

## The Ehrenhaft-Mikhailov effect described as the behavior of a low energy density magnetic monopole-instanton.

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ABSTRACT. Mikhailov has confirmed Ehrenhaft's earlier experiments on ferromagnetic aerosols which exhibit magnetic monopole-like behavior with respect to test magnets. The following letter suggests that the behavior is due to global ordering of electron spins within the aerosol particles which become low energy density magnetic monopole-instantons, rather than high energy monopoles. The instantons are formed by compactification of degrees of freedom as a result of (i) large spin exchange energies and (ii) spherical boundary conditions. The instantons or pseudoparticles are minimum action solutions of  $SU(2)$  Yang-Mills fields on the four-sphere  $S^4$ , or the conformal compactification of the Minkowski space-time  $R^4$  by the conversion of field equations in  $R^4$  into a complex analytic geometry (fiber bundles) on the complex projective 3-space  $P_3$ .

*RÉSUMÉ* Mikhailov a confirmé les anciennes expériences d'Ehrenhaft sur les aérosols ferromagnétiques, qui montrent un comportement du type monopole magnétique par rapport aux aimants d'expérience. Le texte suivant suggère que ce comportement est dû au réarrangement global des spins des électrons des particules de l'aérosol, qui deviennent des instantons-monopoles magnétiques de basse énergie. Les instantons se forment par compactification des degrés de liberté provenant (i) de grandes valeurs d'énergie d'échange de spin et (ii) des conditions aux limites sphériques. Les instantons ou pseudoparticules sont des solutions de moindre action des champs  $SU(2)$  de Yang-Mills sur la 4-sphère  $S^4$ , ou la compactification conforme de l'espace-temps  $R^4$  de Minkowski par la conversion des équations de champ dans  $R^4$  en une géométrie analytique complexe (fibrés) dans l'espace projectif complexe  $P_3$ .

The intention of the following letter is to suggest that the interesting behavior exhibited by ferromagnetic aerosol particles, which has

been demonstrated by Ehrenhaft and Mikhailov, is the behavior of a low energy density magnetic monopole-instanton, (rather than of a high energy density magnetic monopole). Low energy density solutions of Yang-Mills field equations [1] are permitted as instantons or pseudoparticles in  $SU(2)$  reconstituted symmetries [2]-[4]. Instantons or pseudoparticles are the minimum action solutions of  $SU(2)$  Yang-Mills fields - originally formulated in Euclidean  $R^4$  but extrapolated here to Minkowski  $R^4$  - by a conversion into a complex analytic geometry on the complex projective 3-space  $P_3$  [5]-[10]. These solutions can be embedded in a self-dual Yang-Mills hierarchy which is related to the celebrated Kadomtsev-Petviashvili hierarchy [11]-[14].

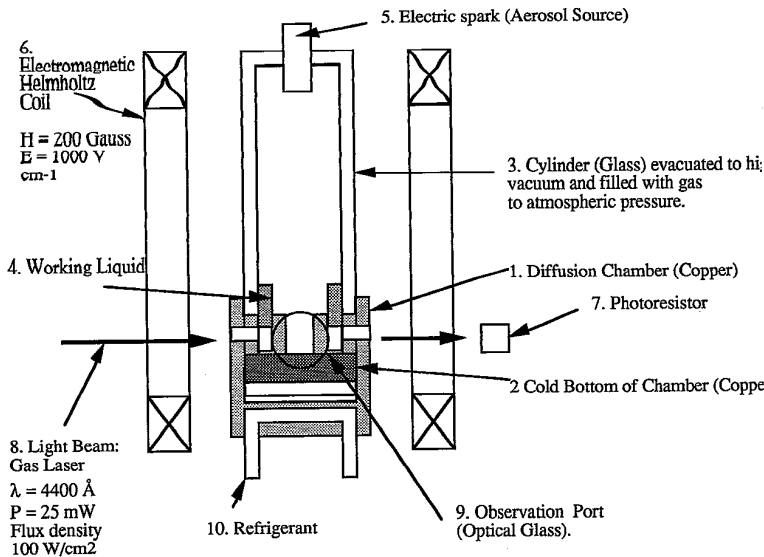
The effect has a history. In some sixty papers, 1930-1951, Ehrenhaft reported magnetic monopole-like behavior and magnetic currents in aerosols. These reports were disputed on the grounds of the complexity of the experimental conditions, the multicomponent nature of the chemical solutions used, and the fact that the magnetic poles did not obey the quantization conditions.

However, similar experiments have recently been carried out. Mikhailov, 1982-1991, has shown that gas suspended ferromagnetic ( $Fe_3O_4$ ) aerosol particles of submicron size and falling in a gaseous medium move in a magnetic field similarly to objects possessing a magnetic charge [15]-[24]. That is, under intense light beam illumination ( $1 \text{ kW/cm}^2$ ) the magnetic particles move in a magnetic field along the lines of force (along coaxial trajectories with respect to the current). Reversal of the magnetic field,  $H$ , causes a reversal of particle motion (which is not the case with magnetic dipoles). Other major features of the experiments are: reduction of the light field causes the particles to stop moving; an increase in field strength or light intensity causes a rise in particle velocity, while a decrease results in reduced particle velocity; the number of particles moving in the direction of the magnetic field appears to equal the number of particles moving in the opposite direction; and the aerosol particles are close to spherical form with diameter  $10^{-5} - 10^{-6} \text{ cm}$ , appearing to carry a magnetic charge  $g \approx \alpha e$ , where  $\alpha$  is the fine-structure constant and  $e$  is the electron charge.

A description of the experimental procedure is as follows (cf. Fig. 1). An airtight chamber with a flask containing the ferromagnetic solution is evacuated until a high vacuum is achieved. The chamber is then filled with a working gas (argon) up to atmospheric pressure. Aerosols are obtained when the solution "dusts" an electromagnetic current interrupter spark contacts. The gas together with aerosols is transported to

the working chamber. A He Ne laser is used to illuminate the particles with about 1 Watt and the beam is focused on the field of view of a microscope (cf. Fig. 2). The light flow density in the focus is about  $10^2 \text{ W/cm}^2$ . Having passed through the chamber the light is measured by a photoresistor. Another light source is used to observe the effect under the influence of an electromagnet. The range of fields used is:  $H = 0 - 200$  Gauss,  $E = 0 - 1000$  V  $\text{cm}^{-1}$ .

In the first experiments on this effect, Ehrenhaft [25] separated aerosol particles, which were dispersed by arc electrodes. In the presence of an intense light beam and a homogeneous magnetic field the particles moved as if carrying a magnetic charge (as in Fig. 2). Besides the Ehrenhaft and the recent Mikhailov experiments, the effect has also been reproduced by Benedict & Leng [26].



**Figure 1.** A schematic of Mikhailov's apparatus. After Mikhailov [20].

Calculations of the magnitude of the magnetic charge vary. Mikhailov's early derivations of the relationship of the empirical results to the magnetic charge [16]-[18],  $g_M$ , show it to be much less than Dirac's magnetic monopole charge,  $g_D$ . The early derived relationship was  $g_M \approx \alpha^2 g_D$ , where  $\alpha = 1/137$  is the fine structure constant, and values of  $g_M$  were obtained of  $10^{-11} - 10^{-14}$  Gauss  $\text{cm}^2$ , which are an order

of magnitude less than the monopole of Dirac [27],[28] ( $g_D = 3.29 \times 10^{-8}$  Gauss cm<sup>2</sup>).

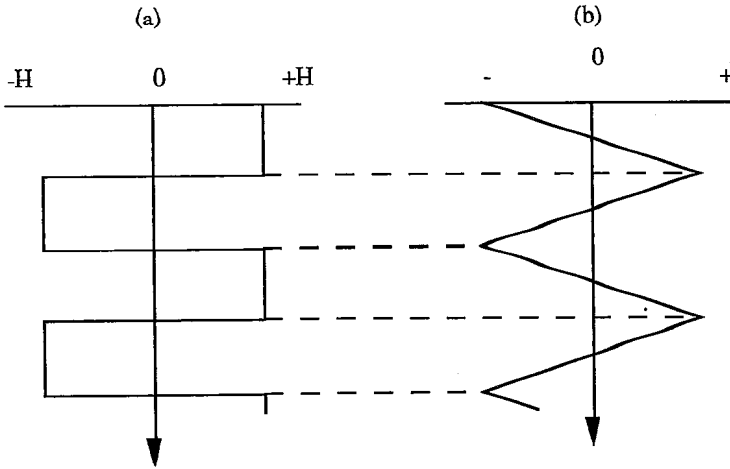
In later derivations [19],[20], the obtained magnetic charge was shown to be on the order of Dirac's monopole:  $g_M \approx g_D$ . This later derivation uses the force relationship:  $F = g_M H$ , and Stokes' law to obtain:

$$g_M = (18\pi/H)\sqrt{[\eta^3/2\gamma\rho]}V_H\sqrt{V_S}, \quad (1)$$

where  $V_S$  is the velocity of the droplet fall =  $[2a^2\gamma\rho/9\eta]$ ;  $V_H$  is the drop speed in the direction of  $H$ ;  $a$  is the droplet radius;  $\gamma$  is the free fall acceleration;  $\eta$  is viscosity;  $\rho$  is the droplet density. However,  $g_D$  is a mean value of a distribution of obtained values which has prompted the suggestion that the obtained values can be multiples of  $g_D$  [20].

Structural nonuniformities on the surface of the particles are not related to the effect, because Mikhailov [20] has shown that water droplets condensed on ferromagnetic particles cannot contain any structural nonuniformities, and yet the effect still occurs. Introducing gaseous  $H_2O, I_2, Br_2, HCl, HNO_3, NH_4OH$  into the working chamber and the exchange of argon for air, does not suppress the observable effect, so the effect is not determined by a particle surface condition.

The effect is found with ferromagnetic particles (iron, nickel and cobalt and thirty other substances), but not with paramagnetic particles. This is an important aspect of the effect because ferromagnetism is an extreme form of paramagnetism with large exchange energies between spins. Because of this and other aspects, the explanation for the effect offered here is that it is occasioned by the occurrence of electron spin within the particles, the large exchange energies and the aerosol spherical symmetry boundary conditions. Only (i) with the freedom for spins to realign within the structure of spherical boundary conditions, and (ii) with cooperative long-range effects predominating (resulting from large exchange energies, as in ferromagnetic but not paramagnetic particles), can degrees of freedom be compactified (within the particle) and an  $SU(2)$  global symmetry compactified in  $P_3$  be obtained.



**Figure 2.** (a) time diagram of the magnetic field; (b) observed trajectory of a particle falling from top to bottom. Schematic after Mikhailov [20].

The  $SU(2)$  group is a Lie algebra such that for the angular momentum generators,  $J_i$ , the commutation relations are  $[J_i, J_j] = i\varepsilon_{ijk}J_k, i, j, k, = 1, 2, 3$  [53]-[56]. A Minkowski space-time compactified is a space in which a gauge is formed by choosing two regions whose union covers the whole space and by specifying a transition function on the overlapping region [57]. The relevance of the  $SU(2)$  symmetry form is in the Yang [58] generalization of Dirac's monopole to  $SU(2)$  gauge fields in four-dimensional spherical space in which the generalized fields have  $SO(5)$  symmetry. In Yang's original formulation, the four dimensions are the three dimensions of space and the angular coordinates. (The radial and angular dependences are separate and there is only field dependence on the angular coordinates). However, in the present instance the four dimensions are the three spatial dimensions and the time dimension (Minkowski space-time) for the following reasons.

The transformations of the spherical boundary conditions of an aerosol particle for broad, rather than for pencil, beams are described by the  $SU(2)$  matrix in the stereographic projection using Cartan's concept of an isotropic vector [64]. The well-known  $SU(2)$  matrix relating the Euler angles of  $O(3)$  and the complex parameters of  $SU(2)$  is:

$$\begin{pmatrix} \cos(\beta/2)\exp[i(\alpha + \gamma)/2] & \sin(\beta/2)\exp[-i(\alpha - \gamma)/2] \\ -\sin(\beta/2)\exp[i(\alpha - \gamma)/2] & \cos(\beta/2)\exp[-i(\alpha + \gamma)/2] \end{pmatrix} \quad (2)$$

where  $\alpha, \beta$  and  $\gamma$  are the Euler angles. It is also well known that a homomorphism exists between  $O(3)$  and  $SU(2)$ , so that the elements of  $SU(2)$  can be associated with rotations in  $O(3)$ ; and  $SU(2)$  is the covering group of  $O(3)$ . Therefore, it is easy to show that  $SU(2)$  can be obtained from  $O(3)$ . Thus the four dimensions are the three Euclidean spatial dimensions and time (or Minkowski space-time). As the effects reported by Mikhailov only occur in ferromagnetic aerosols, i.e., with spin exchanges, and, as, *unlike rotational transformations, spin transformations are realized in time*, the effects examined here are based on Minkowski space-time in  $R^4$ .

For example, an isotropic parameter,  $w$ , can be defined:

$$w = (x - iy)z^{-1},$$

where  $x, y, z$  are the spatial coordinates. If  $w$  is written as the quotient of  $\mu_1$  and  $\mu_2$ , or the homogeneous coordinates of the bilinear transformation, then [64]:

$$|\mu'_1 \mu'_2\rangle = \begin{array}{cc} \cos(\beta/2)\exp[i(\alpha + \gamma)/2] & \sin(\beta/2)\exp[-i(\alpha - \gamma)/2] \\ -\sin(\beta/2)\exp[i(\alpha - \gamma)/2] & \cos(\beta/2)\exp[-i(\alpha + \gamma)/2] \end{array} |\mu_1 \mu_2\rangle \quad (3)$$

Therefore, as is well-known, spins can be defined in terms of  $SU(2)$  and that definition can be associated with  $O(3)$ . The crucial observations are then: (1) The spherical aerosol particle, *qua* cavity, (with ferromagnetic particles entrapped) produces changes in both rotations *and* spins. (2) Rotations are coordinate transformations and have no history and no time. Therefore, only the three spatial dimensions are required for these transformations. Spin, on the other hand, is a process realized in time. Therefore, the three spatial dimensions and time are required.

The sourceless condition is that the action

$$L = \oint \oint \oint \oint f_{\mu\nu}^i f_{\mu\nu}^i d(\text{surface}) = \text{an absolute minimum}, \quad (4)$$

where the integral is taken over a closed four-dimensional surface. The solutions to Eq. [4] are gauge fields (nonintegrable phase factors), e.g.,  $m_1$  and  $m_2$ , confined to any sphere  $S_4$ , in which  $m_1$  is self-dual and orthogonal everywhere, and  $m_2$  is self-antidual and orthogonal everywhere. The absolute minimum condition represented in Eq. [4] indicates that

the monopole formed can be a low energy instanton monopole, not necessarily the high energy form. This minimum was also obtained in [2] and is related to the topology of the boundary conditions confining the fields. This kind of spontaneous compactification of extra dimensions has been shown to occur in arbitrary space-time dimensions when the gauge group is  $SO(N)$  [59-62].

The compactification results in a globally self-organized structure of minimum action which achieves dynamic stability. This suggested model is in contrast to any statistical explanation. The compactification of degrees of freedom due to the interaction of two gauge fields,  $m_1$  and  $m_2$ , is analogous to Onsager's turbulent systems of interacting line vortices [63] which have no energy except the energy of interaction, i.e., no kinetic energy. The global system is thus highly nonlinear. In the present case, spherical boundary conditions of the aerosol particle rather than turbulent forms are the determining factors. This model predicts that the magnetic field lines be parallel resulting in a force-free structure.

The present suggestion of globally coherent ordering is compatible with the fact that the Dirac equation admits a second minimal coupling associated with a chiral gauge which is only valid for a massless particle, but satisfies all the symmetry laws of a monopole [29]. The projection of the total angular momentum on the symmetry axis of the system formed by the monopole and the electric charge is  $eg/c$ . Monopoles and antimonopoles have opposite helicities, but not opposite charges. Therefore an aether can be constructed of monopole-antimonopole pairs (or twistors [30],[31]).

Mikhailov [17],[20] has shown that the effect of the light beam is purely kinetic, i.e., the particles do drift downwards due to gas-kinetic interactions, but the side attraction repulsion effect itself is not at all dependent on the light. In order to induce energy-exchange between similar  $SU(2)$  -  $SU(2)$  processes, light-induced movement may be necessary (resulting in an  $SU(2)$ -current). For example, Mikhailov & Mikhailov [24] have shown that which can be interpreted as, from the present perspective:  $SU(2)$ -radiation and  $SU(2)$ -matter coupling in a superconductor-ferromagnetic aerosol experiment. These investigators demonstrated that a magnetometer, which measures the superconductor current, shows no change in the superconducting coil region under the influence of ferromagnetic aerosols or light beams individually, but the ferromagnetic aerosol particles in conjunction with an illuminating beam produces a decaying current in the coil. As the gap fluxon of a

superconductor is an  $A$  vector potential and representative of the mathematical entity known as a soliton, which is of  $SU(2)$  symmetry form [32], the *moving* ferromagnetic aerosol particle is an  $SU(2)$  (Noether) current in the vicinity of an  $SU(2)$  (superconducting) current. As these currents are of the same ( $SU(2)$ ) symmetry form, interaction can occur. If the aerosol particles are not moving under the influence of the light beam, then there is no  $SU(2)$  aerosol current, merely static  $SU(2)$  charge, and therefore no interaction with the superconducting  $SU(2)$  current is to be expected or was found.

On the basis of this interaction between the magnetic charged aerosols with the magnetic field of the superconducting coil, a relation was derived ([24], Eq. 13):

$$H = H_0 \exp\{-[3\pi\sigma l_0/KR^3]ng^2t\}, \quad (5)$$

where  $H$  is magnetic field strength;  $g$  is magnetic charge;  $n$  is the average density of particles possessing that charge;  $\sigma$  is the cross section of the light beam;  $K = 6\pi\eta r$ ;  $r$  is the radius of the particle;  $\eta$  is the viscosity of the medium. Therefore, an exponential decrease in the magnetic field strength of the coil is predicted, with a time constant:

$$\chi = [3\pi\sigma l_0/KR^3]ng^2t. \quad (6)$$

The experimental data [24] are in approximate agreement with this prediction.

In a second experiment, ferromagnetic aerosol particles with a magnetic charge in interaction with a superconducting surface were demonstrated to produce an inverse square law repulsive force (Meissner effect). The particles moved normal to the superconducting surface. The repulsive force of interaction (between a magnetic charge and its image) was found to be ([24], Eq. 25):

$$F = G^2/4y^2 \quad (7)$$

where  $G$  is a point charge;  $y$  is the distance between the superconducting surface and the charged particle. However, attractive forces were also noted. A possible explanation of the two forces may be as follows. The  $A$  vector gap potential field of a superconductor is well-known to be of  $SU(2)$  symmetry form, but the group symmetry of conventional magnets is of  $U(1)$  symmetry. Therefore, the exclusively repulsive forces of the



Meissner effect may be due to  $SU(2)$  -  $U(1)$  field interaction. In the Mikhailov & Mikhailov experiment the ferromagnetic aerosol particles, (according to the present view), are of  $SU(2)$  symmetry. Therefore, the repulsive *and* attractive effects may indicate an unconventional  $SU(2)$  -  $SU(2)$  field interaction Meissner-type effect.

In summary, from the present perspective, the magnetic monopole produced in the particles is not a high energy monopole in the sense of Dirac. It is, rather, an *instanton*, in the sense of Belavin *et al* [2]. That is, the behavior of the particle is due to compactification of degrees of freedom resulting from its boundary conditions and spin exchange energy coupling possibilities, rather than a compactification resulting from the high energy of the particle. An instanton has the degrees of freedom of a high energy particle, but without the spatial dimensions of a high energy particle or the energy-density. Stability is achieved by the substitution of spherical boundary conditions for the high energy density within restricted spatial dimensions [33]. The instanton created is, therefore, a “pseudoparticle”. Nonetheless, it is of  $SU(2)$  form and the dynamic behavior is caused by the degrees of freedom of the particle with behavior similar to that of the true high energy density monopole. It is the degrees of freedom which determine the behavior, not the high intrinsic energy *per se*.

The Ehrenhaft-Mikhailov monopole-instanton is not even necessarily a Dirac monopole-instanton [27],[28]. There are a number of competing descriptions of monopoles. On the one hand, the Dirac magnetic monopole is an anomalously-shaped (string) magnetic dipole at a singularity [27],[28], and the Schwinger magnetic monopole is essentially a double singularity line [34],[35]. On the other hand, if the magnetic monopole is theoretically justified by a relationship to  $A$  vector potentials, which are the local representation of global constructs, then the existence of *isolated* monopoles is precluded. They may exist globally in *any* situation with the requisite energy conditioning. Such situations are described in Ref.s [36]-[42].

Examples of  $SU(2)$  symmetry fields can appear under forms of energy conditioning other than that for monopole-instantons. For example, whereas conventional Maxwell theory fields are in  $U(1)$  symmetry form, an electromagnetic field which is polarization modulated is in  $SU(2)$  symmetry form [43]-[47]. Therefore, the “Maxwell equations” can be extended into higher symmetry, e.g.,  $SU(2)$  and other symmetry forms [44], [48]-[52]. These higher electromagnetic forms are low-energy density Yang-Mills fields.

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