About a recent paper of P.Y. Chu, an old formula of Planck and Laue, and de Broglie's hidden thermodynamics*

G. Lochak

Fondation Louis de Broglie,
23, Quai de Conti, F- 75006 Paris

ABSTRACT. It is shown that a question recently raised by P.Y.Chu in this journal on "Lagrange function, de Broglie's model and relativity" [1] is close to old results of Planck and Laue on relativistic dynamics and to de Broglie's Hidden Thermodynamics.

RÉSUMÉ. On montre qu'une question récemment soulevée par P.Y.Chu dans cette revue, sur "La fonction de Lagrange, le modèle de de Broglie et la relativité" [1], est étroitement liée à des résultats anciens de Planck et Laue et à la thermodynamique cachée de Louis de Broglie.

In an interesting paper recently published in this journal [1], P.Y.Chu raised a question about the physical significance and the relativistic variance of the Lagrange function of a free particle[1]:

$$L = -m_0c^2(1-\beta^2)^{\frac{1}{2}} \tag{1}$$

From this formula, Chu is led to another one, which is easily proved :

$$mc^2 = m_0c^2(1-\beta^2)^{\frac{1}{2}} + mv^2 \tag{2}$$

where m is the relativistic mass :

$$m = m_0(1-\beta^2)^{-\frac{1}{2}} \tag{3}$$

* Given that my short paper is a comment of a paper written in English, I shall try to write it in the (approximately!) same language.

1 We define β as v/c, as is commonly admitted. Chu defines it as v^2/c^2.

Chu proposes that “this equation is another fundamental equation of special relativity”. He is right, but this formula was found already for a long time in the problem of relativistic thermodynamics, which is presently almost forgotten, which explains that Chu, just as most physicists, apparently does know about it. De Broglie called this formula the “Planck-Laue formula” [2], [3]. It was introduced at the very beginning of the theory of relativity, when Einstein, Planck and Laue have solved the problem of relativistic variance for thermodynamical quantities. They have found that entropy must be a relativistic invariant and that the variance of heat and temperature is :

$$Q = Q_0(1-\beta^2)^{\frac{1}{2}} \quad , \quad T = T_0(1-\beta^2)^{\frac{1}{2}} \tag{4}$$

Their theory may be found in the famous treatise of Max von Laue [5]. Because of the law (4), de Broglie considered the first term in the decomposition (2) of a total amount of energy, as an internal heat and the second term as a kinetic energy : the “pseudo-kinetic energy”, as he said, because this is not the formula commonly given by the theory of relativity. And he suddenly remarked, in the fourties, that the variance of of heat and temperature is the same as the variance of a clock frequency, and thus of the *internal frequency* ν_c of a particle that he had introduced at the beginning of researches that led him to the idea of wave mechanics. This frequency was soon eclipsed by the success of matter waves and remained useless in the theory, forgotten by all the theoreticians, including de Broglie himself during a long time. So, quite later, he tried to find the sense of the analogy between formulae (4) and his old formula :

$$\nu_c = \nu_0(1-\beta^2)^{\frac{1}{2}} \quad \text{, where} \quad h\nu_0 = m_0c^2 \tag{5}$$

Chu quoted this frequency in his paper and he suggested that the first term of the second member of the decomposition (2) of Planck-Laue, could be a “hidden kinetic energy” : he was not so far from the idea of de Broglie who suggested that it is an “internal hidden heat” of the particle. At the same time, de Broglie introduced a theorem of Boltzmann, which was at the origin of the Ehrenfest theory of adiabatical invariants. This theorem claims that a very slow variation of a parameter, in a classical periodic mechanical system, may be interpreted as the exchange, between the system and the external world, of a quantity of heat equal to :

$$\delta Q = \nu_c A_0 \tag{6}$$

ν_c is the frequency of the periodic system and A the Maupertuis action. But, in our case, ν_c is the de Broglie internal frequency (5), the relativistic variance of which is the same as the variance of the quantity of heat δQ. The index *zero* added in the formula (6) to the Maupertuis action A signifies that it is taken in the *proper system* of the particle. Thus, arguing of the analogy between this Boltzmann formula and the Carnot-Clausius formula :

$$\delta Q = T\,\delta S \tag{7}$$

he suggested to define, for an *isolated particle*, a temperature and an entropy by the following formulae:

$$kT = h\nu_c \quad , \quad hS = kA_0 \tag{8}$$

This led de Broglie to the hypothesis that these thermodynamical properties or elementary particles are related to a vacuum property that he called the *hidden thermostat.* This theory was widely developed by de Broglie himself and some pupils (among which the author of the present note). Several surveys were given by de Broglie in references quoted above and in [6] and [7] (the last is not yet published). I gave a general and quite simple survey in the reference [8].

It must be recognized that, despite our efforts, we have not yet been able to deduce, from de Broglie's thermodynamics, new experimental predictions. Perhaps it is not a theory for our times, as it was the case for Huygens wave theory of light, but I am profoundly convinced - and more important, de Broglie was too - that this idea is of the same importance as the one of matter waves.

The aim of this theory is the following. We know that wave mechanics started from an analogy between classical mechanics and wave optics, at the level of geometrical optics, i.e. when the principle of Fermat is equivalent to the principle of Maupertuis. But the strength of this analogy appeared when the idea was extended by de Broglie in his thesis, and soon by Schrödinger in his famous memoirs, to more general cases, when these two principles are no more equivalent because the principle of least action looses his significance (at least in its classical form) : mechanics becomes a wave mechanics. In the same manner, it must be stressed that formulae (8) are limited to the domain of *thermodynamical equilibrium.* And *de Broglie postulated that, for a reversible process, at the geometrical optics limit, the principles of Maupertuis, Fermat and Carnot are equivalent.* But, out of this kind of summit of the three great theories -

classical mechanics, wave optics and thermodynamics - the equivalence is not true : just as waves prevail over classical mechanics at the atomic level, "wave-thermodynamics" must prevail over ordinary wave mechanics for irreversible processes, principally for *quantum transitions* which could thus be described by this theory, as extremely rapid processes, giving up the vue of magic instanteneous tranformations, "indescribable in the frame of space and time".

Without developing the whole theory, I would like to add three remarks :

1) In my paper [8], I quoted a nice reasoning that de Broglie gave me in this occasion (without publishing it elsewhere), in order to explain the interpretation of Planck-Laue formula :

Consider a kind of crystal : a "piece of matter" made of a great number N of harmonic oscillators with the same proper frequency ν_0. The centers of the oscillators are at rest in a certain proper system R_0, in which the values of their energies are $nh\nu_0$, where n is a whole number or half a whole number, i.e. an invariant. The total energy is :

$$W_0 = M_0c^2 = \sum_{k=1}^{N} n_k h\nu_0 \tag{9}$$

which is valid even when the oscillators are exchanging quanta $h\nu_0$ between them. This a way of describing internal chaotic motions in the piece of matter : the energy W_0 is a quantity of internal heat Q_0. Now, let us consider the same system from another reference frame R, which is moving whith respect to R_0 with the velocity βc. Each oscillator may be considered as a clock, the frequency of which, in the system R, is equal to :

$$\nu = \nu_0(1-\beta^2)^{\frac{1}{2}} \tag{10}$$

As far as numbers N and n_k are invariants, we have the following expression for the internal energy W_i observed from R :

$$W_i = \sum n_k h\nu_0(1-\beta^2)^{\frac{1}{2}} = M_0c^2(1-\beta^2)^{\frac{1}{2}} = Q_0c^2(1-\beta^2)^{\frac{1}{2}} \tag{11}$$

But on the other side, we know that the total energy W in R is :

$$W = M_0c^2(1-\beta^2)^{-\frac{1}{2}} = Q_0c^2(1-\beta^2)^{-\frac{1}{2}} \tag{12}$$

with the other sign of the exponent 1/2 ! This is not astonishing because W is not equal to W_i, but to : W_i *plus* the global translation energy W_t of the piece of matter in R. W_t is the difference between the previous expressions. We find :

$$W_t = W - W_i = M_0 v^2 (1 - \beta^2)^{-\frac{1}{2}} \tag{13}$$

and, owing to (11), this equallity may be written as :

$$W = W_i + W_t = Q_0 (1 - \beta^2)^{\frac{1}{2}} + M_0 v^2 (1 - \beta^2)^{-\frac{1}{2}} \tag{14}$$

This is just the Planck-Laue formula.

2) Despite the prestigious signatures and numerous proofs of the variance of temperature and heat, some good physicists have contested this strange relativistic variance because "it must be the same as for an energy, due to the first principle of thermodynamics". In my opinion, the proof given above is (among others) quite convincing, but I shall give another simple argument that I have suggested to de Broglie, who made use of it in several occasions, so that the argument was published only by him as a quotation [9].

Let us consider a gas, the molecules of which have chaotic velocities v_{kx}, v_{ky}, v_{kz} with respect to the proper reference frame defined by the vessel containing the gas. And let us look at the same gas from another reference frame moving with a velocity V (say parallel to x) with respect to the first one. The new velocities are :

$$\begin{aligned} v'_{kx} &= \frac{v_{kx} + V}{1 + v_{kx} V/c^2} \quad , \\ v'_{ky} &= \frac{v_{ky}(1 - V^2/c^2)^{\frac{1}{2}}}{1 + v_{ky} V/c^2} \quad , \quad v'_{kz} = \frac{v_{kz}(1 - V^2/c^2)^{\frac{1}{2}}}{1 + v_{kz} V/c^2} \end{aligned} \tag{15}$$

We see that :

$$v'_{kx} \to c \quad , \quad v'_{ky} \to 0 \quad , \quad v'_{kz} \to 0 \quad \text{when} \quad v \to c \tag{16}$$

In other words, mesured by the new galilean observer, the velocities of all molecules tend to the velocity of light and become parallel to the translation velocity of the observer with respect to the vessel containing the gas. The observer will see only a translational energy of molecules without chaotic motions which could be interpreted as an internal heat. For him, the whole energy is translational, and the amount of heat is equal to zero.

This is not a proof that heat and temperature obey the law (4), but this is a proof that this law is possible, while a law like (3) is not. In addition, we understand where is the origin of the strange relativistic variance of heat : when the velocity of a system increases, it tends to "absorb" the other velocities and the chaotic velocities become negligible by comparison with the translational one.

3) At last, let us give, following de Broglie, the relation between his guidance law (la *formule du guidage*) and the Planck-Laue formula. Consider the Klein-Gordon equation :

$$\Box\psi + \kappa^2\psi = 0 \tag{17}$$

We know that, if we introduce the amplitude and the phase of ψ, we get two equations, one of which is the following :

$$\frac{1}{c^2}\Big(\frac{\partial\phi}{\partial t}\Big)^2 - (\nabla\phi)^2 = m_0^2c^2 + \frac{4\pi^2}{h^2}\frac{\Box a}{a} \tag{18}$$

This is the relativistic Jacobi equation of a particle with a variable proper mass :

$$M_0 = \Big(m_0^2 + \frac{4\pi^2}{h^2c^2}\frac{\Box a}{a}\Big)^{\frac{1}{2}} \tag{19}$$

Now, by analogy with classical formulae, we put :

$$M_0c^2(1-\beta^2)^{-\frac{1}{2}} = \frac{\partial\phi}{\partial t} \quad , \quad M_0v(1-\beta^2)^{-\frac{1}{2}} = -\nabla\phi \tag{20}$$

These formulae give the guidance law that was previously proved by other ways:

$$v = -c^2\frac{\nabla\phi}{\partial\phi/\partial t} \tag{21}$$

This is the velocity of a singularity in the wave, i.e. the velocity of the particle. Now, we can see that if, during an interval of time dt the particle covers an interval dl on its trajectory, the variation of the wave phase is :

$$d\phi = \frac{\partial\phi}{\partial t}dt + \frac{\partial\phi}{\partial l}dl = \Big(\frac{\partial\phi}{\partial t} + v.\nabla\phi\Big)dt \tag{22}$$

Owing to (20) and (22), we get :

$$d\phi = \left[M_0 c^2 (1-\beta^2)^{-\frac{1}{2}} - M_0 v^2 (1-\beta^2)^{-\frac{1}{2}}\right] dt \tag{23}$$

Or :

$$d\phi = M_0 c^2 (1-\beta^2)^{\frac{1}{2}} dt \tag{24}$$

But with a variable mass M_0, the frequency of the internal *clock* of the particle is :

$$\nu_0 = \frac{1}{h} M_0 c^2 \tag{25}$$

Thus, the variation of its internal phase, seen by an observer with the velocity βc will be (due to the law of variance of a clock frequency) :

$$d\phi_i = M_0 c^2 (1-\beta^2)^{\frac{1}{2}} dt \tag{26}$$

This is exactly the formula (24), which means that the internal clock has the same phase variation as the wave : it is the law of phase accordance (la loi de l'accord des phases). Finally, let us identify the second members of the equalities (24) and (25), and we find the Planck-Laue formula.

References

[1] P.Y. Chu, Annales de la Fondation Louis de Broglie, **18**, N° 1, p. 25 (1993).
[2] L. de Broglie, Annales de l'Institut Henri Poincaré, **9**, N° 2, p. 89 (1968).
[3] L. de Broglie, Foundations of quantum mechanics (in French), Academic Press, N.Y. (1971); reprinted (in English) in Annales de la Fondation Louis de Broglie, **12**, N° 4, p. 399 (1987).
[4] L. de Broglie, La réinterprétation de la mécanique ondulatoire, Gauthier-Villars, Paris (1971).
[5] M. von Laue, Die Relativitätstheorie, Vieweg, Braunschweig (1919); La théorie de la relativité, Gauthier-Villars, Paris (t. 1, 1922 ; t. 2, 1926)
[6] L. de Broglie, La thermodynamique de la particule isolée, Gauthier- Villars, Paris (1964).
[7] L. de Broglie, Cours de 1948 : Questions de thermodynamique et de mécanique ondulatoire (en préparation).
[8] G. Lochak, *in*: Louis de Broglie, sa conception du monde physique, Gauthier-Villars, Paris (1973).
[9] L. de Broglie, Comptes rendus de l'Académie des Sciences, **264**, Série B, p. 1173 (1967).
[10] L. de Broglie, IJTP, **1**, N° 1, p. 1 (1968).