Sommerfeld fine structure constant α and its physical interpretation

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ABSTRACT. Sommerfeld explained the fine structure terms of hydrogen atomic spectra considering relativistic mass variation of electron in the K-shell. In his expression he introduced the term "Fine Structure Constant $\alpha = 2\pi e^2/hc$" which is a dimensionless number. The α is also used in other physical theories. Salam, Glashow, Weinberg and others successfully used it in unifying electromagnetic and weak forces of Nature. The inverse of α is nearly equal to 137, a "prime number". It has been shown in this paper that 137 is the maximum atomic number upto which an atom can be built up in our Universe. If Z suddenly increases beyond 137 the K-shell of the atom will collapse and with it probably all the upper atomic shell structures will collapse one by one. It may be dangerous to try to build element artificially beyond $Z = 137$. This may have far reaching consequences in physics and astrophysics.

Further, the number 137 is very very fundamental. So, out of the three presently known fundamental constants e, $\hbar$ and c related to α one should be a "derived constant". It has been discussed in this paper which one is possibly a "derived constant" and which two are fundamental constants.

1. Introduction.

The "Fine Structure Constant α" was originally introduced by Sommerfeld [1] while accounting for the fine structure terms of hydrogen atomic spectra. It is given by

$$\alpha = \frac{2\pi e^2}{hc} = \frac{e^2}{\hbar c} = \frac{1}{137 \cdot 03602(\pm 00021)} \tag{1.1}$$

where

e = unit of atomic charge

h = Planck constant

$h/2\pi = \hbar$ = Dirac constant

c = velocity of electromagnetic wave in vacuum

α is a dimensionless constant. It predicts a deeper relation between electrodynamics and quantum theory, Born [2]. Its numerical value poses a challenge to physics. Attempts were made to deduce the above numerical value theoretically. One such attempt was made by Eddington who gave $1/\alpha = (1/2)n^2(n^2+1)+1$, for $n = 4$ it gives exactly 137. Fine structure constant is extensively used in high energy physics, Perkins [3]. It has been successfully used in the unification of electromagnetic and weak forces in nature by Salam [4,5], Glashow [6], Weinberg [7], and see also their Nobel lectures [8]. Adler [9] provided three speculations for determination of α one of the important fundamental constants that controls phenomena from atomic physics to quantum electrodynamics : (1) from cosmological boundary condition, (2) from microscopic interplay between electromagnetic interaction and other interactions like strong, weak or gravitational type, and (3) from microscopic properties of electromagnetic interaction alone. He put aside the first two as still to-day they are less understood physically. The third one is a well established theory and stood the test of experiment and time. Adler discussed the determination of α from perturbation theory and renormalization, Schweber [10], Jost and Luttinger [11] and nonperturbation theory of Gell-Mann and Low [12].

Section 2 deals with electromagnetic interaction, Bohr quantum condition and a simple way to deduce the relation given in eqn.(1.1). The physical significance of α is interpreted in Section 3. Section 4 discusses the four physical constants α, e, $\hbar$ and c.interconnected through eqn.(1.1). We feel, if the nearest to $1/\alpha = \hbar c/e^2$ is Nature's choice of a fundamental limit then out of e, $\hbar$ and c two will also be Nature's choice of two limits and fundamental. The third one then must be a derived constant. Concluding remarks are given in Section 5.

2. Electromagnetic theory and the atomic model.

All of us are aware that Nature works in a simple way through simple models and simple mathematical principles. As the system grows,

complexity increases and so also the mathematical method. Many experiments have proved Faraday's electromagnetic force and Rutherford-Bohr hydrogen atomic model. Following Bohr [13], taking first hydrogen orbit as circle and his quantum condition we obtain

$$\frac{e^2}{R_{1(1)}^2} = \frac{m_0 v_1^2}{R_{1(1)}} \tag{2.1}$$

$$m_0 v_1 R_{1(1)} = \hbar \tag{2.2}$$

where m_0 = rest mass of electron, $R_{1(1)}$ = radius of first hydrogen orbit of Bohr, $v_{1(1)}$ = velocity of electron in the above first orbit, subscript : first = principal quantum number, 1, second = nuclear charge, (1). Combining eqn.(2.1) with eqn.(2.2) we get

$$v_{1(1)} = \frac{e^2}{\hbar}$$

Dividing both sides by c

$$\frac{v_{1(1)}}{c} = \frac{e^2}{\hbar c} = \alpha \tag{2.3}$$

It is the simplest way to deduce Sommerfeld fine structure constant α. Further, it also gives a powerful, although simple physical insight into the phenomena behind α. The quantum mechanical model of an atom with Ze charge in the nucleus and the outer electrons having four quantum numbers n, l, m_l and m_s will be difficult to visualize. However as Nature follows simple rules, there will be no harm if Bohr's circular model is extended to nth orbital electron in an atom with Ze nuclear charge. Equations (2.1) and (2.2) will become

$$\frac{Ze^2}{R_{n(Z)}^2} = \frac{m_0 v_{n(Z)}^2}{R_{n(Z)}} \tag{2.4}$$

$$m_0 v_{n(Z)} R_{n(Z)} = n\hbar \tag{2.5}$$

They will lead to

$$v_{n(Z)} = \frac{Ze^2}{n\hbar}$$

Dividing both sides by c

$$\frac{v_{n(Z)}}{c} = \frac{Ze^2}{n\hbar c} \tag{2.6}$$

For the first orbit with $n = 1$, we get

$$\frac{v_{1(Z)}}{c} = \frac{Ze^2}{\hbar c} = Z\alpha \tag{2.7}$$

There is more physical possibility for the first orbit being circular. Hence, eqn.(2.7) is physically more correct. Further, the highest speed of the orbital electron comes from K-shell i.e. when $n = 1$. Equation (2.7) shows that the velocity $v_{1(Z)}$ will be equal to the limiting velocity c of our Universe when the atomic charge attains $Z = 137.03602(\pm 00021) = 1/\alpha$.

3. Physical meaning of fine structure constant α.

We feel that the real and physical meaning of Sommerfeld fine structure constant α is that the inverse of it, which is also a dimensionless number, is the maximum atomic number beyond which we cannot build an atom in our Universe, presented by author [14].

$$\frac{1}{\alpha} = \frac{\hbar c}{e^2} = 137.03602(\pm 00021) \tag{3.1}$$

If Z is suddenly made 138 the velocity of electrons in the K-shell would likely to cross the velocity c and will naturally disintegrate. With the disintegretion of K-shell i.e. the first base of atomic structure, the atomic structure itself will probably collapse step by step. Upto the beginning of 1985 and also till the end of 1991 we have the information from Lawrence Berkeley Laboratory, U.S.A. the artificially built atom has the highest $Z = 109$ and $A = 266$, Katz, Seaborg and Morss [15], Loveland and Seaborg [16].

Interestingly $1/\alpha$ is nearly equal to 137 which is a prime number. The highest Z for naturally occuring element to-day is 92, for uranium. The ratio 92/138 is 2/3, showing the Nature's choice for the stability of the element. What She wants to do with the remaining 1/3 of 138 we may try to explore.

TABLE - 1. Distribution of electrons in the atomic shells and subshells. Maximum electrons in : $s = 2$, $p = 6$, $d = 10$, $f = 14$, $g = 18$ etc

Shells and subshells																		
Atoms	At.N°	$4f$	$5s$	$5p$	$5d$	$5f$	$5g$	$6s$	$6p$	$6d$	$6f$	$7s$	$7p$	$7d$	$7f$	$8s$	$8p$	
Pd	(1s to 4d) 46 electrons																	
.....																		
Ba	56	0	2	6	0	0	0	2										
La	57	0	2	6	1	0	0	2										
Ce	58	1	2	6	1	0	0	2										
.....																		
Lu	71	14	2	6	1	0	0	2										
.....																		
Ra	88	14	2	6	10	0	0	2	6	0	0	2						
Ac	89	14	2	6	10	0	0	2	6	1	0	2						
Th	90	14	2	6	10	1	0	2	6	1	0	2						
.....																		
Lr	103	14	2	6	10	14	0	2	6	1	0	2						
.....																		
Xa	120	14	2	6	10	14	0	2	6	10	0	2	6	0	0	2		
Xb	121	14	2	6	10	14	0	2	6	10	0	2	6	1(0)	0	2	0(1)	
Xc	122	14	2	6	10	14	0(1)	2	6	10	0	2	6	1	0	2	1(0)	
.....																		
Xk	137	14	2	6	10	14	16(15)	2	6	10	0	2	6	1	0	2	0(1)	
.....																		
Xz	138	14	2	6	10	14	18	2	6	10	0	2	6	-	-	2	-	

From binding energy relation of electrons in the atomic shell structure, considering nonrelativistic speed of electron, Born [2], Loveland

and Seaborg [16] have given arrangement of elements in the Periodic Table. To study the chemistry of heaviest elements, specially transuranic artificially built elements, Loveland and Seaborg [16] gave relativistic orbitals. They mentioned, liquid drop models of nucleus which predicts instantaneous fission when Coulomb energy E_c becomes two times Surface energy E_s of the nucleus. On this account they expect stability of the nucleus upto the element with $Z = 125$. They also suggested, with Dirac-Fock relativistic calculation, Periodic Table with elements upto $Z = 168$. But, we feel from our arguments given above that naturally or artificially an atom cannot be built in our Universe beyond $Z = 137$.

Periodic Table of "Free Neutral Atoms of Elements" with atomic number (Z) is given in Table-1. Specially electronic sub-shell configuration for borderline elements e.g. La(57), Ce(58), Lu(71) ; Ac(89), Th(90), Lr(103) ; Xa(120), Xb(121), Xk(137) are shown. From Ce(58), to Lu(71), called Lanthanides, we get the rare earth elements when $4f$ sub-shell gets filled up with 14 electrons. Possibly from atom Th(90) to Lr(103), called Actinides, we get elements similar to rare earth elements when $5f$ sub-shell gets filled up again with 14 electrons. Also possibly from atom Xb(121) to atom Xz(138) through atom Xk(137) we probably get analogous to rare earth elements when $5g$ sub-shell gets filled up with 18 electrons.

4. Four physical constants α, e, $\hbar$ and c.

The relation between the four physical constants α, e, $\hbar$ and c has been given in eqn.(2.3) as

$$\alpha = \frac{e^2}{\hbar c} \tag{2.3}$$

We have shown that the dimensionless $1/\alpha$ is really a basic physical constant giving the maximum atomic number upto which we can build an atom in our physical world. It is obvious, as Dirac [18] had said that out of three constants e, $\hbar$ and c two can be fundamental and the third one should be derived from them and $1/\alpha$. Let us examine all three one by one.

(1) c - Velocity of light in vacuum

According to Einstein [19] c is the maximum velocity a material body can attain. It is an inseparable part of the theory of relativity. It is also the maximum velocity with which energy can be carried. Experiments in accelerating machines have shown that mass of a material

particle increases with c as upper limit and as part of a factor. Similarly the decay time of a fast moving cosmic ray μ-meson increases. It plays a fundamental role relating space and time. Our intuition also says that physical velocity cannot go on increasing indefinitely. There must be an upper limit. With so many factors in its favour, we feel that c the velocity of light or electromagnetic phenomenon in vacuum is a fundamental quantity.

(2) e - Unit of atomic charge

There is a possibility that the unit of atomic charge e may not be a fundamental entity. Present evidences show that protons and neutrons are built from three quarks having fractional charges $+2/3e$ and $-1/3e$, Perkins [3]. (1.1) shows

$$e^2 = \alpha\hbar c = \frac{\hbar c}{137.03602(\pm 00021)} \tag{4.1}$$

The charge e has a possible velocity component and it comes as a squareroot of $\hbar c$. When the electron in the K-shell suddenly attains the velocity c with Z suddenly becoming 137.03602 ($\pm$00021), not only the electron mass vanishes but also the electronic charge vanishes. We also know, when an electron and a positron come together their masses are converted into energy and charges neutralize each other. In the reverse phenomenon a high energy quanta under a strong field can be converted into an electron and a positron i.e. creation of masses and charges. There is a possibility that electronic mass as well as charge may not be amongst Nature's fundamental quantities. But, Dirac [18] feels e has to be explained as squareroot of $\hbar$. As squareroots do not occur in basic equations e may be a fundamental quantity.

(3) h - Planck constant, $h/2\pi$ - is called Dirac constant, $\hbar$

We are now left with the above third and last one. Action must have some limit like c, and in this case a lower limit. Planck [20] obtained h as a small constant for the energy equation of oscillators in blackbody radiation as

$$E = h\nu \tag{4.2}$$

Bohr [13] used h to quantize the angular momentum for the first and subsequent orbit as follows

$$m_0 v_1 R_1 = \hbar \tag{4.3}$$

De Broglie [21] gave the wave length of matter wave as

$$\lambda = h/(m_0 v) \tag{4.4}$$

Heisenberg [22] gave the uncertainty relation connecting the amount of uncertainty in momentum and position by

$$(\Delta p)(\Delta x) \geq \hbar \tag{4.5}$$

From heuristic argument Kundu [23] has calculated the amplitude of de Broglie's matter wave as

$$r = \lambda\!\!\!^{-} = \hbar/(m_0 v) \tag{4.6}$$

Kundu [24] has also discussed the motion of electron in Bohr orbit following de Broglie [21] wave motion. He fixed the frequency of matter wave. Its amplitude was given same as shown in eqn.(4.6).

So $\hbar$ is related to many phenomena as mentioned above. The underlyind idea is the association of 2π with h. We know that 2π is associated with circle. Hence $h/2\pi$ points towards a circular motion, Kundu [24]. We have reasons to believe that K-shell of atoms with atomic number Z, at least, will be circular in nature. This brings us to Figure - 1, where (a), (b) and (c) show Z equals to 1, 10 and 137 respectively, and one electron in the K-shell.

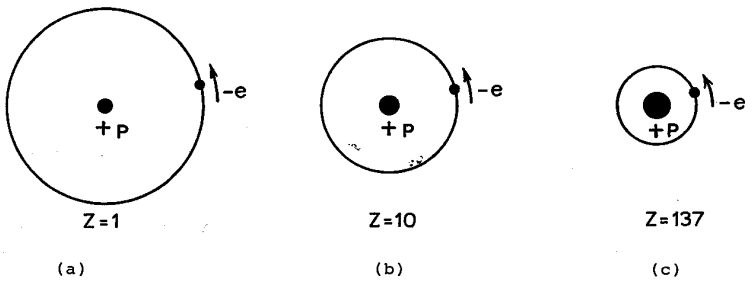

Figure 1. K-shell of atoms with $Z = 1, 10, 137$.

The circle for the K-shell electron gradually reduces as Z increases. With atomic number Z the action relation for the electron motion is

$$m_0 v_{1(Z)} R_{1(Z)} = \hbar \tag{4.7}$$

This along with the balancing forces required for the motion gives eqn.(2.7)

$$\frac{v_{1(Z)}}{c} = \frac{Ze^2}{\hbar c} \tag{2.7}$$

The K-shell electron vanishes when Z suddenly attains 137, that is what Nature ordains

$$\hbar = \frac{137e^2}{c} \tag{4.8}$$

But, probably, she missed it by 0.0264 percent. Also, may be, K-shell a little deviated from circle.

5. Discussion and concluding remarks.

We understand that when Nature chooses a few quantities as the limiting values and then goes on building the physical world by physical laws, those a few quantities become the "Fundamental Constants of Nature". [1]

At present four such fundamental constants α, c, $\hbar$ and e are related through eqn.(1.1). The $1/\alpha$ is a dimensionless pure number. Its present value on the basis of current values of c, $\hbar$ and e is $137.03602(\pm 00021)$, and it is approaching 137, which is a "prime number". The eqn.(2.7) clearly shows that the velocity, or motional velocity, of electron in the K-shell of the atom will certainly reach the limiting velocity c of our Universe when Z the nuclear charge suddenly attains the above value $137.03602(\pm 00021)$. We feel, Nature has put an upper limit of 137 upto which atoms can be built up in our physical Universe, 137 is a prime number and a "Fundamental Constant" of Nature. We understand that this is the meaning of Sommerfeld fine structure constant α. Naturally occuring element with highest Z, at present, is Uranium with $Z = 92$

[1] N.B. Philosophically, a physical phenomenon can be explained through the cause-and-effect. The historical example is the demonstration of "electromagnetic induction" by Michael Faraday himself before a distinguished gathering of members of the Royal Society of London. This was shown by the movement, either way, of a "light spot" in a lamp-and-scale arrangement with a galvanometer connected to a solenoid through which Faraday moved a permanent magnet. (The dignitary laughed at the light-spot movement, and Faraday retorted it would be taxed one day !). If we go behind the cause-and-effect we find permanent magnet movement-emf-current-torque-light spot movement, and so on. Then we come to a stage and cannot answer anymore. We accept Nature ordains like that.

which is $2/3rd$ of 138. Probably after the Big Bang Nature chose to build up our physical world that way.

When Z suddenly crosses 137 the K-shell of the atom collapses and with it possibly all other upper electronic shell structure with collapse one after another. It may be dangerous to build in the laboratory artificial element with Z beyond 137. The α, also connecting electromagnetic and weak forces, in all possibility points towards the decay of atom and going back to primordial stage. This may have far reaching consequences in physics and astrophysics including nuclear force, creation of black-holes, release of enormous energy etc. The origin of enormous amount of energy in a quaser may not be due to the gravitational collapse of gigantic mass billion times our sun, a near impossibility, but due to the collapse of atomic structure as Z approaches 138. This creates gradually the chargeless massive blackhole and release of huge amount of energy due to the mass-energy conversion and the gravitational collapse. Thus, it supports the theory of upper limit of Z developed in this paper. In all probability Nature wanted the equation (3.1) to read like

$$\frac{1}{\alpha} = \frac{\hbar c}{e^2} = 137 = Z_m = K \tag{5.1}$$

where, $Z_m = K$ = maximum atomic number upto which an atom can be built-up in our physical Universe.

So the above prime number 137 plays a very important role in the development-decay-development of our Universe.

The three physical constants c, $\hbar$ and e are intimately related to 137 through fundamental electromagnetic processes. One of them is a constant, but not a fundamental constant, and should be derived from the other two and α. The velocity of electromagnetic phenomenon in vacuum is c. Its deep involvement in various physical theories including the theory of relativity points that it is a fundamental constant. Dirac [18] feels that any fundamental theory is unlikely to give e as a square-root. So if the electronic charge e is considered a fundamental constant, unlike its counterpart electronic rest mass m_0, then we have to accept $\hbar$ as a derived constant as given in eqn.(4.8). The form of Heisenberg [22] uncertainty relation, anyway, has to change to some extent, Kundu [23,24].

But, the real nature of e the electronic negative charge, and the protonic positive charge, is yet to be determined. According to electrostatic

force and present day dimensional analysis it seems e has a velocity component and a quantity which is squareroot of mass and length. There is a distinct possibility that Nature planned to create as well as destroy the atomic world with c as the fundamental upper limit of velocity, $\hbar$ as the fundamental lower limit of action and $Z = 137$ as the fundamental upper limit of atomic nuclear charge. Further, probably Nature planned to create and destroy e through the electromagnetic action by the fundamental natural relation $e^2 = \hbar c/Z$, where Z should have been 137.

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