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Probing Gauge Boson Compositeness and Violations of the Exclusion Principle Using Discrete Time Quantum Mechanics

C. Wolf

Department of Physics, North Adams State College,
North Adams, MA (01247) U.S.A.

ABSTRACT. By constructing a two particle wave function (for two fermions) we discuss possible probes to both gauge boson compositeness and violations of the exclusion principle using discrete time Quantum theory.

RÉSUMÉ. Après avoir construit une fonction d'onde à deux particules (pour deux fermions), nous discutons des tests possibles à la fois du caractère composite du boson de jauge et de violations du principe d'exclusion en utilisant une théorie quantique à temps discret.

Introduction.

Many of the central problems of particle physics may be well hidden beneath a scale that cannot be probed by ordinary experiments or present high energy accelerators. One of those problems is the origin of Lorentz invariance and the possible grainy and discrete nature of space and time at small scales. Finkelstein has suggested that space-time emerges from the primitive structure of a quantum net where continuous space-time and conventional field theory emerge after passing to an averaging process [1]. Wheeler [2] has also suggested such discrete ideas with the known physics of the continuum being rooted in a combinatoric sequence of yes-no choices. In a practical sense both Snyder [3] and t'Hooft [4] have suggested the presence of a discrete space-time latice to eliminate the divergences in field theory and quantum gravity. In order to better understand the evaluation of path intervals T.D. Lee has introduced a discrete time variable in order to dispense with the arbitrary value of the measure in path integrals [5]. On very fundamental grounds Bombelli et. al. have adopted a novel approach to the fundamental structure of space-time, they have through algebra and topology derived the Lorentz signature $(+1, -1, -1, -1)$ starting with a discrete causal set of points [6]. A slightly different path to discreteness has been pioneered by Caldirola where he replaces time derivatives by discrete time differences in both the equation of radiation reaction and quantum theory [7,8]. The fundamental reason for this stems from the belief that at some scale there exists a microscopic uncertainty principle forbidding the response of a particle's wave function arbitrarily close to the point of application of the Hamiltonian. Recami has reasoned that this uncertainty principle emerges from the fact that each particle has its own sense of time (particles microuniverse) and the interaction of the particle with the world of synchronous observers generates a width in response time that we perceive as a discrete time interval leading to a microscopic uncertainty principle for the response time in the frame of synchronous observer [9]. The Lie Admissible structure of the Caldirola approach has been discussed by Santilli et. al. [10] with interactions carried by the isotopy of the theory. We have applied this approach to electron spin resonance [11], electron spin polarization precession [12] and spectral shifts in hydrogen [13] for the discrete time difference case. For the case of discrete spatial differences we have studied neutron interferometry suggesting a range of discrete spatial lengths that may be measured [14]. We have also applied the Caldirola approach to the case when quantum numbers within a particle might be hidden and the discrete temporal effects alone can probe these hidden quantum numbers [15]. In combination with the central limit theorem we have also proposed a method for looking for compositeness in the lepton spectrum using discrete time difference spin polarization precession [16]. In the following note we propose to study two fundamental questions, can discrete time difference quantum theory be used to probe the composite structure of gauge bosons and secondly if there are any violations of the exclusion principle for the constituent (preons) of gauge bosons can discrete time quantum theory probe for these violations. With regard to the composite structure of gauge bosons (w^+, w^-, z), the measured values of the magnetic dipole moment and the electric quadrapole moment of (w^+, w^-, z) can be used to probe for deviations from standard model predictions of these quantities, these deviations are also suggestive of a composite gauge boson structure [17,18]. In what follows we discuss a simple model of gauge boson composite structure and discuss how the structure may be probed using discrete time quantum theory.

With regard to the exclusion principle there have been modifications to the usual statistics that allow for more than one fermion in a symmetric state or more than one boson in an anti-symmetric state, such modifications termed parastatistics have been discussed by Greenberg et. al. [19,20] and Greenberg [21]. The approach of the above authors is to modify the usual commutation and anti-commutation relations to include tri-linear relations that generate the violations to the exclusion principle. Jannussis et al. [22], Santilli [23] and Myung et. al. [24] have also discussed violations of the exclusion principle within the context of hadronic mechanics. The exclusion principle has been well tested for atomic physics with extremely small violations being allowed from a study of atomic levels [25], however for weak interactions in elementary particle phenomena relatively poor tests exist to test for its validity [26]. The great interest in bosonization in field theory [27] may suggest that particle statistics are related to topological dynamics that may violate causality. In other words a fermion's topological structure (spin structure) may feel another fermion's topological structure over space like intervals and we just use this fact in a phenomenological manner to construct symmetric or anti-symmetric wave functions. If discrete time effects exist in nature we suggest that they may be a probe to test for violations of the exclusion principle by basically suggesting that over short time intervals a fermion structure or boson structure may be uncertain. This would allow for a small fraction of a two fermion system to have a symmetric wave function. We study this possibility and look for observable consequences of a state of mixed symmetry within the context of discrete time quantum theory.

2. Composite Structure of Gauge Bosons and Discrete Time Quantum Theory.

We begin our analysis by modeling the internal structure of a gauge boson by the non-relativistic hamiltonian for two preons (spin 1/2) in an infinite potential well in the presence of an external magnetic field as,

$$H = M_0 C^2 + \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + \frac{e}{m} S_{z1} B + \frac{e}{m} S_{z2} B + g S_1 \cdot S_2 \qquad (2.1)$$

here the preons have heavy masses (m) so as to justify the non-relativistic approximation. Such a non-relativistic model of gauge bosons has been previously discussed by Grosser et. al. [28]. Also S_{z1} and S_{z2} are spin operators for the two fermions each of charge $e = e_0/2$, (e_0 = electronic charge), P_1, P_2 are momentum operators, B = external z component magnetic field and M_0C^2 is rest energy parameter relative to which the gauge boson has excitation energy due to the motion of the preons and the spin-spin coupling specified by g. The gauge bosons coupling to the external magnetic field is specified by the terms containing B in Eq.(2.1). For the discrete time modified Schrodinger equation we have (Ref. 10)

$$\Big[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}+\frac{e}{m}(S_{z_1}+S_{z_2})B+g(S_1\cdot S_2)+M_0c^2\Big]\psi = i\hbar\Big[\frac{\psi(t+\frac{\tau}{2})-\psi(t-\frac{\tau}{2})}{\tau}\Big] \tag{2.2}$$

(τ = discrete time interval), (ψ = total wave function).

We separate the equation for a spin up state (for both fermions) by writing

$$\psi = U(x_1,x_2)\alpha\alpha T(t) \tag{2.3}$$

here α = spin up for one fermion, β = spin down for one fermion). Eq.(2.2) and Eq.(2.3) give

$$\Big[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}+M_0C^2\Big]U(x_1,x_2)=E_1U(x_1,x_2) \tag{2.4}$$

and

$$\Big[\frac{e}{m}(S_{z_1}+S_{z_2})B+gS_1\cdot S_2\Big]\alpha\alpha = E_2\alpha\alpha \tag{2.5}$$

with

$$ET(t)=(E_1+E_2)T(t)=i\hbar\frac{\big[T(t+\frac{\tau}{2})-T(t-\frac{\tau}{2})\big]}{\tau} \tag{2.6}$$

We further separate Eq.(2.4) as

$$\Big[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}+M_0C^2\Big]U_1(x_1)U_2(x_2)=E_1U_1(x_1)U_2(x_2) \tag{2.7}$$

and

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}U_1[\frac{1}{U_1}]=\alpha^2 \tag{2.8}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}U_2[\frac{1}{U_2}]=\beta^2 \tag{2.9}$$

with

$$E_1 = \alpha^2 + \beta^2 + M_0C^2 \tag{2.10}$$

We next choose the fermions to be confined to within $(0, L)$ with $V = \infty$ for $x_1, x_2 < 0$, $x_1, x_2 > L$. This gives

$$E_1 = M_0C^2 + \frac{n_1^2h^2}{8mL^2} + \frac{n_2^2h^2}{8mL^2}(n_1 \neq n_2) \tag{2.11}$$

$$U_1(x_1) = \sqrt{\frac{2}{L}}\sin\frac{n_1\pi x_1}{L} \quad , \quad U_2(x_2) = \sqrt{\frac{2}{L}}\sin\frac{n_2\pi x_2}{L} \tag{2.12}$$

Also, Eq.(2.5) gives

$$E_2 = \frac{e}{m}(\hbar B) + \frac{g\hbar^2}{4} \tag{2.13}$$

where S_1, and S_2 are spin 1/2 operators for particle $1, 2$ respectively.

For the solution of Eq.(2.6) we have for the temporal part of the wave function

$$T(t) = Ce^{-\frac{2}{\tau}[\sin^{-1}\frac{(E_1+E_2)\tau}{2\hbar}]it} \tag{2.14}$$

For the total anti-symmetric wave function in the two fermion (preon) state representing the internal structure of the gauge boson we have for $(S_z = 1)$

$$\begin{aligned}\psi = \frac{2}{L}\frac{1}{\sqrt{2}}&[\sin\frac{n_1\pi x_1}{L}\sin\frac{n_2\pi x_2}{L} - \sin\frac{n_1\pi x_2}{L}\sin\frac{n_2\pi x_1}{L}] \\ &\times(\alpha\alpha e^{-\frac{2}{\tau}[\sin^{-1}\frac{(E_1+E_2)\tau}{2\hbar}]it})\end{aligned} \tag{2.15}$$

where

$$E_+ = E_1 + E_2 = M_0C^2 + \frac{n_1^2h^2}{8mL^2} + \frac{n_2^2h^2}{8mL^2} + \frac{e\hbar B}{m} + \frac{g\hbar^2}{4} \tag{2.16}$$

In a similar manner for the state $S_2 = -1$ we have

$$\begin{aligned}\psi = \frac{2}{L}\frac{1}{\sqrt{2}}&[\sin\frac{n_1\pi x_1}{L}\sin\frac{n_2\pi x_2}{L} - \sin\frac{n_1\pi x_2}{L}\sin\frac{n_2\pi x_1}{L}] \\ &\times(\beta\beta e^{-\frac{2}{\tau}[\sin^{-1}\frac{(E_{-}\tau}{2\hbar}]it})\end{aligned} \tag{2.17}$$

where

$$E_- = M_0C^2 + \frac{n_1^2h^2}{8mL^2} + \frac{n_2^2h^2}{8mL^2} - \frac{e\hbar B}{m} + \frac{g\hbar^2}{4} \tag{2.18}$$

and for $S_z = 0$ we have

$$\psi = \frac{2}{L}\frac{1}{\sqrt{2}}[\sin\frac{n_1\pi x_1}{L}\sin\frac{n_2\pi x_2}{L} - \sin\frac{n_1\pi x_2}{L}\sin\frac{n_2\pi x_1}{L}] \\ \times ((\frac{\alpha B + \beta\alpha}{\sqrt{2}})e^{-\frac{2}{\tau}(\sin^{-1}\frac{E_0\tau}{2\hbar})it}) \tag{2.19}$$

where

$$E_0 = M_0C^2 + \frac{n_1^2h^2}{8mL^2} + \frac{n_2^2h^2}{8mL^2} + \frac{g\hbar^2}{4} \tag{2.20}$$

To study gauge boson spin polarization precession in an external magnetic field B we choose the initial state to be such that

$$\langle S_x\rangle_{t=0} = \hbar$$

the $S_x = \hbar$ state at $t = 0$ that is a linear combination of Eq.(2.15), Eq.(2.17) and Eq.(2.19) is

$$\psi = [\frac{\alpha\alpha}{2} + \frac{\beta\beta}{2} + \frac{1}{2}(\alpha\beta + \beta\alpha)] \\ \frac{1}{\sqrt{2}}[\sin\frac{n_1\pi x_1}{L}\sin\frac{m_1\pi x_2}{L} - \sin\frac{n_2\pi x_2}{L}\sin\frac{n_2\pi x_1}{L}]\frac{2}{L} \tag{2.21}$$

Defining

$$\begin{aligned} a_1 &= +\frac{2}{\tau}\sin^{-1}\left(\frac{E_+\tau}{2\hbar}\right) \quad \text{for} \quad S_z = 1 \quad \text{from Eq.(2.16)} \\ a_2 &= +\frac{2}{\tau}\sin^{-1}\left(\frac{E_+\tau}{2\hbar}\right) \quad \text{for} \quad S_z = -1 \quad \text{from Eq.(2.18)} \\ a_3 &= +\frac{2}{\tau}\sin^{-1}\left(\frac{E_0\tau}{2\hbar}\right) \quad \text{for} \quad S_z = 0 \quad \text{from Eq.(2.20)} \end{aligned} \tag{2.22}$$

we have for any time t using the linear combination corresponding to Eq.(2.21) at time t

$$\psi = [\frac{\alpha\alpha e^{-ia_1t}}{2} + \frac{\beta\beta e^{-ia_2t}}{2} + \frac{(\alpha\beta + \beta\alpha)e^{-ia_3t}}{2}] \\ \times (\frac{1}{\sqrt{2}}\frac{2}{L}[\sin\frac{n_1\pi x_1}{L}\sin\frac{n_2\pi x_2}{L} - \sin\frac{n_1\pi x_2}{L}\sin\frac{n_2\pi x_1}{L}]) \tag{2.23}$$

For the x component spin polarization at time t we use Eq.(2.22), Eq.(2.23) and $S_x = S_{x_1} + S_{x_2}$

$$\langle S_x \rangle = \int_0^L \int_0^L \psi^+ (S_{x_1} + S_{x_2}) \psi \, dx_1 dx_2 \tag{2.24}$$

In Eq.(2.22) if

$$a_1 \simeq \frac{E_+}{\hbar} \quad , \quad a_2 \simeq \frac{E_-}{\hbar} \quad , \quad a_3 \simeq \frac{E_0}{\hbar} \tag{2.25}$$

in the limit of $\tau \to 0$ then Eq.(2.23) gives

$$\langle S_x \rangle = \hbar \cos(\frac{eB}{m})t \tag{2.26}$$

which is the usual expression for the expectation value of $\langle S_x \rangle$ for a spin one particle composed of two spin 1/2 preons simultaneously precessing with the same rate about a z component magnetic field. For Eq.(2.26) to agree with the formula for a gauge boson of (mass M_w), charge $e_e = 1.6 \times 10^{-19}$ coul., of spin 1 precessing about a z component magnetic field we must set $2m = M_w$, $(e = e_e/2)$. This essentially states that the effective preon mass must be about 1/2 the value of the composite gauge boson mass (M_w) since the standard model predicts the precession rate to be [29]

$$\omega = \frac{e_e B}{M_w} \quad , \quad (e_e = \quad \text{electronic charge} \quad)$$

Also, since it has been long known that either a chiral symmetry or super symmetry must protect composite particles from acquiring large masses it is not unnatural that two preons, each of mass $M_w/2$ can bind together through hypercolor type binding forces to generate a composite mass of approximate value M_w. These binding factors due to hypercolor type forces are all lumped into $M_0 C^2$ in Eq.(2.1). We also expect the other terms in Eq.(2.1) to produce excitation energies that are small compared to $M_0 C^2$ since the known uncertainty in the mass of the gauge bosons is small. In Eq.(2.24) we will thus approximate

$$m \simeq \frac{M_w}{2}$$

Eq.(2.24) gives for the composite system

$$\langle S_x \rangle = \frac{\hbar}{2} \cos(a_1 - a_3)t + \frac{\hbar}{2} \cos(a_3 - a_2)t \tag{2.27}$$

$$\langle S_x \rangle = \frac{\hbar}{2}\cos\omega_1 t + \frac{\hbar}{2}\cos\omega_2 t \tag{2.28}$$

where $\omega_1 = a_1 - a_3$, $\omega_2 = a_3 - a_2$. Expanding ω_1, ω_2 using Eq.(2.22), the expansion

$$sin^{-1}x \cong x + \frac{x^3}{3!}$$

and Eq.(2.16), Eq.(2.18) and Eq.(2.20) we have for the average x spin polarization

$$\begin{aligned}\langle S_x \rangle = &\frac{\hbar}{2}\cos(\frac{eB}{m}t + \frac{\tau^2}{24\hbar^3}(E_+^3 - E_0^3)t) \\ &+ \frac{\hbar}{2}\cos\Big(\frac{eB}{m}t + \frac{\tau^2}{24\hbar^3}(E_0^3 - E_-^3)t\Big)\end{aligned} \tag{2.29}$$

We thus expect two terms in the x spin polarization that vary according to Eq.(2.29). A careful study of the time dependence of the x spin polarization precession along with a comparison with Eq.(2.29) could serve to place limits on the discrete time interval as well as the parameters M_0C^2, g in Eq.(2.16), Eq.(2.18) and Eq.(2.20). A deviation of the spin polarization precession rate from

$$\omega = \frac{e_e B}{M_\omega}$$

would be a positive signature for both gauge boson compositeness as well as discrete time effects in quantum theory.

We now turn to the question of how the Pauli Principle might be violated for a two fermion bound state, it turns out that for a symmetric spin state, the spin polarization precession rate is insensitive of whether the spatial state is symmetric or anti-symmetric. However, if a two fermion state is in a state of mixed symmetry

$$\begin{aligned}\psi = A((&\sqrt{\frac{2}{L}})^2 \sin\frac{n_1\pi x_1}{L}\sin\frac{n_2\pi x_2}{L} - (\sqrt{\frac{2}{L}})^2 \sin\frac{n_1\pi x_2}{L}\sin\frac{n_2\pi x_1}{L} \\ &(\alpha\alpha e^{-(\frac{2}{\tau}\sin^{-1}\frac{E_+\tau}{2\hbar})it}) \\ + \overline{A}((&\sqrt{\frac{2}{L}})^2 \sin\frac{n_1\pi x_1}{L}\sin\frac{n_2\pi x_2}{L} + (\sqrt{\frac{2}{L}})^2 \sin\frac{n_1\pi x_2}{L}\sin\frac{n_2\pi x_1}{L} \\ &(\alpha\alpha e^{-(\frac{2}{\tau}\sin^{-1}\frac{E_+\tau}{2\hbar})it})\end{aligned}$$

we would find the effective size of the two fermion system (representing a gauge boson) would be different than the pure total anti-symmetric state. Also for electric dipole transitions the expectation value of the operator between two states where one state is in a state of mixed symmetry is different than when the final and initial states are both in an anti-symmetric state. Thus if other input data could be used to place limits on M_0, g and L then a study of electric dipole transitions of gauge bosons from one state to another could test for the presence of Exclusion Principle violations. It is also important to point out that the standard model of

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

only allows for a magnetic dipole moment and electric quadrapole moment of the ω^+, ω^- (Ref. 17). If more elaborate composite models could be constructed then a calculation of the electric dipole moment of a two fermion system and magnetic quadrapole moment could be made and compared with experimental deviations from the Standard model prediction for the ω^+, ω^-, γ vertex interaction.

3. Conclusion.

Our simplified model of a gauge boson as being a composite of two fermions in a potential well has at least led to a provisional test for both compositeness and discrete time effects in gauge boson polarization precession experiments. Since the exclusion principle has been poorly tested for weak interactions, a study of internal transitions of gauge bosons with final and initial states having mixed symmetry might suggest experimental situations where comparison with experiment can be made. As mentioned in the introduction, the exclusion principle is well tested for Atomic physics, however in the domain of weak, strong and possibly super-strong interactions it might be violated. Transition rates, effective radii and binding mechanisms are all sensitive to a state of mixed symmetry and possible discrete time effects might reveal violations of the exclusion principle that otherwise lay hidden beneath the dull phenomenalogical world of the continuum.

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