Observation of the squeezed light and quantum description of macroscopic body movement

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ABSTRACT. Possibility of a nondemolition measurement (observation) of macroscopical objects in widespread quantummechanical states arises from the fact of the squeezed light observation. Macroscopical bodies - objects of classical mechanics - are usually in states with narrow wave packets. It is shown that the absence of macroscopical bodies in widespread states can be due to the focusing influence of the body's gravity field on its wavepacket. An evidence that the gravity is essential in the classic limit of quantum mechanics is given.

RÉSUMÉ La possibilité d'une mesure non destructive (observation) d'objets macroscopiques dans des états quantiques étalés résulte de l'observation en lumière comprimée ("squeezed"). Les corps macroscopiques - objets de la mécanique classique - sont habituellement dans des états avec des paquets d'ondes étroits. On montre que l'absence de corps macroscopiques dans de états étalés peut être due à l'influence focalisante du champ de gravité du corps sur son paquet d'ondes. On donne une preuve que la gravité est essentielle dans la limite classique de la mécanique quantique.

In this paper we want to attract some attention to the consequences for some basic questions of quantum theory which are following from the observation of squeezed light. This observation is one of the most important achievements in optics in the last years and it will have many scientific continuations and practical applications. But we think that the most important consequence of the squeezed light observation will be some changes in our understanding of quantum mechanics. In essence, radical changes in the quantum measurement procedure of macroscopic quantum systems appeared in the observation of squeezed light (the squeezed light is a macroscopic effect since it occurs at great energy or large number of photons) and these changes appeared imperceptibly.

Indeed, the observation of the squeezed light showed that the widespread quantum mechanical wave packets can be measured in such a way that the object of measurement (the field in the cavity of the parametric oscillator) remains in the same state as before the measurement. It means that for macroscopic objects the measurements are possible without a wave packet reduction, which is assumed usually in quantum mechanics.

Quantum descriptions of the electromagnetic and mechanical oscillators are fully identical and, consequently, the widespread wave packets for macroscopic material bodies can be also observable. Narrow wave packets have no any advantages thereat in compare with the widespread; therefore a reason of the absence of the macroscopic bodies with the widespread wave packets in the surrounding us world must be clarified.

One possible explanation of this situation is given at the end of the paper. This explanation is based on the idea that the gravitational field of the macroscopic bodies, being equivalent to the space distortion, leads to the selffocusing of the body's wave packet.

1. Quantum description of the macroscopic body movement.

We concern only one problem - the quantum description of the macroscopical bodies movement. The macroscopical body is a body of significant mass, for example, 1 gram (later we define more exactly this value). So we consider the possibility of quantum description of body movement, which obeys, as is well known, to the laws of classical mechanics. Let us consider a simple example - a uniform movement of macroscopical body with mass m and velocity $\bar{v}$ in a free space. This movement is described by the wave packet [1]

$$\Psi(\bar{r},t) = \frac{C}{(1+\mathrm{it}/\tau)^{3/2}} \exp\left[-\frac{\bar{r}^2 - 2\bar{v}\bar{r}\tau + \bar{v}^2\tau t}{2a^2(1+it/\tau)}\right] \qquad (1.1)$$

with a characteristic size a, connected with the packet decay time τ by relation

$$\tau = ma^2/\hbar \qquad (1.2)$$

For a macroscopic body ($m \approx 1$g) and for a characteristic packet size of the order of the size of an atom ($a \approx 10^{-8}$cm), the packet (1.1) describes a well-defined rectilinear trajectory over great time interval, $\tau \approx 10^{11}$s($\approx 3 * 10^3$years).

However, quantum mechanics does not prohibit large values of the parameter a, comparable, say, to the optical wavelength or even to the geometric size of the macroscopic body at any its mass. Narrow wave packets, many times smaller than the geometric size of the body or even smaller than the typical optical wavelength (10^{-6}m), can be naturally attributed to macroscopical classical objects, since in this case the existence of the wave packet can be neglected generally considering it as a point.

However, to what can we attribute wide wave packets? No one macroscopical classical object with great quantummechanical uncertainty of its position was observed in the world surrounding us. If such objects could be seen we would not have such a science as a classical mechanics with its precisely definite trajectories. This fact, peculiar to a quantum mechanics, was observed long ago and gave initiative to a worldwide interpretation of the widespread solutions of the Schrodinger equation. These solutions were interpreted in an ensemble, statistical sense. It was acknowledged that in the measurement of the coordinate of some definite body, similar to that which described by packet (1.1), one can obtain any value, but if one executes many measurements on bodies in identical states then the distribution of the probability of coordinate will be described by the squared module of the wave function $\Psi(\bar{r}, t)$. For example in the textbook of A.Messiah [2] in the discussion of the widely distributed solutions of the Schroedinger equation it is said: "In the classical approximation the function Ψ describes the "liquid" of classical noninteracting particles of mass m(statistical ensemble) ...". A similar interpretation of the distributed solutions is given in the textbook of L.Schiff [1].

After the laboratory observation of the squeezed light such ensemble, statistical interpretation of the widespread wave packets of the macroscopic bodies becomes doubtful. We shall discuss it later and now show some additional examples of the widespread states of the macroscopic bodies.

2. Mechanical oscillator.

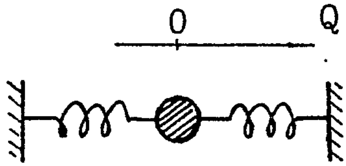

Figure 1. A mechanical oscillator.

In quantum mechanics the hamiltonian [3]

$$H = \frac{1}{2m}P^2 + \frac{1}{2}\kappa Q^2 \tag{2.1}$$

describes a mechanical oscillator (fig.1) with operators of coordinate Q and conjugate momentum P subjected to the commutation relation

$$QP - PQ = i\hbar, \tag{2.2}$$

m is a mass and κ is an elasticity coefficient of the oscillator spring. Introducing variables $p = P/\sqrt{m}$, $q = Q\sqrt{m}$ transforms a hamiltonian to the more usual form

$$H = \frac{1}{2}(p^2 + \omega^2 q^2), (\omega^2 = \kappa/m). \tag{2.3}$$

Operators of creation a^+ and annihilation a of the elementary oscillator excitations

$$a^+ = \frac{1}{\sqrt{2\hbar\omega}}(\omega q - ip) \quad , \quad a = \frac{1}{\sqrt{2\hbar\omega}}(\omega q + ip) \tag{2.4}$$

are important in the oscillator theory. In particular the coherent state of the oscillator $|z\rangle$ (fig.2a) is an eigenstate of the annihilation operator $a|z\rangle = z|z\rangle$; it is a solution of the Schroedinger equation at $z = z_0 e^{-i\omega t}$ and in a coordinate representation it is described by the wave function

$$\Psi_{\text{coh}}(q) = A \exp\Big[\frac{-\omega}{2\hbar}\Big(q - \sqrt{\frac{2\hbar}{\omega}}z\Big)^2\Big]. \tag{2.5}$$

The distribution for this state is shown on fig.2a; it is going periodically from right to left and back according to the harmonic law with amplitude

E_0 and frequency ω. Since the distribution width (or dispersion) of this state is essentially smaller than the oscillation amplitude at $|z| \gg 1 (Q \approx 1\text{cm}, \Delta Q/Q \approx 10^{-14})$ and going to zero with $\hbar \to 0$, this state naturally describes classical oscillations of the mechanical oscillator. Coherent state is a typical state with a narrow wave packet.

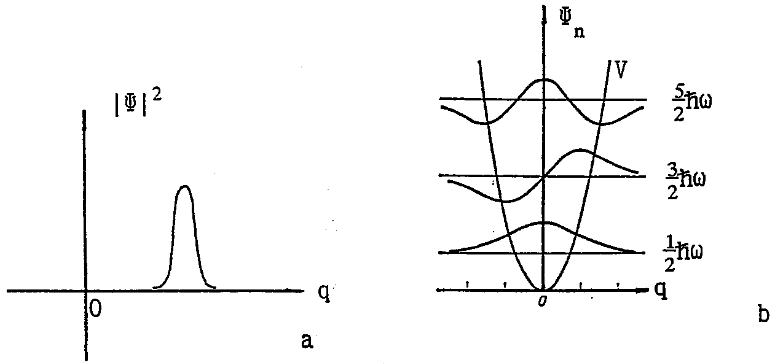

Figure 2. Quantummechanical states of oscillator: a. Coherent state ; b. stationary states

However there are also the stationary states $|n\rangle$, (fig.2b, n – whole numbers) which are the eigen states of the number particle operator (the number of the oscillator excitations): $a^{+}a|n\rangle = n|n\rangle$. The state $|n(t)\rangle = e^{-in\omega t}|n\rangle$ is also a solution of the Schroedinger equation and in the coordinate representation is described by the wave function

$$\Psi_{\text{stat}}(q) = AH_n(\sqrt{\omega/\hbar}q)\exp(-\frac{1}{2}\omega q^2/\hbar), \tag{2.6}$$

where H_n is the Hermite polynome of degree n. The coordinate uncertainty in a coherent state is equal to $\Delta Q_{\text{coh}} = \sqrt{\hbar/2m\omega}$ and in a stationary state is equal to $\Delta Q_{\text{stat}} = \sqrt{(2n+1)\hbar/2m\omega}$.

Uncertainty in stationary state

$$\Delta Q \approx \omega^{-1}\sqrt{E/m}$$

is of the order of the oscillation amplitude in the coherent state at the equal energy ($n = |z|^2$), i.e. it is very great at the great enough energy. So the stationary states at large n are typical widespread states. They are also macroscopical since they exist at high energies ($n \gg 1$) and great masses m. In classical mechanics there is no appropriate movement of some particular object and the stationary states demand an ensemble, statistical interpretation which we concerned above.

3. A macroscopic body movement in the gravitational field.

The movement of the material point of the mass m in the field $U(\bar{r})$, which changes slowly in space, can be described by the gaussian wave packet [4]

$$\Psi(\bar{r},t) = C(t)\exp\left[-(\bar{\rho},F\bar{\rho}) + \frac{i}{\hbar}(\bar{p}_0(t)\bar{\rho} + E(t))\right], \tag{3.1}$$

where $\bar{\rho} = \bar{r} - \bar{r}_0(t)$ and $\bar{r}_0(t), \bar{p}_0(t)$ are solutions of the classic hamiltonian equations

$$\dot{\bar{r}}_0 = \bar{p}_0/m \quad , \quad \dot{\bar{p}}_0 = -\mathrm{grad}_{\mathbf{r}_0} U(\bar{r}_0); \tag{3.2}$$

$3*3$ matrix $F(t)$ is symmetric and has complex elements, which are functions of time (real part of $F(t)$ defines geometric dimensions of the wave packet).

This wave packet is a solution of the Schroedinger equation if in the potential energy expansion near the $\bar{r}_0$ point

$$U(\bar{r}_0+\bar{\rho}) = U(\bar{r}_0) + (\bar{\rho}\ \mathrm{grad}_{\mathbf{r}_0} U(\bar{r}_0)) + \frac{1}{2}(\bar{\rho}, U"\bar{\rho}) + \dots . \tag{3.3}$$

the third order terms marked by dots can be neglected. The $3*3$ matrix $U"$ of the second derivatives is also symmetric. Its real elements

$$U"_{\alpha\beta}(t) = \frac{\partial^2 U(\bar{r})}{\partial x_\alpha \partial x_\beta}\ |_{\bar{r}=\bar{r}_0(t)} \tag{3.4}$$

are time-dependent through the vector $\bar{r}_0(t)$.

The wave packet (3.1) is moving along a classical trajectory (3.2) until its dimensions are so small that the third order terms in the potential energy (3.3) can be neglected. The matrix $F(t)$, which defines in particular the wave packet dimensions, must satisfy the matrix equation

$$i\hbar\dot{F} = \frac{2\hbar^2}{m}F^2 - \frac{1}{2}U", \tag{3.5}$$

which is similar to the Ricatti equation. The values $C(t)$ and $E(t)$ are equal to

$$\begin{aligned} C(t) &= C_0 \exp\left[-\frac{i\hbar}{m}\int_0^t dt(F_{11}+F_{22}+F_{33})\right], \\ E(t) &= \int_0^t dt\left[\frac{\bar{p}_0^2}{2m} - U(\bar{r}_0)\right]. \end{aligned} \tag{3.6}$$

For macroscopic bodies the gravitational field is the most important. As is well-known the classic mechanics appeared from the observations on the macroscopical bodies - planets movement in the field of Sun. In this case the trajectory (3.2) does not depend on mass, since $\bar{p}_0$ and $U(\bar{r}_0)$ both are proportional to m. Introducing in (3.5) an evident dependence $U(\bar{r}_0)$ on mass m, the equation

$$i\dot{\Phi} = 2\Phi^2 - \frac{1}{2}V, (\Phi = F\hbar/m, mV = U''), \tag{3.7}$$

in which all quantities does not depend on mass m, can be obtained. So one can see, that F is growing with the growth of mass m and it means that the wave packet dimensions are decreasing and the wave packet concentrates itself more and more near the classic trajectory (3.2). Consequently the wave packet (3.1) describes naturally in the frames of quantum mechanics the macroscopic body movement.

But it is not the only way to quantum description of the macroscopic body movement. Another way is connected with a well-known eiconal method [2]. When the mass m is great enough and consequently when the deBroglie wavelength $\lambda = \hbar/\sqrt{2mE}$ is small (E- energy of state) the stationary solution of the Schroedinger equation can be written in the form

$$\Psi(\bar{r}, t) = A(\bar{r}) \exp[i(S(\bar{r})/\lambda - \text{ Et}/\hbar)], \tag{3.8}$$

where $S(\bar{r})$ - the eiconal function, which changes a little on the length of the order of the wavelength λ and which satisfies the equation

$$(\text{gradS}(\bar{r}))^2 = 1 - (U/E). \tag{3.9}$$

Amplitude $A(\bar{r})$ also changes a little on the length of the order of the wavelength λ and satisfies the equation

$$(\overrightarrow{\text{grad}}\, S(\bar{r})\, \overrightarrow{\text{grad}}\, A(\bar{r})) + \frac{1}{2}A\Delta S = 0. \tag{3.10}$$

In a gravitational field U and E both proportional to the mass m and then S does not depend on m. Curves orthogonal to the wave surfaces $S(\bar{r})$=const are the classical trajectories of the material body.

In this section basic elements of quantum description of the macroscopical body movement in a gravitational field [2,4] are given. As in the mechanical oscillator, there are two types of the quasiclassical solutions. The narrow gaussian wave packet solution (3.1) naturally describes a classical movement of the macroscopical body. But there are also widespread solutions of the eiconal type. The eiconal function $S(\bar{r})$ is widespread as in a longitudinal, so in the transversal directions to the trajectories. In the transversal direction it can be bound only by possible caustic surfaces (see, for instance, fig.3). If such state is considered as a state of one definite body then there is no anything in classic mechanics corresponding to it.

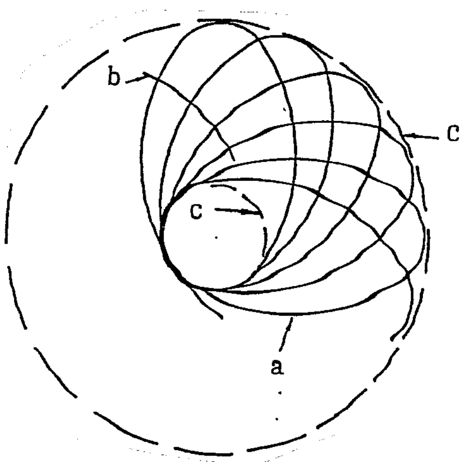

Figure 3. An orbit system for the gravitational field which forms an eiconal solution of the Schroedinger equation: a.trajectory, b.wave front c.caustic lines

In conclusion of this section it should be noted that in quantum mechanics the transition to a classic limit is not such automatic as, say, in the theory of relativity. There it is enough the body velocity divided by the light velocity to be small for the appearance of the classic mechanics laws from the relativity laws. But in quantum mechanics only part of possible states, namely narrow wave packets gets into the classical movements with the growth of mass. Another part of possible states - widespread solutions - can be interpreted in a classical style only in the ensemble, statistical sense. Narrow wave packets can be also interpreted in the ensemble, statistical sense of course, but for them this interpretation is not necessary.

4. Squeezed light and its observation.

Let us look now at a squeezed light. As a laser light it can be excited in an optical cavity. The cavity field is a superposition of the fields of the partial modes or resonances. It is well-known from practice that in a cavity one mode field excitation is possible. Since the space distribution of the mode field is defined by the boundary conditions, so the mode field can be considered as a system with one degree of freedom which coordinate is an electric field of the mode in some chosen point of the cavity. Then the quantum theory of the mode field coincides with the quantum theory of the mechanical oscillator. This field has the same hamiltonian (2.3), where coordinate q must be replaced by the mode electric field E in the chosen point of the cavity. In particular different states - coherent and stationary, which were mentioned above - are possible for the mode field.

But these two state types are not only possible in mechanical or electromagnetic oscillators. Squeezed state joints in itself the properties of the narrow and widespread states [5,6]. In this state the electric field value distribution is described by the gaussian wave packet (fig.2a) as in the coherent state but the dispersion of this distribution is different from the dispersion of the coherent (or vacuum) state. It can be shown that in this case the dispersion changes periodically with the double frequency of the oscillator from the value smaller than the dispersion of the coherent state to the value greater than it.

Basic elements of the squeezed state theory are given below. Squeezed states of light [5,6] are the eigen states $|\zeta\rangle$ of the operator $\mu a + \nu a^+$

$$(\mu a + \nu a^+)|\zeta\rangle = \zeta|\zeta\rangle, (|\mu|^2 - |\nu|^2 = 1), \tag{4.1}$$

where μ, ν, ζ - complex numbers. If μ and ν depend on time $\mu = \mu_0 e^{i\omega t}, \nu = \nu_0 e^{-i\omega t}$, then the states $|\zeta\rangle$ are the solutions of the Schroedinger equation. The most expressive representative of the squeezed states is the squeezed vacuum state corresponding to $\zeta = 0$. In coordinate representation it is described by the wave function

$$\Psi_\zeta(q) = A \exp\left[-\frac{\omega q^2}{2\hbar}\frac{\mu + \nu}{\mu - \nu}\right]. \tag{4.2}$$

The squeezed state dispersion

$$D^2 = \frac{\hbar}{2\omega}\left[|\mu|^2 + |\nu|^2 - 2|\mu| * |\nu| \cos(\Psi_0 + 2\omega t)\right] \tag{4.3}$$

oscillates with the double oscillator frequency. It can be smaller than the coherent state dispersion $D^2_{\rm min} = \frac{\hbar}{2\omega}(|\mu| - |\nu|)^2 < D^2_{\rm coh} = \frac{\hbar}{2\omega}$ and can be greater than it $D^2_{\rm max} = \frac{\hbar}{2\omega}(|\mu| + |\nu|)^2 > D^2_{\rm coh} = \frac{\hbar}{2\omega}$ (fig.4). The instants when the dispersion is small gave initiative to the term - squeezed state. A squeezing coefficient

$$K = D_{\rm coh}/D_{\rm min} = \sqrt{D_{\rm max}/D_{\rm min}} = |\mu| + |\nu| = (|\mu| - |\nu|)^{-1} \qquad (\ 4.4)$$

is a characteristic of the squeezed state. There is an energetic limitation for the squeezing coefficient. At the average photon number N the maximum squeezing coefficient is equal to

$$K_{\rm max} = \sqrt{N+1} + \sqrt{N}. \qquad (4.5\ )$$

The qualitative picture of the squeezed state can be accepted from fig.4. Fig.4a gives the coordinate oscillations versus time in the coherent state. The constant dispersion of this state is shown by the thickness of sine line. At the usual laser intensity, say, at the 1 joule energy, stored in the laser cavity, the ratio of the dispersion to the oscillation amplitude is small, $\approx 10^{-9} - 10^{-10}$.

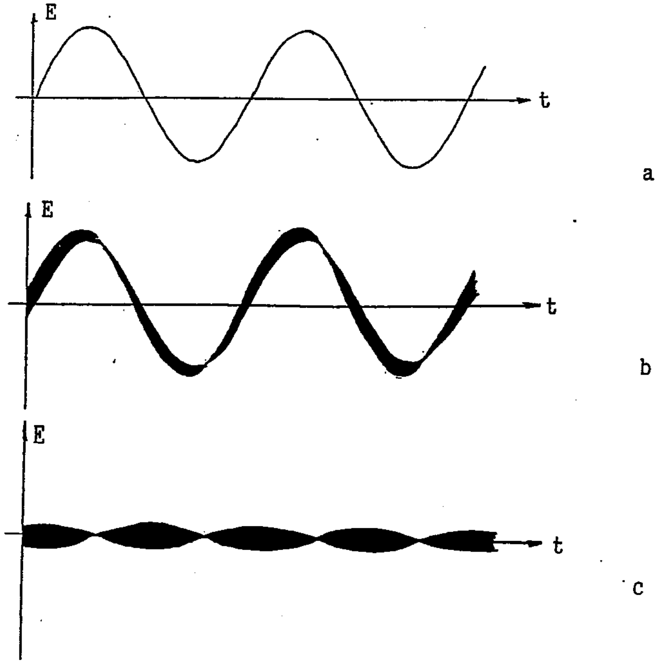

Figure 4. Field oscillations versus time: a.coherent state b.squeezed state c.squeezed vacuum state

Fig.4*b* shows oscillations in the squeezed state. Here the dispersion changes with time and at some instants is of the same order of value as amplitude. The points, where the dispersion is minimal, initiated the name of states as squeezed. Fig.4*c* shows "oscillations" in squeezed vacuum state. Inverted commas are used to pay attention that now there are no any oscillations with ω frequency, but only the dispersion changes with the double frequency. Emphasize that although the oscillations are absent the squeezed vacuum is a high excited, macroscopical state with great energy.

Parametric excitation of the oscillator is described by the Schroedinger equation

$$i\hbar\frac{\partial\Psi(q,t)}{\partial t} = \frac{1}{2}\left[p^2 + \Omega^2(t)q^2\right]\Psi(q,t).$$

As can be seen a parameter of the oscillator - its resonant frequency Ω - depends on time. The solution of this equation, similar to the squeezed vacuum state, is

$$\Psi(q,t) = A(t) * \exp\left[\frac{i\dot{\epsilon}(t)}{2\hbar\epsilon(t)}q^2\right], \tag{4.6}$$

where $\epsilon(t)$ is the solution of the equation

$$\ddot{\epsilon}(t) + \Omega^2(t)\epsilon(t) = 0 \tag{4.7}$$

which satisfies initial conditions $\epsilon(0) = 1$, $\epsilon(0) = i\Omega_0$ (Ω_0 is the average oscillator frequency). The dispersion of the state (4.6) equals

$$D = |\epsilon(t)|\sqrt{\hbar/2\Omega_0}.$$

The squeezing coefficient grows with time if $\epsilon(t)$ is not bound solution of the equation (4.7).

To this time squeezed light was observed in some laboratories [7,8]. One of the possible devices was realized by a group of investigators from the Texas University and is given on fig.5. Although this experiment was not the first one, it is the most fruitful from the engineering point of view. It consists of three major parts - the source of parametric pumping, the parametric generator and the detector-analyzer of squeezed states. The first two parts in major features are similar to that which were used in the first experiments on the parametric generation [9,10]. The source of the parametric pumping power is the solid-state ring laser with a YAG:Nd^{3+} and with an intracavity excitation of the second harmonic in the nonlinear crystal. The second harmonic of the Nd-laser is used for pumping the parametric generator, where the first harmonic appears again but now in the squeezed vacuum state. The receiver-analyzer compares the signals from laser and parametric generator and measures the dispersion of the squeezed light in the instants close to the maximums of the amplitude of the laser signal.

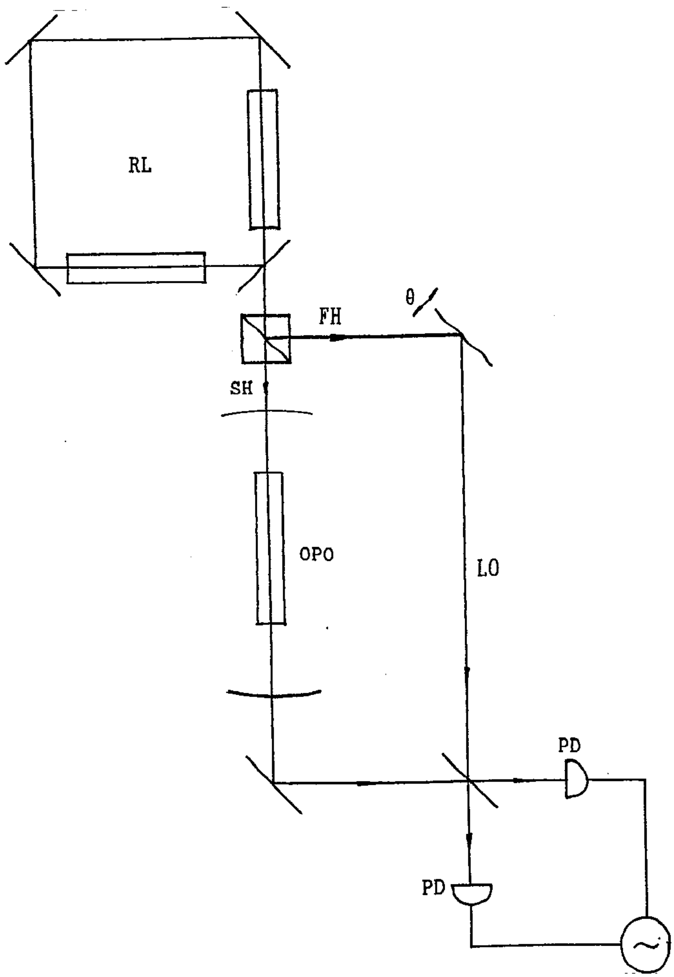

Figure 5. A lab device for observation of the squeezed light: *RL* - ring *Nd*-laser with intracavity generation of the second harmonics, OPO - optical parametric oscillator, LO - local oscillator, *PD* - photodiode, *FH* - first harmonics, *SH* - second harmonics

The results of observation are shown in fig.6. As one can see the parametric signal dispersion changes twice for a period of the laser signal: its period is π, but not 2π. One can see also that at some phases θ the squeezed light dispersion becomes smaller than the dispersion of the coherent light (dashed line). It is just the evidence that the light is in the squeezed state.

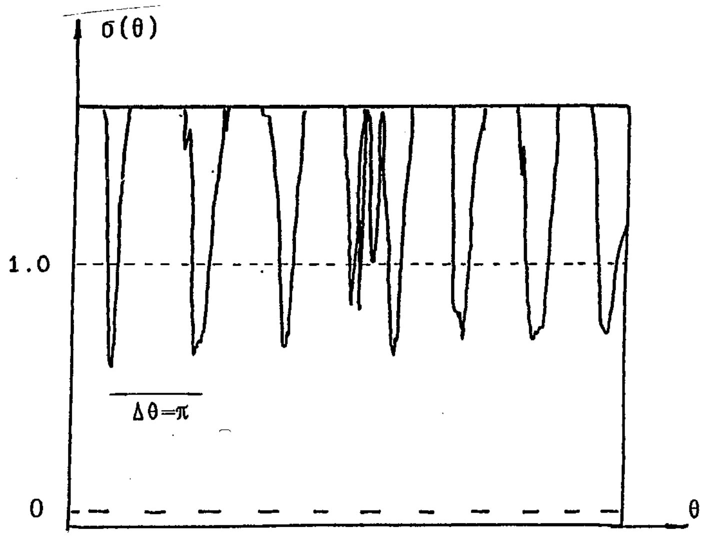

Figure 6. A dispersion of the squeezed light versus the phase of the coherent signal

Take a notice on the unusual character of the measurement procedure. There are no in it partial measurements of the field strength, which in usual quantummechanical measurement procedure form the distribution $|\Psi(E)|^2$. The result of a measurement now is just the state parameter, in our case - the state dispersion. Another peculiarity of measurement is its nondemolition (or nondisturbing) character. Indeed the object of measurement is the light beam, coming from the parametric generator cavity. This beam does not come back into the cavity independently if the measurement was made with it or not. If such measurement was made the information about it cannot reach the cavity and therefore the field state in the cavity cannot be changed. But the results of measurement of the light beam, coming from the cavity, give us an information about the state of the field in the cavity.

Note that due to the dynamic equilibrium between pumping and losses the parametric generator field is stationary. Therefore if we made measurement one time, we can be sure that the field in the cavity is, say, in the squeezed vacuum state for a long time. So the squeezed light observation tells us that there is a macroscopical (number of photons is great) quantummechanical object - the electromagnetic field of the picked out cavity mode - which dispersion we can measure, leaving the object itself in the state in which it was before measurement.

The time instances when the dispersion of the field is smaller than the dispersion of the coherent state extremely attracted attention of the investigators. It is natural since the quantummechanical uncertainties, just as the noise, prevent the precise measurement of corresponding quantities. The smaller this uncertainty the more precisely corresponding quantity can be measured and small uncertainty is an attractive feature of the light squeezed states. But from the point of view of the researcher the states with a great uncertainty are more interesting; they are less classical and more quantummechanical. For this paper they are important since its observation, taking into account that was said above about the squeezed light measurements, shows that the measurement of the widespread state dispersion is possible without disturbing of this widespread state.

Naturally if such measurements are possible with electromagnetic oscillator, they are possible also with a mechanical oscillator. This idea we develop in the next section.

5. Macroscopic bodies in widespread states and their non demolition measurement

So it was shown something more in experiments with squeezed light than its generation possibility. In addition it is shown that the quantummechanical state of one macroscopical object (field of a particular cavity mode) can be learned in a nondisturbing manner. In particular it can be learned that this object is in the widespread state when the uncertainty of its coordinate is a macroscopical value approximately equal to the oscillation amplitude in the coherent state at equal energies of both states.

If such learning is possible with electromagnetic oscillator it is also possible with a mechanical oscillator and even with a macroscopic body freely moving in the space since it can be seen as an oscillator with a zero frequency. Such conclusion is a natural consequence of a similar description of the electromagnetic and mechanical oscillators (see also below).

One can ask - why there are no macroscopic bodies in widespread states in the world surrounding us. Quantum mechanics does not prohibit such states (see first section), and in some cases they are preferable.

One possible explanation of it is concluded in supposition that at the observation of such bodies its wave packet reduction occurs. To exclude such possibility we considered the electromagnetic signal reflection from the body (mirror) which is in the widespread state [11]. Major results of this consideration are given below.

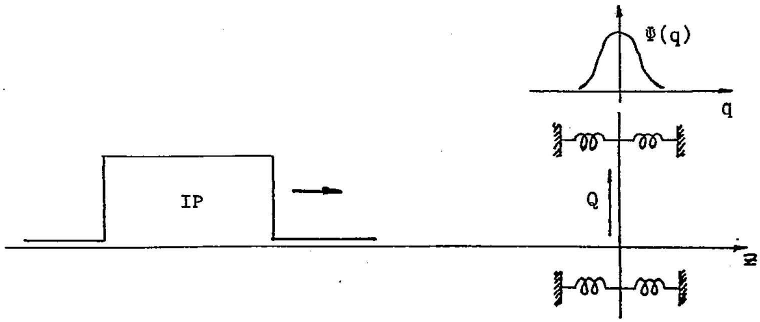

Figure 7. A reflection of light from the mirror, which quantum mechanical state is widespread

We suppose [11] that the mirror has two degrees of freedom (fig.7). The first degree of freedom, transversal oscillator describes the motion of the negative charges (bounded electrons) along the mirror (displacement Q, conjugate momentum P, surface mass density ρ, surface charge density σ). This motion leads to the reflection of the electromagnetic signal. The second degree of freedom, longitudinal oscillator describes the motion of the mirror along the direction of the electromagnetic wave propagation (coordinate q, conjugate momentum p, surface mass density μ). It is supposed also that there are only waves of normal incidence to the mirror. The mirror is infinitely thin and perpendicular to the z-axis along which waves propagate. The electrical field and the charge displacement directed along x-axis. The region of space occupied by the field has the same section form as a form of the mirror and its area is equal to s. Then quantum system mirror+field is described by the hamiltonian

$$
\begin{aligned}
H = & \frac{1}{2\rho s}\,(\,P - \tfrac{\sigma s}{c}A(q)\,)^2 + \frac{1}{2}sKQ^2 \\
& + \frac{1}{2}s\int dz\,[\,\tfrac{1}{4\pi}(\tfrac{\partial A}{\partial z})^2 + 4\pi c^2\Pi^2\,] + \frac{1}{2\mu s}p^2 + \frac{1}{2}s\kappa q^2
\end{aligned} \tag{5.1}
$$

where K and κ - the transversal and longitudinal oscillator elasticities, A(z) - vector-potential of the electromagnetic field and $\Pi(z)$ - its canonically conjugate momentum.

The field oscillators (plane waves) in the initial instant are in coherent states with such phases that the field forms a rectangular pulse filled with a high frequency oscillations

$$\langle E_{\text{in}}(z,t)\rangle = E_0 \sin\omega_0\tilde{t}\,\Pi(\tau_0,\tilde{t}), \tag{5.2}$$

where $\Pi(\tau_0,\tilde{t})$ - a function describing the rectangular form of the in-pulse of the $2\tau_0 = 4\pi n_0/\omega_0$ duration ($2n_0$ - number of wavelength in the pulse), $\tilde{t} = t - \bar{t} - z/c$ and $\bar{t}$ - the instant of the arriving of the pulse at the origin. The transversal oscillator initially is in the ground or vacuum state and the longitudinal oscillator is in a squeezed state described by the wavefunction

$$\Psi(q) = (2\pi\bar{q}^2)^{-1/4}\exp(-q^2/4\bar{q}^2), \tag{5.3}$$

where $\bar{q}^2 = \hbar/(2s\mu\nu\sqrt{K})$ and K is the squeezing coefficient. The additional parameter in the wavefunction (5.3) - the coefficient of squeezing - provides the possibility to change the longitudinal oscillator parameters μ,ν in future without changing its distribution; in particular the transition to the motion in the free space will become possible.

The investigation of this problem shows that the field averaged value in the reflected signal in the stationary conditions and in the resonance ($\Omega = \omega_0$) is

$$\langle E_R(z,t)\rangle = E_0 e^{-2\bar{q}^2/\lambda^2}\sin\Omega(\tau - \bar{t}), \tag{5.4}$$

where $\lambda = c/\Omega$ and $\tau = t + z/$c. The average value of the squared field in the reflected signal is

$$\langle E_R^2(z,t)\rangle = \frac{1}{2}E_0^2\left[1 - e^{-8\bar{q}^2/\lambda^2}\cos 2\Omega(\tau - \bar{t})\right]; \tag{5.5}$$

this value is proportional to the electrical energy density. As can be seen when the position uncertainty of the mirror is small ($\bar{q} \ll \lambda$) the reflected signal keeps the properties of the coherent state; in particular $\langle E_R \rangle^2 \simeq \langle E_R^2 \rangle$. When the position uncertainty is large ($\bar{q} \gg \lambda$) the field average value is close to zero but the average value of the squared field is not close to zero and keeps the finite value; only double frequency oscillations of energy density are close to zero. One can say that the amplitude reflection coefficient goes to zero when the position uncertainty grows, but the intensity reflection coefficient keeps finite value in the same case. It means that the reflected signal being a macroscopical one is in essentially quantum state, as only in such states the relation $(\langle E \rangle^2) < (\langle E^2 \rangle)$ is possible.

It can be shown also that the length of the reflected signal greater than the length of the incident signal approximately on q̄.

Consequence of these results is the conclusion that it is possible to find out experimentally the macroscopical body (mirror) to be in a widespread quantummechanical state as a result of one pulse reflection from this body without essential change of its state. Indeed the reflected pulse, as we could see, bears the information about the widespread state of the mirror and being the macroscopical one can be analyzed with the existing experimental means such as used for example to analyze the squeezed light [7,8]. At the same time it can be shown [13] that the reflection process does not change the mirror state essentially. For macroscopic objects such possibility contradicts to the usual interpretation of the widespread state as describing an ensemble of objects [1,2].

We call attention also to the lengthening of the reflected pulse $\langle \langle E_R^2 \rangle \rangle$ compared with the incident one, due to the partial reflection of the incident pulse from different layer of the mirror distribution over the longitudinal coordinate. The human eye with its low temporal resolving power cannot, of course, notice such a lengthening (of the order $\bar{q}$) of the pulse. For oblique incidence of light from a point source on a mirror, however, this partial reflection from different distribution layers is converted into an angular distribution of different rays, and the angular resolution of the human eye is high enough. A naked eye, therefore, will see bodies with large quantum-mechanical uncertainty simply as blurred. Consequently, were macroscopic bodies with large quantum-mechanical uncertainty to exist in the world surrounding us, they could be simply seen with the naked eye.

6. Why there are no macroscopic bodies in widespread states.

The absence of macroscopic bodies in widespread states could be explained by their formation in the concentrated state at some initial time and their subsequent gradual diffusion. But there are no reasons why they should be formed just in the concentrated state at the initial time.

The concentration of the macroscopic body wavepackets cannot be explained in terms of their interaction with some fields. In fact, since bodies consist of charged particles, they must interact with the electromagnetic field. However, such interaction leads only to the formation of an associated (nonradiative) field, since all particles in uniform rectilinear motion have such fields. A similar argument also applies to other fields. In all cases, the packet (1.1) describes the motion of the center of inertia of the macroscopic body and its associated fields, and consequently, the problem of large values of the parameter a cannot be removed in this way.

An exception is the interaction of the macroscopical body with the gravitational field [12]. According to the general theory of relativity, the gravitational field created by the body can be considered as a deformation of space, a deviation it from Euclidian, with this deformation occurring both in the region occupied by the body and in its vicinity. If the mass related to some part of the wave packet deforms space, the remaining parts of the packet move in this deformed space. In general, any part of the wave packet moves in the space deformed by this part itself and by all the other parts. Consequently, there is an effect of the wave packet on itself, or a self-effect of the wave packet caused by its interaction with the gravitational field. As is shown below this self-effect leads to the formation of an effective potential well within which the wave packet preserves permanently its concentrated form. Moreover, the formation of such a potential well is obviously energetically favored, and this explains the concentration of the wave packets of macroscopic bodies.

Since we are dealing with macroscopic bodies of ordinary density (of the order of 1 g/cm^3), the gravitational potential is weak, and therefore the Newtonian expression for the potential is sufficient for calculation.

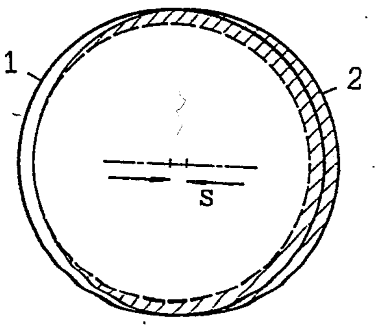

Figure 8. Attraction of two mutually penetrating balls.

Let us examine a uniform spherical body of radius R and density ν. Since we will calculate the gravitational effect of one part of the wave packet on another, and since the wave packet is much smaller than the geometric size of the body, we will first examine the attraction of two mutually penetrating massive spheres whose centers are displaced by a distance s which is much smaller than R (fig.8). As can be seen, the attraction between the spheres can be calculated by taking into account the attraction by the first sphere of a layer of thickness $2s$ which cover half of the first sphere. Considering only the projection of the forces of attraction of unit masses along the line joining centers of the spheres, we obtain the following expression for the force of attraction of the spheres:

$$F = \frac{16\pi^2}{3}\nu^2 R^3 s \int_0^{\pi/2} d\theta \sin\theta \cos\theta = \frac{8\pi^2}{3} G\nu^2 R^3 \mathrm{s}. \tag{6.1}$$

Consequently, the potential energy of the interaction of the two spheres is

$$U_s = \beta s^2, \beta = \frac{4}{3}\pi^2 G\nu^2 R^3. \tag{6.2}$$

Taking into account this expression for the potential energy, we obtain a Schroedinger equation describing the wave packets of the macroscopic bodies

$$i\hbar \frac{\partial \Psi(\bar{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi(\bar{r},t) + U(\bar{r})\Psi(\bar{r},t), \tag{6.3}$$

where

$$U(\bar{r}) = \beta \int d\bar{r}'(\bar{r}-\bar{r}')^2 |\Psi(\bar{r},t)|^2. \tag{6.4}$$

This equation shows that the gravitational self-effect of the wave packet of a macroscopic body is similar to the optical self- focusing effect. The only difference is that self-focusing occurs in the two directions perpendicular to the direction of wave propagation, whereas the gravitational self-effect occurs in all three directions.

Let us find the steady-state, spherically symmetric solution of Eq.(6.3). For $\Psi = \Psi(\bar{r})$ and $\mathrm{Im}\Psi = 0$, we have, according to (6.4),

$$U(\bar{r}) = \alpha + \beta r^2, \tag{6.5}$$

where $\alpha = \beta \int dr'^2 |\Psi(r')|^2$ is an unimportant constant. Thus, to determine $\Psi(r)$, we have the equation

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right) + \beta r^2\Psi = E\Psi, \tag{6.6}$$

whose solution is

$$\Psi(r) = \frac{1}{\sqrt[4]{\pi^3}\sqrt{r_0^3}} \exp\left[-\frac{r^2}{2r_0^2}\right], \tag{6.7}$$

where

$$r_0 = \sqrt[4]{\frac{\hbar^2}{2\beta m}} = \sqrt[4]{\frac{2\hbar^2 R^3}{3Gm^3}} \tag{6.8}$$

is the characteristic size of the wave packet due to the gravitational self-effect. This concentrated wave packet can be considered as a soliton solution of the Schroedinger equation (6.3). Some values are presented in the next Table.

m, g	R, cm	r_0, cm
10^{28}	$1.3 * 10^9$	$1.25 * 10^{-26}$
10^{12}	$6.2 * 10^3$	$1.25 * 10^{-18}$
1	0.62	$1.25 * 10^{-12}$
10^{-12}	$6.2 * 10^{-5}$	$1.25 * 10^{-6}$
10^{-15}	$6.2 * 10^{-6}$	$4 * 10^{-5}$.
10^{-27}	10^{-13}	0.03.

The mass, geometric size of the macroscopic body with density $\nu \approx 1\mathrm{g/cm}^3$ and size of its wave packet are given in the respective columns.

Data for a typical macroscopic body ($m = 1$g) is given in the middle row of Table. The top row presents data for a body with a mass of the order of the mass of the Earth, while the last row is that for a mass of 10^{-15}g, for which the geometric size is comparable to the size of the wave packet. The size corresponding to the condition $r_{\rm cr} = r_0 = R$ can be determined from (6.8):

$$r_{\rm cr} = \sqrt[10]{\frac{9\hbar^2}{32G\pi^3\nu^3}} \tag{6.9}$$

The condition (6.9) gives a quantitative criterion for dividing bodies according to their size into macroscopic and microscopic. A criterion for such division by mass can also be obtained from (6.9). Bodies with mass greater than

$$m_{\rm cr} = \sqrt[10]{\frac{32\pi\nu\hbar^6}{81G^3}} \tag{6.10}$$

should be considered macroscopic.

Thus, for masses greater than 10^{-12}g, the size of the wave packet is negligible not only compared to the geometric size of these masses, but also compared to the typical optical wavelength ($10^{-4} - 10^{-5}$cm).

This discussion explains why there are no macroscopic bodies in states with large quantum-mechanical uncertainty of their center of mass. But this explanation was reached by the very high prize, namely by refusion from the superposition principle and the linearity of the Schroedinger equation [13], as can be seen from (6.3). However the violation of the superposition principle is not very profound. In particular it will not influence the microscopical, atomic effects - the basic field of the quantum mechanics application.

It may be that the refusing from the superposition principle is temporary. After the creation of the quantum theory of gravity our considerations can appear as similar to a semiclassical theory in quantum electrodynamics in which the medium is described quantummechanically and the radiation - classically. In quantummechanical description of both the gravity and medium superposition principle can be restored.

The potential well discussed above is not very deep. By expending some energy a macroscopic body can therefore be changed to a widespread state with a large wave packet. Study of the widespread states of macroscopic bodies would be of great scientific value.

7. Conclusion.

Considerations stated above show that in mechanics of macroscopical bodies the gravity can play an important role. Due to gravity the wave packets of macroscopical bodies are narrow and the classical mechanics is mechanics of narrow but not widespread packets.

These considerations shows also that macroscopical bodies can be in the widespread states and it is a matter of importance to observe them practically. Bodies in such state will be new objects of physical investigation. A natural way to excite such state is a parametric excitation of the oscillator. An oscillator excited in this way two times for a period becomes quantummechanical widespread.

It is clear also why the widespread quantum states were observed first for the electromagnetic field but not for mechanical system. The electromagnetic field mass is very small at usual intensities and it cannot deform the space and besides that the space of the field states is not our 3-dimension space but is abstract Hilbert one. So the narrow wave packets of the field have no an advantage over the widespread states.

In connection with said above it is correct to refer the A.Einstein paper [14], who was the first raised a question about the correct description of the macroscopic body movement.

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