Rest frame compatible wave mechanics II wave equations and fields

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ABSTRACT. The relationships between the rest-frame-compatible (RFC) particle waveforms and equations of the Klein-Gordon and Dirac types is explored. It is observed that these equations, which apply to de Broglie waves in quantum mechanics, have similar forms for RFC waves, but do not specify the RFC components individually, instead giving their product. Wave equations are also obtained corresponding to the "carrier" parts of RFC waveforms; these equations are inferred to have quantum mechanical analogues yielding the momentum eigenvalues of the particle.

Using the RFC picture, it is found possible to develop a wave-mechanical description of an experiment of the type proposed by Einstein, Podolsky and Rosen, that provides a logical explanation for the apparently paradoxical results.

RÉSUMÉ. On examine, dans le cas d'une particule, les relations entre les formes ondulatoires compatibles avec le référentiel propre (RFC) et les équations des types Klein-Gordon et Dirac. On observe que ces équations, qui s'appliquent aux ondes de de Broglie en la mécanique quantique, ont des formes semblables dans le cas des ondes RFC, mais ils ne précisent pas les éléments RFC individuellement, donnant leur produit à la place. En outre, on obtient des équations d'ondes qui correspondent aux éléments "porteurs" des formes ondulatoires RFC; on en déduit que ces équations ont des analogues en la mécanique quantique qui donnent les valeurs propres d'impulsion de la particule.

A l'aide de la représentation précédente, c'est possible de donner une description, quant à la mécanique ondulatiore, d'une expérience du type proposé par Einstein, Podolsky et Rosen, qui fournit une explication logique des résultats apparemment pardoxaux.

1. Introduction.

In an earlier work [1], it was shown that a new particle representation can be developed that has a number of attractive properties. Three of these are: the free particle can be associated with a stable wave packet in all Lorentz frames; the "collapse of the state function" need not appear in the description of particle behaviour; and the representation can be made fully relativistic. The formalism also implies the existence of another type of matter wave, termed the carrier wave, which is not detectable in most experiments on systems of atomic dimensions.

An extended form of de Broglie's hypothesis was proposed as the basis for the particle concept; the hypothesis identifies a massive particle in its rest frame with a stable standing-wave packet. This packet transforms, in frames moving with respect to the rest frame, to produce a stable packet composed of de Broglie waves combined with carrier waves of very short, nearly constant wavelength.

In this paper, the approach is augmented to accommodate electromagnetic fields, and the relationship between the resulting rest-frame-compatible (RFC) particle waveforms and relativistic wave equations is explored. It is inferred from this relationship that the Klein-Gordon and Dirac equations are represented in the RFC picture, not only as they appear in quantum mechanics with de Broglie wavefunctions as solutions, but also with carrier wavefunctions as solutions.

As an illustration of its utility, the RFC wave picture is used to develop a causal description of an Einstein-Podolsky-Rosen (EPR) type experiment. The description provides a logical explanation of the apparently paradoxical results of the experiment.

For clarity, we use the term "de Broglie waves" to apply generically to the particle waves described by conventional quantum mechanics, and "RFC waves" in refering to the waves of the RFC picture. We also reserve the word "trajectory" for the pathway of a particle in space-time, and use "path" for the curve followed in space. Also, the term "state function" will include spinors where necessary.

As discussed in reference [1], the RFC approach assumes that between interactions, a free stable particle in any reference frame occupies a fixed trajectory, and has fixed attributes (mass, charge, momentum, spin state, etc.), that are well defined in all inertial reference frames, including its rest frame. These free-particle attributes are those compatible with the interactions terminating its trajectory.

2. RFC waveforms and world momentum vectors.

We will assume in what follows that we can obtain wave equations in the RFC picture in a manner analagous to a method used in quantum mechanics. This method is adapted from reference [2], and involves identifying relativistic invariants of the world momentum vectors associated with the RFC waves. To make the identification, we adopt complex exponentials for the RFC waves, associating terms of the exponents with the world momenta of the counter-propagating complex waves. We then construct RFC wave equations, using the standard definitions of the quantum mechanical operators of the Schrödinger representation.

Based on the arguments presented in reference [1], we adopt the following complex exponential, counter-propagating waves to represent a free particle moving at velocity v along the z-axis:

$$\begin{aligned} G &= Z[\exp -i\gamma K(1+\beta)(ct-z) + \exp -i\gamma K(1-\beta)(ct+z)] \\ &\equiv Z[g{\rightarrow} + g{\leftarrow}]\,, \text{ where } \beta = v/c\,, \text{ and } K = mc/\hbar. \end{aligned} \tag{1}$$

This is equivalent to assuming that a particle can be associated with the following combination of real and complex travelling waves:

$$\begin{aligned} G =& 2Z\{\cos\gamma K(ct-\beta z)\cos\gamma K(\beta ct - z) \\ & - i\sin\gamma K(ct-\beta z)\cos\gamma K(\beta ct - z)\} \\ =& Z\exp\{-i\gamma K(ct-\beta z)\}.\cos\gamma K(\beta ct - z). \end{aligned} \tag{2}$$

That is, the de Broglie wave in G is complex, while the carrier wave is real. We then write the world momenta for the forward and backward waves, $g{\rightarrow}$ and $g{\leftarrow}$, in analogy to the world momenta used in quantum mechanics for de Broglie waves. That is, $g{\rightarrow}$ and $g{\leftarrow}$ have the associated 4-momentum vectors:

$$\{P_0{\rightarrow}, -P{\rightarrow}\}\ , \text{ and } \{P_0{\leftarrow}, -P{\leftarrow}\} \tag{3}$$

In the case of the free particle of equation (1), these momenta are:

$$\begin{aligned} P_0{\rightarrow} &= \gamma K\hbar(1+\beta); \quad P_0{\leftarrow} = \gamma K\hbar(1-\beta); \\ P{\rightarrow} &= \gamma K\hbar(1+\beta); \quad P{\leftarrow} = -\gamma K\hbar(1-\beta). \end{aligned}$$

In the general case, the derivatives of $g{\rightarrow}$ and $g{\leftarrow}$ are related to the components of the world momenta by:

$$i\hbar\partial_k g{\rightarrow} = P_k{\rightarrow} g{\rightarrow}, \quad i\hbar\partial_k g{\leftarrow} = P_k{\leftarrow} g{\leftarrow}, \quad \text{where } k = 0,1,2,3.$$

These derivatives are then identified with the Schrödinger operators for the components of the world momenta of $g\rightarrow$ and $g\leftarrow$, again in analogy to the same association with de Broglie waves in quantum mechanics [2].

As is evident from a comparison of equations (1) and (2), the particle energy and momentum are given by:

$$E = \frac{1}{2}[E\rightarrow + E\leftarrow], \quad \text{and} \quad \mathbf{P} = \frac{1}{2}[\mathbf{P}\rightarrow + \mathbf{P}\leftarrow] \quad , \tag{4}$$

where $E\rightarrow = cP_0\rightarrow$ and $E\leftarrow = cP_0\leftarrow$.

3. Klein-Gordon type equations.

In exploring the relations of the RFC particle waveforms to the quantum mechanical wave equations and their solutions, we start from the relativistic invariant from which the original Klein-Gordon equation can be obtained. A similar invariant is then applied to RFC waves; the resulting wave equation is found to provide information only about the product of the components of the RFC particle waveform. We will refer to the RFC wave equations obtained in this case as the "RFC analogues" to the original equations.

4. RFC analogue to the Klein-Gordon equation.

The original Klein-Gordon equation for a massive particle can be obtained by substituting the Schrödinger operators for the components of the particle world momentum, P_k, in the invariant relation:

$$[P_k + (q/c)\phi_k][P^k + (q/c)\phi^k] = (mc)^2 \,, \tag{5}$$

where the world momentum components for the particle are identified as:

$$P_k = mcu_k - (q/c)\phi_k \,, \quad \text{with } k = 0, 1, 2, 3.$$

Here, u_k are the covariant components of the particle 4-velocity: $u^k = dx^k/ds$; q is the charge; and ϕ_k are the components of the electromagnetic 4-potential,$\{-V, A_x, A_y, A_z\}$, at the location of the particle [2]. Using the metric:

$$g^{rt} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} , \quad \text{and the Schrödinger operators,}$$

the operator equation corresponding to equation (5) becomes:

$$g^{rt}[i\hbar\partial_r + (q/c)\phi_r][i\hbar\partial_t + (q/c)\phi_t]\Psi = (mc)^2\Psi \qquad (6)$$

a form of the Klein-Gordon equation, whose scalar solutions, Ψ, represent quantum mechanical states of spinless particles.

From an examination of equation (4), it is easy to show that, if we assume the corresponding relation between the RFC world momenta and the electromagnetic potentials, there should be an invariant expression directly analagous to equation (5) for RFC waveforms:

$$\left[\frac{1}{2}(P{\rightarrow}_k + P{\leftarrow}_k) + (q/c)\phi_k\right]\left[\frac{1}{2}(P{\rightarrow}^k + P{\leftarrow}^k) + (q/c)\phi^k\right] = (mc)^2. \qquad (7)$$

Making the Schrödinger operator substitutions gives the relation:

$$g^{rt}[i\hbar\partial_r + (q/c)\phi_r][i\hbar\partial_t + (q/c)\phi_t](G{\rightarrow}G{\leftarrow}) = 4\,(mc)^2(G{\rightarrow}G{\leftarrow}), \qquad (8)$$

where $G{\rightarrow}$ and $G{\leftarrow}$ are to represent general RFC waveforms.

Note that $G{\rightarrow}$ and $G{\leftarrow}$ appear as the product, $(G{\rightarrow}G{\leftarrow})$, in the solutions to equation (8). Hence neither this equation, nor its associated Dirac equation, permit determination of $G{\rightarrow}$ or $G{\leftarrow}$ individually. This is to be expected; in the absence of fields, $G{\rightarrow}$ and $G{\leftarrow}$ have zero length, so only combinations of $G{\rightarrow}$ and $G{\leftarrow}$ could be described by wave equations with a mass term.

Note also that for the free particle of equation (1), the solution $(G{\rightarrow}G{\leftarrow})$ has the form:

$$(g{\rightarrow}g{\leftarrow}) \sim \exp -2i\gamma K(ct - \beta z),$$

with twice the exponent of the free-particle de Broglie wave solution,Ψ, of the original Klein-Gordon equation (6). (Also, since the RFC analogue wave equation is of the same form, it presents the same difficulties as those perceived in the original, with regard to the interpretation of its operators and solutions [3,5].)

An examination of the range of conditions for which equation (8) is valid shows that, like the original, it retains its validity for stable bound states, that is, it has solutions for states in which the momentum eigenvalues are imaginary, with the time-dependent portion periodic, so that the particle energy can be a real, conserved quantity. Hence there is a kinematically acceptable Klein-Gordon type wave equation (8) for bound and unbound states in the RFC picture.

From the relation between the quantum mechanical free-particle solution,Ψ, and the free-particle solution for $(G{\to}G{\leftarrow})$, it is reasonable to infer that solutions of the RFC analogue Klein-Gordon equation are equivalent in information content to the corresponding solutions of the original equation. If this is true, $(G{\to}G{\leftarrow})$ provides essentially the same information as the quantum mechanical wavefunction regarding probabilities and physical properties of possible states of spinless particles, plus some information regarding the RFC waves involved. We will explore this inference further in section 5.

5. Dirac equation.

Using the procedure of reference [2], we can obtain the RFC analogue of the Dirac equation. That is, using Γ-factors, whose multiplication rules are:

$$\Gamma^r\Gamma^t + \Gamma^t\Gamma^r = -2g^{rt}.1,$$

we can write coupled linear operator equations for the solutions, $(G{\to}G{\leftarrow})$, which are now assumed to comprise 4-component spinors, describing the states of a spin-1/2 particle.

In this way, assuming the equivalence of the quantum mechanical and RFC analogue representations, we obtain the RFC analogue Dirac equation:

$$\Gamma^r(\partial_r + i\epsilon\phi_r)(G{\to}G{\leftarrow}) = 2K(G{\to}G{\leftarrow}), \tag{9}$$

where $\epsilon \equiv -q/(\hbar c)$. The Γ-factors can be chosen to be:

$$\Gamma^r = \sqrt{2}\begin{bmatrix} 0 & \sigma^r \\ \sigma^{r*} & 0 \end{bmatrix}, \quad \text{where}$$

$$\sigma^0 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \sigma^1 = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\sigma^2 = \frac{1}{\sqrt{2}}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}, \quad \sigma^3 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

and the asterisk denotes complex conjugate.

In the previous section, it was deduced that the solutions to the RFC analogue Klein-Gordon equation contain the same information as in the original quantum mechanical equation, along with information regarding the RFC waves comprising the particle. To support this conjecture, we will compare the quantum mechanical and RFC solutions of the Dirac equation for an unbound state, the free particle, and for a bound state, the ground state of the hydrogen atom.

(1) The Free Dirac Particle

In this case, the RFC analogue Dirac equation is

$$\Gamma^r(\partial_r)(G{\rightarrow}G{\leftarrow}) = 2K(G{\rightarrow}G{\leftarrow}). \tag{10}$$

The solutions can be expressed as:

$$(G{\rightarrow}G{\leftarrow})_i = Xu_i \exp -2i\gamma K(ct-\beta z) \quad \text{in positive-energy states, (11}a)$$
$$\text{and,} \quad = Xu_i \exp +2i\gamma K(ct-\beta z) \quad \text{in negative-energy states, (11}b)$$

where u_i are spinors, and $i = 1, 2$ in (11a); $3, 4$ in (11b).

If we define $g{\rightarrow}'$ and $g{\leftarrow}'$ to have exponents of the opposite sign to those of $g{\rightarrow}$ and $g{\leftarrow}$ in equation (1), it is apparent that the exponential parts of $(G{\rightarrow}G{\leftarrow})_i$ are compatible with the following RFC waveform:

$$G_i = Z(g{\rightarrow} + g{\leftarrow})\, v_i \quad \text{and} \quad Z(g{\rightarrow}' + g{\leftarrow}')\, v_i \tag{12}$$

for positive- and negative-energy states, respectively, where v_i are the corresponding free-particle spinors in the RFC picture.

Now if we substitute $M = 2m$ in equation (10), the form of the equation corresponds to that of the original Dirac equation, and the free-particle solutions should also be identical in form. For example, if we choose $i = 1$, and let X be a constant, we should have the corresponding free-particle solution to the original Dirac equation:

$$(G{\rightarrow}G{\leftarrow})_1 = X \exp -2i\gamma K(ct-\beta z). \begin{bmatrix} 1 \\ 0 \\ cP_z/(Mc^2+E_+) \\ 0 \end{bmatrix} \tag{13}$$

Here, we use the Dirac α-matrices [3] instead of the Γ's, P_z is proportional to the z-derivative of $(G{\rightarrow}G{\leftarrow})$ and E_+ to the time-derivative of $(G{\rightarrow}G{\leftarrow})$ for positive-energy solutions. For $(G{\rightarrow}G{\leftarrow})_1$:

$$cP_z = 2\hbar\gamma K\beta c \quad \text{and} \quad E_+ = +2\hbar\gamma Kc.$$

Substituting these values in equation (13) reduces the spinor in $(G{\rightarrow}G{\leftarrow})_1$ to one of the quantum mechanical spinor solutions of the Dirac equation for a free particle. Hence we can express the spin-1/2 RFC free-particle waveform with this spin state as:

$$G_1 = Z(g{\rightarrow} + g{\leftarrow}).\begin{bmatrix} 1 \\ 0 \\ cP_z/(mc^2 + E_+) \\ 0 \end{bmatrix} \tag{14}$$

where now P_z and E_+ refer to the particle momentum and energy, as in the corresponding quantum mechanical spinor. The same procedure can, of course, be applied to the other spinor solutions, giving all the original spinors.

(2) The State Functions of the Hydrogen Atom

As another example, we compare the state functions of the hydrogen atom in the RFC and quantum mechanical pictures.

In the usual procedure for obtaining the hydrogen state functions, they are assumed to be of the form:

$$\Psi_{nks} = \frac{z_s}{r} Y_k \exp -(iEt + Pr)/\hbar, \quad s = 1, 2, \tag{15}$$

where z_s are real power series in r whose terms are characteristic of the energy levels, Y_k is the angular dependence term, and s distinguishes the two spinor components of each state function. The magnitude of k is 1/2 plus the total angular momentum quantum number, j.

As in the free particle case, we can retain the original form for the solutions of the RFC analogue wave equation for the hydrogen atom if we substitute $2m$ for m in the original equation. The only resulting changes from the original solutions, other than in multiplicative constants, are that the exponent of the final term in equation (15) is doubled and the terms of Z_s increase; all other terms remain the same. The ratio of the spinor components remains unchanged, so the RFC spinors in the solutions $(G{\rightarrow}G{\leftarrow})$ are those of the original state functions.

Hence the analogue Dirac equation yields the products, $(G{\rightarrow}G{\leftarrow})$, which for an electron bound in the hydrogen atom is similar in functional dependence to the original solutions except that the term in the exponential is doubled and the series terms increase, and comprises the same spinors. For a free particle, the term in the exponential of $(G{\rightarrow}G{\leftarrow})$ is also twice that of the corresponding de Broglie wavefunction, and is compatible with the use of the G in equation (1) to represent a free spin-1/2 particle, when combined with the spinors of the quantum mechanical solution.

These observations support the previous inference that for bound and unbound particles, the solutions $(G{\rightarrow}G{\leftarrow})$ carry the same physical information as the solutions to the original equation, combined with some information regarding their forward and backward RFC components.

6. RFC Analogue Equations For Carrier Waves

There also exist RFC analogue wave equations similar to those obtained above, which involve the carrier wave component of the RFC particle, through quotients of $G{\rightarrow}$ and $G{\leftarrow}$.

Examination of equation (1) shows that, under the previous assumptions, there should also exist invariant expressions for the world momenta of a particle that involve quotients of $G{\rightarrow}$ and $G{\leftarrow}$. These invariants will be seen to be simply related to the those obtained for the product $(G{\rightarrow}G{\leftarrow})$, in that the difference, rather than the sum, is taken of the world momentum components:

$$\left[\frac{1}{2}(P{\rightarrow}_k - P{\leftarrow}_k) + \frac{q}{c}\phi_k\right]\left[\frac{1}{2}(P{\rightarrow}^k - P{\leftarrow}^k) + \frac{q}{c}\phi^k\right] = -(mc)^2 \quad (16)$$

The corresponding operator equations, the RFC analogue Klein-Gordon equations for carrier waves, are:

$$g^{rt}[i\hbar\partial_r + (q/c)\phi_r][i\hbar\partial_t + (q/c)\phi_t](G{\rightarrow}/G{\leftarrow}) = -4(mc)^2(G{\rightarrow}/G{\leftarrow})$$
$$g^{rt}[i\hbar\partial_r + (q/c)\phi_r][i\hbar\partial_t + (q/c)\phi_t](G{\leftarrow}/G{\rightarrow}) = -4(mc)^2(G{\leftarrow}/G{\rightarrow}) \quad (17)$$

We will proceed directly to the corresponding analogue Dirac equations, to examine their solutions.

RFC Analogue Dirac Equations for Carrier Waves

Under the previous assumptions, we can also use the Γ-factors to write linear operator equations for the components of the spinors $(G{\rightarrow}/G{\leftarrow})$ and $(G{\leftarrow}/G{\rightarrow})$ representing the carrier wave components of a spin-1/2 particle.

The resulting RFC analogue Dirac equations for carrier waves can be expressed as:

$$\Gamma^r(\partial_r + i\epsilon\phi_r)(G{\rightarrow}/G{\leftarrow}) = 2iK(G{\rightarrow}/G{\leftarrow}),$$
and
$$\Gamma^r(\partial_r + i\epsilon\phi_r)(G{\leftarrow}/G{\rightarrow}) = 2iK(G{\leftarrow}/G{\rightarrow}) \tag{18}$$

In the case of the free particle, $\phi_r = 0$. With the definitions of $G{\rightarrow}$ and $G{\leftarrow}$ of equation (1), the two quotients appearing in equations (18) are mutually reciprocal complex exponentials. Since the equations are linear, we can combine them to produce real solutions for the free particle case, supporting the choice of waveform in equation (1), which has a real carrier-wave component. It is easy to show that the solutions are the quantum mechanical spinors for the free Dirac particle with E and cP interchanged, multiplied by the carrier waveform of equation (2).

The quantum mechanical equivalent of equations (18) the eigenvalue equation for cP, given values of E, for a spin-1/2 Dirac particle. In the formalism of reference [3], it can be written for a free particle as:

$$\{E(\alpha.\mathbf{r}) + i\beta mc^2\}\Phi_p = cP\Phi_p\,, \tag{19}$$

where r is unit vector in the direction of motion, and Φ_p is the momentum spinor of the free particle.

This pattern would be repeated in the case of the electron bound in the hydrogen atom; the corresponding RFC analogue carrier wave equation has as solutions the imaginary momentum eigenvalues corresponding to the stable states of the atom. That is, applying the quantum mechanical equivalent of equations (18) to the hydrogen atom would result in the momentum eigenfunctions, along with an expression for the momentum eigenvalues, which are, approximately:

$$(cP)^2 \approx -\{\alpha mc^2\}^2/[(\mu+s)^2+\alpha^2]\,, \tag{20}$$

where α is the fine-structure constant and $\mu + s$ is the highest exponent of the power series in r of the wavefunction for a particular state. Converting these to energy values would give the standard expression for the hydrogen energy eigenvalues [3]:

$$E^2 = \{mc^2\}^2(\mu+s)^2/[(\mu+s)^2+\alpha^2]. \tag{21}$$

Hence we infer that whenever there is an eigenvalue equation for de Broglie waves in a bound or unbound system, there is a corresponding eigenvalue equation for the carrier waves in that system, whose eigenvalues are the momenta complementary to the energy eigenvalues found for the de Broglie waves. That is, there are quantum mechanical wave equations for the particle momentum eigenfunctions, that are directly analagous to the carrier wave equations of the RFC picture.

7. RFC analogue Dirac equation for de Broglie waves

If we now return to the RFC analogue Dirac equation for ($G\!\rightarrow G\!\leftarrow$), we can use its solutions to give the de Broglie wave part of the RFC waveform. As noted in Section 5, these solutions are consistent with the choice of waveforms of equation (1). Hence we can write down the waveforms for the Dirac particle using the formalism of reference [3], with the $g\!\rightarrow$ and $g\!\leftarrow$ of equation (1) supplemented by their counterparts, $g'\!\rightarrow$ and $g'\!\leftarrow$, with exponents of the opposite sign. Using the representation based on Dirac γ matrices in the usual coordinate system, the eight independent RFC solutions can be combined so that forward and backward pairs have spin components $+1/2\hbar$ and $-1/2\hbar$ along the z-axis (the effects of the carrier equations have been absorbed in A, B, C and D):

$$G_1 = A(g\!\rightarrow + g\!\leftarrow)\begin{bmatrix} 1 \\ 0 \\ cP_z/(mc^2+E_+) \\ 0 \end{bmatrix}; \qquad (22a)$$

$$G_2 = B(g\!\rightarrow + g\!\leftarrow)\begin{bmatrix} 0 \\ 1 \\ 0 \\ -cP_z/(mc^2+E_+) \end{bmatrix}; \qquad (22b)$$

$$G_3 = C(g\!\rightarrow' + g\!\leftarrow')\begin{bmatrix} cP_z/(mc^2-E_-) \\ 0 \\ 1 \\ 0 \end{bmatrix}; \qquad (22c)$$

$$G_4 = D(g\!\rightarrow' + g\!\leftarrow')\begin{bmatrix} 0 \\ -cP_z/(mc^2-E_-) \\ 0 \\ 1 \end{bmatrix}; \qquad (22d)$$

where now P_z, E_- and E_+ refer to the particle momentum and energy, with

$$g\rightarrow = \exp\{-i\gamma K(1+\beta)(ct-z)\}\,; \quad g\leftarrow = \exp\{-i\gamma K(1-\beta)(ct+z)\}\,;$$

$$g\rightarrow' = \exp\{+i\gamma K(1+\beta)(ct-z)\}\,; \quad g\leftarrow' = \exp\{+i\gamma K(1-\beta)(ct+z)\}\,.$$

Equations (22) include a set of waves ($g\rightarrow'$ and $g\leftarrow'$) for which the energies of the waves, that is the eigenvalues of the time component of the Schrödinger operator, are negative. (This is to be expected from the method of obtaining the RFC analogue Dirac equation.) Since the physical particle energy is the algebraic sum of the wave energies (equation 4), it also becomes negative for these solutions, under the assumptions made. Hence the same questions arise here, in connection with the non-positive-definite energy (and the positive-definite charge) of the particle, as those encountered in connection with the original Dirac theory [3,4,5,6].

8. Massive Dirac particle in the RFC picture.

As described, for example, in reference [3], we can interpret the negative-energy solutions ($g\rightarrow'$ and $g\leftarrow'$) of a free particle for $v << c$ as representing states with the opposite charge and positive energy, the massive free anti-particle. Hence we can characterise a massive free Dirac particle or anti-particle moving at speeds much less than c by a linear combination of G_1 and G_2 of equations (22), i.e., both $\mathbf{P}$ and E reversed in sign for an anti-particle.

9. Construction of the wave packet.

As described in reference [1], the RFC particle can be localized in a wave packet, using a function of space variables to modulate the standing wave in the rest frame. (Assuming that the particle is stable, the function should not be time-dependent.) The modulating function selected is the Gaussian, which, when included in the expression for G, gives:

$$G = \exp -a^2(z-vt)^2(G_1+G_2)\,, \tag{23}$$

with a being constant in a given inertial frame [1].

We now examine a version of the EPR experiment [7] that was analysed in reference [8]. In this case, we need not distinguish particles from anti-particles, so the spinors along the z-axis (a selected space orientation) can be represented simply by:

$$\begin{bmatrix}1\\0\end{bmatrix} \quad \text{or} \quad \begin{bmatrix}0\\1\end{bmatrix}.$$

10. Analysis of the EPR experiment

In the type of EPR experiment studied here, a system with zero total angular momentum, composed of two massive spin-1/2 particles, breaks up in a process that does not disturb the correlation of the spins of the particles [8]. The particles then separate an arbitrarily great distance before spin component measurements are performed on both, for a chosen orientation of the spins in space. In principle, the particles can separate so far that the measurements could be completed before a light signal could traverse the distance between the particles. In this case, from the conventional viewpoint, no information about the values measured could be exchanged by the particles in the time interval between the measurements.

Quantum theory predicts that the measurements will show the spin correlation to persist, regardless of the particle separation distance or the orientation chosen for the measurements. While the RFC picture makes the same prediction, the additional information about the underlying mechanism leads to the use of RFC state functions different from the corresponding quantum mechanical ones to describe the system behaviour.

(1) Quantum Mechanical Description

The quantum theory uses a coherent superposition of spin component eigenfunctions to represent the spin state of the pair of free particles after breakup of the spin-0 system [8]. We denote the free-particle region of space as that in which the magnetic field, H, of the Stern-Gerlach devices used to measure the spin components has a value less than δ, the value above which the spin state function would be significantly affected. If we denote as $\Psi(\sigma)$ the spin part of the state function for particles 1 and 2 in the free-particle region, we have:

$$\Psi(\sigma)[free,\ H < \delta] = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix}_1 \begin{bmatrix}0\\1\end{bmatrix}_2 - \begin{bmatrix}1\\0\end{bmatrix}_2 \begin{bmatrix}0\\1\end{bmatrix}_1\right). \qquad (24)$$

In the deflection region, where $H > \delta$, the spin state function, $\Phi(\sigma)$, is a space-dependent sum of these eigenfunctions:

$$\Phi(\sigma)[defl.,\, H > \delta] = (\frac{1}{\sqrt{2}}) \left\{ \exp(i\mu) \begin{bmatrix} 1 \\ 0 \end{bmatrix}_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_2 + \exp(i\epsilon) \begin{bmatrix} 1 \\ 0 \end{bmatrix}_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}_1 \right\}, \tag{25}$$

where μ and ϵ are independent functions of the space coordinates. That is, the fields are considered to have the effect of multiplying the separate terms of the spin state function by uncontrollable, independent phase factors. The particles do not have separable spin state functions until the result of one spin measurement is known. Hence in the quantum mechanical picture, only one of the particles (the "last" to be measured) ever has an independent spin state function, upon collapse of the 2-particle spin state function following the "first" spin measurement.

(2) RFC Description

In the RFC picture, the information about the results of both spin measurements is available everywhere on the trajectories of both particles, by virtue of the counter-propagating waves and their associated spins. Hence we are free to consider each particle to be in the spin component eigenstate that is compatible with both the correlation constraint and the spin measurement.

In the free-particle region, we can select the RFC state function for particle 1 to have spin up, implying that the measurement on the particle in detector 1 gives this result. Then we can express its (non-relativistic) RFC free-particle state function by:

$$G_1(t,z,\sigma) = Z \exp -a^2(z-vt)^2(g\rightarrow + g\leftarrow) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{26}$$

Further, we can ensure continuity of the value and slope of the waves in G_1 with those of the state function of the deflected particle, $F_1(t,z',\sigma)$, where z' represents the space coordinates of the particle in the deflection region of detector 1. Suppose that the field reaches the value δ at the point $z = L$, equivalent to $z' = L'$; then these continuity requirements can be represented approximately by the relations:

$$G_1(t,L,\sigma)[free,\, H = \delta] = F_1(t,L',\sigma)[deflected,\, H = \delta], \quad \text{and}$$

$$\frac{dG_1}{dz}(t,L,\sigma)[free,\, H = \delta] = \frac{dF_1}{dz'}(t,L',\sigma)[deflected, H = \delta] \tag{27}$$

The quantum mechanical analysis of reference [8], synopsized in equations (24) and (25), can then be interpreted in the RFC picture as showing that the spin part of F is that of G multiplied by a phase factor which is a function of z'. If we denote the spin parts of G and F by $G(\sigma)$ and $F(\sigma)$ respectively, then this can be expressed for either particle as:

$$F(\sigma)[deflected,\, H > \delta] = G(\sigma)[free,\, H < \delta].\exp i\alpha\,(z'), \tag{28}$$

Hence we can, in principle, generate solutions to the RFC analogue Dirac equations (9) for products of the forward and backward RFC components of both particles that are well-behaved over the trajectories from breakup of the spin-0 system to impact with the detectors at the end of the flight paths in the spin measurement devices.

Note that in the RFC picture there is no collapse of the 2-particle spin state function to a single-particle spin state as a result of the detection process, as is part of the quantum mechanical algorithm for calculating probabilities of the spin states of the particles. Rather, the spin-0 system breakup is viewed as the transformation of a 2-particle spin state function with zero total angular momentum to a separate spin component eigenfunction for each particle. Each eigenfunction satisfies the criteria implied by the interactions at the ends of its trajectory: zero total angular momentum, and the result of the spin component measurement.

From the form of the argument used here, it is clear that regardless of the type of detectors used, their results would still represent constraints on the choice of the particle spin eigenfunctions. The RFC picture of the experiment would be essentially unchanged.

Note also that we were obliged to choose a result of one spin measurement to identify a specific spin component eigenfunction for each particle. The experiment is describable using the RFC waveforms, but its outcome is only predictable in the sense that the spin correlation is required to persist regardless of the choice of orientation or time delay for the measurements. To obtain probabilities for various outcomes of the experiment, the quantum mechanical algorithm must be used.

11. Conclusion

In this paper, we have obtained relations between the RFC particle waveforms and wave equations of the Klein-Gordon and Dirac types. It was observed that the quantum mechanical wave equations for de

Broglie waves appear to be associated with RFC waveforms, but do not specify the backward and forward RFC components individually, instead giving their product. Wave equations were also obtained for the carrier-wave components of RFC waveforms, which were inferred to have quantum mechanical analogues. The solutions to the analagous quantum mechanical equations would give momentum eigenvalues of stable states of a system.

It was then possible, using RFC waveforms and equations, to develop a wave-mechanical description of an EPR-type experiment which provides a logical explanation for the experimental results predicted by quantum theory. It also became apparent, from the relation deduced between the quantum mechanical and RFC wave equations, why conventional quantum theory cannot provide this explanation.

It is noteworthy that the mechanism presented here is compatible with an analysis by Bell of possible conditions whereby the predicted quantum mechanical results could arise in such experiments [9].

The quantum mechanical conception of time has been the subject of controversy since the early days of the theory [10]. The comparison of the RFC and quantum mechanical pictures helps to clarify the significance of the time asymmetry inherent in the quantum mechanical algorithm used to calculate probabilities. This asymmetry is an important feature of the algorithm, and its removal (here, in changing to the RFC picture) impairs the capacity to estimate probabilities, as noted in reference [11].

The RFC picture can also be applied in a similar manner to explain more elaborate EPR-type experiments, for example that discussed in reference [12].

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