Inter-theory relations and foundations of physics: the case of statistical thermodynamics and classical thermodynamics

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ABSTRACT. The relation existing between classical (pure) thermodynamics and statistical thermodynamics is analysed at a metatheoretical level by means of a network of inter-theory relations involving formal analogies and theory reductions. On this basis, we obtain a conceptual framework by which we discuss an alternative definition of thermodynamic limit and the complementarity existing between the statistical and dynamical descriptions of macroscopic systems. The topic is also considered from the epistemological and historical point of view.

RÉSUMÉ. On analyse la relation qui existe entre la thermodynamique classique (pure) et la thermodynamique statistique au niveau métathéorique, à travers un réseau de relations inter-théoriques qui comporte des analogies formelles et des réductions de théories. Il est donc possible d'obtenir une structure conceptuelle par laquelle on peut examiner une définition alternative de la limite thermodynamique et la complémentarité entre la description dynamique et celle statistique des systèmes macroscopiques. Le suyet est analysé aussi du point de vue épistémologique et historique.

Introduction.

In the second half of the 19th century the atomistic interpretation of the laws of classical thermodynamics (CT) led to the birth of statistical thermodymamics (ST). This theory has two fundamental characteristics: on the one hand it deprives the second law of CT of its **absolute** character (and therefore corrects it) while, on the other, it reveals the impossibility of obtaining an **integral** reduction of CT to either classical mechanics (CM) or quantum mechanics (QM).

ST is situated in fact at an intermediate level between a purely macroscopic description and a microscopic description of thermodynamic systems; this level is characterized by the presence of fluctuations around mean values of the macroscopic variables of a system in equilibrium, due to the microscopic activity of the system itself. Thus it appears clear that the study of the fluctuations is essential to clarify the relation that exists between ST and CT, with the aim of establishing: a) what are the physical hypotheses and mathematical structures that characterize these theories; b) in what sense can ST be interpreted as a generalization of CT and be approximated by it under suitable assumptions in certain extreme cases; c) what relation exists between ST, CT and other theories such as CM and QM.

The crux of the question is the possibility of obtaining, within the framework of the phenomenological theory of fluctuations, some correlation relations between thermodynamic quantities which are completely similar to the uncertainty relations of QM. These ST uncertainty relations have already been obtained and discussed in previous literature. Here we will limit ourselves to presenting the fundamental conditions for obtaining them, and discuss how they can be interpreted and what is their significance for our inquiry.

Through such analysis it will be possible to construct a network of inter-theory relations, based on formal analogies and on the reduction of certain theories to others, which will enable us to develop a number of considerations on the use of analogies, on the correspondence principles and on the concept of complementarity (which already occupies a central role in the interpretation of QM), by re-examining the historical genesis of ST and QM. From the start we draw attention to the terminology: when a theory T' can be obtained from T as an extreme case ($T \to T'$ for $a \to b$) we say that T' is reduced to T.

1. Uncertainty relations in statistical thermodynamics.

The fluctuations and correlations of thermodynamic quantities can be computed on the basis of the phenomenological theory developed by Einstein [1], Szilard [2], Mandelbrot [3], Tisza and Quay [4], with the explicit objective of building a ST based on principles analogous to those of CT. This approach is based on the inversion of the Boltzmann relation between entropy and probability of thermodynamic states, according to which, for a state near to equilibrium, with entropy S and thermodynamic probability W, we have

$$S - S_0 = k \ln(W/W_0) \tag{1}$$

where S_0 and W_0 are, respectively, the entropy and the thermodynamic probability of the state of equilibrium. According to Landau and Lifshitz [5], by means of (1), the second order moments can be calculated exactly starting from the Gaussian distribution

$$W = W_0 \exp{(S - S_0)/k} \tag{2}$$

where

$$S - S_0 = -\frac{1}{2} \sum_{i,j=1}^{N} \Phi_{ij} x_i xj \tag{3}$$

The entropy S is a function of the instantaneous values of the extensive variables x_i and x_j $(i, j = 1, \ldots, N)$. If we put $x_i = 0$, then the deviations from the equilibrium are given by $\delta x_i = x_i$, while $\Phi_{ij} = (\partial^2 S/\partial x_i \partial x_j)_0$. The intensive variables conjugate to the x_i are defined by the relations $X_i = -\partial S/\partial x_i$.

In this manner the following correlation relations are obtained:

$$\overline{x_i x_j} = k \Phi_{ij}^{-1} \tag{4a}$$

$$\overline{X_i X_j} = k \Phi_{ij} \tag{4b}$$

$$\overline{x_i X_j} = k \delta_{ij}. \tag{4c}$$

In the case of a small part of a closed and finite system in equilibrium, the relations (4c) for the conjugate variables take the following form:

$$\triangle U \triangle (1/T) \approx k \quad , \tag{5a}$$

$$\triangle V \triangle (P/T) \approx k \quad , \tag{5b}$$

(Gilmore [6], Caianiello and Noce [7], Kreuzer [8]). In these relations Boltzmann's constant plays a similar role to that played by Planck's constant in QM uncertainty relations.

Rosenfeld [9], [10], interpreted the uncertainty relations of ST, (5a) and (5b), as the formal counterpart of the complementarity existing between the dynamical and statistical descriptions which can be obtained for every macroscopic system.

The eq.(5a), in particular, clarifies the relation existing between the concepts of energy and temperature: they can be referred to the above descriptions and are operationally defined by means of mutually exclusive procedures. In fact, in order to be able to specify the energy of a thermodynamic system it must be isolated from all outside influence, while definition of the temperature assumes the system to be interacting with a thermostat having infinite thermal capacity. These two ideal situations are represented, in the theory of statistical ensembles, by the microcanonical and the canonical systems respectively. In fact they correspond to the two extreme cases which, assuming the existence of limit conditions, such as complete isolation or interaction with an infinite system, are based on a principle other than the phenomenological, according to which, by contrast, the subsystem being studied interacts weakly with a finite system (Kreuzer [8], Landau and Lifshitz [5]).

In the following paragraphs, referring back to considerations made by Rosenfeld and extending results presented in a previous paper (Neri [11]), we shall see how the meaning of the (5a) and (5b) can provide indications on the relation existing between ST, CT and QM and, therefore, how they permit us, at a metatheoretical level, to clarify the relation between the mechanical description and the thermodynamic description of physical systems.

2. Atomism, classical thermodynamics and reductionism.

In the second half of the 19th century an extensive debate developed on atomism and the possibility of its utilization in interpreting the laws of thermodynamics which had recently been discovered. On this problem, the cultural climate of the time presented widely differing views (Brush [12]). Some conceptions resorted to a mechanistic interpretation, although within their confines differing interpretations of the relation between CT and CM could be identified. Other conceptions rejected contamination of the phenomenological laws of CT with models and hypotheses on the atomic structure of matter, and accepted in a general sense the interpretation of heat as a form of motion, or attributed physical significance only to phenomenological concepts (empiriocriticism), or assigned to energy the status of fundamental concept (energetics). These last two tendencies were based on a radical rejection of mechanical explanation. While some physicists took a firm stand for or against atomism (Boltzmann for, Mach and Oswald against), others provided contributions which were sometimes in harmony with one tendency, sometimes with the other; this shows how difficult it is to describe the general situation schematically.

Within this general pattern ST appears, under various aspects, to be the by-product of the attempt to obtain a complete reduction of CT to CM (Brush [13], Daub [14]). This is particularly evident if we consider the intellectual itinerary followed by Boltzmann (Klein [15]). ST in fact was born of the recognition (by Maxwell and subsequently by Boltzmann himself) of the necessarily statistical nature of the law of entropy. The birth of ST revealed that, on one hand, the laws of CT did not possess the absolute validity which upholders of pure thermodynamics wished to attribute to them; and on the other, that the laws of CM - and with them the purely dynamical description - were not sufficient to interpret the laws of CT on the basis of the atomic theory. The search for mechanical analogies, furthermore, revealed the existence of a number of purely formal similarities between CT and CM (Klein [16]).

All these facts point to the existence of a far more complex, but less substantial, $CT - CM$ relation, than that originally posited by reductionists, and one which involves other theories, such as ST and, as we will see, QM.

3. Inter-theory relations: formal analogy and reduction.

The metatheoretical pattern which we intend to analyse links the four theories named so far by inserting them into a structure having the form of an arithmetic proportion, given by

$$CM : QM \approx CT : ST \quad , \tag{6}$$

which can be defined as a "inter-theory relation of the second order" since, through a series of binary formal relations between theories, it allows us to analyse relations between relations (Strauss [17]).

It is easy find examples in the history of physics where the application of a type (6) structure provided a fundamental heuristic instrument for discovering new theories. Suffice it to mention the discovery of wave mechanics, obtained by Schrödinger starting from the pattern:

wave mech.: class. mech. $\approx$ wave optics: geom.optics.

The inter-theory relations which appear in (6) belong to two types: a) formal analogies, deriving from Hamiltonian formalism, which link deterministic ($CM - CT$) or statistical ($QM - ST$) theories; b) correspondence principles, which allow us to obtain certain theories, such as

CM or CT, from others, such as QM or ST, taking the limit of some characteristic constant ($QM \to CM$, $ST \to CT$).

The use of analogies in science is a much debated point in modern epistemology. As far as the role they play in scientific explanation is concerned, very different positions are to be found in the literature (Hesse [18], Bunge [19]). There is however a general consensus that analogies perform an important function in the formation of theoretical ideas. Distrust of them springs from the absence of a logic of analogy, from the frailty of this conceptual instrument compared with the solidity of the perfect equivalence. However, notwithstanding their failings, they produce scientific knowledge, and permit the transfer of the solution of mathematical problems in one sector of inquiry to another, leading to the development of new physical hypotheses.

In the literature, greatly differing approaches are also to be found on the problem of the reduction of theories. These approaches are based on differing definitions (Nagel [20], Schaffner [21], Kemeny and Oppenheim [22], Sklar [23], Nickles [24], Popper [25]), or even the rejection of the possibility of establishing logical connections between different theories, on account of their presumed incommensurability (Kuhn [26], Feyerabend [27]). In the light of our investigation a viewpoint which acquires particular interest was formulated by Schaffner. He attempts to extend the concept, introduced by Nagel, of reduction of a theory T' to a more general theory T as a relation of derivability of T' from T. According to Schaffner, instead of conceiving the reduction as a pure logical deduction of T' from T ($T \to T'$), it must be interpreted as the product of a double relation: deduction from T of a new theory T" ($T \to T$") and formal analogy between T" and the reduced theory T' (T" $\approx T'$). This would enable us to avoid the criticism raised against the Nagel model of reduction because of its rigidity with respect to the meaning change of the terms, which according to Schaffner could take place in a partial, but not arbitrary manner, as on the contrary the upholders of the idea of incommensurability affirm.

The general problem of the use of analogies and the procedures of reduction in science is in any case still an open question, and it would not be possible here to provide a overall answer to the questions raised by these conceptual instruments. Instead, we will attempt to show that in the cases which interest us, these forms of inter-theory relation play a constructive role and have close and important connections. In particular, it will become clear that the pattern of reduction proposed by

Schaffner is very well suited to describe the relations $QM \to CM$ and $ST \to CT$, which will be expressed by means of suitable asymptotic conditions and appropriate formal analogies.

4. The $CM - CT$ relation.

The formal analogy between CM and CT has already been widely analysed in the literature (Corben and Stehle [28], Peterson [29], Rey de Luna and Zamora [30]). In this paragraph we shall restrict ourselves to establishing the relations of correspondence which exist between CM and CT quantities or equations, relations which we will meet again, in other forms, when considering the analogy between QM and ST.

The common foundation of CM and CT consists of Hamiltonian formalism. By means of this, considerable correspondences can be established between the two theories.

a) In both CM and in CT it is possible to identify canonical conjugate variables. In CM they are linked to action by the relation

$$dA = pdq - Hdt \quad , \tag{7}$$

while in CT they are linked to entropy by the fundamental equation

$$dS = (1/T)dU + (P/T)dV \quad . \tag{8}$$

b) A conservative mechanical system with N degrees of freedom can be described by means of an extended phase space with $2N + 2$ dimensions, in which time and energy are assimilated to all the other canonical variables (Synge [31]). For $N = 1$, in particular, we obtain the following Poisson brackets:

$$[q, p] = 1 \quad , \quad [E, t] = 1 \quad , \tag{9}$$

with $H(q, p) = E$.

Similarly, in a closed thermodynamic system in equilibrium, we may define the generalized Jacobian

$$[\alpha, \beta] = \frac{\partial\alpha}{\partial U}\frac{\partial\beta}{\partial(1/T)} - \frac{\partial\alpha}{\partial(1/T)}\frac{\partial\beta}{\partial U} + \frac{\partial\alpha}{\partial V}\frac{\partial\beta}{\partial(P/T)} - \frac{\partial\alpha}{\partial(P/T)}\frac{\partial\beta}{\partial V} \quad . \tag{10}$$

Then we have

$$[U, 1/T] = 1 \quad , \quad [V, P/T] = 1 \quad . \tag{11}$$

c) The motion of the mechanical system is limited to one surface of the extended phase space with $2N + 2$ dimensions, defined by the equation of energy

$$\Omega(x, y) = 0 \quad . \tag{12}$$

In this equation x and y indicate, respectively, the set of the $N + 1$ coordinates ($x_1 = q_1, x_2 = q_2, \ldots, x_{N+1} = t$) and of the $N + 1$ momenta ($y_1 = p_1, y_2 = p_2, \ldots, y_{N+1} = -E$) of the system.

The analogue of (12) for CT represents the equation of state of the system.

In general, the following series of correspondences can be established:

CM	CT
mech.conjugate variables	thermod.conjugate variables
equation of energy	equation of state
action	entropy
Poisson brackets	Jacobians
canonical transformations	Legendre transformations
equations of motion	Maxwell equations

Particularly interesting is the relation between action (in CM) and entropy (in CT). Besides establishing the conjugation relations between the canonical variables of the respective theories, they are subject to variational principles defining the conditions of evolution of the system, and are adiabatic invariants. In CT, furthermore, for entropy it is possible produce an equation analogous to CM's Hamilton-Jacobi equation. From this viewpoint, retrospective analysis of a work by Ehrenfest [32] on Planck's blackbody law (Neri [33]) is interesting, in which the adiabatic invariance properties of entropy and action are studied.

5. The $CM - QM$ relation.

QM and CM are theories which refer to the same domain of physical phenomena (particle mechanics) and they use concepts concerning the same descriptions, but placed in different mathematical structures.

As we have seen in the previous paragraph, CM allows us to obtain conjugation relations expressed by means of Poisson brackets (9). Each pair of quantities contains a variable (q in the first case, t in the second) which refers to the space-time description, and a variable (p and

E respectively) which refers to the causal description, based on the energy and momentum conservation laws. The classical deterministic ideal would be to be able to attribute both descriptions simultaneously to the system, with no limitation whatsoever.

The situation differs in QM, where the values corresponding to the conjugate variables of (9) are linked by the Heisenberg relations, given by

$$\triangle q \triangle p \approx h \quad , \quad \triangle E \triangle t \approx h \quad . \tag{13}$$

Relations (13) establish the existence of correlations dependent on the value of a universal constant (h) having the dimensions of the action.

These relations have been interpreted (Bohr [34]) as the quantitative expression of the complementarity relation existing in QM between causal and space-time description of a physical system, the union of which characterizes classical theory. The term complementarity indicates a relation between descriptions which are mutually exclusive and, at the same time, necessary to obtain a complete representation of the physical situation. It provides a conceptual instrument which makes it possible to overcome the evident contradictions which would arise if we attempt to attribute unlimited validity to mutually exclusive concepts. By introducing complementarity we generalize the conceptual structure into which the concepts borrowed from classical physics have to be introduced, and we define the type of limitation to which their definition is subject.

The canonical conjugate variables of CM correspond therefore to the correlated variables of QM. Thus the theory based on the notion of complementarity constitutes a rational generalization of classical deterministic theory. A quantitative formulation of the link existing between the two theories can be expressed by the condition

$$h \to 0 \quad , \tag{14}$$

by means of which we can define the relation of reduction of CM to QM ($QM \to CM$). Condition (14), applied to eq. (13), is equivalent to ignore the correlations between conjugate variables and the limitations which complementarity imposes on the description of the system.

The introduction of (14) requires some clarification.

a) Firstly, the need to safeguard full consistency of quantum results with the classical limit obliges us to introduce the additional condition according to which the characteristic quantum numbers of the system tend to infinity, in such a manner that the products of these numbers with Planck's constant, corresponding to the classical action variables, remain constant. Hassoun and Kobe [35] have put forward important cases featuring applications of this definition of the classical limit. Take, for example, the case of the harmonic oscillator, whose energy spectrum, according to QM, is given by

$$E_{quant} = (n + \frac{1}{2})\frac{h\omega}{2\pi} \quad . \tag{15}$$

If the conditions $h \to 0$, $n \to \infty$ are imposed, so that $nh = J = \pi A^2 \omega m =$ constant, we obtain

$$E_{quant} \to \frac{1}{2} m\omega^2 A = E_{class} \quad . \tag{16}$$

The zero-point energy , which has no classical analogue, disappears completely.

b) The condition (14), obviously, has a purely metaphorical meaning, since in reality Planck's constant has a very precise value. However, we don't agree with the objections raised against this procedure (Rohrlich [36]). In fact, the introduction of this condition has great importance at the metatheoretical level, where the procedure just illustrated, in which physical constants are treated like variable quantities, enables us to analyse the logical relation between different theories.

6 The $CT - ST$ relation.

Like QM and CM, ST and CT too are theories which refer to one single domain of phenomena (those characterising macrosystems in equilibrium). The fundamental difference between ST and CT is that they are based on different conceptions about the structure of matter. From these theories derive essentially different descriptions of phenomena, expressed using the same concepts, set however in different formal structures. In the transition from one theory to another some properties of these concepts and their domain of definition are modified, but the modification is always guided by the relation that can be established between the formal structures of the theories by means of a procedure of passage to the limit (Rohrlich [37]). Indeed ST can be interpreted as a generalization of CT, and the formal structure of CT shows itself to be a special case of that of ST, obtainable from the latter when certain extreme conditions are imposed, analogous to those which permit us to define the relation between QM and CM. This is particularly evident in the realm of the already mentioned phenomenological approach, within which the uncertainty relations (5a) and (5b) can be obtained directly.

In CT the thermodynamic conjugate variables are linked by Jacobians (11). Each pair contains an intensive variable, which refers to the system as a whole and characterising the macroscopic state of equilibrium, and an extensive variable. In ST, by contrast, the conjugate variables are correlated through relations (5a) and (5b), which can be interpreted in terms of complementarity between the dynamical and statistical descriptions. The significance of this complementarity relation has already been discussed in the literature (Bohr [38], Rosenfeld [9], [10]), but it is worthwhile stressing that this idea, applied to ST, is fundamental in solving a number of longstanding questions:

a) firstly, by clarifying the relation between dynamical and statistical description, it excludes the possibility of obtaining an integral reduction of the laws of CT to the laws of CM;

b) consequently, it allows us to attribute to the statistical description a role which is not secondary to the dynamical description, thus allowing a direct objective interpretation of the probabilities employed in ST. In fact, it must be pointed out that if we were to regard the probabilities employed in ST as a mere consequence of the subjective non-knowledge of the dynamical state by the observer, the paradoxical conclusion would follow that all CT - whose laws doubtless possess an objective character - has on the contrary a value dependent on the observer's ignorance.

The link existing between ST and CT, definable by means of the thermodynamic limit, can be expressed in complete analogy with the classical limit of QM by means of the condition

$$k \to 0 \tag{17}$$

(Tisza and Quay [4], Neri [11], Compagner [39]). Like the condition (14), the limit (17) must be integrated by supplementary asymptotic conditions, imposed on the number of particles and on all the microscopic quantities relative to the single constituent elements of the system, in such a manner that all its macroscopic properties remain unvaried.

As an example, let us consider a perfect gas made up of N particles with mass m and thermal wavelength λ, whose entropy is expressed, at high temperatures, by the Sackur-Tetrode formula

$$S(N,V,T) = Nk\ln(V/\lambda^3 N) + 5Nk/2. \tag{18}$$

Within the limit $k \to 0$, such that $N \propto N_A \to \infty$, $\lambda \to 0$, $m \to 0$ (where $N_A k = R = \text{constant}$, $Nm = \text{constant}$, $N\lambda^3 = \text{constant}$), entropy is unvaried.

The fact that Avogadro's number N_A tends, within this limit, to infinity, indicates that CT is the continuum limit of ST, in a manner coherent with the fact that, while the latter is explicitly based on the atomist conception, the former rejects all hypotheses on the discrete structure of matter.

As Compagner has demonstrated, the thermodynamic limit defined by (17) and by the supplementary conditions is substantially identical to the thermodynamic limit usually defined by means of the conditions

$$N \to \infty \quad , \quad V \to \infty \quad , \quad N/V = \text{constant}, \tag{19}$$

where the microscopic variables and intensive macroscopic quantities remain constant, while the extensive macroscopic variables diverge, such as entropy in (18).

The difference between the two approaches consists in a different transformation of scale imposed upon the real system. Definition (17), which treats Boltzmann's constant and Avogadro's number as variables of a metatheoretical language, has however the advantage of showing more clearly the logical relation between ST and CT. In this connection, certain considerations by Boltzmann on the relation between continuous and discrete in the framework of his finitistic conceptions in mathematics and atomistic conceptions in physics are illuminating (Boltzmann [40], Dugas [41]).

The condition $k \to 0$ obviously has the effect of annulling the fluctuations and correlations of the thermodynamic variables caused by the discrete microscopic structure of the system. As a consequence the limits imposed by complementarity on the definition of the thermodynamic concepts also disappear.

7. The $QM - ST$ relation.

The formal analogy between QM and ST is based on the possibility of obtaining, in both theories, uncertainty relations between the conjugate (correlated) variables. These variables represent the statistical correspondent of the conjugate variables of the Hamiltonian theories of which ST and QM are the generalization. The following diagram represents the sets of relations which exist:

CM $[q,p] = [E,t] = 1$	CT $[U, 1/T] = [V, P/T] = 1$
QM $\triangle q \triangle \approx \triangle E \triangle t \approx h$	ST $\triangle U \triangle (1/T) \approx \triangle V \triangle (P/T)$

Having identified a formal similarity between theories or simple equations does not mean we have established complete equivalence of the mathematical structures considered. It is therefore better to clarify at this point that we do not wish to claim that these theories are completely isomorphic: it would be possible, in fact, to highlight differences as well as similarities (such as the fact that QM conjugate variables are linked by Fourier transforms, while those of ST are linked by Laplace transforms) and there are problems that might be treated in order to examine closely the relation between these theories (the role of probability, the theory of measurement). The existence of differences between the objects we are comparing is implicit in the use of analogies, and is why they are utilised with caution. But the damage caused by negative prejudice towards analogical procedures could be as serious as that produced by overestimation. On the other hand, the formal relations described so far seem sufficient to clarify the main points of the paper and show that the conditions (14) and (17) characterize a general relation existing between deterministic and statistical theories, independently of the role played by probability in the statistical theories.

The real problem facing us when studying relations of this type is to establish if the positive analogy (i.e. the set of similar properties which the objects compared display) comprises all the relevant properties of the objects. If that is the case, the analogy spontaneously suggests that we extend to one of the two objects (the less known) the properties which we already know as belonging to the other. If that is not the case, i.e. if certain relevant properties of one object are not shared by the other, then the analogy must be rejected. In any case, however, it is an effective instrument for increasing scientific knowledge (Turner [42]).

Application of these general considerations to the relations in the diagram above permits us to establish in a more explicit manner the link between QM and ST and between ST and CT, and to clarify some questions regarding the actual foundations of ST, on condition that we interpret (13) and (5) in terms of complementarity.

The diagram also suggests additional reflections:

a) The Planck's and Boltzmann's constants, which appear in the uncertainty relations of QM and ST, have the same physical dimensions as the adiabatic invariants of CM and CT, and are the expression of their quantization.

b) The extreme conditions which allow us to pass from QM to CM and from ST to CT, expressed respectively by (14) and (17), correspond to the conditions in which, with the disappearance of the limitations linked to the complementarity relations, a univocal description of the systems under study is obtained: CM attributes pure corpuscular characteristics to particles, while in CT there is no trace of the dynamical description of the microscopic constituents. Vice versa, in the case of small quantum numbers, or of systems made up of small numbers of particles, the corpuscular or the thermodynamic concepts are subject to serious limitations of definition (Feshbach [43]).

c) One last striking aspect of the analogy existing between QM and ST and between the respective complementarity relations is the role that the classical concepts play in them (Bohr [44]; Folse [45], Sklar [23]). In both cases in fact the classical theory (CM or CT) is generalized by a statistical theory which corrects it. But while the classical theory gives erroneous results which are at variance with experimental data, it continues to supply the fundamental concepts through which the new theory describes phenomena, although the latter sets these concepts in a new formal structure and interprets them by means of complementarity.

d) An important difference, which leads us to hold that the two complementarity relations are mutually independent, consists of the different type of physical systems to which they refer: while QM complementarity refers to single systems, ST complementarity describes the properties of systems composed of large numbers of elementary constituents.

Conclusion.

We can summarize the network of inter-theory relations analysed as follows:

$$\begin{array}{ccccc}
CM & \longleftarrow & \text{formal analogy} & \longrightarrow & CT \\
\Big\uparrow & & & & \Big\uparrow \\
\begin{array}{c}\text{reduction of}\\ CM \text{ to } QM\end{array} & & & & \begin{array}{c}\text{reduction of}\\ CT \text{ to } ST\end{array} \\
\Big\uparrow & & & & \Big\uparrow \\
QM & \longleftarrow & \text{formal analogy} & \longrightarrow & ST
\end{array}$$

The horizontal relations are formal analogies between theories which refer to different phenomena, and which cannot therefore be directly reduced one to the other. This applies both to the relation between CM and CT and the relation between QM and ST. Vertical relations, which consist of correspondence principles, are expressed by asymptotic conditions which can be interpreted at a metatheoretical level. The link between the mathematical structures of the theories involved is founded on formal analogies, while the physical interpretations of the concepts and the limits to which their definition is subject remain differentiated. The transition from one theory to another is in any case amenable to rational reconstruction, since the meaning of the concepts is not modified in an arbitrary manner, as is highlighted by the role that the classical concepts play in the context of the complementarity relations.

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