreover, the frequency of emitted radiation comes as a difference of the frequencies of de Broglie circuital waves in those two orbits. Table-1 gives remarkable agreement between our deduction from above, eqn. (4.9) and the conventional deduction i.e. the energy difference between two orbits divided by Planck constant h, eqn. (4.5). Further, there is good agreement, Table-2 between "transition probability" calculation of hydrogen atomic spectra by quantum mechanical method and by our method considering the amplitude of de Broglie wave, eqn. (5.5).

Association of 2π with Planck constant h to form another constant i.e. Dirac constant $\hbar$ points towards a circular motion at some place in the atomic and wave phenomena!

The circuital motion conceived in this paper may be compared with spin motion of electron in the orbit. The positive and negative spin can be easily identified. The circuital and orbital motion of electron is united together through a single motional velocity v, Fig.3. Thus, it can explain close spin-orbital coupling. Further, in this model an electron need not take an uncertain jump between two orbits for the emission of radiation. Because, for all atoms $r_1 = R_1$ there may be exchange of first orbit electrons with nucleus component, and interesting results may follow.

In a previous paper author [6] gave the proof of existence of inherent change of potential energy in a moving material particle, eqn. (1.3). This is a "Velocity Dependent Potential Energy". This seems to be a "Hidden Variable" not suggested by anyone so far. The cause of de Broglie wave motion is probably due to this hidden variable. There is a distinct possibility that a continuous alternate change between increase of kinetic energy "$E_2 - E_1$" and decrease of potential energy "$E_1 - E_3$"is taking place, similar to continuous change of energy between electromagnetic and electrostatic energy in a lossless L-C circuit. In this context we may possibly have to reinterpret uncertainty principle, and other deeper consequences may follow.

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